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A RELATIVISTIC DESCRIPTION OF THE FERMI
MOTION EFFECTS ON DEUTERIUM TARGETS*

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A RELATIVISTIC DESCRIPTION OF THE FERMI
MOTION EFFECTS ON DEUTERIUM TARGETS

by
DARMADI KUSNO

APPROVED:

Michael J. Moravcsik
Michael J. Moravcsik, Advisor

A DISSERTATION

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and the Graduate School of the University of Oregon
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A comprehensive analysis of the inconsistencies of the conventional, non-relativistic approach, which has been used so far in the extraction of neutron data from deuterium targets, is given. A new approach dealing with the smearing effects, due to the nucleon's Fermi motion inside the deuteron, is developed as an alternative to the conventional one. This new approach is a spin-less, relativistic, simple and consistent approach. In our approach the normalization condition for the relativistic deuteron vertex function arises naturally in hadronic scattering discussions, and furthermore has a physical meaning. The sum-rules we obtain in the leptonic scattering agree with the quark-parton model expectations. We find that the West β -correction is absent in hadronic scattering, but it is still there for leptonic scattering. The form of the correction for leptonic case, is however, different in our calculations from that given by the conventional approach. We show that within the so-called "off-shell kinematics"---"on-shell dynamics" formalism, the electroproduction expression is consistent, i.e., it leads to a vanishing σ_L^d (the absorption cross

section of scalar virtual photons) in the $q^2 = 0$ limit. Hence, we do not need the off-shell corrections (as far as this consistency problem is concerned) which only unnecessarily complicate the problem. In analyzing the recent high energy cross section data of πD^- , πN^- , and ND^- , NN^- processes, we find that even at such high energies the conventional approach leads to non-negligible smearing correction σ_s . On the other hand our approach, by using a simple ansatz for the relativistic vertex function which gives a good fit to the elastic deuteron form factor, leads to a vanishing σ_s as we expect. We show that the inclusion of a spin degree of freedom in our formalism for the leptonic scattering, where the smearing effects are very important, does not affect our previous, spin-less results. Thus even though the new formalism uses an ansatz for the relativistic vertex function (which eventually will have to be checked either against a full relativistic calculation or against other processes, e.g. pion electroproduction from deuteron, etc.) and even though the new formalism includes spin complications only in an average sense, we claim that it is likely to be superior to the previous methods of making smearing corrections to experimental data, and hence, should be used for that purpose. A new covariant model of the elastic electromagnetic form factors of the deuteron in the impulse approximation is also presented. The treatment includes spin and allows for a possibility of determining completely the two elastic structure functions.

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I. INTRODUCTION

Since the turn of the century, beams of monochromatic electromagnetic particles have played a leading role in the investigation of the inner structure of complex systems.¹ Common examples are the study of the structure of crystals and large organic molecules by the use of X-rays and low energy electron scattering. Rutherford's low energy alpha particles allowed him to "see" atomic structure, revealing the presence of a charged point-like core, the nucleus. Electron scattering by Hofstadter and co-workers during the 1950's established the size of nuclei and the nucleon charge density in nuclei.

In these experiments the fundamental interaction used to probe the target structure was electromagnetism. Due to the comparatively weak coupling of photons to charged particles (characterized by a coupling constant $\alpha_e = 1/137$), this type of probe can penetrate unaffected deep inside a target particle. The electron interacts with matter primarily through electromagnetism and does not participate in strong interactions (characterized by a coupling constant of approximately 15), hence the problem of strong interactions masking substructure is not present. With the availability of electron energies in the multi GeV range at SLAC (Stanford Linear Accelerator Center), it has become possible to study the inner structure of the proton and the neutron. Beams of protons and mesons have been also used to study nucleon structure. Difficulties arise in the interpretation of the results, since the strong interactions are not understood anywhere nearly as well as the electromagnetic interactions.

Studies of the structure of the proton have been done by elastic and inelastic electron proton scattering experiments. The measurements of the proton elastic form factors have provided information on the charge and magnetic moment distributions in the proton. Inelastic electron proton scattering has opened a new channel of information. A big surprise came in the discovery that this inelastic cross section was very large. As the elastic cross section died away with increasing momentum transfer, the relative inelastic cross section increased. The total scattering strength at a fixed angle and incident energy of all inelastic states was quite large and suggested the existence of a substructure of charged point-like constituents within the nucleon that was responsible for the total scattering strength of this system. The most striking feature of the data is that they exhibit the "Bjorken scaling" phenomena, which says that at very high energy and momentum transfers the role of the mass parameters is unimportant. These facts led to the formulation of the quark-parton model,² which is the zeroth-order contribution to QCD (Quantum Chromodynamics).³ Nowadays many high energy physicists consider QCD as the best candidate for a strong interaction theory.

No study of nucleon structure can be complete without a corresponding study of the structure of the neutron. Unbound protons are easily available in the form of a liquid hydrogen target. There are no free neutron targets available, furthermore neutron beams are difficult to work with. These facts compel us to use the deuteron as a neutron target. There has in fact been considerable progress in the past few years in the use of high-energy neutron beams, but only neutron-proton and neutron-nucleus

reactions can be studied in this way. Experiments utilizing the scattering of neutron beams from atomic electrons have been done, but kinematic considerations limit such experiments to very low momentum and energy transfers. People have been thinking about experiments between an electron beam and reactor thermal neutrons, but flux considerations render this type of experiments unfeasible. Thus reactions involving a neutron and any non-nuclear particles are best studied using deuterium targets, at least until the time that colliding-secondary-beam techniques are developed. Of course the utilization of deuterium introduces some complications which will be discussed later. But this is the price that one has to pay in order to make available something which is analogous to free neutron targets.

The scattering of high energy particles from deuterium is perhaps our most important source of information on the structure of the neutron. The accurate extraction of such information, however, is far from straightforward. Consider, for example, a measurement of the total cross section for the scattering of some incident particle, such as a pion, from deuterium. The neutron cross sections are usually found by using the equation

$$\sigma_d(\nu) = \sigma_n(\nu) + \sigma_p(\nu) - \sigma_s(\nu) - \sigma_G(\nu), \quad (1.1)$$

where ν is the laboratory energy of the incident particle, σ_G is the Glauber correction for shadowing of one nucleon by the other in the deuterium nucleus, and σ_s is the smearing correction due to the fact that the bound nucleons undergo internal or "Fermi" motion. There are some other corrections to (1.1) like off-mass-shell effects, isobar effects, etc., but most of these corrections are incalculable even to the extent

of making a reasonable and believable estimate of their magnitude.⁴ Although intuition may tell us that the "incalculable" corrections are probably quite small ($\leq 1\%$) we should be cautious for intuition can sometimes lead one astray; model calculations of some of these corrections would be a very useful contribution in helping us confirm our "intuition".

It is expected, due to the fact that the deuteron has a small binding energy (~ 2.2 MeV), that σ_s gives a negligible contribution to (1.1) for hadronic scattering in the high energy region. Thus in practice one considers only σ_G in the extraction of neutron total cross sections.⁵ However, we have an opposite situation for leptonic scattering, i.e. the σ_G (which is of order α_e^4 for electron case, and clearly is smaller for neutrino one) is usually neglected and one considers only the Fermi motion effects in the extraction of the electromagnetic neutron structure functions.⁶

In the past, calculations of the deuterium smearing correction have been essentially "non-relativistic", in that they used the non-relativistic wave function of the deuteron rather than the relativistic vertex function. The method, which now has been adopted by experimentalists, was first proposed by Faldt and Ericson⁷ for hadronic scattering in 1968. They showed that if the Fermi motion inside the deuteron was properly taken into account in the analyzing of pion scattering data (note that there were a number of prominent resonances in the pion-nucleon system in the energy range they were considering), then the discrepancy between the theoretical and experimental value of σ_G , which was debated at that time, can be solved. This method was subsequently improved by West,^{4,8} who first introduced the phase-space restrictions (due to the fact that the

neutron is not free, but is part of a bound state), and was also extended to electron scattering in 1972, at the time when everyone was excited by the deep inelastic lepton-hadron processes. From now on we will refer to this method as the conventional or standard approach.

Recently this conventional method has been criticized^{9,10,11} for its inconsistencies, apart from the question of the validity of using non-relativistic wave functions. The smearing correction is most substantial in regimes where σ_p and σ_n vary most rapidly, because then σ_d receives significant contributions from parts of the kinematic region to be integrated over in which σ_p and σ_n are very different from their values in the middle of the region. A particularly important example of such a situation is deep-inelastic lepton scattering, for the case where one is exploring the structure function of the neutron that is embedded in the deuteron at values of the Bjorken scaling variable ω close to 1. This is of very considerable theoretical importance,¹² in that the ratio σ_n/σ_p in deep-inelastic electron or muon scattering gives information about the relative shapes of the momentum distributions of u(up) and d(down) quarks in the nucleon. Deep-inelastic scattering at large q^2 is a situation where a proper relativistic treatment of the corrections is surely essential, and it is likely that the corrections are very large near $\omega=1$.

The aim of this thesis is to develop a simple, consistent, and covariant derivation of the smearing correction. This correction is perhaps the simplest of those corrections that need to be considered in the extraction of neutron data from deuterium targets. Although such effects can be quite large in the leptonic scattering, it is remarkable

that so little explicit attention has been paid to them in the literature.

The body of this thesis begins by reviewing the conventional approach to the smearing correction, first through its applications in hadronic scattering, and then in the electron scattering. We will limit ourselves to the discussion of the (hadronic) total cross sections and the (leptonic) structure functions throughout this thesis. The inconsistencies in this method will be pointed out. A simple covariant approach to this problem will be proposed as an alternative to the conventional one. We will show explicitly that our approach is consistent and avoids some of the approximations made in the earlier contribution to this problem.

II. THE CONVENTIONAL APPROACH

This section basically contains a review of the results of the conventional approach to the Fermi motion effects on deuterium targets.¹³

We shall present the results in two cases:

- A. Hadronic scattering, where the incident particle has a small but finite mass.
- B. Electron scattering, where the incident particle has a virtual mass which can become large.

In part C we will discuss the identification of the virtual scattering amplitude.

As far as practical calculations are concerned we can present the conventional approach in a non-relativistic way. This is what we intend to do in the beginning of section II.A. (hadronic scattering), so that we can understand how the idea evolved. After some discussions we will switch to the relativistic presentation of the conventional approach for the following reasons. First of all, the mathematical expressions are the same whether we use the non-relativistic or relativistic way. Secondly, there are some subtleties of the smearing problem that can be understood more clearly if we present it in a relativistic way. Finally, in connection with the inconsistencies problem of the conventional approach which will be discussed in the next section, it is more convenient to present the conventional approach in a relativistic way.

A. Hadronic Scattering

A.1. Non-Relativistic

The non-relativistic approach has been used by experimentalists to extract the neutron data from deuterium targets. The easiest way to take account the Fermi motion effects is just simply by smearing the cross sections.¹⁴ Thus in hadronic scattering,

$$\sigma_n^{\text{smeared}}(v) + \sigma_p^{\text{smeared}}(v) = \sigma_m(v) + \sigma_p(v) - \sigma_s(v) = \int d^3k [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] \times [\sigma_n(v') + \sigma_p(v')], \quad (2.1)$$

where $f_s(|\vec{k}|)$ represent the S- and D-wave components of the deuteron wave function. The S-wave component is

$$f_s(|\vec{k}|) = (2\pi^2)^{-1/2} \int_0^\infty j_0(|\vec{k}|r) u(r) r dr = (4\pi|\vec{k}|^2)^{-1/2} u(|\vec{k}|), \quad (2.2)$$

and the D-wave component is given by

$$f_d(|\vec{k}|) = (2\pi^2)^{-1/2} \int_0^\infty j_2(|\vec{k}|r) w(r) r dr = (4\pi|\vec{k}|^2)^{-1/2} w(|\vec{k}|), \quad (2.3)$$

where $j_2(|\vec{k}|r)$ is the spherical Bessel function of order 2:

$$j_2(z) = z^2 \left(-\frac{1}{z} \frac{d}{dz} \right)^2 \left(\frac{\sin z}{z} \right). \quad (2.4)$$

In (2.1) the invariants v and v' are given by

$$v = \frac{P \cdot q}{M_d}, \quad v' = \frac{P \cdot q}{M}. \quad (2.5)$$

p and P are defined in Fig. 1 and are the nucleon and deuteron momenta, respectively; M and M_d are their masses.

By using (2.5) we can rewrite (2.1) as

$$\sigma^{\text{smeared}}(v) = \frac{2\pi M}{(v^2 - q^2)^{1/2}} \int_0^\infty [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] |\vec{k}| d|\vec{k}| \times \int_{v_-}^{v_+} \sigma(v') dv', \quad (2.6)$$

where σ here means $\sigma_n + \sigma_p$. The limits on the v' integration are given by

$$M v'_\pm = p_0 v \pm (v^2 - q^2)^{1/2} |\vec{p}|, \quad (2.7)$$

where we have yet to specify the relationship between p_0 and $|\vec{p}|$. We will assume that the nucleon spectator is on its mass shell, and the interacting nucleon is considered to be a virtual nucleon (i.e., not on its mass-shell). Thus

$$\begin{aligned} p_0 &= M_d - k_0 \\ &= M_d - (|\vec{k}|^2 + M^2)^{1/2}, \\ \vec{p} + \vec{k} &= \vec{0} \end{aligned} \quad (2.8)$$

However, this relation implies v'_- can be negative, and hence, it is necessary to impose a threshold condition on $\sigma(v')$, i.e., $\sigma(v') = 0$ when $v' < 0$, in (2.6). This condition can be regarded also as the phase-space restriction due to the fact that the nucleons are not free, but are

FIG. 1 Kinematics. The broken line represents the incident particle of four-momentum q ; the single solid lines are the nucleons of momenta p (the interacting particle) and k (the spectator). The double line represents the deuteron of four-momentum P .

part of a bound state. The case which is based on (2.6) with the threshold condition will be called case 1.

Fäldt and Ericson⁷, and West⁸ have suggested that one should smear the invariant matrix element instead of the cross section. Thus instead of (2.1) we have

$$\sigma_{\text{smeared}}(\nu) = \int d^3k \left(\frac{\nu'^2 - q^2}{\nu^2 - q^2} \right)^{1/2} [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] \sigma(\nu'). \quad (2.9)$$

By using (2.5) we rewrite (2.9) as

$$\sigma_{\text{smeared}}(\nu) = \frac{2 M \pi}{\nu^2 - q^2} \int_0^\infty [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] |\vec{k}| d|\vec{k}| \times \int_{\nu_-}^{\nu_+} (\nu'^2 - q^2)^{1/2} \sigma(\nu') d\nu'. \quad (2.10)$$

The case which is based on (2.10) with the threshold condition will be called case 2. It is known also as the conventional approach.

The above two cases give different results for the smearing process. As an illustration they are shown in Figs. 2 and 3. From the graphs it is clear that the flux factor depletes the cross section. For a certain deuteron wave function (e.g., Hamada Johnston for the case 1 and Hulthén Sugawara soft-core for the case 2), both cases can lead to a small smearing correction σ_s , as we expect. Thus it is difficult to show the superiority of one over the other. Notice that in the non-relativistic framework it is not clear whether we should smear the cross section or the invariant matrix element. We will show later that the relativistic description of

FIG. 2. Graph of $\sigma_{\text{smeared}}(\nu)$ of pion scattering for the two cases 1 and 2 mentioned in the text, using the Hulthén-Sugawara soft-core deuteron wave function (See Section III D). The dashed line is the unsmeared cross section, i.e. $\sigma_{\pi^-p} + \sigma_{\pi^+p}$. The experimental data are for π^-D total cross sections.

FIG. 3. Graph of $\sigma_{\text{smeared}}(\nu)$ of nucleon scattering for the two cases 1 and 2, using the Hamada-Johnston deuteron wave function. The dashed line is the unsmeared cross section, i.e. $\sigma_{pp} + \sigma_{np}$. The experimental data are for pD total cross sections.

this problem leads to a result, which favors case 2, or the conventional approach, over case 1.

A.2. Relativistic

We will consider again the previous subject in a relativistic way. Before we continue the discussion we would like to state explicitly the basic assumptions which were used by the previous authors^{4,10} and also in this thesis.

The basic assumptions are as follows:

1. The deuteron is considered as a (p,n) bound state. We do not consider the isobar admixture, the six-quarks degree of freedom, and the meson exchange current contribution in the deuteron.
2. We work in the (incoherent) impulse approximation. This approximation excludes shadowing corrections.
3. We use "off-shell kinematics" -- "on-shell dynamics" formalism. Stated plainly, we use the on-shell amplitude but off-shell kinematics. This is not a bad approximation if the interacting nucleon is not far off the mass shell, which may be true for a weakly bound system like the deuteron.
4. To illustrate the physical point without obscuring the issue with algebra we will neglect the spin of the particles involved in this and the next two sections. The spin effects will be discussed in section V.

In the incoherent impulse approximation the imaginary part of the forward scattering amplitude is given by an incoherent sum of the contributions from Fig. 1 (which is considered now as a Feynman graph), where the lower, spectator nucleon is in turn a neutron and a proton. Thus, the smeared amplitude is given by^{9,15}

$$\text{Im } A^{\text{smeared}}(\nu) = \int \frac{d^4 k}{(2\pi)^4} \frac{\phi^2(p^2, k^2)}{(p^2 - k^2 + i\epsilon)^2} \text{Im } A(\tilde{\nu}) \times 2\pi \delta(k^2 - M^2) \theta(s' - M^2), \quad (2.11)$$

where A here means $A_p + A_n$, and ϕ is the n - p - d vertex function. We parametrize the A 's in terms of $\tilde{\nu}$ instead of ν' , which was adopted in the non-relativistic approach (See part C of this section). The invariants s' and $\tilde{\nu}$ are given by

$$s' = (p + q)^2 \equiv M^2 + q^2 + 2M\tilde{\nu}. \quad (2.12)$$

In (2.11) the off-shell dependences of the single nucleon scattering amplitudes are neglected (assumption 3). We incorporate the θ function explicitly in (2.11), which is due to the threshold condition on $\text{Im } A_1(\tilde{\nu})$. Admittedly, there is an ambiguity in this condition; since the imaginary part of the forward scattering amplitude includes both elastic and inelastic contributions, there is no single "right" quantity to put in the argument of the θ in (2.11), and $M + m_\pi$ would have been as good an approximation to use as M . At very high energies, however, either $M + m_\pi$ or M can be safely neglected with respect to s' . For simplicity, therefore, the

former choice will be adopted.

In the LAB system,

$$p_o = M_d - k_o = M_d - (|\vec{k}|^2 + M^2)^{1/2}, \quad \vec{p} + \vec{k} = \vec{0}. \quad (2.13)$$

In terms of the total cross sections, we rewrite (2.11) as

$$\sigma^{\text{smeared}}(\nu) = \int d^3 k |f(\vec{k})|^2 \left(\frac{\tilde{\nu}^2 - q^2}{\nu^2 - q^2} \right)^{1/2} \sigma(\tilde{\nu}) \times \theta(s' - M^2), \quad (2.14)$$

where we had defined

$$|f(\vec{k})|^2 \equiv \frac{M}{2(2\pi)^3 M_d k_o} \frac{\phi^2(p^2, k^2=M^2)}{(p^2 - M^2 + i\epsilon)^2}. \quad (2.15)$$

The $\phi(p^2, k^2=M^2)$ in (2.15) is known as the truncated n - p - d vertex function (truncated simply means that one of the nucleons, here the spectator, is on shell). In the high energy limit (2.14) takes a simpler form,

$$\sigma^{\text{smeared}}(\nu) = \int d^3 k |f(\vec{k})|^2 \frac{\nu'}{\nu} \sigma(\nu') \theta(\nu'). \quad (2.16)$$

The problems we are facing now are how we should interpret $|f(\vec{k})|^2$, and how we use these formulas in practice since we do not know the exact ϕ (no one has solved the bound state problem yet). We will address ourselves to these questions in the next section. Comparison of this part A.2 with part A.1 shows that we can recover the non-relativistic results by just making a simple correspondence between $|f(\vec{k})|^2$, or more precisely $\phi^2(p^2, k^2=M^2)$, and $f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)$. Notice that in

the case 1, which can give a small σ_s for a certain deuteron wave function,

$$|f(\vec{k})|^2 \frac{v'}{v} \equiv f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|). \quad (2.17)$$

However, there is a serious problem here because the LHS of (2.17) is not definite positive throughout the whole region of $|\vec{k}|$, and hence, should not be set equal to the usual non-relativistic deuteron wave function. Due to this undesirable feature, we can say now that the case 1 is ruled out, and we are left only with the case 2 or the conventional approach, where

$$|f(\vec{k})|^2 \equiv f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|) \quad (2.18)$$

B. Electron Scattering

The case of electron scattering is a little trickier than the above due to the fact that one is dealing with an incident particle that is virtual (we always work in the one-photon exchange approximation; see Fig. 4). A virtual photon of imaginary mass $(q^2)^{1/2}$ is absorbed on the target producing an arbitrary number of undetected final states. Experimentally only the energy and scattering angle of the final electron are detected. The electromagnetic structure of the target can be described by the tensor,¹⁶

$$W_{\mu\lambda}(q^2, \nu) = \sum_N \langle P | J_\mu(0) | N \rangle \langle N | J_\lambda(0) | P \rangle (2\pi)^3 \delta^4(P_N - P - q), \quad (2.19)$$

FIG. 4. Impulse approximation graph for the electron scattering case illustrating the one-photon exchange; l and l' represent the initial and final electrons, respectively.

where J_μ is the electromagnetic current operator, P the four-momentum of the target. Here q represents the four-momentum of the incident virtual photon. An average over the polarization in the initial state is implied in ϵ_N , and N is supposed to indicate an arbitrary final state. The most general form for $W_{\mu\lambda}$ consistent with Lorentz and gauge invariance, and conservation of parity is

$$W_{\mu\lambda}(q^2, \nu) = - \left(g_{\mu\lambda} - \frac{q_\mu q_\lambda}{q^2} \right) W_1(q^2, \nu) + \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left(P_\lambda - \frac{P \cdot q}{q^2} q_\lambda \right) \frac{W_2(q^2, \nu)}{M_T^2} \quad (2.20)$$

where $\nu = (S - M_T^2 - q^2)/2M_T$, M_T is the target mass and the W_i are scalar functions of the independent variables q^2 and ν (and in principle P^2 ; for a free target P^2 is of course fixed). In terms of these W_i the doubly differential cross section is

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha_e^2}{q M_T} E'^2 \cos^2 \frac{\theta}{2} \left[W_2 + 2W_1 \tan^2 \frac{\theta}{2} \right] \quad (2.21)$$

where E' is the energy of the scattered electron, θ its scattering angle (in the LAB system), and $d\Omega$ the elemental solid angle; α_e is the fine-structure constant ($\approx 1/137$).

