132
11/5/78

LA-7484-MS
Informal Report

LA 681

The Limb-Darkening Opacity Experiment Using a Laser-Heated Plasma

University of California

LOS ALAMOS SCIENTIFIC LABORATORY

Post Office Box 1663 Los Alamos, New Mexico 87545

MASTER

MASTER

REPRODUCTION OF THIS DOCUMENT IS UNLIMITED

LA-7484-MS
Informal Report
UC-34
Issued: October 1978

The Limb-Darkening Opacity Experiment Using a Laser-Heated Plasma

Nelson M. Hoffman
Leston W. Miller
Joseph M. Mack

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

THE LIMB-DARKENING OPACITY EXPERIMENT USING A LASER-HEATED PLASMA

by

Nelson M. Hoffman
Leston W. Miller
Joseph M. Mack

ABSTRACT

We review the limb-darkening technique, a method for measuring monochromatic opacity information, which has had successful astrophysical applications. We discuss the application of the technique to laser-produced plasmas in materials and regimes of temperature and density of interest to weapons designers, and estimate the magnitude of the limb-darkening effect in such situations. Finally, we describe an experimental study, now in progress, to evaluate the feasibility of this approach.

I. INTRODUCTION

The current body of opacity information stems almost entirely from theoretical calculations. Experimental determinations are rare, and have been expensive to perform and difficult to interpret. However, the advent of high-power pulsed lasers may allow relatively inexpensive and more easily interpretable opacity experiments to be performed in a laboratory environment. The newest generation of lasers, capable of delivering 10 kJ of radiant energy in a 1-ns pulse, will allow such opacity experiments to reach regimes of temperature and density that are of direct interest to the weapons community.

This report describes the limb-darkening technique, a method for obtaining monochromatic opacity information, whose feasibility is now under study at the Los Alamos Scientific Laboratory (LASL). In Sec. II, we set forth the theoretical basis of the limb-

darkening technique; Sec. III describes the special problems attendant in using a laser as an energy source; Sec. IV describes the magnitude of the limb-darkening effect one might expect in various situations; and the current feasibility study is outlined in Sec. V.

A review of all published techniques of opacity measurement is beyond the scope of this report, but we note briefly two other interesting approaches. One approach, discussed by Larson, Spillman, Earnes, Hafer, and Fickett,¹ would employ high-power lasers to determine Rosseland mean opacities. The method described by Zel'dovich and Raizer² gives monochromatic opacity information and might work with a laser driver. It may be possible to combine the limb-darkening technique with these two approaches and increase the information output of an experiment. We shall not pursue this idea in this report, but it bears further study.

II. THE THEORETICAL BASIS OF THE LIMB-DARKENING EXPERIMENT

The specific intensity I_ν of radiation at frequency ν emerging from an emitting layer of plasma usually varies as a function of polar angle θ , the angle between the direction of observation and the normal to the layer (Fig. 1). This variation (referred to as "limb darkening" in observations of the sun) depends on the run of temperature T vs optical depth τ_ν within the layer. One can see deep into the layer near $\theta = 90^\circ$, but highly oblique views near $\theta = 0^\circ$ show only the outer parts of the layer. If T increases inward, the plasma will appear brighter when viewed along the normal than when viewed obliquely. Limb-darkening observations can be used to determine $T(\tau_\nu)$; observations at different frequencies can be used to find the ratio of opacity at these frequencies in the layer. The technique has been used successfully by Pierce and Waddell³ in the case of the solar atmosphere, for example.

In more detail, the technique proceeds as follows. The emergent specific intensity $I_\nu(\mu)$, where $\mu = \cos \theta$, is given by the formal solution to the radiative transfer equation

$$I_\nu(\mu) = \int_0^\infty B_\nu(\tau_\nu) e^{-\tau_\nu/\mu} \frac{d\tau_\nu}{\mu} \quad (1)$$

Here the source function B_ν is the Planck function. The plasma layer is assumed to be thick enough that the upper limit of integration is taken to be infinity. Equation (1) shows that $I_\nu(\mu)$ is related to the Laplace transform of $B_\nu(\tau_\nu)$,

$$I_\nu(\mu) = \frac{1}{\mu} L[B_\nu]$$

where $L[f]$ denotes the Laplace transform of f . Determining the function $B_\nu(\tau_\nu)$ is equivalent to finding the inverse Laplace transform of $\mu I_\nu(\mu)$. In practice, this is sometimes hard to do if the data are noisy or sparse. An alternative approach is to assume a model for the source function's dependence on τ_ν , and then to fit the parameters in the model by least squares.⁴

Once $B_\nu(\tau_\nu)$ has been found, it is solved for temperature to give $T(\tau_\nu)$. Suppose this procedure is carried out at two frequencies ν_1 and ν_2 , giving $T(\tau_{\nu_1})$

Fig. 1.