One should note that the choice of $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ as structure functions is not unique. In general the cross section, when one photon exchange is used, will have the form¹⁷

$$\sigma = \sigma_{\text{point}} \left[a(q^2, \nu) + b(q^2, \nu) \tan^2 \frac{\theta}{2} \right], \quad (2.22)$$

where σ_{point} is the cross section for scattering from a point target with no structure. But obviously, other form factors $a'(q^2, \nu)$ and $b'(q^2, \nu)$ can also be chosen such that,

$$\sigma = \Gamma(q^2, \nu, \theta) [a'(q^2, \nu) + b'(q^2, \nu) \epsilon(q^2, \nu, \theta)], \quad (2.23)$$

Γ and ϵ being known functions not depending on the target. One such representation invented by Hand,^{1,18} is

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma_T [T_T(q^2, \nu) + \epsilon \sigma_L(q^2, \nu)], \quad (2.24)$$

where¹⁶

$$\sigma_{T,L}(q^2, \nu) = \frac{2\pi\alpha_e^2}{M_T K} \sum_N |\langle N | J_{T,L} | P \rangle|^2 (2\pi)^4 \delta^4(P_N - P - q), \quad (2.25)$$

$$\Gamma_T = \frac{\alpha_e^2}{2\pi^2} \frac{K E}{Q^2 E' (1 - \epsilon)}, \quad (2.26)$$

$$K = \frac{W^2 - M_T^2}{2M_T}, \quad (2.27)$$

$$\epsilon = \left[1 + 2 \left(1 + \frac{\nu^2}{Q^2} \right) \tan^2 \frac{\theta}{2} \right]^{-1}. \quad (2.28)$$

In Hand's picture σ_T and σ_L can be interpreted as the total cross sections for transversely and longitudinally polarized virtual photons (on the target). The transverse and longitudinal directions are defined with respect to the direction of $\vec{q}$ in the LAB frame. K is the energy

of an equivalent real photon capable of producing a final state of mass W . ϵ is known as the photon polarization parameter and Γ_L is interpreted as the effective flux of virtual photon. Dividing this photon flux factor into the data yields a total virtual photo-absorption cross section, $\sigma_T + \epsilon \sigma_L$.

We can determine the relationship between the W 's and the σ 's by looking in the LAB frame. If we define the z -axis along the direction of q then $W_{xx} = W_1$; so, comparing (2.20) with (2.25) we find¹

$$W_1(q^2, \nu) = \frac{K M_T}{4\pi^2 \alpha_e} \sigma_T(q^2, \nu). \quad (2.29)$$

A similar procedure for the z components leads to

$$W_2(q^2, \nu) = \frac{K M_T}{4\pi^2 \alpha_e} \frac{q^2}{q^2 - (P \cdot q / M_T)^2} [\sigma_T(q^2, \nu) + \sigma_L(q^2, \nu)]. \quad (2.30)$$

In the impulse approximation, as shown in Fig. 4,

$$W_{\mu\lambda}^d(q^2, \nu) = \frac{M_d}{M} \sum_{i=n,p} \int d^3k |f(\vec{k})|^2 W_{\mu\lambda}^i(q^2, \tilde{\nu}) \theta(s' - M^2), \quad (2.31)$$

where⁹ (see Appendix A)

$$W_{\mu\lambda}^i = - (g_{\mu\lambda} - \frac{q_\mu q_\lambda}{q^2}) W_1^i(q^2, \tilde{\nu}) + \tilde{p}_\mu \tilde{p}_\lambda \frac{W_2^i(q^2, \tilde{\nu})}{M^2},$$

$$\tilde{p}_\mu = p_\mu - \frac{p \cdot q}{q^2} q_\mu.$$

Since in the electron scattering the Glauber correction is small⁶ and hence can be neglected, we have $W_{\mu\lambda}^d = (W_{\mu\lambda}^p + W_{\mu\lambda}^n)^{\text{smeared}}$.

The tensor equation (2.31) yields 16 separate dependent equations, one for each component. The dependence arises from the current conservation $q^\mu W_{\mu\lambda} = W_{\mu\lambda} q^\lambda = 0$. If the z axis is defined to be along the direction of the virtual photon, an examination of the various tensorial components leads to the following two equations for the smearing of the W_1 and the W_2 structure functions:

$$W_1^d(q^2, \nu) = \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 \theta(s' - M^2)$$

$$\times [W_1(q^2, \tilde{\nu}) + \frac{k_x^2}{M^2} W_2(q^2, \tilde{\nu})], \quad (2.32)$$

$$W_2^d(q^2, \nu) = \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 \mathcal{F} W_2(q^2, \tilde{\nu}) \theta(s' - M^2), \quad (2.33)$$

$$\mathcal{F} = \left(\frac{\nu'}{\nu} \right)^2 \left[\left(1 - \frac{p_z q^2}{M \nu' |\vec{q}|} \right)^2 - \frac{p_x^2}{M^2} \left(\frac{\nu}{\nu'} \right)^2 \frac{q^2}{|\vec{q}|^2} \right], \quad (2.34)$$

where W_1 on the right hand sides means $W_1^p + W_1^n$. The p^2 - dependences of the W 's have been neglected since we know nothing about the off-shell behavior of the form factors.

It has become conventional to introduce two new structure functions

$$F_1(q^2, \omega_T) = W_1(q^2, \nu),$$

$$F_2(q^2, \omega_T) = M_T^{-1} \nu W_2(q^2, \nu), \quad (2.35)$$

where $\omega_T = -2M_T v/q^2$. In terms of these new variables and functions we can rewrite (2.32) and (2.33) in the form

$$F_1^d(q^2, \omega_d) = \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 \theta(\tilde{\omega}_N - 1) \times [F_1(q^2, \tilde{\omega}_N) - \frac{2p_x^2}{\tilde{\omega}_N q^2} F_2(q^2, \tilde{\omega}_N)], \quad (2.36)$$

$$F_2^d(q^2, \omega_d) = \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 \theta(\tilde{\omega}_N - 1) \times \frac{\tilde{\omega}_N}{\omega_d} F_2(q^2, \tilde{\omega}_N) \tilde{f}^p, \quad (2.37)$$

where

$$\omega_d = -\frac{2M_d v}{q^2} = \frac{M_d}{M} \tilde{\omega}_N, \quad (2.38)$$

$$\tilde{\omega}_N = -\frac{2Mv}{q^2}, \quad (2.39)$$

$$\tilde{f}^p = \left[1 + \frac{p^2 - M^2}{q^2 \tilde{\omega}_N} \right]^2 \left\{ \left[1 - \frac{4 M_d x F_2}{\omega_d (\tilde{\omega}_N q^2 + p^2 - k^2)} \right]^2 - \frac{4 x^2 q^2 p_x^2}{(\tilde{\omega}_N q^2 + p^2 - M^2)^2} \right\}, \quad (2.40)$$

$$r = \frac{v}{|\vec{q}|} \left(1 - \frac{q^2}{v^2} \right)^{-1/2} = \left(1 - \frac{4M_d^2}{\omega_d^2 q^2} \right)^{-1/2}. \quad (2.41)$$

In the deep inelastic limit the equations take on a very simple form, for in that case both r and $\tilde{f}^p$ tend to unity and we obtain

$$F_1^d(q^2, \omega_d) = \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 F_1(q^2, \omega_N') \theta(\omega_N' - 1), \quad (2.42)$$

$$F_2^d(q^2, \omega_d) = \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 \frac{\omega_N'}{\omega_d} F_2(q^2, \omega_N') \theta(\omega_N' - 1), \quad (2.43)$$

where $\omega_N' = -2Mv'/q^2$.

Note that in our discussion it is not necessary to assume Bjorken scaling, so that we retain the possibility of a q^2 dependence in the structure functions. The expressions given in (2.14), (2.16), (2.36), (2.37), (2.42) and (2.43) form the basis for the conventional, "non-relativistic" approach to the Fermi motion effects on deuterium targets.¹⁹ By "non-relativistic" we mean the use of

$$|f(\vec{k})|^2 = F_s^2(|\vec{k}|)^2 + f_d^2(|\vec{k}|).$$

C. Identification of Virtual-Particle Cross-Sections

In both electromagnetic and strong-interaction scattering an ambiguity arises as to what one should use for the total cross sections

for scattering from a virtual particle. In this thesis we will adopt the common view that these are the same as the real particle total cross sections. Following Bodek⁹ we will also use the q^2 and $\tilde{\nu}$ as the relevant variables.

We shall now present a threshold argument for choosing these particular variables.^{8,9} Suppose we artificially separate W_2 into elastic and inelastic contributions,

$$W_2 = W_2^{\text{el}} \delta((p+q)^2 - M^2) + W_2^{\text{inel}} \theta((p+q)^2 - M_{\text{th}}^2), \quad (2.44)$$

where the pion threshold is $M_{\text{th}} = M + M_\pi$. Now if the nucleon is real (free), the argument of the δ function is

$$(p+q)^2 - M^2 = q^2 + 2p \cdot q, \quad (2.45)$$

while, if it is virtual (bound), the argument is

$$(p+q)^2 - M^2 = q^2 + 2p \cdot q + p^2 - M^2. \quad (2.46)$$

If we use the variable $\tilde{\nu}$ (See (2.12)) then (2.45) and (2.46) have the same form, i.e. $q^2 + 2M\tilde{\nu}$, in the two cases. The same result follows for the argument of the θ function. Hence in order to ensure the correct threshold behavior in a simple way,²⁰ it is obviously convenient to consider the W 's as functions of q^2 and $\tilde{\nu}$.

III. THE INCONSISTENCIES OF THE CONVENTIONAL APPROACH

In this section we will discuss the inconsistencies of the conventional approach. We shall present the arguments in four cases:

- A. The normalization condition, where the identification problem of $|f(\vec{k})|^2$ will be discussed.
- B. The sum-rules, which expressions do not agree with the naive quark-parton model interpretations.
- C. The West β -correction, which is the smearing correction if the total cross sections were constant.
- D. The smearing corrections, which in the high energy hadronic processes turn out to be non-negligible, and hence, they are in disagreement with what we expect.

A. The Normalization Condition

The relation between $|f(\vec{k})|^2$ and the truncated n-p-d vertex function is given by (see Section II-A.2.)

$$|f(\vec{k})|^2 = \frac{N}{2(2\pi)^3 M_d K_0} \frac{\phi^2(p^2, k^2 = M^2)}{(p^2 - M^2 + i\epsilon)^2}. \quad (3.1)$$

The problem we are facing now is how to normalize $|f(\vec{k})|^2$, or more precisely how we should interpret it.

The easiest way is just simply interpret $|f(\vec{k})|^2$ as the probability of finding a nucleon with momentum $\vec{k}$ in the deuteron rest-system, and

hence

$$\int d^3k |f(\vec{k})|^2 = 1 \quad (3.2)$$

This view has been adopted in the conventional approach, where one identifies $|f(\vec{k})|^2$ as the usual non-relativistic deuteron wave function. In other words, in this approach one defined the vertex function as

$$\phi^2(p^2) = 2 (2\pi)^3 k_0 \frac{M_d}{M} (p^2 - M^2)^2 [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)]. \quad (3.3)$$

Clearly there is an ambiguity in this kind of identification. This choice will be labeled as case A.

There is another way to proceed, namely to apply the relation of the elastic electromagnetic form factor at $q^2 = 0$ to the total charge.²¹

In terms of the truncated vertex function the impulse approximation model for the electromagnetic form factor is shown in Fig. 5; the cross on one of the internal lines is to indicate that the (spectator) nucleon is to be taken on the mass shell (spectator approximation).²²

The elastic form factor is given by

$$\begin{aligned} (2P - q)_\mu F_d(q^2) &= \sum_{i=p,n} \frac{1}{(2\pi)^3} \int \frac{d^3k}{2k_0} \frac{\phi(p^2)}{p^2 - M^2 + i\epsilon} \\ &\times \frac{\phi((p+q)^2)}{(p+q)^2 - M^2 + i\epsilon} \\ &\times \left\{ (2p+q)_\mu F^i(q^2) + q_\mu \frac{(p+q)^2 - p^2}{q^2} [1 - F^i(q^2)] \right\}. \quad (3.4) \end{aligned}$$

FIG. 5. The impulse approximation model for the elastic form factor. The cross on the lower line indicates that the particle is to be treated on the mass shell.

We show in Appendix A that this form satisfies the gauge-invariance condition. Going to the static ($q = 0$) limit in the deuteron rest-frame, we have

$$\frac{1}{(2\pi)^3} \int \frac{d^3k}{2k_0} \frac{p_0}{M_d} \frac{\phi^2(p^2)}{(p^2 - M^2 + i\epsilon)^2} = 1 \quad (3.5)$$

It is tempting, based on this result, to define $\phi(p^2)$ such that

$$\frac{p_0}{2(2\pi)^3 k_0 M_d} \frac{\phi^2(p^2)}{(p^2 - M^2 + i\epsilon)^2} \equiv |\psi(|\vec{k}|)|^2 \quad (3.6)$$

could be interpreted as the probability to find a nucleon with momentum $\vec{k}$ inside the deuteron. However this attempt must be abandoned because $p_0 = M_d - k_0 = M_d - (|\vec{k}|^2 + M^2)^{1/2}$ in the deuteron rest-system, which implies that $|\psi(\vec{k})|^2$ has negative values for $|\vec{k}| \geq (M_d^2 - M^2)^{1/2}$, and hence, cannot be interpreted as a probability function.

This analysis, however, induced one to define the vertex function as follows (we use a different symbol here to differentiate from the previous one in case A):

$$\phi^2(p^2) = 2 (2\pi)^3 M_d (p^2 - M^2)^2 [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] \quad (3.7)$$

This kind of choice will be labelled as case B.

B. The Sum-Rules

In Section II we had shown that in the deep inelastic limit (DIL) the deuteron electromagnetic structure functions take on a very simple form, i.e.

$$F_1^d(q^2, \omega_d) = \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 F_1(q^2, \omega_N') \theta(\omega_N' - 1),$$

$$F_2^d(q^2, \omega_d) = \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 \frac{\omega_N'}{\omega_d} F_2(q^2, \omega_N') \theta(\omega_N' - 1).$$

A simple direct consequence of these equations is the following sum-rules.

Let's consider

$$\int_1^\infty \frac{d\omega_d}{\omega_d^\alpha} F_2^d(q^2, \omega_d) = \left(\frac{M}{M_d}\right)^{\alpha-1} \int_{M/M_d}^\infty \frac{d\omega_N'}{\omega_N'^\alpha} F_2^d(q^2, \omega_N')$$

$$= \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 \left(\frac{p_0 - p_3}{M_d}\right)^\alpha$$

$$\times \int_{(p_0 - p_3)/M_d}^\infty \frac{d\omega_N'}{\omega_N'^\alpha} F_2(q^2, \omega_N') \theta(\omega_N' - 1), \quad (3.8)$$

where we had used the fact that

$$\frac{\omega_N'}{\omega_d} = \frac{v'}{v} \xrightarrow{\text{DIL}} \frac{p_0 - p_3}{M} \quad (3.9)$$

The threshold condition, $\theta(\omega_N' - 1)$, allows us to rewrite (3.8) as

$$\int_1^{\infty} \frac{d\omega_d}{\omega_d^\alpha} F_2^d(q^2, \omega_d) = \frac{M_d}{M} \int d^3k |\vec{f}(\vec{k})|^2 \left(\frac{p_0 - p_3}{M_d} \right)^\alpha \times \int_1^{\infty} \frac{d\omega_N'}{\omega_N'^\alpha} F_2(q^2, \omega_N'). \quad (3.10)$$

The sum rules for $\alpha \leq 0$ certainly do not converge, for the experimental data show that as $\omega \rightarrow \infty$, $F_2(q^2, \omega) \rightarrow \text{constant}$ (See Fig. 6).²³ On the other hand, some of the sum rules for $\alpha > 0$ certainly do. Notice that we do not require α to be an integer.

Consider the $\alpha = 1$ sum rule:

$$\int_1^{\infty} \frac{d\omega_d}{\omega_d} F_2^d(q^2, \omega_d) = \int d^3k |\vec{f}(\vec{k})|^2 \frac{p_0 - p_3}{M} \times \int_1^{\infty} \frac{d\omega_N'}{\omega_N'} F_2(q^2, \omega_N') \quad (3.11)$$

The result seems to show us that it is not a wave-function independent sum rule. Also it disagrees with the quark-parton model² expectation, that is

$$\int_1^{\infty} \frac{d\omega_d}{\omega_d} F_2^d(q^2, \omega_d) = \int_1^{\infty} \frac{d\omega_N'}{\omega_N'} F_2(q^2, \omega_N'), \quad (3.12)$$

which states that the (infinite) sum of the squares of the charges of the quarks and anti-quarks in the deuteron is obtained by adding together the corresponding sums for the proton and neutron separately. Eq. (3.12) is obviously a necessary feature of the impulse approximation.²⁴

In the conventional approach (or case A here), where one identifies

$$|\vec{f}(\vec{k})|^2 = f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|),$$

equation (3.11) can be rewritten as

$$\int_1^{\infty} \frac{d\omega_d}{\omega_d} F_2^d(q^2, \omega_d) = \int d^3k [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] \frac{p_0}{M} \times \int_1^{\infty} \frac{d\omega_N'}{\omega_N'} F_2(q^2, \omega_N'), \quad (3.13)$$

where $p_0 = M_d - (|\vec{k}|^2 + M^2)^{1/2}$.

In writing (3.13) we had used the fact that

$$\int d^3k [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] |\vec{k}| \cos\theta = 0 \quad (3.14)$$

FIG. 6. νW_2^p and νW_2^d vs. $\omega' = \omega_N + \frac{M^2}{Q^2}$. Data are from the region $Q^2 > 1.0 \text{ GeV}^2$, $W > 2.0 \text{ GeV}$. $R_p = R_d = 0.18$, where $R = \sigma_L/\sigma_T$. Errors shown are statistical only. In most cases the error bars are smaller than the symbols.

Notice that in the conventional approach, we are still facing the problem we discussed before.

A similar approach can also be applied to $F_1^d(q^2, \omega_d)$. For example we have

$$\int_1^\infty \frac{d\omega_d}{\omega_d^2} F_1^d(q^2, \omega_d) = \int d^3k |f(\vec{k})|^2 \frac{p_0 - p_3}{M} \times \int_1^\infty \frac{d\omega_N'}{\omega_N'^2} F_1(q^2, \omega_N'). \quad (3.15)$$

The sum rules given by (3.11) and (3.15) are consistent with the so-called Callan-Gross relations:²

$$2F_1^d(q^2, \omega_d) = \omega_d F_2^d(q^2, \omega_d),$$

$$2F_1^i(q^2, \omega_N') = \omega_N' F_2^i(q^2, \omega_N'); \quad i = p, n. \quad (3.16)$$

C. The West β -Correction

We had shown in Section II that in the high energy limit, the hadronic smeared total cross-section is given by

$$\sigma^{\text{smeared}}(\nu) = \int d^3k |f(\vec{k})|^2 \frac{\nu'}{\nu} \sigma(\nu') \delta(\nu'). \quad (3.17)$$

In the deuteron rest-frame we can write

$$v' - v = -\frac{v}{M} (\epsilon + T - |\vec{k}| \cos\theta), \quad (3.18)$$

by using

$$|\vec{q}| \approx q_0 = v, \quad M_d = 2M - \epsilon,$$

where ϵ is the deuteron binding energy and by defining $T = (M^2 + |\vec{k}|^2)^{1/2} - M$ which is the relativistic kinetic energy. The angle θ above is the angle between $\vec{k}$ and $\vec{q}$.

For slowly varying cross sections we can expand $\sigma(v')$ around $v' = v$.

Then from (3.17) we obtain

$$\begin{aligned} \sigma^{\text{smeared}}(v) &= \int d^3k |f(\vec{k})|^2 \theta(v') \\ &\times \left\{ \sigma(v) - \frac{\epsilon + T - |\vec{k}| \cos\theta}{M} \frac{1}{dv'} [\nu' \sigma(v')] \right. \\ &\quad \left. + \dots \right\} \quad (3.19) \end{aligned}$$

If we define

$$\sigma^{\text{smeared}}(v) = \sigma(v) - \sigma_S(v), \quad (3.20)$$

then

$$\sigma_S(v) = \beta_W \sigma(v) + (\beta_W - \beta) v \frac{d}{dv} \sigma(v) \Big|_{v'=v} + \dots, \quad (3.21)$$

where

$$\beta_W = \beta \left(1 - \frac{\epsilon}{M}\right) + \frac{\epsilon + \langle T \rangle - \gamma}{M}, \quad (3.22)$$

$$1 - \beta = \int d^3k |f(\vec{k})|^2 \theta(v'), \quad (3.23)$$

$$\langle T \rangle = \int d^3k |f(\vec{k})|^2 T \theta(v'), \quad (3.24)$$

$$\gamma = \int d^3k |f(\vec{k})|^2 |\vec{k}| \theta(v') \cos\theta. \quad (3.25)$$

If $\sigma(v')$ were constant, then

$$\sigma_S = \beta_W (\sigma_p + \sigma_n) \equiv \sigma_W. \quad (3.26)$$

The correction c_W is known as the West β -correction.²⁵ To get a feeling of how big β_W is, we calculate it for the two cases, i.e., for two kinds of identification for $|f(\vec{k})|^2$, we mentioned in the previous section. For the deuteron non-relativistic wave function we (arbitrarily) choose four among the very many wave functions:²⁶ Hamada-Johnston, Reid (hard- and soft-) core and Hulthén-Sugawara soft-core. The picture of these wave functions is shown in Figs. 7-10. For practical purposes we need to rewrite (3.23), (3.24) and (3.25) in the more convenient forms, which are given explicitly in the Appendix B. The results are shown in Table 1.

From Table 1 we learn that the West β -correction is not small. Furthermore the table indicates that the ambiguity in the identification

FIG. 7. 3S co-ordinate space wave functions for Hamada-Johnston—, Reid HC—, Reid SC —, and Hulphén-Sugawara SC—-. The normalization is $\int_0^\infty [u^2(r) + w^2(r)] dr = 1$, where r is expressed in pion Compton wavelengths.

FIG. 8. 3S momentum-space wave functions for the potentials of FIG. 7. The normalization is $\int_0^\infty [u^2(k) + w^2(k)] dk = 1$, where $k = |\vec{k}|$ is expressed in pion (π^+) masses.

FIG. 9. ^3D co-ordinate space function $W(r)$ for the potentials of FIG. 7.

FIG. 10. ^3D momentum-space wave functions for the potentials of FIG. 7.

TABLE 1

Calculated Values for β , $\langle T \rangle$, γ and β_W

WAVE FUNCTION	$\beta \times 10^5$		$\langle T \rangle$ (MeV)		γ (MeV)		$\beta_W \times 10^5$	
	A	B	A	B	A	B	A	B
H.J.	7,924	10,692	19.49	15.36	-5.48	-3.38	10.783	12,900
R.H.C.	5,025	7,889	19.94	15.87	-5.19	-3.23	7,915	10,141
R.S.C.	4,048	6,869	19.63	15.56	-5.47	-3.41	6,939	9,110
H.S.S.C.	5,978	7,597	12.48	10.49	-2.54	-1.57	7,787	9,101

The above table shows the physical quantities β , $\langle T \rangle$, γ and β_W for the two cases A and B as discussed in the text. The labels H. J. (Hamada Johnston), R.H.C. (Reid hard-core), R.S.C. (Reid soft-core) and H.S.S.C. (Hulthén-Sugawara soft-core) mean the wave functions used for the calculation.

of $|f(\vec{k})|^2$ in terms of the usual non-relativistic deuteron wave-function can introduce a large uncertainty into the smearing.

Experimentally,⁶ the electromagnetic structure functions F_i are relatively slowly varying functions of ω (see Fig. 6) so an expansion of the type used for $\sigma(\nu)$ should be a reasonable approximation here. Let us concentrate on F_2 ; we shall consider F_1 later. Recall, in the deep inelastic limit,

$$F_2^d(q^2, \omega_d) = \frac{M_d}{M} \int d^3k |f(\vec{k})|^2 \frac{\omega_N'}{\omega_d} F_2(q^2, \omega_N') \theta(\omega_N' - 1) \quad (3.27)$$

In the deuteron rest frame we have, by using (3.9),

$$\omega_N' - \frac{M}{M_d} \omega_d = - \frac{\omega_d}{M_d} (\epsilon + T - |\vec{k}| \cos \theta). \quad (3.28)$$

Expanding $F_2(q^2, \omega_N')$ around $\omega_N' = \frac{M}{M_d} \omega_d$ we obtain

$$F_2^d(q^2, \omega_d) = F_2(q^2, \frac{M}{M_d} \omega_d) [1 - \beta_W(\omega_d)] - [\beta_W(\omega_d) - \beta(\omega_d)] \frac{M}{M_d} \omega_d \frac{d}{d\omega_N'} F_2(q^2, \omega_N') + \dots, \quad (3.29)$$

$\omega_N' = \frac{M}{M_d} \omega_d$

where

$$\beta_W(\omega_d) = (1 - \frac{\epsilon}{M}) \beta(\omega_d) + \frac{\epsilon + \langle T(\omega_d) \rangle - \gamma(\omega_d)}{M}, \quad (3.30)$$

$$1 - \beta(\omega_d) = \int d^3k |f(\vec{k})|^2 \theta(\omega_N' - 1), \quad (3.31)$$

$$\langle T(\omega_d) \rangle = \int d^3k |f(\vec{k})|^2 T \theta(\omega_N' - 1), \quad (3.32)$$

$$Y(\omega_d) = \int d^3k |f(\vec{k})|^2 |\vec{k}| \theta(\omega_N' - 1) \cos\theta. \quad (3.33)$$

The reason that the above integrals are ω_d -dependent is that the threshold constraint here (leptonic case) leads to a constraint that

$$\cos\theta \leq \frac{\omega_d P_0 - M_d}{\omega_d |\vec{k}|}, \quad (3.34)$$

which is relevant only when

$$|\vec{k}| \geq \frac{M_d^2 (1 - \omega_d)^2 - M_d^2 \omega_d^2}{2\omega_d M_d (\omega_d - 1)}. \quad (3.35)$$

There are two important points to note concerning this condition: (i) it depends only upon ω_d and (ii) it becomes increasingly more important as $\omega_d \rightarrow M_d/(M_d - M) \approx 2$. The resulting depletion thus scales and is expected to increase as ω_d approaches 2. Because of the scaling character of (3.35), the integrals that have to be performed after expanding $\omega_N' F_2(q^2, \omega_N')$ in (3.27) will now be ω_c dependent. Note that $\beta(\omega_d)$ has its minimum value ($= \beta$) at $\omega_c \rightarrow \infty$ and increases with decreasing ω_d (For more details see Appendix B).

Similarly for $F_1(q^2, \omega_d)$ we get

$$\begin{aligned} F_1^d(q^2, \omega_d) &= \frac{M_c}{M} F_1(q^2, \frac{M}{M_d} \omega_d) [1 - \beta(\omega_d)] \\ &- [F_1(\omega_c) - \beta(\omega_d)] \omega_d \frac{d}{d\omega_N'} F_1(q^2, \omega_N') \Big|_{\omega_N' = \frac{M}{M_d} \omega_d} \\ &+ \dots \end{aligned} \quad (3.36)$$

I. The Smearing Corrections

In this section we will limit ourselves to σ_s , the smearing correction, in hadronic scattering.²⁷ In the previous sections we had shown that the conventional approach, where one used (3.3) for the definition of the vertex function, leads to sum-rules which are in disagreement with the quark-parton model expectation and to the existence of the (non-negligible) West β -correction (σ_W). Unfortunately these predictions cannot be tested experimentally, for we do not have free neutron targets. However, in hadronic scattering there are two cases where all the relevant cross sections are measurable: (i) pion-nucleon scattering (isospin invariance: $\sigma_{\pi+p} = \sigma_{\pi-n}$) and (ii) nucleon-nucleon scattering (isospin invariance: $\sigma_{np} = \sigma_{pn}$). Thus the aim of this section is to "test" this conventional approach by using both processes mentioned above.

In the conventional approach the smearing cross section in the LAB frame is given by, in the high energy limit, (see section II.A)

$$\begin{aligned}\sigma^{\text{smeared}}(v) &\equiv \sigma(v) - \sigma_s(v) \\ &= \int d^3k \frac{v'}{v} [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] \\ &\quad \times \sigma(v') \theta(v') .\end{aligned}\quad (3.37)$$

It is sometimes more convenient to use the parameter $\beta_s(v)$ to characterize the Fermi motion effects. It is defined by

$$\sigma_s(v) = \beta_s(v) \sigma(v) , \quad (3.38)$$

and hence,

$$\sigma^{\text{smeared}}(v) = \sigma(v) [1 - \beta_s(v)] . \quad (3.39)$$

For practical purposes we will use (2.5) to rewrite (3.37) as

$$\sigma^{\text{smeared}}(v) = \sigma_1(v) + \sigma_2(v) , \quad (3.40)$$

where²⁸

$$\sigma_1(v) = \frac{2\pi M}{v^2} \int_0^{k_1} [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] |\vec{k}| d|\vec{k}| \int_{v'_-}^{v'_+} v' \sigma(v') dv' , \quad (3.41)$$

$$\sigma_2(v) = \frac{2\pi M}{v^2} \int_{k_1}^{\infty} [f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|)] |\vec{k}| d|\vec{k}| \int_0^{v'_+} v' \sigma(v') dv' , \quad (3.42)$$

and

$$k_1 = (M_d^2 - M^2) / 2M_d , \quad (3.43)$$

$$v'_\pm = \frac{1}{N} [M_d - (|\vec{k}|^2 + M^2)^{1/2} \pm |\vec{k}|] . \quad (3.44)$$

We will use (3.41) and (3.42) to calculate $\sigma(v)$, and hence, $\beta_s(v)$ for pion and nucleon beams by using the recent high energy cross sections data (50 - 370 GeV)^{5,29}. For our convenience we parameterize the cross section data as

$$\sigma(v) = a + b \ln^2(v/c) , \quad (3.45)$$

where a , b and c are the fitting parameters. They are given in Table 2, see Figures 11 and 12. As the non-relativistic deuteron wave function we use the same wave functions that are used in the calculation of the quantity β_W . The complete formula for the calculation of this smeared cross sections, σ^{smeared} , is given in Appendix C. The result is shown in Figures 13 and 14 for the smeared cross sections, $\sigma^{\text{smeared}}(v)$, and Figures 15 and 16 for the smearing corrections, $\sigma_s(v)$. We also calculate the $\beta_s(v)$ and compared it to the β_W we obtained before (See Table 3). The $\beta_s(v)$ is practically constant throughout the region we considered.

TABLE 2

Fitting Parameters for Hadronic Total Cross-Sections

	a (mb)	b(mb)	c (GeV)	# Data N	$\overline{\chi^2} = \chi^2 / (N-3)$
pD	73.16	0.78	70.00	15	0.72
pp	38.24	0.37	48.00	20	0.76
np	38.91	0.42	50.00	6	0.74
π^- D	45.56	1.00	80.09	16	0.95
π^- p	24.00	0.64	94.00	29	2.60
π^+ p	23.14	0.47	59.00	20	0.72

FIG. 11. σ_{π^-D} , σ_{π^-p} , and σ_{π^+p} . The curve is the result of the fit (3.45) and the data are from: $\bullet$ Carroll *et al.*; $\circ$ Denisov *et al.*; $\times$ Ayres *et al.*; $\circ$ Akopdjanov *et al.*; $\blacksquare$ Berger *et al.*; $\triangle$ Fong *et al.*; ∇ Bogert *et al.*; $\blacktriangle$ Firestone *et al.*; $\blacktriangledown$ Apokin *et al.*

FIG. 12. σ_{pD} , σ_{pp} and σ_{np} . The curve is the result of the fit (3.45) and the data are from: $\bullet$ Carroll *et al.*; $\times$ Ayres *et al.*; $\square$ Denisov *et al.*; $\circ$ Longo *et al.*

FIG. 13. Graph of $\sigma_{\text{smeared}}(\nu)$, the smeared cross section, versus the incident pion energy for various wave functions. The labels, RSC (Reid soft-core), RHC (Reid hard-core), HSSC (Hulthén-Sugawara soft-core) and HJ (Hamada Johnston) mean the wave functions used for the smearing by using (3.40). The dashed line is the unsmeared cross section, i.e. $\sigma_{\pi-D} + \sigma_{\pi+p}$. The experimental data are for π -D total cross sections.

FIG. 14. Graph of σ_{smeared} of nucleon scattering for various wave functions. The dashed line is the unsmeared cross section, $\sigma_{pp} + \sigma_{np}$. The experimental data are for pD total cross section.

FIG. 15. Graph of $\sigma_s(\nu)$, the smearing correction, versus the incident pion-energy, for various wave functions. The dashed line is the (relativistic) result of using (4.19) and (4.63), which is discussed in Section IV. The experimental data are $\sigma_{\pi-p} + \sigma_{\pi+p} - \sigma_{\pi-D}$, which expression is usually taken as σ_G , the Glauber correction.

FIG. 16. Graph of $\sigma_s(\nu)$ of nucleon scattering of various wave functions. The dash-dot line supposedly represents the $\sigma_{pp} + \sigma_{np} - \sigma_{pD}$, which expression is usually taken to be σ_G .

TABLE 3
Calculated Values for β_s

WAVE FUNCTION	$\beta_s \times 10^{+5}$		$\beta_w \times 10^{+5}$
	PION	NUCLEON	
H.J.	2,165 $\pm$ 7	2,202 $\pm$ 3	10,783
R.H.C.	-1,080 $\pm$ 7	-1,040 $\pm$ 3	7,915
R.S.C.	-1,920 $\pm$ 7	-1,881 $\pm$ 3	6,939
H.S.S.C.	-157 $\pm$ 4	-123 $\pm$ 2	7,787
Relativistic Hulthén	-70 $\pm$ 22	-39 $\pm$ 9	0

The above table shows the physical quantities β_s for the pion- and nucleon-processes, as discussed in the text, by using the conventional approach. The last line is based on the new approach (See Section IV). For a comparison we show also the β_w . (See Table 1.).

The results we get show that the shape of $\sigma_s(v)$, and hence of $\beta_s(v)$ is essentially wave-function-, energy- and process-independent, whereas its magnitude is rather strongly dependent upon the wave function. This might not be surprising for it is to be expected that σ_s will be sensitive to the tail of the momentum distribution and this varies considerably from one wave function to another.⁴ The fact that σ_s is energy and process-independent merely reflects the fact that total cross sections have similar shapes in the high energy region (50 - 370 GeV) we considered.

Intuitively we expect the σ_s can be neglected in the region we consider. This view has been adopted by experimentalists.⁵ It turns out, however, that the smallness of σ_s is not realized by the conventional approach. It gives a non-negligible σ_s , which in fact can even have the same order of magnitude as σ_C , the Glauber correction, for some wave functions. In the region we analyze, the σ_s is about 0 to 3 percent, depending on which deuteron wave function we use. It is surprising that the Hamada Johnston wave function and the Reid hard-and soft-core wave functions, which are in the best agreement with the measured elastic deuteron form factors,³⁰ give a large value of σ_s . On the other hand, the Hulthén-Sugawara soft-core, which is in poor agreement with those measurements, gives essentially zero results (within the experimental errors). Thus, there is a contradiction here.

The previous calculations are based on (3.3) for the definition of the vertex function. We also take a similar calculation by using (3.7) instead of (3.3), i.e. case B. The difference of the results will give

us some measure of the ambiguity we discussed before. The comparison is shown in Figures 17 and 18 for an illustration. We learn that the ambiguity can have a profound effect. It is clear that case B is worse than the case A (conventional approach); it gives a larger σ_s and furthermore it can lead to a nonsense result!

Before we close this section we would like to give a remark concerning the so-called "off-shell correction ambiguity". Bodek⁹ had pointed out that the electroproduction expression of the conventional approach in the $q^2 = 0$ limit does not yield a vanishing σ_L^d (the absorption cross section of scalar virtual photons). He showed that one can force σ_L^d to vanish in the $q^2 = 0$ limit by including off-mass-shell corrections to the electroproduction structure functions. However, these constraints lead to several possible forms for off-mass-shell corrections. Notice that this ambiguity can introduce a larger uncertainty into the smearing than the one introduced by the uncertainty in the deuteron wave function.

We disagree with Bodek's argument. We will argue that as long as we stay within our basic assumptions, especially the so-called "off-shell kinematics"—"on-shell dynamics" formalism, the electroproduction expression is consistent, i.e., it leads to a vanishing σ_L^d in the $q^2=0$ limit. Hence, we do not need the off-shell corrections (as far as this problem is concerned) which only unnecessarily complicate the problem. The details of the discussion will be given in the next section.

FIG. 17. Graph of $\sigma^{\text{smeared}}(\nu)$ of pion scattering for the two cases A and B mentioned in the text by using the Hamada Johnston deuteron wave-function.

FIG. 18. Graph of σ^{smeared} of nucleon scattering for the two cases A and B. We use here the Hulthén-Sugawara soft-core deuteron wave function.

IV. THE NEW APPROACH

In the previous sections we had shown how difficult it is to construct a simple and consistent formalism of the Fermi motion effects on deuterium targets. The usual relativistic approach has the following problems:

1. The normalization condition on $|f(\vec{k})|^2$.

The attempt to use the common procedure, i.e., the relation of the elastic electromagnetic form factor at $q^2 = 0$ to the total charge, for the normalization condition fails since it leads to a probability function which can have negative values for some values of $|\vec{k}|$. One, of course, can choose

$$\int d^3k |f(\vec{k})|^2 = 1,$$

but there is no physical reason for it, except simplicity.

2. Whatever convention we use for the normalization condition of $|f(\vec{k})|^2$ we will not get sum-rules which agree with the quark-parton model expectations.
3. The existence of the West 3-correction.
4. How do we carry out this formalism in practice? Remember that we really do not know what is the "right" form for $\phi(p^2, k^2 = M^2)$.

As far as problem 3 is concerned we cannot say anything because it cannot be tested experimentally (we do not have any evidence yet that

the hadronic total cross section has reached a constant value). The problem 4 is not a new problem; it has something to do with the bound-state problem which no one has solved yet, and so nothing can be said about it. Thus, we are left with the consistency problem in the formalism.

In the conventional approach, which has been used so far to extract the neutron data from deuterium targets, one sets

$$|f(\vec{k})|^2 = f_s^2(|\vec{k}|) + f_d^2(|\vec{k}|).$$

There is no physical reason for this, except perhaps simplicity. Once one accepts this identification, in principle the problems 1 and 4 have been "solved". The problem 2 is still there, and when we calculate β_W (problem 3), it turns out to be non-negligible.

Recently this conventional approach has been criticized.¹⁰ Unfortunately this criticism is basically focused only on the problem 3, which is meaningless if it is seen from the experimental point of view. Hence, the criticism missed the real issue, i.e. the "right" descriptions which are "free" from the problems we mentioned before. One of the purposes of this thesis is to "correct" this situation.

We think the main objection to the conventional approach is the use of the non-relativistic deuteron wave functions. We doubt if the wave functions, which are adjusted to fit the phase shifts between 0 and 350 MeV/c, can be successfully used to momenta larger than 1 GeV/c. Furthermore this approach leads to a non-negligible smearing correction

σ_s for the high energy hadronic total cross section.

In this section we will discuss a new approach, that was recently proposed by us,^{11,27} to this problem. We will show that this new approach is simple and consistent. Furthermore it is also a relativistic approach.

A. The Brodsky Parameterization Method

It is surprising that the common procedure fails to give the normalization condition for $|f(\vec{k})|^2$. This failure is probably due to the use of the spectator approximation,²² apart from the assumption that ϕ is an analytic function in k_0 -plane, in the calculation of the elastic form factor. In this section we propose another approximation which gives better results.

Consider Fig. 5 without the cross on the spectator line. The elastic deuteron form factor (spin-less case) is given by

$$\begin{aligned} \Gamma_\mu &= (2P + q)_\mu F_d(q^2) \\ &= \sum_{i=n,p} \int \frac{d^4k}{(2\pi)^4 i} \frac{\phi(p^2, k^2)}{p^2 - M^2 + i\epsilon} \frac{\phi((p+q)^2, k^2)}{(p+q)^2 - M^2 + i\epsilon} \frac{1}{k^2 - M^2 + i\epsilon} \\ &\quad \times \left\{ (2p+q)_\mu F^1(q^2) + q_\mu \frac{(p+q)^2 - p^2}{q^2} [1 - F^1(q^2)] \right\}. \end{aligned} \quad (4.1)$$

We show in Appendix A that this form satisfies the gauge-invariance condition. In (4.1) the p^2 - and $(p+q)^2$ -dependences of the F^1 , have been neglected since we know nothing about the off-shell behavior of the form factors.

For our convenience we will use the Brodsky parameterization method³¹ to parameterize the four momenta as follows:

$$\begin{aligned} P_\mu &= \left(\rho + \frac{M_d^2}{4\rho}, \vec{0}_\perp, \rho - \frac{M_d^2}{4\rho} \right), \\ p_\mu &= \left(\alpha \rho + \frac{p^2 + \vec{k}_\perp^2}{4\alpha\rho}, \vec{k}_\perp, \alpha \rho - \frac{p^2 + \vec{k}_\perp^2}{4\alpha\rho} \right), \\ k_\mu &= \left((1-\alpha)\rho + \frac{k^2 + \vec{k}_\perp^2}{4(1-\alpha)\rho}, -\vec{k}_\perp, (1-\alpha)\rho - \frac{k^2 + \vec{k}_\perp^2}{4(1-\alpha)\rho} \right), \\ q_\mu &= \left(\frac{P \cdot q}{2\rho}, \vec{q}_\perp, -\frac{P \cdot q}{2\rho} \right). \end{aligned} \quad (4.2)$$

Here $\rho = \frac{1}{2}(P_0 + P_3)$ is an arbitrary parameter and notice that all invariants are independent of ρ . It need not be large, e.g. in the deuteron rest-frame $\rho = \frac{1}{2}M_d$. Also $\alpha = (p_0 + p_3)/(P_0 + P_3) = (p_0 + p_3)/2\rho$. The momentum conservation, $P_\mu = p_\mu + k_\mu$, gives

$$p^2 = \alpha M_d^2 + k^2 - \frac{k^2 + \vec{k}_\perp^2}{1-\alpha} \quad (4.3)$$

The great merit of the Brodsky parameterization is the simple factorization of the k^2 integration:

$$\int d^4k = \int d^2\vec{k}_\perp \int_{-\infty}^{\infty} \frac{d\alpha}{2|1-\alpha|} \int_{-\infty}^{\infty} d k^2 \quad (4.4)$$

Since $F_d(q^2) = (r_0 + r_3)/4\rho$, by using (4.2) and (4.4) we can rewrite (4.1) as

$$F_d(q^2) = \sum_{i=n,p} F^i(q^2) \int d^2\vec{k}_\perp \int_{-\infty}^{\infty} \frac{\alpha d\alpha}{2(2\pi)^3 |1-\alpha|} \times \int_{-\infty}^{\infty} \frac{dk^2}{2\pi i} \phi(p^2, k^2) \phi((p+q)^2, k^2) H^{-1}, \quad (4.5)$$

where

$$H = (\alpha M_d^2 - M^2 - \frac{\alpha k^2 + \vec{k}_\perp^2}{1-\alpha} + i\epsilon) \times \left\{ \alpha M_d^2 - M^2 - \frac{\alpha k^2 + [\vec{k}_\perp + (1-\alpha)\vec{q}_\perp]^2}{1-\alpha} + i\epsilon \right\} (k^2 - M^2 + i\epsilon). \quad (4.6)$$

We will now make the following assumptions for ϕ :

1. ϕ is an analytic function in the k^2 -plane.³²
2. The α -dependences of ϕ are such that

$$\lim_{\alpha \rightarrow 0} \alpha \phi^2 = \text{finite and} \quad \lim_{\alpha \rightarrow 1} \phi = 0 \quad [\text{See Section IV E}]. \quad (4.7)$$

The third denominator in (4.5) represents a simple pole in the lower k^2 half-plane, whereas the first two denominators lead to poles in the upper or lower k^2 half-plane, depending on whether $(1-\alpha)/\alpha$ is positive or negative. In the second case we can close the integration contour in the upper k^2 half-plane, pushing a semicircle to infinity and we end up with zero. Thus α is restricted to the interval $0 < \alpha < 1$ [Notice that, due to the constraints (4.7), $\alpha = 0$ or 1 gives zero contribution to (4.5)]. If we close the contour in this case, we can do it in the upper or lower k^2 half-plane picking up two poles or one pole, respectively. The two expressions, of course, have to be equal. The result is³³

$$F_d(q^2) = \sum_{i=n,p} F^i(q^2) \int d^2\vec{k}_\perp \int_0^1 \frac{\alpha d\alpha}{2(2\pi)^3 (1-\alpha)} \psi(\alpha, \vec{k}_\perp) \times \psi(\alpha, \vec{k}_\perp + (1-\alpha)\vec{q}_\perp), \quad (4.8)$$

where

$$\psi(\alpha, \vec{k}_\perp) \equiv \frac{\phi(p^2, k^2 = M^2)}{p^2 - M^2} = \frac{1}{\alpha} \frac{\phi(\alpha, \vec{k}_\perp)}{M_d^2 - M^2(\alpha, \vec{k}_\perp)}, \quad (4.9)$$

and

$$M^2(\alpha, \vec{k}_\perp) = \frac{M^2 + \vec{k}_\perp^2}{\alpha(1-\alpha)} \quad (4.10)$$

is the invariant mass of the two-nucleon system $(p+k)^2$, where

$$p^2 = \alpha M_d^2 - \frac{\alpha M^2 + \vec{k}_\perp^2}{1 - \alpha} \quad (4.11)$$

B. Absence of the West β -Correction in Hadronic Scattering

In this section we will consider hadronic scattering and limit ourselves to the case where the incident particle has a small but finite mass, such as pion-deuteron scattering. See Fig. 1 and the equation (2.11). We will use again the Brodsky parameterization [See Eq. (4.2)] with the on-shell

$$q_\nu = \left(\rho' + \frac{m^2}{4\rho}, \vec{0}_\perp, -\rho' + \frac{m^2}{4\rho} \right) \quad (4.12)$$

Notice that p_ν and q_ν have been defined in a general set of frames along the interaction axis. A specific frame in this set is selected by relating ρ and ρ' . For example, the deuteron rest frame is defined by the conditions

$$\rho = \frac{1}{2} M_d \text{ and } \nu = \rho' + \frac{m^2}{4\rho} \quad (4.13)$$

where ν is the incident particle energy.

In terms of experimentally measured total cross sections³⁴

$$\sigma^{\text{smeared}}(\nu) = \sum_{i=n,p} \int d^2\vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) \left(\frac{\tilde{\nu}^2 - q^2}{\nu^2 - q^2} \right)^{1/2} \times \sigma_1(\tilde{\nu}) \theta(s' - M^2), \quad (4.14)$$

where

$$G(\alpha, \vec{k}_\perp) = \frac{M}{2(2\pi)^3 M_d (1 - \alpha)} \frac{\phi^2(p^2, k^2 = M^2)}{(p^2 - M^2)^2} = \frac{M}{2(2\pi)^3 M_d \alpha^2 (1 - \alpha)} \frac{\phi^2(\alpha, \vec{k}_\perp)}{[M_d^2 - M^2(\alpha, \vec{k}_\perp)]^2}, \quad (4.15)$$

and

$$\begin{aligned} 2M\tilde{\nu} &= 2M\nu' + p^2 - M^2, \\ s' &= (p + q)^2 = p^2 + m^2 + 2M\nu', \\ M\nu' &= p \cdot q = \alpha M_d \nu + \frac{m^2}{4M_d \nu} \left(M_d^2 - \frac{M^2 + \vec{k}_\perp^2}{1 - \alpha} \right) \end{aligned} \quad (4.16)$$

In the high energy limit,

$$\tilde{\nu} \approx \nu' \approx \alpha \frac{M_d}{M} \nu, \quad (4.17)$$

then

$$\sigma^{\text{smeared}}(\nu) = \frac{M_d}{M} \int_0^1 d\alpha \alpha G(\alpha) \sigma\left(\alpha \frac{M_d}{M} \nu\right), \quad (4.18)$$

where

$$G(\alpha) = \int d^2\vec{k}_\perp G(\alpha, \vec{k}_\perp), \quad (4.19)$$

and the α on RHS of (4.18) means $\alpha_n + \alpha_p$.