Specific intensity I_ν varies with viewing angle θ for planar plasma whose temperature varies with optical depth τ_ν .

and $T(\tau_{\nu_2})$. Since $d\tau = \kappa \rho dx$ (where κ , ρ , and x denote opacity, density, and normal coordinate, respectively), we see that

$$\left. \frac{dT/d\tau_{\nu_1}}{dT/d\tau_{\nu_2}} \right|_T = \frac{dT}{\kappa_{\nu_1} \rho dx} \cdot \frac{\kappa_{\nu_2} \rho dx}{dT} = \frac{\kappa_{\nu_2}}{\kappa_{\nu_1}} \bigg|_T \quad (2)$$

Observations at many frequencies within an interval give the relative shape of the opacity coefficient over the entire frequency interval, as well as its variation with temperature through the layer.

Ideally, the experimentally determined relative monochromatic opacity curves should be assigned to points in the (ρ, T) -plane. The limb-darkening technique necessarily furnishes the temperature values with which the curves are associated [as seen in Eq. (2)], but it provides no density diagnostics. One might resort to a numerical simulation of the plasma to calculate its density structure; the simulation must also reproduce the observed $T(\tau_\nu)$ structure, of course, and this would give greater assurance of its reliability. It would be preferable, however, to measure experimentally the run of density through the plasma, possibly by holographic interferometry, for example.

The limb-darkening technique alone does not measure the absolute value of the opacity. In combination with the Zel'dovich-Raiser technique or the technique of Larson et al., it might do so. In fact, any means of measuring dT/dx independently would allow an absolute measurement of the

product $\kappa_r \rho$, since $\kappa_r \rho = (dT/dx)/(dT/d\tau_r)$. But relative opacities themselves are useful for comparison to theoretical calculations. For example, the relative height of a photoionization edge and its temperature dependence may be determined from limb-darkening observations in spectral bands on either side of the edge. Also, relative opacity data can reveal line merging and bumps or dips due to configuration interaction or other mechanisms just below photoionization edges.

The limb-darkening technique depends on the existence of local thermodynamic equilibrium (LTE) in the plasma under observation. If LTE exists, then the source function in Eq. (1) is a Planckian and the opacity κ has its LTE value. Laser-produced plasmas are notorious for being driven far from equilibrium by the energy-deposition processes, so careful consideration must be given to establishing LTE. One possibility is to use the laser to generate a disturbance, which in turn heats the plasma under observation in a manner that is closer to equilibrium than direct laser heating. Another possibility is to use a very brief laser pulse such that LTE is established after the pulse has ended but while the plasma is still hot. Ideally, the existence of LTE should be verified by both experimental diagnostics and theoretical considerations of the rates of microscopic processes; neither approach alone is infallible.

III. THE LIMB-DARKENING TECHNIQUE USING A LASER ENERGY SOURCE

The powerful pulsed lasers used in fusion research are promising tools for the opacity experimenter. Their primary virtue is the ability to deliver energy into a very small volume in an extremely short period of time. Large energy densities are thereby produced, and plasmas can be formed with temperatures of hundreds or thousands of electron volts and densities near solid density. Peculiar problems are associated with the use of a laser to produce a plasma, and we shall now discuss some of them as they relate to opacity experiments.

The interaction of the laser beam with its target is complex, with many different physical processes occurring simultaneously. One result of this interaction is the production of suprathermal electrons that stream rapidly through the target and preheat it.

Such preheating is a problem in the fusion program where it disturbs the target pellet in advance of the compression wave. In an opacity experiment, however, it need not be a problem, because the purpose of the laser is simply to produce a heated region; whether the region is heated by suprathermal particles, a shock wave, or a combination of several mechanisms is of secondary interest.