The above result, Eq. (4.18), suggests us that we interpret $G(\alpha, \vec{k}_\perp)$ as the probability of finding the nucleon with α along the direction of the deuteron and transverse momentum $\vec{k}_\perp$ which is perpendicular to $\vec{P}$. For the normalization of $G(\alpha, \vec{k}_\perp)$ we will use the condition

$$\int d^2\vec{k}_\perp \int_0^1 d\alpha \alpha G(\alpha, \vec{k}_\perp) = \frac{M}{M_d} = \frac{1}{2}, \quad (4.20)$$

which expresses the fact that the sum of the fractional momenta of the proton and the neutron is the total (fractional) momentum of the deuteron.

In the above discussion we have implicitly used isospin symmetry, i.e.,

$G_{p/d}(\alpha, \vec{k}_\perp) = G_{n/d}(\alpha, \vec{k}_\perp) \equiv G(\alpha, \vec{k}_\perp)$. Since the interacting nucleon (spectator) is off (on) the mass shell, neither ϕ nor G are necessarily symmetric around $\alpha = \frac{M}{M_d} \approx \frac{1}{2}$. However, for the interacting nucleon which is not far off the mass shell it is not a bad approximation to assume ϕ , and hence, G to have such property. If one adopt such a view, then

$$G(\alpha, \vec{k}_\perp) = G(1-\alpha, \vec{k}_\perp)$$

Notice that if G has this property, the condition (4.20) will be equivalent to

$$\int d^2\vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) = 1,$$

which means that the number of particles is fixed. The deuteron "structure function" in α is given in terms of $\frac{M_d}{M} \alpha G(\alpha)$. It is not difficult to see that Eqs. (4.18) and (4.20) lead to the absence of the West β -correction in hadronic scattering. For a slowly varying cross sections we can expand $\sigma(\alpha \frac{M_d}{M} \nu)$ around $\alpha = \frac{M}{M_d} \approx \frac{1}{2}$. Notice that $G(\alpha)$ is expected to be strongly peaked at α around $\frac{M}{M_d}$ (see Section IV E). It turns out that the smearing corrections $\sigma_s(\nu)$, and hence $\beta_s(\nu)$, can be expanded in powers of δ , where

$$\delta = \left[\left(\frac{M}{2M-\epsilon} \right)^2 - \frac{1}{4} \right]^{1/2} \approx \left(\frac{\epsilon}{4M} \right)^{1/2},$$

with ϵ the binding energy of the deuteron and M the mass of the nucleon (so that $\delta \approx 1/40$). The details of the discussion will be given in Appendix D.

C. Leptonic Scattering

We now consider the case where the "projectile" is a virtual (off-shell) particle (proton or W-boson) of four momentum q . See Fig. 4.

C.1. Electron Case.

In the impulse approximation we can write³⁴

$$W_{\lambda_C}^{ed}(q^2, \nu) = \frac{M_d}{M} \sum_{i=n,p} \int d^2\vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) W_{\lambda_C}^{ei}(q^2, \tilde{\nu}) \times \theta(s' - M^2), \quad (4.21)$$

where the $W_{\lambda c}$ is the usual forward virtual Compton amplitude (see Appendix A and Section IV D)

$$W_{\lambda\rho}^{ei} = - \left(g_{\lambda\rho} - \frac{q_\lambda q_\rho}{q^2} \right) W_1^{ei}(q^2, \tilde{\nu}) + \left(p_\lambda - \frac{p \cdot q}{q^2} q_\lambda \right) \left(p_\rho - \frac{p \cdot q}{q^2} q_\rho \right) M^{-2} W_2^{ei}(q^2, \tilde{\nu}), \quad (4.22)$$

$$Nv' = \alpha M_d v - \vec{q}_\perp \cdot \vec{k}_\perp, \quad (4.23)$$

and similarly for $W_{\lambda\rho}^{ed}$ by changing $p_\lambda \rightarrow P_\lambda$ and $M \rightarrow M_d$. This form satisfies the gauge-invariance condition.

To project out the $W_{1,2}^{ed}$ from $W_{\lambda\rho}^{ed}$ we can use the projection operators $P_{1,2}^{\lambda\rho}$ with the property

$$P_1^{\lambda\rho} W_{\lambda\rho}^{ed} = W_1^{ed}, \quad i = 1, 2. \quad (4.24)$$

It is not difficult to verify that

$$P_1^{\lambda\rho} = -\frac{1}{2} g^{\lambda\rho} + \frac{q^2}{2M_d^2} \frac{p^\lambda p^\rho}{q^2 - v^2},$$

$$P_2^{\lambda\rho} = -\frac{q^2}{2(q^2 - v^2)} \left(-g^{\lambda\rho} + \frac{3q^2}{M_d^2} \frac{p^\lambda p^\rho}{q^2 - v^2} \right), \quad (4.25)$$

will satisfy (4.24).

Applying (4.25) to (4.21) we get

$$W_1^{ed}(q^2, \nu) = \frac{M_d}{M} \int_0^1 d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) \theta(s' - M^2) \times [W_1^e(q^2, \tilde{\nu}) + A(\alpha, \vec{k}_\perp) W_2^e(q^2, \tilde{\nu})], \quad (4.26)$$

$$W_2^{ed}(q^2, \nu) = \frac{M_d}{M} \int_0^1 d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) B(\alpha, \vec{k}_\perp) \times W_2^e(q^2, \tilde{\nu}) \theta(s' - M^2), \quad (4.27)$$

where the W^e 's on RHS means $W^{ep} + W^{en}$. The A and B functions are given in Appendix E.

In the deep inelastic limit

$$\nu \rightarrow \infty, \quad q^2 \rightarrow -\infty, \quad \omega_d = -\frac{2M_d \nu}{q^2} \text{ fixed,}$$

then

$$W_1^{ed}(q^2, \nu) = \frac{M_d}{M} \int_0^1 d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) \theta(\omega' - 1) \times [W_1^e(q^2, \nu') + \frac{\vec{k}_\perp^2}{2M^2} W_2^e(q^2, \nu')], \quad (4.28)$$

$$W_2^{ed}(q^2, \nu) = \left(\frac{M_d}{M} \right)^3 \int_0^1 d^2 \vec{k}_\perp \int_0^1 d\alpha \alpha^2 G(\alpha, \vec{k}_\perp) W_2^e(q^2, \nu')$$

$$\times \theta(\omega' - 1), \quad (4.29)$$

where

$$\omega' = - \frac{2 M_d'}{q^2} \quad (4.30)$$

Defining

$$\begin{aligned} W_1^{ed} &= F_1^{ed}, & \omega_d^{-1} W_2^{ed} &= F_2^{ed}, \\ W_1^{ei} &= F_1^{ei}, & \omega_d^{-1} W_2^{ei} &= F_2^{ei}, \quad i = p, n, \end{aligned}$$

$$\omega_N = - \frac{2 M_N}{q^2} = \frac{M}{M_d} \omega_d, \quad (4.31)$$

we can rewrite (4.28) and (4.29) as³⁵

$$F_1^{ed}(q^2, \omega_d) = \frac{M_d}{M} \int_0^1 d\alpha G(\alpha) F_1^e(q^2, \alpha \frac{M_d}{M} \omega_N) \theta(\alpha \frac{M_d}{M} \omega_N - 1), \quad (4.32)$$

$$\begin{aligned} F_2^{ed}(q^2, \omega_d) &= \frac{M_d}{M} \int_0^1 d\alpha \alpha G(\alpha) F_2^e(q^2, \alpha \frac{M_d}{M} \omega_N) \\ &\times \theta(\alpha \frac{M_d}{M} \omega_N - 1). \end{aligned} \quad (4.33)$$

A similar expansion around $\alpha = \frac{2}{q^2} \frac{M_d}{M}$ can also be carried out here as the one in hadronic scattering (see Appendix B). Notice that in our discussion it is not necessary to assume Bjorken scaling, so that we retain the possibility of a q^2 -dependence in the structure functions.

In terms of $x_d = \frac{1}{\omega_d}$ (or $x_N = \frac{1}{\omega_N} = \frac{M_d}{M} x_d$), Bjorken scaling variable,

$$F_1^d(q^2, x_d) = \frac{M_d}{M} \int_{x_d}^1 d\alpha G(\alpha) F_1^e\left(\frac{M}{M_d} \frac{x_N}{\alpha}\right), \quad (4.34)$$

$$F_2^d(q^2, x_d) = \frac{M_d}{M} \int_{x_d}^1 d\alpha \alpha G(\alpha) F_2^e\left(\frac{M}{M_d} \frac{x_N}{\alpha}\right). \quad (4.35)$$

This is a familiar form in the quark-parton model of the hadron.²

A simple direct consequence of Eqs. (4.32), (4.33) and the normalization condition (4.20) are the following sum-rules:

$$\begin{aligned} \int_1^\infty \frac{d\omega_d}{\omega_d^p} F_2^{ed}(q^2, \omega_d) &= \left(\frac{M}{M_d}\right)^{p-1} \int_{M/M_d}^\infty \frac{d\omega_N}{\omega_N^p} F_2^{ed}(q^2, \omega_N) \\ &= \frac{M_d}{M} \int_0^1 d\alpha \alpha^p G(\alpha) \\ &\times \int_\alpha^\infty \frac{d\omega_N'}{\omega_N'^p} F_2^e(q^2, \omega_N') \theta(\omega_N' - 1). \end{aligned} \quad (4.36)$$

The threshold condition allows us to rewrite this as

$$\int_1^{\infty} \frac{d\omega_d}{\omega_d^p} F_2^{ed}(q^2, \omega_d) = \langle \alpha^p - 1 \rangle \int_1^{\infty} \frac{d\omega_N}{\omega_N^p} F_2^e(q^2, \omega_N), \quad (4.37)$$

where

$$\langle \alpha^p - 1 \rangle = \frac{M_d}{M} \int_0^1 d\alpha \alpha^p G(\alpha). \quad (4.38)$$

The sum-rules for $p \leq 0$ certainly do not converge, but some of the sum-rules for $p > 0$ certainly do. Notice that p need not necessarily be an integer:

The interesting one is the $p = 1$ sum-rule:

$$\int_1^{\infty} \frac{d\omega_d}{\omega_d} F_2^{ed}(q^2, \omega_d) = \int_1^{\infty} \frac{d\omega_N}{\omega_N} F_2^e(q^2, \omega_N), \quad (4.39)$$

which is a wave-function-independent sum-rule. This sum-rule agrees with the quark-parton model² expectation.

Similarly for F_1^{ed} we obtain

$$\int_1^{\infty} \frac{d\omega_d}{\omega_d^{p+1}} F_1^{ed}(q^2, \omega_d) = \langle \alpha^p - 1 \rangle \int_1^{\infty} \frac{d\omega_N}{\omega_N^{p+1}} F_1^e(q^2, \omega_N). \quad (4.40)$$

Notice that the sum-rules (4.37) and (4.40) are consistent with the Callan-Gross relations [see Eq. (3.16)].

C.2. Neutrino Case

In this section we will consider deep-inelastic neutrino scattering off the deuteron, say $\nu_\mu + d \rightarrow \mu^- + X$. Consider again Fig. 4, where k, k' and q now are the four-momenta of the incoming neutrino, outgoing lepton and W-boson, respectively.

The general form of $W_{\lambda\rho}^{\nu d}$ is given by³⁶

$$\begin{aligned} W_{\lambda\rho}^{\nu d}(q^2, \nu) = & -g_{\lambda\rho} W_1^{\nu d} + P_\lambda P_\rho M_d^{-2} W_2^{\nu d} \\ & - i\epsilon_{\lambda\rho\beta\gamma} P^\beta q^\gamma (2M_d^2)^{-1} W_3^{\nu d} \\ & + q_\lambda q_\rho M_d^{-2} W_4^{\nu d} \\ & + (P_\lambda q_\rho + q_\lambda P_\rho) (2M_d^2)^{-1} W_5^{\nu d}, \end{aligned} \quad (4.41)$$

where the W 's on RHS are scalar functions of q^2 and ν , and similarly

for $W_{\lambda\rho}^{\nu p, \nu n}$ by changing $P \rightarrow p$ and $M_d \rightarrow M$.

To project out the W 's from $W_{\lambda\rho}^{\nu d}$ we need the projection operators $P^{\lambda\rho}$:

$$P_i^{\lambda\rho} W_{\lambda\rho}^{\nu d} = W_i^{\nu d}, \quad i = 1, \dots, 5. \quad (4.42)$$

We find

$$\begin{aligned}
 P_1^{\lambda\rho} &= [2 M_d^2 (q^2 - v^2)]^{-1} \left\{ -M_d^2 (q^2 - v^2) g^{\lambda\rho} + q^2 P^\lambda P^\rho \right. \\
 &\quad \left. + M_d^2 q^\lambda q^\rho - M_d v (P^\lambda q^\rho + q^\lambda P^\rho) \right\}, \\
 P_2^{\lambda\rho} &= [2 M_d^2 (q^2 - v^2)^2]^{-1} \left\{ -q^2 M_d^2 (q^2 - v^2) g^{\lambda\rho} + 3 q^4 P^\lambda P^\rho \right. \\
 &\quad \left. + M_d^2 (2 v^2 + q^2) q^\lambda q^\rho - 3 q^2 M_d v (P^\lambda q^\rho + q^\lambda P^\rho) \right\}, \\
 P_3^{\lambda\rho} &= -i (q^2 - v^2)^{-1} \epsilon^{\lambda\rho\beta\gamma} F_{\beta\gamma}, \\
 P_4^{\lambda\rho} &= [2 M_d^2 (q^2 - v^2)^2]^{-1} \left\{ -M_d^2 (q^2 - v^2) g^{\lambda\rho} + 3 M_d^4 q^\lambda q^\rho \right. \\
 &\quad \left. + M_d^2 (2 v^2 + q^2) P^\lambda P^\rho - 3 M_d^3 v (P^\lambda q^\rho + q^\lambda P^\rho) \right\}, \\
 P_5^{\lambda\rho} &= [M_d^2 (q^2 - v^2)^2]^{-1} \left\{ M_d^3 v (q^2 - v^2) g^{\lambda\rho} - 3 M_d v q^2 P^\lambda P^\rho \right. \\
 &\quad \left. - 3 M_d^3 v q^\lambda q^\rho + M_d^2 (2 v^2 + q^2) (P^\lambda q^\rho + q^\lambda P^\rho) \right\}. \quad (4.41)
 \end{aligned}$$

Applying these projection operators to the equation

$$W_{\mu\nu}^{\lambda\rho}(q^2, v) = \frac{M_d}{M} \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) W_{\lambda\rho}^{\mu\nu}(q^2, \tilde{v}) \theta(s' - M^2), \quad (4.42)$$

we obtain

$$W_1^{\lambda\rho}(q^2, v) = \frac{M_d}{M} \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) [W_1^{\lambda\rho}(q^2, \tilde{v}) + A W_2^{\lambda\rho}(q^2, \tilde{v})] \theta(s' - M^2),$$

$$W_2^{\lambda\rho}(q^2, v) = \frac{M_d}{M} \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) B W_2^{\lambda\rho}(q^2, \tilde{v}) \theta(s' - M^2),$$

$$W_3^{\lambda\rho}(q^2, v) = \frac{M_d}{M} \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) C W_3^{\lambda\rho}(q^2, \tilde{v}) \theta(s' - M^2)$$

$$\begin{aligned}
 W_4^{\lambda\rho}(q^2, v) &= \frac{M_d}{M} \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) [D W_2^{\lambda\rho}(q^2, \tilde{v}) + \frac{M_d^2}{M^2} W_4^{\lambda\rho}(q^2, \tilde{v}) \\
 &\quad + E W_5^{\lambda\rho}(q^2, v)] \theta(s' - M^2),
 \end{aligned}$$

$$\begin{aligned}
 W_5^{\lambda\rho}(q^2, v) &= \frac{M_d}{M} \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) [F W_2^{\lambda\rho}(q^2, \tilde{v}) + G_0 W_5^{\lambda\rho}(q^2, \tilde{v})] \\
 &\quad \times \theta(s' - M^2), \quad (4.45)
 \end{aligned}$$

where A, B, C, D, E, F and G_0 are functions of α and $\vec{k}_\perp$. Their complete forms are given in Appendix E. In the deep inelastic limit these complicated functions have simple forms. In fact,

$$W_1^{\lambda\rho}(q^2, v) = \frac{M_d}{M} \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) [W_1^{\lambda\rho} + \frac{\vec{k}_\perp^2}{2M^2} W_2^{\lambda\rho}] \theta(\omega' - 1),$$

$$W_2^{\lambda\rho}(q^2, v) = \left(\frac{M_d}{M} \right)^3 \int d^2 \vec{k}_\perp \int_0^1 d\alpha \alpha^2 G(\alpha, \vec{k}_\perp) W_2^{\lambda\rho} \theta(\omega' - 1),$$

$$W_3^{\lambda\rho}(q^2, v) = \left(\frac{M_d}{M} \right)^3 \int d^2 \vec{k}_\perp \int_0^1 d\alpha \alpha G(\alpha, \vec{k}_\perp) W_3^{\lambda\rho} \theta(\omega' - 1),$$

$$W_4^{vd}(q^2, \nu) = \left(\frac{M_d}{M}\right)^3 \int_0^1 d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) W_4^v(\omega' - 1),$$

$$W_5^{vd}(q^2, \nu) = \left(\frac{M_d}{M}\right)^3 \int_0^1 d^2 \vec{k}_\perp \int_0^1 d\alpha \alpha G(\alpha, \vec{k}_\perp) W_5^v(\omega' - 1). \quad (4.46)$$

By defining

$$W_1^{vd} = F_1^{vd}$$

$$\nu M_d^{-1} W_1^{vd} = F_1^{vd}, \quad i = 2, \dots, 5,$$

and similarly for $W^{vp, \nu n}$ by changing $\nu \rightarrow \nu'$ and $M_d \rightarrow M$, we can rewrite (4.46) as³⁵

$$F_{1,3,5}^{vd}(q^2, \omega_d) = \frac{M_d}{M} \int_0^1 d\alpha G(\alpha) F_{1,3,5}^v(q^2, \alpha \frac{M_d}{M} \omega_N) \theta(\alpha \frac{M_d}{M} \omega_N - 1),$$

$$F_2^{vd}(q^2, \omega_d) = \frac{M_d}{M} \int_0^1 d\alpha \alpha G(\alpha) F_2^v(q^2, \alpha \frac{M_d}{M} \omega_N) \theta(\alpha \frac{M_d}{M} \omega_N - 1),$$

$$F_4^{vd}(q^2, \omega_d) = \frac{M_d}{M} \int_0^1 \frac{d\alpha}{\alpha} G(\alpha) F_4^v(q^2, \alpha \frac{M_d}{M} \omega_N) \theta(\alpha \frac{M_d}{M} \omega_N - 1). \quad (4.47)$$

Simple direct consequences of (4.47) and of the normalization condition (4.20) are the wave-function-independent sum-rules for the neutrino scattering:

$$\int_1^\infty \frac{d\omega_d}{\omega_d^2} F_{1,3,5}^{vd}(q^2, \omega_d) = \int_1^\infty \frac{d\omega_N'}{\omega_N'^2} F_{1,3,5}^v(q^2, \omega_N'),$$

$$\int_1^\infty \frac{d\omega_d}{\omega_d} F_2^{vd}(q^2, \omega_d) = \int_1^\infty \frac{d\omega_N'}{\omega_N'} F_2^v(q^2, \omega_N'),$$

$$\int_1^\infty \frac{d\omega_d}{\omega_d^3} F_4^{vd}(q^2, \omega_d) = \int_1^\infty \frac{d\omega_N'}{\omega_N'^3} F_4^v(q^2, \omega_N'). \quad (4.48)$$

The higher-moment sum-rules can also be easily obtained.

D. The Off-Shell Correction Ambiguity

Bodek⁹ had pointed that there is an inconsistency in the electroproduction expression of the conventional approach in the $q^2 = 0$ limit, i.e. it does not yield a vanishing σ_L^d (the absorption cross section of scalar virtual photons) as $q^2 \rightarrow 0$. He suggested the application of an off-shell correction to correct this defect. However, there is some ambiguity in this method, due to the fact that the constraints imposed on the electroproduction amplitudes in the $q^2 = 0$ limit lead to several possible forms for off-mass shell corrections to the nucleon electromagnetic structure functions. Furthermore this ambiguity may introduce a larger uncertainty into the smearing than the one which is introduced

by the uncertainty in the deuteron wave function.

This problem is basically related to the (general) problem: How to relate the amplitudes for scattering from a virtual (or off-shell) particle to the one from a real (or on-shell) particle. Any formalism for the Fermi motion effects on deuterium targets, whether it is non-relativistic or relativistic, will face this kind of problem.

We disagree with Bodek's argument. We will show that as long as we stay within the basic assumptions (see Section II A.2), the electroproduction expression is consistent, i.e., it leads to a vanishing σ_L^d in the $q^2 = 0$ limit.

Recall that

$$W_{\lambda\rho}^d(q^2, \nu) = \frac{M_d}{M} \int_0^1 d^2\vec{k}_1 \int_0^1 da \, G(a, \vec{k}_1) W_{\lambda\rho} \, \theta(s' - M^2).$$

The $W_{\lambda\rho}$ on RHS is the forward virtual (the incoming photon is off-shell) Compton amplitude from the virtual (interacting) nucleon. In general we really do not know what is $W_{\lambda\rho}$. However if the interacting nucleon is not far off the mass shell, which is presumably true for a weakly bound system like the deuteron, it is not a bad approximation to assume (see Appendix A)

$$W_{\lambda\rho} = - \left(g_{\lambda\rho} - \frac{q_\lambda q_\rho}{q^2} \right) W_1 + \left(p_\lambda - \frac{p \cdot q}{q^2} q_\lambda \right) \left(p_\rho - \frac{p \cdot q}{q^2} q_\rho \right) W_2 M^{-2}.$$

Thus the $W_{\lambda\rho}$ has a form like the usual one given for the real (on-shell) nucleon, but with the one exception, that the scalar form factors W_1 and W_2 depend on the variables q^2 and $\tilde{\nu}$ instead of q^2 and ν = $p \cdot q/M$ (see Section II C). The relation between $\tilde{\nu}$ and ν is given by

$$2 M \tilde{\nu} = 2 M \nu' + p^2 - M^2.$$

Notice that we had neglected the p^2 -dependences of the W 's in the above formula.

Consider Eqs. (4.26) and (4.27). In the $q^2 = 0$ limit these equations become

$$W_1^d(\nu) = \frac{M_d}{M} \int_0^1 d^2\vec{k}_1 \int_0^1 da \, G(a, \vec{k}_1) W_1(\tilde{\nu}) \, \theta(\tilde{\nu}), \quad (4.49)$$

$$W_2^d(\nu) = \left(\frac{M_d}{M} \right)^3 \int_0^1 d^2\vec{k}_1 \int_0^1 da \, a^2 G(a, \vec{k}_1) W_2(\tilde{\nu}) \, \theta(\tilde{\nu}). \quad (4.50)$$

We now express the photo absorption cross sections in terms of W_1 and W_2 (see Section II B):

$$\sigma_T^d(\nu) = \frac{4\pi^2 \alpha_e}{M_d K} W_1^d(\nu). \quad (4.51)$$

$$\sigma_L^d(v) = - \frac{\alpha_e^2}{M_d K} \left[\frac{(P \cdot q)^2}{M_d^2 q^2} W_2^d(\tilde{v}) + W_1(\tilde{v}) \right], \quad (4.52)$$

where

$$\tilde{v} = \frac{\tilde{E} - M_d^2}{2M_d} = v. \quad (4.53)$$

Notice that these two relations, given in (4.51) and (4.52), have been derived as the (interacting) nucleon is on-shell (real). As the previous problem (i.e., the $W_{\lambda\rho}$) we also do not know what the relations should be when the (interacting) nucleon is off-shell (virtual). Consistent with what we have done before, we will also assume here that

$$\sigma_T^{p,n}(\tilde{v}) = \frac{4\pi^2 \alpha_e}{M\tilde{v}} W_1^{p,n}(\tilde{v}), \quad (4.54)$$

$$\sigma_L^{p,n}(\tilde{v}) = - \frac{4\pi^2 \alpha_e}{M\tilde{v}} \left[\frac{(P \cdot q)^2}{M^2 q^2} W_2^{p,n}(\tilde{v}) + W_1^{p,n}(\tilde{v}) \right]. \quad (4.55)$$

By using Eqs. (4.51) and (4.54) we find that (4.49) yields

$$\sigma_T^d(v) = \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) \frac{\tilde{v}}{v} \sigma_T(\tilde{v}) \theta(\tilde{v}), \quad (4.56)$$

which agrees with (4.16). Combining Eqs. (4.49) and (4.50) and using (4.52) we find

$$\sigma_L^d(v) = - \frac{4\pi^2 \alpha_e}{Mv} \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) \theta(\tilde{v}) \times \left[\frac{v^2}{q^2} \frac{M_d^2}{M^2} \alpha^2 W_2(\tilde{v}) + W_1(\tilde{v}) \right]. \quad (4.57)$$

Recall for real particles $\sigma_L = 0$, thus (4.55) implies that

$$W_2(\tilde{v}) = - \frac{M_d^2 q^2}{(P \cdot q)^2} W_1(\tilde{v}). \quad (4.58)$$

Using (4.58) we can rewrite (4.57) as

$$\sigma_L^d(v) = \frac{4\pi^2 \alpha_e}{Mv} \int d^2 \vec{k}_\perp \int_0^1 d\alpha G(\alpha, \vec{k}_\perp) \theta(\tilde{v}) \times \left[\left(\frac{M_d v}{Mv'} \alpha \right)^2 - 1 \right] W_1(\tilde{v}), \quad (4.59)$$

which equals zero because in the $q^2 = 0$ limit $Mv' = \alpha M_d v$ [see Eq. (4.23)].

Thus the electroproduction expression is consistent, i.e., it leads to a vanishing σ_L^d in the $q^2 = 0$ limit. We do not need the off-shell corrections as Bodek suggested (as far as this problem is concerned), and hence, our new approach has no off-shell correction ambiguity either.

E. The Truncated n-p-d Vertex Function

In the previous discussions we had shown the consistency of our simple, new approach. The normalization condition comes out naturally

in hadronic scattering, and it also has a physical meaning. The sum-rules we got agree with the quark-parton model expectations. Thus our new approach is superior than the conventional approach as far as these consistencies are concerned. We have also shown that the electroproduction expression is consistent, and hence, we do not need the off-shell corrections, as the ones Bodek suggested, which only complicate the problem unnecessarily.

Our new approach predicts the absence of the West β -correction in hadronic scattering, but it is still there for leptonic scattering in the deep inelastic limit. The form of the correction for the leptonic case is, however, different in our new approach from that given by the conventional approach, which we claim is not correct since it leads to sum-rules which disagree with quark-parton model expectations. Unfortunately this prediction cannot be tested experimentally since the total hadronic cross sections are not constant, and furthermore free neutron targets are not available. However, we can calculate the smearing corrections β_g , and hence β_s , for pion- and nucleon- processes and see whether they are small in the high energy region as we expect.

To carry out the calculations we need the knowledge of the truncated n-p-d vertex function. In principle this can be obtained by solving the Bethe-Salpeter equation of the two nucleon bound-state problem which, however, is hard to solve in practice. Usually one uses the ladder approximation and also a specific ansatz for the interaction kernel to find an approximate solution. In this thesis we do not attempt to do

this calculation. Instead a phenomenological approach will be followed using a plausible ansatz for the truncated n-p-d vertex function ϕ .

In the construction of $\phi(\alpha, \vec{k}_A)$ we need to take into account the following properties:

$$1. \lim_{\alpha \rightarrow 0} \alpha \phi^2(\alpha, \vec{k}_A) = C, \text{ a finite constant (which can be chosen to be zero),}$$

$$\lim_{\alpha \rightarrow 1} (1 - \alpha) \phi^2(\alpha, \vec{k}_A) = 0$$

$$2. \lim_{|\vec{k}_A| \rightarrow \infty} \phi(\alpha, \vec{k}_A) = 0$$

See footnote 35. Notice that throughout our formalism we assumed that the (interacting) nucleon is not far off the mass shell. By analyzing Eq. (4.11)

$$p^2 = \alpha M_d^2 + M^2 - \frac{M^2 + \vec{k}_A^2}{1 - \alpha}$$

we find that, strictly speaking,

$$0.26 \leq \alpha \leq 0.66 \text{ and } 0 \leq \vec{k}_A^2 \leq 0.14 \text{ (GeV)}^2$$

if the interacting nucleon is near the mass shell. These numbers are obtained by taking $M^2 - p^2 = 2\mu M < p^2 < M^2$, where μ is the pion mass.

If we relax this physical restriction, and take $0 < p^2 < M^2$, then

$$0 < \alpha \leq 0.75 \text{ and } 0 < \vec{k}_A^2 \leq 0.49 \text{ (GeV)}^2.$$

3. Recall that

$$G(\alpha, \vec{k}_1) = \frac{M}{2(2\pi)^3 M_d (1 - \alpha)} \frac{\phi^2(\alpha, \vec{k}_1)}{(p^2 - M^2)^2}$$

Clearly $G(\alpha, \vec{k}_1)$ has a double pole at $p^2 = M^2$.

Because the deuteron binding energy ϵ is small, the function $G(\alpha, \vec{k}_1)$, and hence $\phi(\alpha, \vec{k}_1)$, must be such that a large part of its contribution to the integral comes from values of p^2 not far from M^2 . In terms of α and $\vec{k}_1$, this implies that $G(\alpha, \vec{k}_1)$ is expected to have a peak at $\alpha \approx \frac{M}{M_d}$ and $\vec{k}_1^2 \approx 0$.

4. According to the dimensional-scaling quark-model³⁷ the elastic deuteron electromagnetic form factor should approach a power-law behavior at large q^2 ,

$$F_d \sim (q^2)^{-5}.$$

Experimental data seems to support this prediction (see Fig. 19).

A simple form for $\phi(\alpha, \vec{k}_1)$ which satisfies the properties 2 and 4 is

$$\phi(\alpha, \vec{k}_1) = A(\alpha) [M^2 - p^2 + B(\alpha)]^{-1} [M^2 - p^2 + C(\alpha)]^{-1}$$

Thus

$$G(\alpha, \vec{k}_1) = \frac{M}{2(2\pi)^3 M_d (1 - \alpha)} \frac{A^2(\alpha)}{(M^2 - p^2)^2 [M^2 - p^2 + B(\alpha)]^2} \times \frac{1}{[M^2 - p^2 + C(\alpha)]^2}$$

FIG. 19. The deuteron form factor data compared to $(1 + \frac{q^2}{m_0^2})^{-1} F_N^2(q^2/4)$; this is the quark model prediction of Ref. 37. The points • are from Ref. 39, points o from Refs. 30, 40 and 41. In the figure $m_0^2 = .28 \text{ (GeV)}^2$.

Recall that

$$M^2 - p^2 = (1 - \alpha)^{-1} [M^2(\alpha) + \vec{k}_1^2],$$

where

$$M^2(\alpha) = M^2 - \alpha(1 - \alpha) M_d^2. \quad (4.60)$$

Notice that $M^2(\alpha)$ has a minimum value, which is Mc^2 , at $\alpha = M/M_d$. To satisfy the property 3 in a simple way we will choose

$$B(\alpha) = \frac{\delta^2}{1 - \alpha} \quad \text{and} \quad C(\alpha) = \frac{\gamma^2}{1 - \alpha},$$

where δ^2 and γ^2 are free parameters (their magnitudes must be less than M^2). Thus we can rewrite

$$G(\alpha, \vec{k}_1) = \frac{M A^2(\alpha)}{2(2\pi)^3 M_d} \frac{(1 - \alpha)^5}{[M^2(\alpha) + \vec{k}_1^2]^2 [M^2(\alpha) + \vec{k}_1^2 + \delta^2]^2} \times \frac{1}{[M^2(\alpha) + \vec{k}_1^2 + \gamma^2]^2}$$

Finally to satisfy the property 1 we will choose

$$A(\alpha) = A_0 \alpha^2$$

In summary³⁸

$$\phi(\alpha, \vec{k}_1) = \frac{A_0 \alpha^2 (1 - \alpha)^2}{[M^2(\alpha) + \vec{k}_1^2 + \delta^2] [M^2(\alpha) + \vec{k}_1^2 + \gamma^2]}, \quad (4.61)$$

and

$$G(\alpha, \vec{k}_1) = \frac{M A_0^2}{2(2\pi)^3 M_d} \frac{\alpha^4 (1 - \alpha)^5}{[M^2(\alpha) + \vec{k}_1^2]^2 [M^2(\alpha) + \vec{k}_1^2 + \delta^2]^2} \times \frac{1}{[M^2(\alpha) + \vec{k}_1^2 + \gamma^2]^2}. \quad (4.62)$$

We expect our theory to join, when the energies and momenta are small, onto the familiar non-relativistic treatments. In particular, the G function should be related in this limit to the square of the non-relativistic wave-function. For momenta small with respect to M , then in the deuteron rest-frame

$$1 - \alpha = 1 - \frac{p_0 + p_3}{M_d} = \frac{k_0 + k_3}{M_d} = \frac{M + k_3}{M_d}.$$

Thus

$$M^2(\alpha) = Mc^2 + k_3^2,$$

and hence, the G function becomes

$$G(\alpha, \vec{k}_1) \xrightarrow{\text{N.R. limit}} \frac{C}{(\vec{k}^2 + Mc)^2} \frac{1}{(\vec{k}^2 + Mc + \delta^2)^2} \frac{1}{(\vec{k}^2 + Mc + \gamma^2)^2} \equiv N_d |\psi_{NR}(\vec{k})|^2,$$

where

$$C = \frac{M A_0^2}{2(2\pi)^3 M_d} \left(\frac{M}{M_d} \right)^4 \left(1 - \frac{M}{M_d} \right)^5.$$

The form of the non-relativistic limit of G looks like the Hulthén wave function for deuteron, and hence, we can call the $G(\alpha, \vec{k}_1)$ given by

(4.62) as the "generalized" relativistic Hulthén wave function.

The parameters δ^2 and γ^2 can be found by fitting the deuteron form factor data.^{30,39,40} A fit that can be achieved for our spinless model is given in Fig. 20 for the values³⁸

$$\gamma^2 = 2 \delta^2 = 400 \text{ Me}^2,$$

where the isoscalar form factor was taken to be equal to the usual dipole form for the proton form factor. Once these parameters are determined we can calculate the deuteron "structure function" and we find

$$\begin{aligned} \frac{M_d}{M} \alpha G(\alpha) = A_0^2 \frac{\alpha^5 (1-\alpha)^5}{64 \pi^2 \delta^{10}} \left\{ -3 \ln \left[\frac{M^2(\alpha) + 2 \delta^2}{M^2(\alpha)} \right] \right. \\ \left. + \delta^2 \left[\frac{5 M^2(\alpha) + \delta^2}{M^2(\alpha) (M^2(\alpha) + \delta^2)} + \frac{1}{M^2(\alpha) + 2 \delta^2} \right] \right\}. \end{aligned} \quad (4.63)$$

It has a peak at $\alpha = \frac{M}{M_d}$ and is symmetric around it (see Fig. 21). The normalization constant A_0^2 turns out to be $731.41 (\text{GeV})^{10}$.

By using this deuteron "structure function" we calculate the smearing correction $\sigma_s(v)$ for pion- and nucleon-processes. The results turn out to be very small, essentially zero within the experimental error (see Figs. 15 and 16). In terms of $\beta_s(v)$ they are shown in Table 3. Thus our new approach, by using a simple ansatz for the truncated n-p-d vertex function, leads to a vanishing smearing correction σ_s for high energy hadronic total cross sections, as we expect. Notice that this conclusion

FIG. 20. The deuteron form factor $F_d^2(Q^2)$. It is evaluated in the relativistic impulse approximation by using Eqs. (4.8) and (4.61). The data are from Refs. 30, 39 and 40.

is based on the use of a simple ansatz for ϕ . It is true that this kind of form will not completely describe the deuteron. However, it is unlikely that a "correct" wave function will give completely different results, i.e., large σ_s .

FIG. 21. Graph of the deuteron "structure function" versus α .

V. SPIN EFFECTS

In the descriptions of the Fermi motion effects on the deuterium targets, that were discussed in the previous sections, the spins of the particles were neglected. In this section we will include a spin degree of freedom in our formalism, and we would like to know whether its inclusion will affect the previous (spin-less) results.

For hadronic scattering case we will adopt the current view that the smearing correction σ_s is presumably small, and hence, can be neglected in the high energy scattering. Notice that our spin-less, new approach confirms this view. We shall assume here that the spin effects will not change the (spin-less) conclusion in high energy hadronic scattering. Whether this assumption is valid or not will be investigated later. Hence, we shall consider only leptonic scattering and will limit ourselves to the spin-averaged case.

A. The Spin-Averaged Deuteron Structure Function

See Fig. 4, and Section IV C. In the incoherent impulse approximation,

$$W_{\nu\rho}^d(q^2, \nu) = \sum_{\lambda} \sum_{i=n,p} \int d^2\vec{k}_1 \int_0^1 \frac{d\alpha}{2(2\pi)^3(1-\alpha)} \frac{1}{(p^2 - M^2)^2}$$

$$\times W_{\mu\rho}^1(q^2, \nu) \epsilon_B(\vec{p}, \lambda) \epsilon_\delta(\vec{p}, \lambda) \times \text{Tr}[\bar{\Gamma}^B (\not{p} + M) \Gamma^\delta (-\not{k} + M)] \delta(s' - M^2), \quad (5.1)$$

where ϵ is the deuteron polarization vector and Γ is the truncated n-p-d vertex function. The most general form for Γ is⁴²

$$\Gamma^\delta(p^2) = F'(p^2) \gamma^\delta + \frac{G'(p^2)}{2M} (p - k)^\delta + \frac{H - M}{M} [H'(p^2) \gamma^\delta + \frac{I'(p^2)}{2M} (p - k)^\delta]. \quad (5.2)$$

The relation of $\bar{\Gamma}$ to Γ is given by

$$\bar{\Gamma}^B = C \gamma^5 (\Gamma^B)^T (C \gamma^5)^{-1}, \quad (5.3)$$

where C is the charge conjugation operator,⁴³

$$C = i \gamma^2 \gamma^0 \quad (5.4)$$

and the symbol T means transpose.

Recall that

$$\sum_{\lambda} \epsilon_B(\vec{p}, \lambda) \epsilon_\delta(\vec{p}, \lambda) = -g_{B\delta} + \frac{p_B p_\delta}{M_d^2}, \quad (5.5)$$

and by defining

$$\begin{aligned} \phi^2(\alpha, \vec{k}_1) \equiv & (-g_{B\delta} + \frac{p_B p_\delta}{M_d^2})^T \Gamma^B(\alpha, \vec{k}_1) (\not{p} + M) \\ & \times \Gamma^\delta(\alpha, \vec{k}_1) (-\not{k} + M). \end{aligned} \quad (5.6)$$

we can rewrite (5.1) as

$$W_{\mu\rho}^d(q^2, \nu) = \sum_{i=n,p} \int_0^1 d^2\vec{k}_1 \int_0^1 \frac{d\alpha}{2(2\pi)^3 \alpha^2 (1-\alpha)} \frac{\phi^2(\alpha, \vec{k}_1)}{[M_d^2 - M^2(\alpha, \vec{k}_1)]^2} \times W_{\mu\rho}^i(q^2, \tilde{\nu}) \theta(s' - M^2). \quad (5.7)$$

In terms of $G(\alpha, \vec{k}_1)$, where

$$G(\alpha, \vec{k}_1) = \frac{M}{2(2\pi)^3 M_d \alpha^2 (1-\alpha)} \frac{\phi^2(\alpha, \vec{k}_1)}{[M_d^2 - M^2(\alpha, \vec{k}_1)]^2} \quad (5.8)$$

then

$$W_{\mu\rho}^d(q^2, \nu) = \frac{M_d}{M} \sum_{i=n,p} \int_0^1 d^2\vec{k}_1 \int_0^1 d\alpha G(\alpha, \vec{k}_1) W_{\mu\rho}^i(q^2, \tilde{\nu}) \times \theta(s' - M^2). \quad (5.9)$$

which is (4.21) for electron case, or (4.44) for neutrino case. Notice we recover the spin-less mathematical expressions for the $W_{\mu\rho}^i$, except the spin-less vertex function ϕ changes to spin-dependent vertex function ϕ . Hence, we conclude that all previous (spin-less) results, e.g. the wave-function independent sum-rules, are still valid.

Writing ϕ^2 in terms of the four scalar functions F' , G' , H' and I' , we find

$$\begin{aligned} \phi^2(\alpha, \vec{k}_1) = & [2(1-\alpha)^3 M^4 M_d^2]^{-1} \\ & \times \left\{ Z_1 F'^2 + Z_2 F' G' - M^2 Z_3 G'^2 \right. \\ & + (p^2 - M^2) [Z_4 H'^2 - Z_5 I'^2 + Z_6 H' F' \\ & \left. + Z_7 (H' G' + I' F') + Z_8 G' I'] \right\}, \quad (5.10) \end{aligned}$$

where Z 's are functions of α and $\vec{k}_1^2$. They are given in Appendix F.

From this result we see that the inclusion of spin makes life more difficult for we need now four (basically unknown) scalar functions F' , G' , H' and I' instead of one in the scalar (spin-less) case. In practice it is not a bad approximation to neglect H' and I' , because these form factors appear only in a term multiplied by $p^2 - M^2$, and hence, give presumably small contributions. The candidates for these scalar functions can be tested by using elastic electron-deuteron scattering.

B. The Deuteron Electromagnetic Form Factors

The electromagnetic form factors of nucleons and nuclei measured in electron scattering experiments form some of the basic data from which we derive much of our knowledge of nucleon and nuclear structure. The form factors of the deuteron are particularly important because they serve as the touch stones against which we can compare our most precise microscopic theories. Measurements at large momentum transfer probe these systems with increased resolution and are expected to be sensitive

to such details as high momentum parts of deuteron wave function, to relativistic kinematics, to the effects of meson exchange currents, and eventually to the internal structure of the nucleons.

In Section IV A we developed a model of the deuteron form factor where the spins of the particles involved were neglected. In this section we will extend those ideas by including the spin degree of freedom. This will enable us to describe the two elastic structure functions corresponding to charge-quadrupole and magnetic scattering.

B. 1. Kinematics.

The matrix element of elastic electron deuteron scattering in the one-photon-exchange (OPE) approximation is

$$M = \bar{u}(\vec{k}') \gamma^\mu u(\vec{k}) \frac{e}{2} \langle P' | J_\mu | P \rangle, \quad (5.11)$$

where u and $\bar{u}$ are fermion spinors, k and k' are the 4-momenta of the initial and final electron; P and $P' = P + q$ will be the 4-momenta for the initial and final deuterons (see Fig. 5. without the cross on the spectator line). The deuteron current matrix element writes:

$$\begin{aligned} G_\mu(q^2) &= \langle P' | J_\mu | P \rangle \\ &= 2(P_0 P'_0)^{-1/2} \epsilon_S^{\lambda' *} (P', \lambda') G_\mu^{\beta\alpha}(q^2) \epsilon_\alpha(P, \lambda). \end{aligned} \quad (5.12)$$

Here ϵ and ϵ' are polarization 4-vectors for the incoming and outgoing deuterons, respectively, and satisfy

$$\epsilon \cdot P = \epsilon' \cdot P' = 0. \quad (5.13)$$

We shall write the covariant decomposition of $G_\mu^{\beta\alpha}(q^2)$ as:^{44,45}

$$\begin{aligned} G_\mu^{\beta\alpha}(q^2) &= D_\mu [g^{\beta\alpha} F_1(q^2) - \frac{q^\beta q^\alpha}{2M_d^2} F_3(q^2)] \\ &\quad + [q^\beta g_\mu^\alpha - g_\mu^\beta q^\alpha] F_2(q^2), \end{aligned} \quad (5.14)$$

where

$$D_\mu = P'_\mu + P_\mu = 2P_\mu + q_\mu, \quad (5.15)$$

and F_1 , F_2 and F_3 are Lorentz scalar functions of the momentum transfer q^2 .

To make contact with the experimental data we define the three "physical" form factors, that is the charge, quadrupole and magnetic deuteron form factors, in terms of the F_1 , F_2 and F_3 :⁴⁵

$$\begin{aligned} G_C &= F_1 + \frac{2}{3} \eta G_Q, \\ G_M &= F_2, \\ G_Q &= F_1 - F_2 + (1 + \eta) F_3, \end{aligned} \quad (5.16)$$

where $\eta = -q^2/4M_d^2$, $G_C(0) = 1$, $G_M(0) = \mu_d$, and $G_Q(0) = Q_d$. Here μ_d , Q_d are the deuteron magnetic and quadrupole moments in units of $(2M_d)^{-1}$ and M_d^{-2} respectively.

The elastic structure functions A and B in the Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega} \Big|_{NS} [A(Q^2) + B(Q^2) \tan^2 \theta/2] \quad (5.17)$$

are given by

$$A = G_C^2 + \frac{8}{9} r^2 G_Q^2 + \frac{2}{3} G_M^2, \quad (5.18)$$

$$B = \frac{4}{3} \eta (1 + \eta) G_M^2.$$

In Eq. (5.17)

$$\frac{d\sigma}{d\Omega} \Big|_{NS} = \left(\frac{\alpha_e}{2E} \right)^2 \frac{\cos^2 \theta/2}{\sin^4 \theta/2} \frac{1}{[1 + \frac{2E}{M_d} \sin^2 \theta/2]}, \quad (5.19)$$

where E is the energy of the incoming electron, θ the LAB scattering angle of the electron, and we define $Q^2 = -q^2 > 0$ so that

$$Q^2 = \frac{4E^2 \sin^2 \theta/2}{1 + \frac{2E}{M_d} \sin^2 \theta/2}. \quad (5.20)$$

B.2. The Relativistic Impulse Approximation.