A second difficulty is the generation of contaminating nonthermal photons during the laser pulse. Among several possible sources are harmonics of the fundamental laser frequency produced in the target, and fluorescence,⁴ bremsstrahlung, transition radiation, and plasmons^{5,6} generated by suprathermal electrons (only fluorescence and bremsstrahlung are problems in the x-ray band). To guard against these contaminants, time discrimination of observed signals can be used to separate the possibly contaminated emission, which occurs during the laser pulse, from the thermal emission, which occurs after the pulse ends. Also, the geometry of the observing apparatus can be designed to minimize responsiveness to scattered nonthermal emission.

A third problem is the nonuniform distribution of energy across the laser beam. This distribution is sometimes idealized as being Gaussian, but can be chaotic, at least in the outer edges of the focused laser spot, because of the excitation of various modes in the laser oscillator; however, most of the energy is concentrated in a central peak whose intensity varies smoothly from center to edge. This nonuniform intensity of the laser beam will produce a nonuniformly heated plasma. Detailed account of this nonuniformity must be made by performing spatially resolved observations of the plasma. It is important that the size scale of the intensity variation be much greater than the photon mean free path ℓ in the plasma at the frequency of observation; otherwise, the ray along which the integral in Eq. (1) is performed will pass through regions of differing laser heating, hence differing $T(\tau_r)$, thus vitiating the technique. This requirement is easy to fulfill at optical frequencies, where ℓ in metals is probably very small,⁷ of order 10^{-5} cm; this is to be compared with intensity variation scales of 10^{-3} to 10^{-2} cm for the LASL L-Division Nd:glass laser, whose focused spot size is $\sim 150 \mu\text{m}$. On the other hand, at x-ray frequencies, ℓ may be as large as $\sim 10^{-3}$ cm in hot, high-Z material. Thus, for x-ray

observations more powerful lasers with larger focused spot sizes may be necessary, depending on the material under study. It is possible that enough smoothing may occur as the deposited laser energy is transported throughout the target to give a sufficiently uniform distribution of heating to the resultant plasma. One example of equilibrating the laser energy is apparent in the targets of Larson et al.¹

A fourth consideration is the geometry of the radiating plasma layer. Two possible geometrical configurations are a planar plasma and a spherically symmetric plasma. In either case, care must be taken that the desired geometry is actually produced and maintained until the observation is completed. In the case of a spherically symmetric plasma (for example, the sun), limb darkening is observed by viewing the plasma along a single line of sight; if the plasma is spatially resolved, then all viewing angles are present, ranging from normal at the center of the plasma image to extremely oblique at the edge (the "limb") of the image. A spherically symmetric plasma can be produced by a laser beam after the manner of Larson et al.¹ So long as the plasma expands radially, the spherical symmetry is preserved, and limb-darkening observations can be made over a relatively long interval.

A planar plasma can be produced by focusing the laser beam on a thin metal foil target. If the foil is much thinner than the lateral extent of the laser spot, a planar plasma will exist for a while after the foil is heated. Once the plasma has expanded by an amount comparable to the lateral extent of the laser spot, however, it is no longer planar. For a spot of $\sim 10^{-2}$ -cm extent, a foil of thickness $\sim 10^{-3}$ cm should allow a planar plasma to be formed initially. Measurements by Veaser and Solem⁸ indicate free surface velocities of $\sim 10^6$ cm s⁻¹ at the back side (i.e., the side away from the laser) of the foil, when heated by a 30-J laser pulse of 300-ps FWHM. Thus, the plasma becomes nonplanar after ~ 10 ns, as viewed from the back of the foil. The limb-darkening data must therefore be obtained during an interval of < 10 ns after the pulse.

Consideration of all these peculiarities of the laser environment leads to the conclusion that a spherically symmetric plasma is better suited for the limb-darkening experiment than a planar plasma. Such a spherical plasma could be produced from a target like that proposed by Larson et al.,¹

but with additional design features to suppress suprathermal preheat, to smooth out laser beam nonuniformities, and to suppress the effects of the complicated front-side physics (i.e., non-LTE phenomena, instabilities, and density profile modification).⁹ Although a spherical geometry is superior for efficiency of data gathering and preservation during plasma expansion, we have chosen a planar geometry for our study because a spherical plasma necessarily contains more matter than a planar plasma of similar extent, and a more powerful laser pulse is therefore necessary to heat it. Also, the spherical plasma must be uniformly heated over its surface if the limb-darkening information is to be obtained in a single line of sight, and this could be difficult to verify. Furthermore, the thin foil targets used to produce a planar plasma are inexpensive and easily procured, compared to the more elaborate targets necessary to produce a spherical plasma.