Consider Fig. 5 without the cross on the spectator line. Conventional Feynman rules give for this diagram

$$G_\mu^{Su}(q^2) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{p^2 - M^2 + i\epsilon} \frac{1}{(p+q)^2 - M^2 + i\epsilon} \frac{1}{k^2 - M^2 + i\epsilon} \times \text{Tr} \left\{ \bar{\Lambda}^B((p+q)^2, k^2) (\not{p} + \not{q} + M) F_\mu(q^2) (\not{p} + M) \times \Lambda^A(p^2, k^2) (-\not{p} + M) \right\}, \quad (5.21)$$

where Λ^A is the n-p-d vertex that describes the covariant coupling of the deuteron to two nucleons arbitrarily off mass shell. In the previous equation F_μ is the elastic electromagnetic vertex of the nucleon. To satisfy the gauge-invariance condition⁴⁶ it is necessary to constraint F_μ such that it satisfies the Ward-Takahashi identity (See Appendix A):

$$q^\mu F_\mu = \not{q}. \quad (5.22)$$

To be consistent with the approach we developed in Section IV A, we will choose (for simplicity)

$$F_\mu = \frac{G_{ES} + \tau G_{MS}}{1 + \tau} \gamma_\mu + \frac{G_{MS} - G_{ES}}{2M(1 + \tau)} i \sigma_{\mu\nu} q^\nu + \left[1 - \frac{G_{ES} + \tau G_{MS}}{1 + \tau} \right] \frac{\not{q}}{q^2} q_\mu, \quad (5.23)$$

where

$$G_{ES} = G_E^p + G_E^n, \quad G_{MS} = G_M^p + G_M^n, \quad (5.24)$$

the isoscalar nucleon form factors, and

$$\tau = q^2/4M^2. \quad (5.25)$$

Following the same method as described in Section IV A we can rewrite

(5.21) as

$$G_\mu^{Su}(q^2) = \int d^2 k_\perp \int_0^1 \frac{d\alpha}{2(2\pi)^3(1-\alpha)} \frac{1}{p^2 - M^2} \frac{1}{(p+q)^2 - M^2} \times \text{Tr} \left\{ \bar{\Gamma}^B((p+q)^2) (\not{p} + \not{q} + M) F_\mu(q^2) \times (\not{p} + M) \Gamma^A(p^2) (-\not{p} + M) \right\}, \quad (5.26)$$

where

$$\Gamma^{\alpha}(p^2) = \Lambda^{\alpha}(p^2, k^2 = M^2) \quad (5.27)$$

is the truncated n-p-d vertex function given by (5.2).

To project the F's from $G_{\mu}^{\beta\alpha}$, we will use the projection operators such that

$$(P_1)_{\beta\alpha}^{\mu} G_{\mu}^{\beta\alpha}(q^2) = F_1(q^2), \quad i = 1, 2, 3. \quad (5.28)$$

We find

$$\begin{aligned} (P_1)_{\beta\alpha}^{\mu} &= D^{-4} D_{\beta\alpha}^{\mu} D^{\mu}, \\ (P_2)_{\beta\alpha}^{\mu} &= (6q^2)^{-1} (q_{\beta} g_{\alpha}^{\mu} - g_{\beta\alpha}^{\mu} q^{\alpha}), \\ (P_3)_{\beta\alpha}^{\mu} &= 2M_d^2 (3q^2 D^2)^{-1} \left(g_{\beta\alpha}^{\mu} - 4 \frac{q_{\beta} q_{\alpha}}{q^2} \right) D^{\mu}. \end{aligned} \quad (5.29)$$

Applying these projection operators to (5.26) we obtain

$$\begin{aligned} F_1(q^2) &= \int d^2 \vec{k}_1 \int_0^1 \frac{d\alpha}{2(2\pi)^3 \alpha^2 (1-\alpha)} \frac{1}{M_d^2 - M^2(\alpha, \vec{k}_1)} \\ &\times \frac{1}{M_d^2 - M^2(\alpha, \vec{k}_1)} A_1(\alpha, \vec{k}_1), \quad i = 1, 2, 3, \end{aligned} \quad (5.30)$$

where $M^2(\alpha, \vec{k}_1)$ is given by (4.10), and the symbol - means that the argument $\vec{k}_1$ in the function should be changed to $\vec{k}_1 + (1-\alpha)\vec{q}_1$. The functions $A_1(\alpha, \vec{k}_1)$ are given by the following relations:

$$\begin{aligned} A_1(\alpha, \vec{k}_1) &= \text{Tr} \left\{ \tilde{\Gamma}^{\beta}(\alpha, \vec{k}_1) (\not{p} + \not{q} + M) \right. \\ &\times [(P_1)_{\beta\alpha}^{\mu} F_{\mu}(q^2)] (\not{p} + M) \\ &\times \Gamma^{\alpha}(\alpha, \vec{k}_1) (-\not{k} + M) \left. \right\}, \quad i=1, 2, 3. \end{aligned} \quad (5.31)$$

In terms of the four scalar functions F' , G' , H' and I' we can write A_1 as

$$\begin{aligned} A_1 &= \tilde{F}' F' [F_{1s} E_1(1) + F_{2s} E_1(2)] + \tilde{F}' G' [F_{1s} E_1(3) + F_{2s} E_1(4)] \\ &+ \tilde{G}' F' [F_{1s} E_1(5) + F_{2s} E_1(6)] + \tilde{G}' G' [F_{1s} E_1(7) + F_{2s} E_1(8)] \\ &+ (p^2 - M^2) \left\{ \tilde{F}' H' [F_{1s} E_1(9) + F_{2s} E_1(10)] + \tilde{F}' I' [F_{1s} E_1(11) + F_{2s} E_1(12)] \right. \\ &+ \tilde{G}' H' [F_{1s} E_1(13) + F_{2s} E_1(14)] + \tilde{G}' I' [F_{1s} E_1(15) + F_{2s} E_1(16)] \left. \right\} \\ &+ [(p+q)^2 - M^2] \left\{ \tilde{H}' F' [F_{1s} E_1(17) + F_{2s} E_1(18)] + \tilde{H}' G' [F_{1s} E_1(19) + F_{2s} E_1(20)] \right. \\ &+ \tilde{I}' F' [F_{1s} E_1(21) + F_{2s} E_1(22)] + \tilde{I}' G' [F_{1s} E_1(23) + F_{2s} E_1(24)] \left. \right\} \\ &+ (p^2 - M^2) [(p+q)^2 - M^2] \left\{ \tilde{H}' H' [F_{1s} E_1(25) + F_{2s} E_1(26)] \right. \\ &+ \tilde{H}' I' [F_{1s} E_1(27) + F_{2s} E_1(28)] \\ &+ \tilde{I}' H' [F_{1s} E_1(29) + F_{2s} E_1(30)] \\ &+ \tilde{I}' I' [F_{1s} E_1(31) + F_{2s} E_1(32)] \left. \right\}, \end{aligned} \quad (5.32)$$

where F_{1s} and F_{2s} are the Dirac and Pauli isoscalar form factors:

$$\begin{aligned} F_{1s} &= \frac{G_{ES} + \tau G_{MS}}{1 + \tau}, \\ F_{2s} &= \frac{G_{MS} - G_{ES}}{2M(1 + \tau)}, \end{aligned} \quad (5.33)$$

and the $E_1(j)$'s are complicated functions of a and $\vec{k}_1$, they are given in Appendix G. By using (5.30) we can calculate $F_1(Q^2)$, and hence, $A(Q^2)$ and $B(Q^2)$ which can be compared to the experimental data, once we have the knowledge of the scalar functions F' , G' , H' and I' . This work is still in progress and will be reported soon in another paper.⁴

VI. CONCLUSION AND DISCUSSION

In this thesis we have addressed ourselves to the real issue, i.e. the "right" description of the Fermi motion effects on deuterium targets. By the right description we mean a description which satisfies these following conditions:

1. It is a relativistic approach.
2. It is consistent.
3. It is simple, i.e. easy to be understood, and it can deal with the problem in a general way and does not need to go to the limiting case.

The conventional approach, which has been used so far to obtain the neutron data, satisfies the condition 3 only. This approach is essentially non-relativistic, in that it uses the non-relativistic wave function of the deuteron rather than the relativistic one. Notice that these non-relativistic wave functions are adjusted to fit the phase shifts between 0 and 350 MeV/c. Hence the validity of their use to momenta larger than, say 1 GeV/c should be questioned (apart from the question of which wave function should be used in the calculation). Furthermore it has another problem, i.e. it is inconsistent. One of these inconsistencies in the conventional approach, i.e. the existence of the West β -correction,²⁵ was first pointed out by Frankfurt and Strikman.¹⁰ This was later confirmed by Landshoff and Polkinghorne,¹⁰ and us¹¹ by using different approaches. Recently we have also shown another inconsistency,²⁷ i.e. the existence of a non-negligible smearing correction σ_s for high energy

hadronic total cross section. In this thesis we give a comprehensive analysis for these inconsistencies of the conventional approach, and hopefully this effort will make clear the misunderstanding that may exist in the earlier contribution to this problem.

As an alternative to the conventional approach, we have developed a new approach.^{48,49} This new approach satisfies all the three conditions we mentioned before. We have also put a new ingredient in it, i.e. the spin effects, which have been neglected so far in the discussion of the Fermi motion effects on deuterium targets. It is true that this spin degree of freedom makes life more difficult, but this is a fact that we cannot avoid anyway.

Once the "right" description of the Fermi motion effects on deuterium targets has been set up, the next step will be to find the "correct" relativistic deuteron vertex function, which is necessary to carry out this method in practice. In this thesis we have used a simple ansatz for it, the "generalized" relativistic Hulthén wave function. It leads us to a vanishing σ_s in high energy hadronic total cross sections, besides that it can give a good fit to the elastic deuteron form factor. Even though the new formalism uses an ansatz for the relativistic vertex function and even though the new formalism includes spin complications only in an average sense, we claim that it is likely to be superior to the previous methods of making smearing corrections to experimental data, and hence, should be used for that purpose.⁵⁰ The ansatz, of course, eventually will have to be checked against other processes, e.g. pion

electro-production from deuteron, etc. Other direction to find this vertex function is by developing such a vertex function from a version of the Bethe-Salpeter equation which is consistent with our previous formalism on smearing corrections.

Our new method can also be used to deal with other problems in connection with the deuteron target corrections, namely meson exchange effects, shadow corrections, etc. All these challenging problems make life more interesting, and we are looking forward to reporting our work on these problems in the future.

APPENDIX A

THE GAUGE-INVARIANCE CONDITION

In this appendix we will discuss the gauge-invariance problem for the diagram shown in Figs. 4 and 5. Let's consider Fig. 5 and neglect the spin of the particles involved for a moment. Then

$$\Gamma_\mu(q^2) = \sum_{i=n,p} \int \frac{d^4k}{(2\pi)^4} \phi(k, P' - k) \Delta_F^i(P' - k) f_\mu^i(P' - k, P - k) \times \Delta_F^i(P - k) \phi(P - k, k) \Delta_F^i(-k), \quad (A1)$$

where the ϕ is the n-p-d vertex function, f_μ is the γ -N-N vertex function which satisfies the Ward-Takashi identity ($q_\mu = P'_\mu - P_\mu$):

$$q^\mu f_\mu(P' - k, P - k) = \Delta_F^{-1}(P' - k) - \Delta_F^{-1}(P - k), \quad (A2)$$

and Δ_F^i is the full dressed propagator.

The gauge-invariance condition will be satisfied if the scalar

$$I(q^2) = q^\mu \Gamma_\mu(q^2) = 0. \quad (A3)$$

Using (A.2) we obtain

$$I(q^2) = 2 \int \frac{d^4k}{(2\pi)^4} \phi(k, P' - k) [\Delta_F^i(P - k) - \Delta_F^i(P' - k)] \times \phi(P - k, k) \Delta_F^i(-k). \quad (A4)$$

Notice that the scalar I is invariant under the interchange $P \leftrightarrow P'$; however, RHS of (A.4) changes sign. This implies I must be zero, and hence the gauge-invariance condition is satisfied.

The general form of f_μ is given by

$$f_\mu(P' - k, P - k) = f_\mu(p+q, p) = (2p+q)_\mu F(p^2, q^2, p \cdot q) + q_\mu G(p^2, q^2, p \cdot q). \quad (A5)$$

F and G here are not independent, but are related by (A.2); thus (we approximate the Δ_F^i by the free propagator Δ_F , so that $q^\mu f_\mu = -2k \cdot q$)

$$f_\mu = (2p+q)_\mu F + q_\mu \frac{(p+q)^2 - p^2}{q^2} (1 - F). \quad (A6)$$

Notice that the second term in (A.6) will be zero on the mass shell limit, and thus we get the usual form.

If we include the spin degree of freedom, then we will have

$$\Gamma_\mu(q^2) = \sum_{i=n,p} \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left\{ \bar{\Lambda}^B(k, P' - k) S_F^i(P' - k) \times f_\mu^i(P' - k, P - k) S_F^i(P - k) \Lambda^A(P - k, k) \times S_F^i(-k) \right\} \epsilon_A(P) \epsilon_B^*(P'). \quad (A7)$$

$$q^\mu f_\mu(P' - k, P - k) = S_F^{i-1}(P' - k) - S_F^{i-1}(P - k), \quad (A8)$$

and

$$I(q^2) = 2 \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left\{ \bar{\Lambda}^B(k, P' - k) [S_F'(P - k) - S_F'(P' - k)] \right. \\ \left. \times \Lambda^A(P - k, k) S_F'(-k) \right\} \epsilon_\alpha(P) \epsilon_\beta^*(P') \quad (A9)$$

By taking the transpose of the matrix in curly brackets, and then using the PT invariance⁵¹

$$[S_F'(k)]^T = T S_F'(k) T^{-1}, \\ [\bar{\Lambda}^A(q, p)]^T = T \Lambda^A(p, q) T^{-1}, \quad (A10)$$

we obtain

$$I(q^2) = 2 \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left\{ \bar{\Lambda}^A(k, P - k) [S_F'(P - k) - S_F'(P' - k)] \right. \\ \left. \times \Lambda^B(P' - k, k) S_F'(-k) \epsilon_\alpha^*(P) \epsilon_\beta(P') \right\}. \quad (A11)$$

Notice that under the interchange $P \leftrightarrow P'$ and (index) $\alpha \leftrightarrow \beta$ the RHS of (A.11) will be equal to $-I(q^2)$, and hence, $I = 0$.

Now consider Fig. 4, where q is the four-momentum of the virtual photon. We will use the identity⁵²

$$q^\mu T_{\mu\rho}(p, p; q, q) = \Gamma_\rho(p+q, p) - \Gamma_\rho(p, p-q), \quad (A12)$$

where the graphical representation is shown in Fig. 22. In (A.12) $T_{\mu\rho}$ is the virtual forward Compton scattering amplitude. Recall that $W_{\mu\rho}$ is the imaginary part of $T_{\mu\rho}$. Since we use the off-shell kinematics and on-shell dynamic formalism, i.e., F is a real function of q^2 only in (A6), the imaginary part of RHS of (A12) is equal to zero.

Thus,

$$q^\mu T_{\mu\rho} = 0 \quad (A13)$$

for an interacting nucleon which is (not far) off-shell. Hence, we can use the usual form for the $W_{\mu\rho}$ given by (2.20). This conclusion holds also if we include spin effects in our formalism.

FIG. 22. Graphical representation of the identity Eq. (A12).

APPENDIX B

THE THRESHOLD CONDITION

The phase-space restriction (or the threshold condition) $\theta(S'-M^2)$, which exists in hadronic and leptonic scattering, is due to the fact that the nucleons are not free, but are part of a bound state. In this Appendix we will discuss how this condition will affect our final formulas used for the calculations.

Let's consider the physical quantities β , $\langle T \rangle$ and γ which we encounter in hadronic scattering. Recall that

$$\beta = 1 - \int d^3k |f(\vec{k})|^2 \theta(v'), \quad (B1)$$

where

$$v' = E \cdot q / M. \quad (B2)$$

The threshold condition reads

$$p \cdot q > 0 \quad (B3)$$

Clearly, this relation restricts the region over which the phase-space integral must be performed, since we get

$$\cos \theta \leq p_0 / |\vec{p}| = [M_d - (|\vec{k}|^2 + M^2)^{1/2}] / |\vec{k}|. \quad (B4)$$

Notice that if the RHS of (B4) is larger than one, there will be no restriction, while if it is less than one, $\cos \theta$ must go from -1 to $[M_d - (|\vec{k}|^2 + M^2)^{1/2}] / |\vec{k}| \geq 0$. Therefore, we can rewrite (B1) as

$$\beta = 1 - 4\pi \int_0^{k_1} |\vec{k}|^2 d|\vec{k}| |f(\vec{k})|^2 - 2\pi \int_{k_1}^{k_2} |\vec{k}|^2 d|\vec{k}| |f(\vec{k})|^2 \left\{ 1 + \frac{M_d - (|\vec{k}|^2 + M^2)^{1/2}}{|\vec{k}|} \right\}, \quad (B5)$$

where

$$\begin{aligned} k_1 &= (M_d^2 - M^2)/2 M_d, \\ k_2 &= (M_d^2 - M^2)^{1/2}. \end{aligned} \quad (B6)$$

Similarly, we find that

$$\begin{aligned} \langle T \rangle &\equiv \int d^3k |f(\vec{k})|^2 [(|\vec{k}|^2 + M^2)^{1/2} - M] \theta(v') \\ &= 4\pi \int_0^{k_1} |\vec{k}|^2 d|\vec{k}| |f(\vec{k})|^2 [(|\vec{k}|^2 + M^2)^{1/2} - M] \\ &\quad + 2\pi \int_{k_1}^{k_2} |\vec{k}|^2 d|\vec{k}| |f(\vec{k})|^2 \\ &\quad \times [(|\vec{k}|^2 + M^2)^{1/2} - M] \left\{ 1 + \frac{M_d - (|\vec{k}|^2 + M^2)^{1/2}}{|\vec{k}|} \right\}. \end{aligned} \quad (B7)$$

and

$$\begin{aligned} \gamma &\equiv \int d^3k |f(\vec{k})|^2 \cos \theta |\vec{k}| \theta(v') \\ &= \pi \int_{k_1}^{k_2} |\vec{k}| d|\vec{k}| |f(\vec{k})|^2 \\ &\quad \times \left\{ M_d^2 + M^2 - 2M_d (|\vec{k}|^2 + M^2)^{1/2} \right\}. \end{aligned} \quad (B8)$$

In leptonic scattering the threshold condition has the form (in the deep inelastic limit) $\theta(\omega_N' - 1)$, which leads to a constraint that

$$\cos \epsilon \leq \frac{\omega_d [M_d - (|\vec{k}|^2 + M^2)^{1/2}] - M_d}{\omega_d |\vec{k}|}. \quad (B9)$$

By using a similar analysis as we did before, we find that the integral will appear as

$$\begin{aligned} \int_R &= \int_0^{k_1(\omega_d)} 4\pi |\vec{k}|^2 d|\vec{k}| \\ &\quad + \int_{k_1(\omega_d)}^{k_2(\omega_d)} 2\pi |\vec{k}|^2 d|\vec{k}| \left\{ 1 + \frac{\omega_d [M_d - (|\vec{k}|^2 + M^2)^{1/2}] - M_d}{\omega_d |\vec{k}|} \right\}. \end{aligned} \quad (B10)$$

where

$$\begin{aligned} k_1(\omega_d) &= \frac{M_d^2 (1 - \omega_d)^2 - M^2 \omega_d^2}{2\omega_d M_d (\omega_d - 1)}, \\ k_2(\omega_d) &= [M_d^2 (1 - \omega_d)^2 - M^2 \omega_d^2]^{1/2} / \omega_d. \end{aligned} \quad (B11)$$

APPENDIX C

THE SMEARED HADRONIC TOTAL CROSS SECTION'S FORMULAS

In Section IIID we have shown that

$$\sigma^{\text{smeared}}(\nu) = \sigma_1(\nu) + \sigma_2(\nu), \quad (C1)$$

where

$$\sigma_1(\nu) = \frac{2\pi M}{\nu^2} \int_0^{k_1} |f(\vec{k})|^2 |\vec{k}| d|\vec{k}| \int_{\nu_-}^{\nu_+} \nu' \sigma(\nu') d\nu', \quad (C2)$$

$$\sigma_2(\nu) = \frac{2\pi M}{\nu^2} \int_{k_1}^{\infty} |f(\vec{k})|^2 |\vec{k}| d|\vec{k}| \int_0^{\nu_+} \nu' \sigma(\nu') d\nu', \quad (C3)$$

and

$$k_1 = (M_d^2 - M^2)/2M_d,$$

$$M \nu_{\pm} = \nu [M_d^2 - (|\vec{k}|^2 + M^2)]^{1/2} = |\vec{k}|. \quad (C4)$$

We will parameterize the cross sections in the form

$$\sigma(\nu) = a + b \ln^2(\nu/c). \quad (C5)$$

Substituting (C5) to Eqs. (C2) and (C3), we can write

$$\sigma^{\text{smeared}}(\nu) = a_s + b_s \ln^2(\nu/c_s), \quad (C6)$$

where

$$a_s = D_1 - \frac{D_2^2}{4D_3},$$

$$b_s = D_3,$$

$$c_s = M \exp(-D_2/2D_3), \quad (C7)$$

$$D_i = A_i^{(1)} + A_i^{(2)}, \quad i=1,2,3, \quad (C8)$$

$$A_1^{(1)} = 4\pi \int_0^{k_1} |f(\vec{k})|^2 |\vec{k}| d|\vec{k}| X^{(1)}(|\vec{k}|), \quad (C9)$$

$$X^{(1)} = e_1 |\vec{k}| B - e_2 [D \ln(B + |\vec{k}|) - E \ln(B - |\vec{k}|)]$$

$$- e_3 (D - E),$$

$$X^{(2)} = 2 e_2 (D - E) - e_5 |\vec{k}| B,$$

$$X^{(3)} = 4 e_2 |\vec{k}| B, \quad (C10)$$

$$B = M_d - (|\vec{k}|^2 + M^2)^{1/2},$$

$$D = (B + |\vec{k}|)^2 \ln(B + |\vec{k}|),$$

$$E = (B - |\vec{k}|)^2 \ln(B - |\vec{k}|), \quad (C11)$$

$$A_2^{(1)} = 4\pi \int_{k_1}^{\infty} |f(\vec{k})|^2 |\vec{k}| d|\vec{k}| Y^{(1)}(|\vec{k}|), \quad (C12)$$

$$Y^{(1)} = (B + |\vec{k}|)^2 \left[\frac{1}{4} e_1 - e_3 \ln(B + |\vec{k}|) + e_2 \ln^2(B + |\vec{k}|) \right],$$

$$Y^{(2)} = (B + |\vec{k}|)^2 [-e_3 + 2e_2 \ln(B + |\vec{k}|)],$$

$$Y^{(3)} = e_2 (B + |\vec{k}|)^2, \quad (C13)$$

where the coefficients e 's are given by

$$M e_1 = 2M e_3 + \sum_{j=+,-} (1 + \ln c_j) b_j \ln c_j,$$

$$4M e_2 = \sum_{j=+,-} b_j,$$

$$2M e_3 = 2M e_2 + \sum_{j=+,-} a_j. \quad (C14)$$

In Eq. (C14) the symbol $+$ ($-$) refers to the case where the incoming beams are π^+ (π^-) or $n(p)$ beams, respectively. For the values of a 's, b 's and c 's, see Table 2.

A similar analysis to the relativistic result

$$\sigma^{\text{smear}}(v) = \frac{M_d}{M} \int_0^1 d\alpha \alpha G(\alpha) \left(\frac{M_d}{M} \alpha v \right), \quad (C15)$$

leads to

$$a'_s = F_1 + b'_s (U - T^2),$$

$$b'_s = e'_2,$$

$$c'_s = F_3 \frac{M}{M_d} e^{-T}, \quad (C16)$$

where

$$U = \frac{M_d}{M} \int_0^1 d\alpha \alpha G(\alpha) \ln^2 \alpha,$$

$$T = \frac{M_d}{M} \int_0^1 d\alpha \alpha G(\alpha) \ln \alpha, \quad (C17)$$

and

$$F_1 = e'_1 - \frac{e'_3{}^2}{4e'_2},$$

$$F_3 = \exp(-e'_3/2 e'_2) \quad (\text{in GeV}), \quad (C18)$$

$$e'_1 = \sum_{j=+,-} [a_j + b_j \ln^2 c_j],$$

$$e'_2 = \sum_{j=+,-} b_j,$$

$$e'_3 = -2 \sum_{j=+,-} b_j \ln c_j. \quad (C19)$$

APPENDIX D

EXPANSIONS IN POWERS OF δ

In Section IV we have shown that in the high energy limit, the smeared hadronic total cross section is given by

$$\sigma^{\text{smeared}}(\nu) = \frac{M_d}{M} \int_0^1 d\alpha \alpha G(\alpha) \sigma(\alpha \frac{M_d}{M} \nu), \quad (D1)$$

with the normalization condition

$$\frac{M_d}{M} \int_0^1 d\alpha \alpha G(\alpha) = 1. \quad (D2)$$

Since $G(\alpha)$ is strongly peaked at $\alpha \sim \frac{M}{M_d}$, one has the very approximate relation

$$\sigma^{\text{smeared}}(\nu) \approx \sigma_p(\nu) + \sigma_n(\nu), \quad (D3)$$

which strictly holds only in the limit of zero binding but has a simple physical interpretation. Recall that

$$\sigma^{\text{smeared}}(\nu) \equiv [\sigma_p(\nu) + \sigma_n(\nu)] [1 - \epsilon_s(\nu)]. \quad (D4)$$

In this Appendix we will show that $\epsilon_s(\nu)$, and hence $\sigma_s(\nu)$, can be expanded in powers of δ , where

$$\delta = \left(\frac{M^2}{M_d^2} - \frac{1}{4} \right)^{1/2} = \left(\frac{\epsilon}{4M} \right)^{1/2}, \quad (D5)$$

with ϵ the deuteron binding energy, so that $\delta \approx \frac{1}{40}$.

Recall that

$$G(\alpha) = \int d^2 \vec{k}_1 G(\alpha, \vec{k}_1) \\ = \frac{M}{2(2\pi)^3 M_d (1-\alpha)} \int d^2 \vec{k}_1 \frac{\phi^2(p^2, k^2=N^2)}{(p^2 - M^2)^2}. \quad (D6)$$

Following Landshoff and Polkinghorne,¹⁰ we define

$$\frac{1}{16\pi^2} \frac{\phi^2(p^2)}{(p^2 - M^2)^2} \equiv \frac{d}{dp^2} \chi(p^2), \quad (D7)$$

such that

$$\chi(\infty) = 0.$$

Substituting (D7) to (D6) we obtain

$$G(\alpha) = \frac{M}{M_d} \chi(\alpha M_d^2 - \frac{\alpha M^2}{1-\alpha}). \quad (D8)$$

Because of the definition (D7), $\chi(p^2)$ has a simple pole at $p^2 = M^2$ and also a logarithmic cut starting there. Hence we can write

$$\chi(p^2) = \frac{\xi(M^2 - p^2)}{M^2 - p^2}, \quad (D9)$$

such that ξ has the structure

$$\xi(z) = \xi(0) + \frac{z}{i} \int_0^\infty \frac{dz' \operatorname{Im} \xi(z')}{z' (z' - z)}. \quad (D10)$$

This integral receives also a contribution from an anomalous threshold cut which is quite close to $p^2 = M^2$, at $z' \approx 30$ Me, and from normal thresholds further away.¹⁰

In terms of ξ we can rewrite (D1) as

$$\sigma^{\text{smear}}(v) = \int_0^1 d\alpha \frac{\alpha(1-\alpha)}{M^2(\alpha)} \xi\left[\frac{M^2(\alpha)}{1-\alpha}\right] \sigma\left(\alpha \frac{M_d}{M} v\right), \quad (\text{D11})$$

where

$$M^2(\alpha) \equiv M^2 - \alpha(1-\alpha) M_d^2. \quad (\text{D12})$$

In the α -plane, the pole at $p^2 = M^2$ appears at two points $\alpha = \frac{1}{2} \pm i\delta$, where δ is defined in (D5).

Let $\bar{\alpha} = \alpha - \frac{1}{2}$, then

$$\sigma^{\text{smear}}(v) = \frac{1}{4M_d^2} \int_{-1/2}^{1/2} d\bar{\alpha} \frac{1-4\bar{\alpha}^2}{\bar{\alpha}^2 + \delta^2} \xi\left(\frac{\bar{\alpha}^2 + \delta^2}{\frac{1}{2} - \bar{\alpha}} M_d^2\right) \sigma\left[\left(\bar{\alpha} + \frac{1}{2}\right) \frac{M_d}{M} v\right]. \quad (\text{D13})$$

Landshoff and Polkinghorne¹⁰ had shown that an integral of the form

$$\int_{-1/2}^{1/2} d\bar{\alpha} \frac{R(\bar{\alpha})}{\bar{\alpha}^2 + \delta^2}, \quad (\text{D14})$$

where $g(y)$ is supposed to be well behaved near $y=0$, has the expansion

$$\left(\frac{\pi}{\delta} - 4\right) g(0) + P \int_{-1/2}^{1/2} d\bar{\alpha} \frac{g(\bar{\alpha}) - g(0)}{\bar{\alpha}^2} + (\text{series of positive powers of } \delta). \quad (\text{D15})$$

The symbol P in (D15) means the principal-value operation, necessary to deal with the (simple) pole of the integrand at $y=0$. We apply this to (D13) by setting

$$g(y) = \frac{1-4y^2}{4M_d^2} \xi\left(\frac{y^2}{\frac{1}{2}-y} M_d^2\right) \sigma\left[\left(y + \frac{1}{2}\right) \frac{M_d}{M} v\right]$$

(our neglect of the δ^2 -dependence of ξ gives an error of order δ^2).

We find

$$\begin{aligned} \sigma^{\text{smear}}(v) = & \left\{ \left(\frac{\pi}{\delta} - 4\right) \frac{\xi(0)}{4M_d^2} \sigma(v) \right. \\ & + \frac{1}{4M_d^2} P \int_{-1/2}^{1/2} d\bar{\alpha} \frac{1-4\bar{\alpha}^2}{\bar{\alpha}^2} \left\{ (1-4\bar{\alpha}^2) \xi\left(\frac{2\bar{\alpha}^2}{1-2\bar{\alpha}} M_d^2\right) \sigma\left[\left(\bar{\alpha} + \frac{1}{2}\right) \frac{M_d}{M} v\right] \right. \\ & \left. \left. - \xi(0) \sigma(v) \right\} \right\} \\ & + (\text{series of positive powers of } \delta). \end{aligned} \quad (\text{D16})$$

We apply a similar procedure to the normalization condition (D2):

$$\begin{aligned} 1 = & \left(\frac{\pi}{\delta} - 4\right) \frac{\xi(0)}{4M_d^2} + \frac{1}{4M_d^2} P \int_{-1/2}^{1/2} d\bar{\alpha} \frac{1-4\bar{\alpha}^2}{\bar{\alpha}^2} \left[(1-4\bar{\alpha}^2) \xi\left(\frac{2\bar{\alpha}^2}{1-2\bar{\alpha}} M_d^2\right) - \xi(0) \right] \\ & + (\text{series of positive powers of } \delta). \end{aligned} \quad (\text{D17})$$

Divide it out, the result is (to first order in δ),

$$\beta_S(v) = \frac{\delta}{\pi} P \int_{-1/2}^{1/2} dy \frac{1-4y^2}{y^2} \frac{\xi(\frac{2y^2}{1-2y} M_d^2)}{\xi(0)} \times \left\{ 1 - \frac{\alpha[(\gamma + \frac{1}{2}) \frac{M_d}{M} v]}{\alpha(v)} \right\} \quad (D18)$$

The above result shows that in the new approach expansion of the smearing correction is one in powers of δ , while in the conventional approach [See Eq. (3.21)] it is essentially an expansion in powers of δ^2

In the deep inelastic limit of electron scattering we have found that

$$F_2^d(q^2, \omega_d) = \frac{M_d}{M} \int_0^1 da \alpha G(\alpha) F^2(q^2, \alpha \frac{M_d}{M} \omega_N) \times \theta(\alpha \frac{M_d}{M} \omega_N - 1), \quad (D19)$$

$$F_1^d(q^2, \omega_d) = \frac{M_d}{M} \int_0^1 da G(\alpha) F_1(q^2, \alpha \frac{M_d}{M} \omega_N) \theta(\alpha \frac{M_d}{M} \omega_N - 1). \quad (D20)$$

A similar expansion of (D19) and (D20) in powers of δ can also be carried out here. We obtain

$$F_2^d(q^2, \omega_d) = F_2(q^2, \omega_N) \theta(\omega_N - 1) + \delta \Delta_2(q^2, \omega_N), \quad (D21)$$

$$F_1^d(q^2, \omega_d) = 2F_1(q^2, \omega_N) \theta(\omega_N - 1) + \delta \Delta_1(q^2, \omega_N), \quad (D22)$$

where

$$\Delta_2(q^2, \omega_N) = \frac{P}{\pi} \int_{-1/2}^{1/2} dy \frac{1-4y^2}{y^2} \frac{\xi(\frac{2y^2}{1-2y} M_d^2)}{\xi(0)} \times \left\{ F_2(q^2, \frac{M_d}{M} \omega_N (y + \frac{1}{2})) \theta(y + \frac{1}{2} - \frac{M}{M_d} \frac{1}{\omega_N}) - F_2(q^2, \omega_N) \theta(\omega_N - 1) \right\}, \quad (D23)$$

and

$$\Delta_1(q^2, \omega_N) = 2 \frac{P}{\pi} \int_{-1/2}^{1/2} \frac{dy}{y^2} \frac{\xi(\frac{2y^2}{1-2y} M_d^2)}{\xi(0)} \times \left\{ (1-2y) F_1(q^2, \frac{M_d}{M} \omega_N (y + \frac{1}{2})) \theta(y + \frac{1}{2} - \frac{M}{M_d} \frac{1}{\omega_N}) - (1-4y^2) F_1(q^2, \omega_N) \theta(\omega_N - 1) \right\}. \quad (D24)$$

The above results show that the West β -correction is not absent in leptonic scattering. The form of the correction, is however, different from that given by the conventional approach [See Eqs. (3.29) and (3.36)].

APPENDIX E

THE COEFFICIENT FUNCTIONS OF α AND $\vec{k}_1$ IN DEEP
INELASTIC LEPTONIC SCATTERING

In this Appendix the complete form of the functions A, B, ..., G, discussed in Section IV for leptonic scattering is given.

$$A(\alpha, \vec{k}_1) = [2M_d^2(q^2 - v^2)]^{-1} [-M_d^2 v^2 \vec{k}_1^2 + M_d v(\vec{q}_1 \cdot \vec{k}_1) A_1 + q^2 A_2 + M_d^2 (\vec{q}_1 \cdot \vec{k}_1)^2],$$

where

$$A_1 = (1 - \alpha)^{-1} [(1 - \alpha)^2 M_d^2 - M^2 - \vec{k}_1^2],$$

$$A_2 = [4(1 - \alpha)^2]^{-1} \left\{ \vec{k}_1^4 + 2\vec{k}_1^2 [(1 - \alpha)^2 M_d^2 + M^2] + [(1 - \alpha)^2 M_d^2 - M^2]^2 \right\}.$$

(E1)

$$B(\alpha, \vec{k}_1) = [2M_d^2(q^2 - v^2)^2]^{-1} \left\{ 2\alpha^2 M_d^4 v^4 - 4\alpha M_d^3 v^3 (\vec{q}_1 \cdot \vec{k}_1) \right. \\ \left. + M_d^2 v^2 [q^2 B_1 + 2(\vec{q}_1 \cdot \vec{k}_1)^2] + M_d v q^2 (\vec{q}_1 \cdot \vec{k}_1) B_2 \right. \\ \left. + q^4 B_3 + q^2 M_d^2 (\vec{q}_1 \cdot \vec{k}_1)^2 \right\},$$

where

$$B_1 = (1 - \alpha)^{-1} [2\alpha M^2 - 2\alpha(1 - \alpha^2) M_d^2 + (3\alpha - 1) \vec{k}_1^2],$$

$$B_2 = (1 - \alpha)^{-1} \left\{ -3(M^2 + \vec{k}_1^2) + M_d^2 [4 - (1 + \alpha)^2] \right\},$$

$$B_3 = [4(1 - \alpha)^2]^{-1} \left\{ 3\vec{k}_1^4 + 2\vec{k}_1^2 [3M^2 + (3\alpha^2 - 2\alpha - 1) M_d^2] \right. \\ \left. + 3M^4 + 2M_d^2 M^2 (\alpha^2 + 2\alpha - 3) + M_d^4 (3\alpha^4 - 4\alpha^3 + 2\alpha^2 - 4\alpha + 3) \right\}.$$

(E2)

$$C(\alpha, \vec{k}_1) = [M^2(q^2 - v^2)]^{-1} [-\alpha M_d^2 v^2 + M_d v(\vec{q}_1 \cdot \vec{k}_1) + C_1],$$

where

$$C_1 = q^2 [2(1 - \alpha)]^{-1} [(1 - \alpha^2) M_d^2 - M^2 - \vec{k}_1^2]. \quad (E3)$$

$$D(\alpha, \vec{k}_1) = [2M_d^2(q^2 - v^2)^2]^{-1} [M_d^2 v^2 D_1 + M_d v(\vec{q}_1 \cdot \vec{k}_1) D_2 + q^2 D_3],$$

where

$$D_1 = A_2 + [2(1 - \alpha)]^{-1} M_d^2 [2\alpha M^2 - (1 - \alpha)(3\vec{k}_1^2 + 2\alpha M_d^2)],$$

$$D_2 = 3 M_d^2 A_1,$$

$$D_3 = [4(1 - \alpha)^2]^{-1} M_d^2 [(1 - \alpha)^2 M_d^2 - M^2]^2. \quad (E4)$$

$$E(\alpha, \vec{k}_1) = [2M_d^2(q^2 - v^2)^2]^{-1} [M_d^3 v^3 E_1 - M_d^4 v^2 (\vec{q}_1 \cdot \vec{k}_1) + M_d v q^2 E_2 \\ - 2q^2 M_d^4 (\vec{q}_1 \cdot \vec{k}_1)],$$

where

$$E_1 = A_1 + 3\alpha M_d^2,$$

$$E_2 = -M_d^2 A_1. \quad (E5)$$

$$F(\alpha, \vec{k}_1) = [M^2 M_d^2(q^2 - v^2)^2]^{-1} \left\{ M_d^3 v^3 F_1 + 2 M_d^2 v^2 (\vec{q}_1 \cdot \vec{k}_1) F_2 \right. \\ \left. + M_d v q^2 F_3 - q^2 (\vec{q}_1 \cdot \vec{k}_1) F_4 \right\},$$

where

$$F_1 = (1 - \alpha)^{-1} [\alpha(1 - \alpha)^2 M_d^2 - \alpha M^2 + (1 - 2\alpha) \vec{k}_1^2],$$

$$F_2 = -A_1 + \alpha M_d^2,$$

$$F_3 = -B_3 + (1-\alpha)^{-1} M_d^2 \left\{ \alpha[(1-\alpha^2) M_d^2 - \vec{k}_1^2] - 3(1-\alpha) q^{-2} (\vec{q}_1 \cdot \vec{k}_1)^2 \right\},$$

$$F_4 = 2 M_d^2 q^{-2} C_1.$$

$$G_d(\alpha, \vec{k}_1) = [M_d^2 (q^2 - v^2)^{-1}]^{-1} [\alpha M_d^4 v^4 - M_d^3 v^3 (\vec{q}_1 \cdot \vec{k}_1) + M_d^2 v^2 q^2 G_1 + M_d^3 v q^2 (\vec{q}_1 \cdot \vec{k}_1) + q^4 G_2],$$

where

$$G_1 = -\frac{1}{2} A_1 - 2\alpha M_d^2,$$

$$G_2 = M_d^2 q^{-2} C_1.$$

(E5)

(E7)

APPENDIX F

SPIN-DEPENDENT VERTEX FUNCTION

In Section V we have defined the spin-dependent vertex function

$\Phi(\alpha, \vec{k}_1)$ as

$$\bar{\Sigma}^2(\alpha, \vec{k}_1) = (-g_{\beta\delta} + M_d^{-2} P_\beta P_\delta) \text{Tr} \left\{ \bar{\Gamma}^\beta (\not{p} + M) \Gamma^\delta (-\not{k} + M) \right\}, \quad (F1)$$

where the truncated n-p-d vertex function Γ is given by^{42,43}

$$\Gamma^\beta(c, \vec{k}_1) = F'(c, \vec{k}_1) \gamma^\beta + (2M)^{-1} G'(c, \vec{k}_1) (p-k)^\beta + M^{-1} (\not{p}-M) [H'(c, \vec{k}_1) \gamma^\beta + (2M)^{-1} I'(c, \vec{k}_1) (p-k)^\beta], \quad (F2)$$

and

$$\bar{\Gamma}^\beta = C\gamma^5 (\Gamma^\beta)^\Gamma (C\gamma^5)^{-1}. \quad (F3)$$

In terms of these four scalar functions F' , G' , H' and I' we find

$$\Phi^2(\alpha, \vec{k}_1) = [2(1-\alpha)^3 M_d^4]^{-1} \left\{ Z_1 F'^2 + Z_2 F' G' - M^2 Z_3 G'^2 + (p^2 - M^2) [Z_4 H'^2 - Z_5 I'^2 + Z_6 H' F' + Z_7 (H' G' + I' F') + Z_8 G' I'] \right\}, \quad (F4)$$

where

$$Z_1 = 4(1-\alpha) M^4 \left\{ (M^2 + \vec{k}_1^2)^2 + 2(1-3\alpha) M^2 \vec{k}_1^2 + (1-\alpha) M_d^2 [(2\alpha-5) M^2 - (1-\alpha)^2 (\alpha+2) M_d^2 - (1+2\alpha) \vec{k}_1^2] \right\},$$

$$Z_2 = 8(1-\alpha)^{-1/4} \left\{ (M^2 + \vec{k}_1^2)^2 + (1-\alpha) M_d^2 [(\alpha^2 - \alpha + 1) M_d^2 + (2\alpha-3) M^2 + (1-2\alpha) \vec{k}_1^2] \right\},$$

$$\begin{aligned}
Z_3 &= (1-\alpha)^2 M_d^2 [(1-\alpha)^4 M_d^4 + 2(4\alpha-3) M^2 \vec{k}_1^2 - (8\alpha-7) M^4 \\
&\quad + 3\vec{k}_1^4 + (1-\alpha)^2(4\alpha-5) M^2 M_d^2 + 3(1-\alpha)^2 M_d^2 \vec{k}_1^2] \\
&\quad + (M^2 + \vec{k}_1^2)^2 [\vec{k}_1^2 + (4\alpha-3) M^2], \\
Z_4 &= 4(1-\alpha) M^2 \left\{ (M^2 + \vec{k}_1^2)^2 - (1-\alpha) M_d^2 [(1+2\alpha)\vec{k}_1^2 + (10\alpha-7) M^2 \right. \\
&\quad \left. - (1-\alpha)(\alpha^2 + \alpha - 2) M_d^2] \right\}, \\
Z_5 &= (M^2 + \vec{k}_1^2)^3 + (1-\alpha)^2 M_d^2 [(1-\alpha)^4 M_d^4 + 2M^2 \vec{k}_1^2 - M^4 \\
&\quad + 3\vec{k}_1^4 - (1-\alpha)^2 M_d^2 M^2 + 3(1-\alpha)^2 M_d^2 \vec{k}_1^2], \\
Z_6 &= -4E(1-\alpha)^3 M^4 M_d^2, \\
Z_7 &= 4(1-\alpha) M^2 \left\{ \vec{k}_1^2 [\vec{k}_1^2 + 2M^2 + 2(1-\alpha)^2 M_d^2] \right. \\
&\quad \left. + [M^2 - (1-\alpha)^2 M_d^2]^2 \right\}, \\
Z_8 &= 4(1-\alpha) M^2 \left\{ (1-\alpha)^2 M_d^2 [2\vec{k}_1^2 - 2M^2 + (1-\alpha)^2 M_d^2] \right. \\
&\quad \left. + (M^2 + \vec{k}_1^2)^2 \right\}.
\end{aligned}
\tag{F5}$$

APPENDIX G

THE COEFFICIENT FUNCTIONS OF α AND $\vec{k}_1$ IN ELASTIC ed SCATTERING

See Eq. (5.32). In this Appendix we will give the functions E_1 (j)'s in the form of scalar products of the four-momenta of the problem.