We observe the back side of the foil to avoid the problems associated with front-side physics, although we therefore attain lower plasma temperatures. The observations are made in the optical band during the feasibility study to avoid the expense and difficulty of x-ray observations, although ultimately, it is x-ray opacities that are of greatest interest; if limb darkening can be observed at optical frequencies in a dense metal plasma, we can better justify an effort to observe limb darkening at x-ray frequencies.

Because observations are to be made in the optical band during the feasibility study, a fifth difficulty is the possibility that the plasma has a variable refractive index¹⁰ at the observation frequency. This problem does not arise for x-ray observations because x-ray photons have frequencies greatly exceeding the plasma frequency of even extremely dense, highly ionized plasmas. But for optical observations, the refractive index of the plasma must vary slowly over a photon mean free path l . This condition is likely to be satisfied everywhere except near the critical density for the frequency of observation. Even if the initial plasma is overdense, a small amount of expansion should bury the critical surface deeply enough to be invisible, since l is so small. Observations should be made at the highest possible frequency to avoid approaching the plasma frequency in the visible layers.

IV. THE EXPECTED MAGNITUDE OF THE LIMB-DARKENING EFFECT

In this section, we present some analytic and numerical calculations to demonstrate the magnitude of the limb darkening effect in various cases. The analytic calculations show that the angular distribution of emitted radiation can be very strongly affected by the shape of the target temperature profile. The limb-darkening effect for laser targets may be much larger at angles near normal than it is in the case of visible solar radiation.

The numerical results are for x-ray photons, and we use theoretical opacity data to predict limb darkening.

A. Analytical Results for $h\nu \gg kT$

The attractiveness of the limb-darkening technique of opacity measurement depends on the expected monotonic increase of temperature with increasing optical depth in the outer several photon-mean-free-paths of the source. If the source heating is attributed to a shock wave as discussed by Zel'dovich and Raizer,² the free surface temperature is zero and increases inward. If some other heating mechanism is present, the free surface temperature may be greater than zero but is still expected to increase inward from some value that is low compared to that deep in the source. Thus in the outer layers of the source, kT is small compared to $h\nu$ if the frequency of observed radiation ν is appropriately chosen, and the Planckian source function in Eq. (1) is well approximated by

$$B_\nu(\tau_\nu) = (2h\nu^3/c^2) \exp(-h\nu/kT) \quad (3)$$

where T is a function of τ_ν . When the opacity is high, this approximation is valid over a range of τ_ν much greater than unity. Thus, $B_\nu(\tau_\nu)$ may be a very rapidly increasing function of τ_ν in the observable part of the source. Its increase may be so rapid as to outweigh the attenuation caused by increasing optical depth over a large range. The result is that the main contribution to the emergent intensity comes from optical depths greater than unity.

With the above expression substituted for B_ν in Eq. (1) to give $I_\nu(\mu) = (2h\nu^3/c^2) \int_0^\infty \exp(-h\nu/kT -$

$\tau_\nu/\mu) d\tau_\nu/\mu$, it is easy to show that the integrand has a maximum at the optical depth where

$$T^2(\tau_\nu) = \frac{h\nu\mu}{k} \frac{\partial T}{\partial \tau_\nu} \quad (4)$$

by differentiating the integrand with respect to τ_ν . Most of the contribution to the integral comes from the neighborhood of this maximum, and if the temperature $T(\tau_\nu)$ at the maximum is such that Eq. (3) is still a good approximation, its use is justified. An increase in opacity implies a decrease in $\partial T/\partial \tau_\nu$ for fixed ρ and $\partial T/\partial x$. Thus, Eq. (4) shows that an increase in opacity causes the maximum in the integrand to occur in a lower temperature region. Thus, this approximation is best when the opacity is high.