$$\begin{aligned}
E_1(1) &= 4 D^{-4} \left\{ D^2 [3k \cdot D (p^2 - M^2 + p \cdot q) - p \cdot D (2M^2 + 2p \cdot k + k \cdot q)] \right. \\
&\quad \left. - 4 k \cdot D (p \cdot D)^2 \right\}, \\
E_2(1) &= 4(3q^2)^{-1} [q^2 (3M^2 + 3p \cdot k) - 2p \cdot q k \cdot q], \\
E_3(1) &= 16 M_d^2 (3q^4 D^2)^{-1} \left\{ q^2 k \cdot D (p^2 - M^2 + p \cdot q) \right. \\
&\quad \left. + p \cdot D [k \cdot q (3q^2 + 8p \cdot q) - 2q^2 p \cdot k] \right\}, \\
E_1(2) &= -4D^{-2} q^2 M (p \cdot D - k \cdot D), \\
E_2(2) &= \frac{4}{3} M (p^2 + M^2 - p \cdot q + 4 p \cdot k), \\
E_3(2) &= 16 M_d^2 (3D^2)^{-1} M k \cdot D, \\
E_1(3) &= 2D^{-4} (p \cdot D - k \cdot D) \left\{ 2 p \cdot D (p \cdot D - k \cdot D) \right. \\
&\quad \left. - D^2 (p^2 - M^2 + p \cdot q - k \cdot q) \right\}, \\
E_2(3) &= (3q^2)^{-1} \left\{ k \cdot q [3M^2 - p^2 + 2k \cdot q - 4 p \cdot q - 2 p \cdot k] \right. \\
&\quad \left. + q^2 (p^2 + M^2 - 2 p \cdot k) + p \cdot q [3(p^2 - M^2) + 2p \cdot q] \right\}, \\
E_3(3) &= 4M_d^2 (3q^4 D^2)^{-1} \left\{ q^2 [k \cdot D (p^2 - M^2 + 4 p \cdot q) + p \cdot D (3M^2 + p^2)] \right. \\
&\quad \left. - 4p \cdot q p \cdot D (q^2 + 2p \cdot q) + k \cdot q [4p \cdot D (q^2 + 4p \cdot q - 2k \cdot q) - 4q^2 k \cdot D] \right\},
\end{aligned}$$

$$\begin{aligned}
E_1(4) &= 2(MD^4)^{-1}(p \cdot D - k \cdot D) \left\{ k \cdot q [2(p \cdot D)^2 - D^2(p^2 + M^2)] \right. \\
&\quad \left. + D^2[p \cdot k(q^2 + 2p \cdot q) - q^2 M^2] - 2p \cdot D k \cdot E p \cdot q \right\}, \\
E_2(4) &= (3M q^2)^{-1} \left\{ q^2 [M^2(p^2 + M^2 - 4p \cdot q) + p \cdot k(p^2 - M^2 - 2p \cdot k + 4p \cdot q)] \right. \\
&\quad \left. + k \cdot q [2M^2 k \cdot q + 2q^2(2M^2 - p \cdot k) - (q^2 + 2p \cdot q)(p^2 + M^2)] \right. \\
&\quad \left. + 2(p \cdot q)^2(2p \cdot k - M^2) \right\}, \\
E_3(4) &= 4M_d^2(3q^4 M D^2)^{-1} \left\{ q^2 p \cdot D [q^2(p \cdot k - M^2) - 4k \cdot q(p \cdot q - k \cdot q)] \right. \\
&\quad \left. + q^2(p \cdot D - k \cdot D) [p \cdot k(2p \cdot q + q^2) - M^2 q^2 - k \cdot q(p^2 + M^2)] \right. \\
&\quad \left. + 3q^2(p \cdot q - k \cdot q) [2p \cdot D p \cdot k - k \cdot D(p^2 + M^2)] \right. \\
&\quad \left. - [k \cdot D(q^2 + 2p \cdot q) - 2k \cdot q p \cdot D] [q^2(p^2 - p \cdot k) - 4p \cdot q(p \cdot q - k \cdot q)] \right\}, \\
E_1(5) &= E_1(3) + D^{-2}(p \cdot D - k \cdot D)(q^2 + 2p \cdot q - 2k \cdot q), \\
E_2(5) &= -E_2(3) + (3q^2)^{-1} \left\{ k \cdot q [q^2 + 2p \cdot q + 2(p^2 - p \cdot k)] \right. \\
&\quad \left. + q^2 [5M^2 - p^2 - 2(p \cdot q + 2p \cdot k)] - p \cdot q(q^2 + 2p \cdot q) \right\}, \\
E_3(5) &= E_3(3) + 2M_d^2(3q^2 D^2)^{-1} \left\{ (q^2 + 2p \cdot q)(p \cdot D - k \cdot D) \right. \\
&\quad \left. - 3D^2(q^2 + 3p \cdot q) - 8p \cdot D p \cdot k + k \cdot q(5D^2 + 4k \cdot D) \right\}, \\
E_1(6) &= -E_1(4) + 4q^2(MD^4)^{-1}(p \cdot D - k \cdot D)(p \cdot D k \cdot D - M^2 D^2), \\
E_2(6) &= E_2(4) + (3M)^{-1} \left\{ k \cdot q(p^2 - 3M^2 + 2p \cdot q) \right. \\
&\quad \left. + (2p \cdot q + q^2)(3M^2 + p \cdot k) \right\}, \\
E_3(6) &= -E_3(4) + 4M_d^2(3MD^2)^{-1} \left\{ 3p \cdot q [k \cdot D(p^2 + M^2 - q^2 - 2p \cdot q) - 2p \cdot D p \cdot k] \right. \\
&\quad \left. + q^2 [4M^2 k \cdot D + p \cdot D(2p \cdot k - 5M^2)] + k \cdot q p \cdot D(8k \cdot q - 3q^2 - 2p \cdot q) \right\},
\end{aligned}$$

$$\begin{aligned}
E_1(7) &= (M^2 D^4)^{-1}(p \cdot D - k \cdot D)^2 \left\{ k \cdot D(p^2 - M^2 + p \cdot q) \right. \\
&\quad \left. + p \cdot D(2M^2 - 2p \cdot k - k \cdot q) \right\}, \\
E_2(7) &= (6q^2 M^2)^{-1} \left\{ k \cdot q [k \cdot q(p^2 - 2M^2) - p \cdot q(p^2 - 5M^2 + 2p \cdot k) - q^2(p^2 - M^2)] \right. \\
&\quad \left. + q^2 [M^2(p^2 - M^2 - p \cdot q) + p \cdot k(2p \cdot k - \frac{1}{2}q^2 - p^2 - 3M^2)] \right. \\
&\quad \left. - 2(p \cdot q)^2(M^2 - p \cdot k) \right\}, \\
E_3(7) &= 2M_d^2(3q^4 M^2 D^2)^{-1} T[k \cdot D(p^2 - M^2 + p \cdot q) + p \cdot D(2M^2 - 2p \cdot k - k \cdot q)], \\
E_1(8) &= q^2(MD^4)^{-1}(p \cdot k - k \cdot D)^3, \\
E_2(8) &= (6M)^{-1} [q^2(p^2 + M^2 - 2p \cdot k) - (p \cdot q - k \cdot q)^2], \\
E_3(8) &= -\frac{1}{2} M T E_3(27), \\
E_1(9) &= -(Mq^2)^{-1} E_1(2), \\
E_2(9) &= 4(q^2)^{-1}(q^2 + p \cdot q), \\
E_3(9) &= -(Mq^2)^{-1} E_3(2), \\
E_1(10) &= 4(MD^2)^{-1} [k \cdot D(p \cdot q + q^2) + p \cdot D q^2], \\
E_2(10) &= M^{-1} E_2(1), \\
E_3(10) &= 16 M_d^2(3q^2 MD^2)^{-1} [k \cdot D(p \cdot q + q^2) + 3p \cdot D k \cdot q], \\
E_1(11) &= D^{-2}[D^2(M^2 + k \cdot q + p \cdot k) - 2p \cdot D k \cdot D] E_1(27), \\
E_2(11) &= (3q^2 M^2)^{-1} [k \cdot q(3M^2 - p^2 + 2p \cdot k + 2k \cdot q - 2p \cdot q) \\
&\quad - 2p \cdot q(M^2 + p \cdot k) + q^2(M^2 - p \cdot k)], \\
E_3(11) &= 4M_d^2(3q^4 M^2 D^2)^{-1} \left\{ k \cdot q [p \cdot D(q^2 + 4p \cdot q - 4k \cdot q) - 4k \cdot D(q^2 + p \cdot q)] \right. \\
&\quad \left. + q^2 [2M^2 p \cdot D + k \cdot D(3p \cdot q - p^2 - M^2)] + 4k \cdot D(p \cdot q)^2 \right\},
\end{aligned}$$

$$E_1(12) = -M(p \cdot q + q^2 + k \cdot q) E_1(27),$$

$$E_2(12) = (3M)^{-1} [M^2 - p^2 - 2(p \cdot q - k \cdot q)],$$

$$E_3(12) = 4M_d^2 (3Mq^2 D^2)^{-1} \left\{ k \cdot q (D^2 + 2k \cdot D) - p \cdot q (D^2 + 2p \cdot D) - q^2 (p \cdot D - k \cdot D) \right\},$$

$$E_1(13) = (M^2 - p \cdot k - k \cdot q) E_1(27),$$

$$E_2(13) = (3q^2 M^2)^{-1} [k \cdot q (p^2 - 3M^2 - 2p \cdot q - 3q^2 + 2k \cdot q + 2p \cdot k) + 4(M^2 - p \cdot k)(q^2 + p \cdot q)],$$

$$E_3(13) = 4M_d^2 (3q^4 M^2 D^2)^{-1} \left\{ q^2 [k \cdot D (p^2 - M^2 - q^2) + 2p \cdot D (M^2 - p \cdot k)] + 2(q^2 + 2p \cdot q) [k \cdot q p \cdot D - k \cdot D (p \cdot q + q^2)] + k \cdot q [2p \cdot D (q^2 + 2p \cdot q - 2k \cdot q) + 4k \cdot D (p \cdot q + q^2)] \right\},$$

$$E_1(14) = E_1(12) - 2M^2 E_1(28),$$

$$E_2(14) = (3Mq^2)^{-1} \left\{ q^2 (p^2 + M^2 - 2p \cdot k) - (p \cdot q - k \cdot q)^2 + 3(p \cdot q + q^2 - k \cdot q)^2 \right\},$$

$$E_3(14) = 8M_d^2 (3q^2)^{-1} E_1(14),$$

$$E_1(15) = (Mq^2)^{-1} E_1(8) = E_1(23),$$

$$E_2(15) = (Mq^2)^{-1} E_2(8) = E_2(23),$$

$$E_3(15) = - (Mq^2)^{-1} E_3(8) = E_3(23),$$

$$E_1(16) = (M^3 D^4)^{-1} (p \cdot D - k \cdot D)^2 [k \cdot D (q^2 + p \cdot q) - k \cdot q p \cdot D],$$

$$E_2(16) = (6M^3)^{-1} [k \cdot q (p^2 - p \cdot k + p \cdot q - k \cdot q) + (q^2 + p \cdot q)(M^2 - p \cdot k)],$$

$$E_3(16) = 2M_d^2 (3q^4 M^3 D^2)^{-1} T[k \cdot D (q^2 + p \cdot q) - k \cdot q p \cdot D],$$

$$E_1(17) = - (Mq^2)^{-1} E_1(2),$$

$$E_2(17) = - 4(q^2)^{-1} p \cdot q,$$

$$E_3(17) = - (Mq^2)^{-1} E_3(2),$$

$$E_1(18) = - 4(MD^2)^{-1} [p \cdot q k \cdot D + k \cdot q p \cdot D],$$

$$E_2(18) = 4(3Mq^2)^{-1} [q^2 (3M^2 + 2p \cdot k) - 2p \cdot q k \cdot q],$$

$$E_3(18) = -16M_d^2 (3q^2 MD^2)^{-1} (p \cdot q k \cdot D + 3k \cdot q p \cdot D),$$

$$E_1(19) = (M^2 - p \cdot k) E_1(27),$$

$$E_2(19) = (3q^2 M^2)^{-1} [2(2p \cdot q - k \cdot q)(p \cdot k - M^2) - k \cdot q (p^2 - M^2)],$$

$$E_3(19) = 4M_d^2 (3q^4 M^2 D^2)^{-1} \left\{ q^2 [k \cdot D (p^2 - M^2) + 2p \cdot D (M^2 - p \cdot k)] + 2k \cdot q (p \cdot q D^2 - 2k \cdot q p \cdot D) - 4(p \cdot q)^2 k \cdot D \right\},$$

$$E_1(20) = 2(MD^2)^{-1} (p \cdot D - k \cdot D) (p \cdot q - k \cdot q),$$

$$E_2(20) = (q^2)^{-1} [2E_2(8) + M^{-1} (p \cdot q - k \cdot q)^2],$$

$$E_3(20) = 8M_d^2 (3q^2)^{-1} E_1(20),$$

$$E_1(21) = E_1(11) + M E_1(28),$$

$$E_2(21) = - E_2(11) + 2(3M^2 q^2)^{-1} [2M^2 q^2 + (k \cdot q)^2],$$

$$E_3(21) = E_3(11) + 8M_d^2 (3q^2 M^2 D^2)^{-1} (k \cdot q p \cdot D + 2p \cdot q k \cdot D),$$

$$E_1(22) = (4M^2)^{-1} E_1(2),$$

$$E_2(22) = (6Mq^2)^{-1} [4(k \cdot q)^2 - 4(p \cdot q)^2 + q^2 (2M^2 - 2p^2 + 3q^2)],$$

$$E_3(22) = 2M_d^2(3M_d^2D^2)^{-1} [3q^2D^2 - 2p.q(D^2 + 2p.D) - 2k.q(D^2 + 2k.D)],$$

$$E_1(24) = -E_1(16) - Mq^2 E_1(31)$$

$$E_2(24) = -E_2(16) + (6M^3)^{-1} [k.q(p.q - k.q) + q^2(M^2 - p.k)],$$

$$E_3(24) = 2M_d^2(3q^4M^3D^2)^{-1} T(p.D k.q - k.D p.q),$$

$$E_1(25) = -4(M^2D^2)^{-1} k.D,$$

$$E_2(25) = 0 = E_{1,3}(26) = E_{3,2,3}(32),$$

$$E_3(25) = -(q^2M^3)^{-1} E_3(2),$$

$$E_2(26) = 4M^{-1},$$

$$E_1(27) = 2(M^2D^2)^{-1} (p.D - k.D) = E_1(29),$$

$$E_2(27) = (3M^2q^2)^{-1} (p.q - k.q + 4M^2 - 4p.k),$$

$$E_3(27) = 2M_d^2(3q^2)^{-1} E_1(27) = E_3(29),$$

$$E_1(28) = -M^{-1} k.q E_1(27) = -E_1(30),$$

$$E_2(28) = -(3M^3q^2)^{-1} [2k.q(p.q - k.q) + q^2(p.k - M^2)],$$

$$E_3(28) = -4M_d^2(3M^3q^2D^2)^{-1} [k.D(3p.q - 4k.q) + p.D k.q],$$

$$E_2(29) = (M^2q^2)^{-1} (p.q + q^2 - k.q),$$

$$E_2(30) = E_2(28) - (M^3q^2)^{-1} k.q,$$

$$E_3(30) = -E_3(28) + 4M_d^2(M^3q^2)^{-1} k.D,$$

$$E_1(31) = -(M^4D^4)^{-1} k.D(p.D - k.D)^2,$$

$$E_2(31) = (M^4q^2)^{-1} [k.q.p.q - k.q - q^2(p.k - M^2)],$$

$$E_3(31) = -2M_d^2(3q^4D^2M^3)^{-1} k.D T,$$

where

$$T = q^2.p^2 + M^2 - 2p.k - 3p.q + 3k.q - 4(p.q - k.q)^2.$$

To rewrite the above functions in terms of α and $\vec{k}_1$, we can use these following relations:

$$D^2 = 4M_d^2 - q^2,$$

$$p^2 = \alpha M_d^2 - \frac{\alpha M^2 + \vec{k}_1^2}{1 - \alpha},$$

$$k.D = \frac{1}{2}(1 - \alpha)(2M_c^2 - q^2) + \vec{q}_1 \cdot \vec{k}_1 + \frac{M^2 + \vec{k}_1^2}{1 - \alpha},$$

$$p.D = \frac{1}{2}D^2 - k.D,$$

$$p.q = -\frac{1}{2}\alpha q^2 - \vec{q}_1 \cdot \vec{k}_1,$$

$$k.q = -\frac{1}{2}q^2 - p.q,$$

$$p.k = \frac{1}{2}(1 - \alpha)M_d^2 + \frac{(2\alpha - 1)M^2 + \vec{k}_1^2}{2(1 - \alpha)},$$

$$q^2 = -\vec{q}_1^2.$$

REFERENCES AND FOOTNOTES

1. An excellent review of electron scattering as a probe of the character of matter is given in: G. B. West, Phys. Rep. 18, 263 (1975).
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11. D. Kusno and M. J. Moravcsik, OITS-104, Phys. Rev. D20, (1979), in print.
12. See, for example, the review by P. V. Landshoff and H. Osborn, in Electromagnetic Interactions of Hadrons (ed. A. Donnachie and R. Shaw), Plenum, New York, 1978.
13. W. B. Atwood, SLAC Report No. 185 (1975), unpublished. See also References 4,7,8 and 9.
14. R. L. Cool et al., Phys. Rev. D1, 1887 (1970).

15. Throughout this thesis, amplitudes are given in dimensionless units [See R. J. Eden, High Energy Collisions of Elementary Particles, Cambridge Univ. Press (New Ycrk) 1967]; the optical theorem has the form

$$\text{Im } A_{1T}(\nu) = 2M_T(\nu^2 - m_1^2)^{1/2} \sigma_{1T}(\nu),$$

where M_T and m_1 are the target and incident particle masses, respectively.

16. Our states are covariantly normalized as

$$\langle p|p' \rangle = (2\pi)^3 E \delta^3(\vec{p} - \vec{p}')$$

to make $W_{\mu\lambda}$ dimensionless [See the footnote 15].

17. J. Bernstein, Elementary Particles and Their Currents, W. H. Freeman and Co. (1968), Chapt. 6.
18. L. Hand, Phys. Rev. 129, 1834 (1964).
19. The use of these expressions in the extraction of neutron data from deuteron and proton data had been discussed in great details by Atwood (Ref. 13) and Bodek (the second paper of Ref. 9). Note that our formulas are slightly different from theirs, i.e. we include the factor $p^2 - M^2$ in our formulas.
20. Equivalently one could, of course, consider them as functions of q^2 and s' (Notice that $s' = M^2 + q^2 + 2M \cdot 0$).
21. We realize that the use of the impulse approximation at small q^2 's might be questionable, and hence the usual practice of extrapolating to $q^2 = 0$ for the form factor might not be realistic.

22. F. Gross, Phys. Rev. 142, 1025 (1966) and references contained therein. The spectator approximation means essentially that one can make the following replacement (inside the integral):

$$\frac{1}{k^2 - M^2} \rightarrow i\pi \frac{\delta(k_0 - \sqrt{\vec{k}^2 + M^2})}{\sqrt{\vec{k}^2 + M^2}}.$$

This approximation amounts to restricting ourselves to taking only the contribution of the pole at $k_0 = (M^2 + \vec{k}^2)^{1/2}$ in performing the integral over k_0 . It has been shown by Gross that, for q^2 small, the result one gets by the above replacement coincides with that obtained by computing the imaginary part of the diagram with the Cutkosky rule and then the full amplitude from a dispersion relation. This result is true if in the dispersion relation one keeps only the anomalous-threshold contribution, or equivalently if one assumes that the vertex function is a constant [See L. Bertocchi and A. Cappella, Nuovo Cimento A51, 369 (1967)]. Here one assumes that the above replacement is also allowed with an arbitrary vertex function and for large q^2 .

23. J. S. Poucher et al., Phys. Rev. Lett. 32, 118 (1974).

24. Notice that if we assume

$$|f(\vec{k})|^2 \equiv |f(|\vec{k}|)|^2$$

then (3.11) can be rewritten as

$$\int_1^\infty \frac{d\omega_d}{\omega_d} F_2^d(q^2, \omega_d) = \int d^3k |\psi(|\vec{k}|)|^2 \int_1^\infty \frac{d\omega_N}{\omega_N} F_2^N(q^2, \omega_N).$$

Thus, by using (3.5) and (3.6), we recover the quark-parton model result. Unfortunately $|\psi(|\vec{k}|)|^2$ has no physical meaning, i.e., it is not a probability function. Another attempt could be by defining the vertex function $\phi^2(p^2)$ such that

$$\int d^3k |\phi(\vec{k})|^2 \frac{p_0 - p_3}{M} = 1.$$

However we face the same problem as before, i.e., the quantity

$$|\phi(\vec{k})|^2 = |f(\vec{k})|^2 \frac{p_0 - p_3}{M}$$

cannot be interpreted as the probability function for $p_0 - p_3 = M_d - (|\vec{k}|^2 + M^2)^{1/2} + k_3$ is not definite positive throughout the whole region of $|\vec{k}|$.

25. We define the West β -correction as the smearing correction in the case when σ 's are constant, and this definition is consistent with what implicitly suggested in Ref. 4. We think the one adopted by Ref. 10, i.e. $\sigma = \beta (\sigma_p + \sigma_n)$, should not be interpreted as the West β -correction.
26. T. Hamada and I.D. Johnston, Nucl. Phys. 34, 382 (1962).
R. W. Reic, Jr., Ann. Phys. (N.Y.) 50, 411 (1968).
L. Hulthén and M. Sugawara, in Handbuch der Physik, edited by S. Flügge (Springer, Berlin, 1957), Vol. 39, p.1.
27. D. Kusno and M. J. Moravcsik, OITS - 118 preprint, September 1979, to be published.

28. In the calculation of $\sigma_g(v)$, for our convenience, we use the cut-off $k \sim 3$ GeV/c. We do not think the higher cut-off will change the conclusions.
29. S. P. Denisov *et al.*, Phys. Lett. 36B, 415 (1971); Nucl. Phys. B65, 1 (1973). D. Bogert *et al.*, Phys. Rev. Lett. 31, 1271 (1973). M. J. Longo *et al.*, Phys. Rev. Lett. 33, 725 (1974). G. A. Akopdjanov *et al.*, Nucl. Phys. B75, 401 (1974). E. L. Berger *et al.*, Nucl. Phys. B77, 365 (1974). D. Fong *et al.*, Nucl. Phys. B102, 386 (1976). A. Firestone *et al.*, Phys. Rev. D14, 2902 (1976). V. D. Apokin *et al.*, Nucl. Phys. B106, 413 (1976). D. S. Ayres *et al.*, Phys. Rev. D15, 3105 (1977).
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32. This assumption corresponds to the usual static approximation for the bound state wave function in which the relative time variable is set to zero [See P. M. Fishbane and I. J. Muzinich, Phys. Rev. D8, 4015 (1973)].
33. In the limit $q^2 \rightarrow 0$, (4.8) becomes

$$\int d^2\vec{k}_1 \int_0^1 d\alpha \frac{\alpha}{2(2\pi)^3(1-\alpha)} \frac{\phi^2(p^2, k^2=M^2)}{(p^2-M^2)^2} = 1.$$

Notice that the integrand is definite positive, and hence, in principle one can identify it as the probability function.

The above result, i.e., the definite positiveness of the integrand, shows that our approximation method is better than the spectator approximation.

34. The threshold condition does not rule out the contribution from the region $1 < \alpha < \infty$. In this region, the argument of ϕ becomes large, that is, p^2 is large and positive (for fixed $\vec{k}_1^2$). The fact that p^2 is large means that the interacting nucleon is far off-shell. Since the deuteron is a weakly bound system, it is necessary to require that ϕ must be small when its argument becomes large. Thus presumably the region $1 < \alpha < \infty$ gives small contribution to the integral of (4.14), and hence, will be neglected.
35. In (4.32) we had neglected the term

$$\frac{1}{2Mv} \int d^2\vec{k}_1^2 d^2\vec{k}_1 \int_0^1 \frac{d\alpha}{\alpha} G(\alpha, \vec{k}_1) F_2^e(q^2, \alpha \frac{M_d}{M} \omega_N),$$

$$\times \theta(\alpha \frac{M_d}{M} \omega_N - 1),$$

which is justified since the $|\vec{k}_1|$ -dependence of $\phi(\alpha, \vec{k}_1)$ is such that

$$\lim_{|\vec{k}_1| \rightarrow \infty} G(\alpha, \vec{k}_1) = 0,$$

and hence the integral will be finite (See Section IV E).

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We do not include the term

$$i(2M_d^2)^{-1} W_6^{vd}(q^2, \nu) 4P_{\lambda q_3} - q_{\lambda} p_0$$

here since it gives zero contribution to the cross section. For our convenience we are including the W_4 and W_5 terms in the $W_{\lambda p}$, though they are usually neglected since their contributions to the cross section are $O(m^2)$, where m is the lepton mass in the final state.

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48. Frankfurt and Strikman had also suggested another approach to this problem. They use the so-called dispersion approach developed by Gribov. This approach is relativistic. However, at the present stage (as they show in their papers) it is not clear whether this approach will satisfy the condition 2 and 3. In practice they also use the non-relativistic wave functions. Finally we would like to point out that our new approach uses the usual Feynman diagram approach complemented with the Brodsky parameterization method. This approach should be a familiar one to anybody who had taken a relativistic quantum mechanics course.
49. Landshoff and Polkinghorne's approach is basically similar to ours, except that the mathematical problems we are facing are much easier than theirs. They use the so-called Sudakov-variable method. Due to this kind of difficulty, their approach can only deal with the limiting case, i.e. the high energy limit.
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