These ideas, as well as the magnitude of the limb darkening, may be illustrated by evaluating the integral in Eq. (1) with a simple model for the temperature variation with τ_ν . Let

$$T(\tau_\nu) = T_0 + a \tau_\nu \quad (5)$$

that is, T_0 is the free surface temperature and a is a constant temperature gradient. It is convenient to introduce a brightness temperature $T_\nu(\mu)$, defined by the equation $I_\nu(\mu) \equiv (2h\nu^3/c^2) \exp[-h\nu/kT_\nu(\mu)]$. Then from Eqs. (1) and (3)

$$\exp[-h\nu/kT_\nu(\mu)] = \int_0^\infty \exp(-h\nu/kT - \tau_\nu/\mu) \tau_\nu/\mu \quad (6)$$

If the temperature profile model specified by Eq. (5) is now substituted for T in the integrand and the integration variable is changed to

$$x = (\tau_\nu/\mu + T_0/a\mu)/\sqrt{h\nu/ka\mu} \quad (7)$$

the result is

$$\exp[-h\nu/kT_\nu(\mu)] = \sqrt{h\nu/ka\mu} \exp(T_0/a\mu - 2\sqrt{h\nu/ka\mu}) \times \int_{T_0/\sqrt{h\nu/ka\mu}}^\infty \exp[\sqrt{h\nu/ka\mu}(2-x-1/x)] dx \quad (8)$$

The integral in Eq. (8) can be evaluated in various ways; when $T_0 = 0$ it can be expressed in terms of a Hankel function of the first kind with an imaginary argument. The integrand has a maximum at $x = 1$, at which point

$$T(\tau_V) = T_a \equiv \sqrt{h\nu a \mu / k} \quad (9)$$

as can be seen from Eq. (7). This is also in agreement with Eq. (4), which is model independent. If the lower limit of the integral is less than 1, i.e., if $T_0 < T_a$, the maximum of the integrand is included in the range of integration, and if $\sqrt{h\nu / k a \mu} = T_a / a \mu$ is large compared to 1, the integral is then approximately equal to $(\pi^2 k a \mu / h\nu)^{1/4} = (\pi a \mu / T_a)^{1/2}$. This result was obtained by expanding the exponent in the integrand in a Taylor series about its maximum and dropping terms higher than second order. Thus we obtain

$$\exp[-h\nu / k T_V(\mu)] = (\pi^2 h\nu / k a \mu)^{1/4} \exp(T_0 / a \mu - 2 \sqrt{h\nu / k a \mu}) \quad (10)$$

This can be solved for the brightness temperature and T_a , from Eq. (9), can be introduced to give

$$T_V(\mu) = \frac{T_a}{2 - \frac{1}{2} \frac{a \mu}{T_a} \ln \left(\pi \frac{T_a}{a \mu} \right) - \frac{T_0}{T_a}} \quad (11)$$

The two conditions $T_a \gg T_0$ and $T_a \gg a \mu$ imply that the last two terms in the denominator are probably negligible so the result obtained is

$$T_V(\mu) \approx \frac{1}{2} T_a = \frac{1}{2} \sqrt{h\nu a \mu / k} \quad (12)$$

It is reasonable that the brightness temperature is a fraction of the temperature at the maximum of the integral. This result could be used with Eq. (4) to determine the order of magnitude of $\partial T / \partial \tau$, at a temperature equal to a few times the brightness temperature without measuring limb darkening.

In summary, the limb darkening occurs because a hot region of the source is viewed through an outer,

cooler region. The temperature of the hot region, given by $T \sim T_a \equiv \sqrt{h\nu a \mu / k}$ or by Eq. (4), decreases with decreasing μ . When μ is decreased so that T_a decreases to $\sim T_0$, the limb darkening will stop and the brightness temperature will be $\sim T_0$ for smaller μ . The deeper regions of the target at temperatures large compared to T_a cannot be seen because of the high opacity.

The possible magnitude of the limb darkening is shown in Eq. (10), which is plotted as a function of μ in Fig. 2 for some different values of T_0/a . For these plots, the temperature T_a at the maximum of the integral was such that $k T_a \equiv \sqrt{h\nu k a \mu} = \sqrt{\mu} \times 0.7$ eV, and $h\nu = 2.95$ eV, corresponding approximately to the maximum sensitivity of an S-20 photocathode. This implies a temperature change over unit optical depth given by $k a = 0.166$ eV.

Plots were made for three surface temperatures given by $T_0/a = 0, 2$, and 3. The temperature T_a at the maximum of the integral, defined by Eq. (9), decreases with decreasing μ , and when it approaches

Fig. 2.
Limb-darkening functions determined analytically by Eq. (10). The vertical axis shows specific intensity normalized to its value at $\mu = 1$, where $\mu = \cos \theta$. The parameters T_0 and a characterize the assumed temperature model: $T(\tau_r) = T_0 + a \tau_r$. Dashed lines show where Eq. (10) is no longer valid.

T_0 , Eq. (10) is no longer valid. The dashed portions of the curves indicate where T_a approaches T_0 . If T_a becomes less than T_0 , i.e., the maximum of the integrand lies below the range of integration, the integrand is a decreasing function of τ_a , dominated by the factor $\exp(-\tau_a/\mu)$ throughout the range of integration. If this factor varies rapidly enough with increasing optical depth, the integral may be evaluated by the Eddington-Barbier method, which is discussed by Kourganoff.¹¹ This method has been applied extensively in astrophysical problems. The Eddington-Barbier relation of most direct application to this problem is

$$I_\nu(\mu) \approx B_\nu(\tau_\nu = \mu) \quad .$$

Because $\mu \leq 1$, this relation implies that the brightness temperature $T_b(\mu)$ will be equal to the source temperature at some optical depth less than unity.

In contrast, for the simple model discussed above, the brightness temperature is given by Eq. (11), or by $T_b(\mu) \approx 1/2 T_a$, where T_a is the temperature at the maximum of the integrand, which occurs at optical depth.

$$\tau_a = \frac{T_a - T_0}{a} = \sqrt{\hbar\nu/kT_a} - T_0/a \quad .$$

For the example on which Fig. 2 is based,

$$\tau_a \approx 4.22 \sqrt{\nu} - T_0/a \quad ,$$

which is greater than unity for $\mu \approx 1$.

This discussion shows how the Laplace transform, Eq. (1), can be inverted and a target temperature profile determined by making use of knowledge of the nature of $B_\nu(\tau_\nu)$. If the target surface is cooler than its interior, the radiation intensity will vary strongly with direction for angles near the target normal if the frequency of observed radiation is high enough. Many of the restrictions on this problem are necessary only to simplify the discussion; they are not necessary in general.

B. Numerical Predictions of Limb Darkening

We have performed detailed numerical calculations using tabulated theoretical opacity data to

predict the magnitude of the limb-darkening effect at x-ray energies in metal plasmas. We used the 1D radiation-diffusion Lagrangian-hydrodynamics code FFF to calculate a self-consistent plasma structure. Then the multigroup transport code SNAPD was used to calculate the emergent intensity from the plasma at various viewing angles. At this stage of the study we are not concerned with simulating the details of the laser target interaction; our intent is merely to demonstrate that limb darkening is observable in principle at temperatures and densities likely to occur in laser-produced plasmas. Thus FFF is sufficient to calculate the plasma structure. Later calculations can be done with LASNEX to simulate the actual laser-heated target as well as possible.

The results from one such calculation are shown in Figs. 3-5. The calculation began with a plane aluminum foil 50 μm thick. There were 100 mesh zones through the aluminum; the exterior zones were 0.1 μm thick. The foil was given a uniform temperature of 150 eV and allowed to expand adiabatically.

Figures 3 and 4 show the temperature and density structure, respectively, of the foil at 0.5 ns after the start of the calculation. The foil has expanded symmetrically about its center at $x = 0.0025$ cm. The peak temperature is ~ 100 eV; the peak density is $\sim 1.2 \text{ g cm}^{-3}$. Figure 5 shows the limb-darkening

Fig. 3.

Calculated temperature structure of planar aluminum foil at $t = 0.5$ ns. Foil was initially 5×10^{-3} cm thick, with uniform temperature of 150 eV throughout.

Fig. 4.

Calculated density structure of planar aluminum foil at $t = 0.5$ ns. See caption to Fig. 3.

functions at various photon energies. The open circles are the calculated intensities, normalized to $I_\nu(\mu=1)$; results are shown for viewing angles of 0° , 15° , 30° , 45° , 60° , and 75° . The solid curves are quadratics fit by least squares to the open circles.

The photon energies shown actually represent the centers of the energy bins of nonzero width for which the intensities are computed. The bins centered at 1.75 and 1.25 keV are 0.5 keV wide; the bins centered at 0.8, 0.45, 0.2, and 0.05 keV are, respectively, 0.4, 0.3, 0.2, and 0.1 keV wide.

Figure 5 shows that limb darkening can be quite strong, even at viewing angles close to normal, for 0.45-, 0.80-, and 1.75-keV photons. For 0.05- and 0.20-keV photons the limb darkening is not so strong, both because dT/dr is shallower over the visible region of the plasma (notice the small temperature slope in the outer regions of the plasma in Fig. 3) and because $h\nu \sim kT$ so the source function does not vary rapidly with temperature. For 1.25-keV photons we actually see limb brightening. This

Fig. 5.

Calculated limb-darkening functions for planar aluminum foil whose structure is shown in Figs. 3 and 4. Open circles are the calculated results, shown for $\mu = 1.00, 0.966, 0.866, 0.707, 0.500$, and 0.259 , corresponding to $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ$, and 75° , respectively. Solid curves are best fit quadratics. Results are shown for six photon energies.

is because the foil is not optically thick to these photons, which are just below the aluminum K-edge.

The quadratic fits are excellent for all photon energies except 1.25 keV. To find the inverse Laplace transform of a quadratic is trivial; it is simply another quadratic. Care is required in replacing the individual points with a fitted curve because one is adding information that is not actually present in the set of points.

These results show that limb darkening should be observable in hot dense metal plasmas. One can go further and actually simulate the entire limb-darkening experiment; limb-darkening curves like those in Fig. 5 can be calculated, inverted to find $B_\nu(\tau_\nu)$, solved for $T(\tau_\nu)$, and ratios taken of $dT/d\tau$, at various frequencies to find the opacity ratios, as described in Sec. II. This way, we hope to recover the theoretical opacity data used as input to the simulation. We have done this for several cases, and the results will be presented in a separate report.¹²

V. THE FEASIBILITY STUDY

The feasibility study is an effort to demonstrate that the limb-darkening technique can be successfully applied to laser-produced plasmas. During the study we hope (1) to observe limb darkening at optical frequencies in an aluminum plasma; (2) to determine $T(\tau)$ in a single very broad optical band using an absolutely calibrated observing system; (3) to measure the relative opacity of the aluminum plasma in broad red and blue filter bands, and its dependence on the temperature in the plasma. Attainment of these goals will give us confidence that the technique can be applied at x-ray energies.

We are using the LASL L-Division Nd:glass laser, which has been described elsewhere.^{13,14} The system consists of a mode-locked oscillator that produces a short pulse, and a combination of YAG, rod, and disk amplifiers that strengthens the pulse. We use one of four available separate beam lines. Operation and maintenance of the laser is handled by L Division.

The laser produces a pulse of 1.06- μm -wavelength infrared radiation. The pulse has nominally a Gaussian time profile with a 300-ps FWHM. The total energy in the pulse varies from shot to shot; it may

be as much as 50 J, although we typically use pulses containing between 20 and 30 J. The pulse is focused through an $f/3.5$ lens into a spot of roughly triangular shape with sides 100 to 150 μm long. Thus, typical power densities are near $10^{16} \text{ W cm}^{-2}$. The intensity peaks toward the middle of the spot and falls off toward the edges.

The aluminum foil targets are mounted inside a vacuum chamber $\sim 1\text{-m}$ diam. The chamber is evacuated to a pressure of 1 torr or less to avoid laser breakdown of the air. Questar telescopes coupled to infrared Vidicon cameras and mounted on the chamber give a view of the target; their output is displayed on TV monitors, which allows us to place the targets accurately and to aim and focus the laser beam.

The target mount inside the chamber includes a wheel that can be rotated from outside the chamber and can hold three targets; thus, we can shoot several times without opening the chamber. The mount is oriented so that the axis of the laser beam makes an angle of 15° with the target normal; this enhances the absorption of laser energy, according to the resonance absorption model. The mounting wheel also serves as a light baffle, to prevent scattered light emitted by the front-side plasma from reaching the observing apparatus at the back of the foil.

Most of our work has been done with foils whose thickness is $1/2 \text{ mil} = 12.5 \mu\text{m}$, although some shots have been done with $3/4\text{-}$ and 1-mil foils. Therefore, the laser spot size is much greater than the foil thickness, so that the initial plasma is planar.

Since we have a planar plasma, we must view it along several lines of sight. A straightforward way is to construct several identical observing systems, each with a different viewing angle with respect to the plasma normal, as shown in Fig. 6. Streak cameras provide spatial and temporal resolution, which are required to allow for beam nonuniformity and to discriminate against unwanted light. TV cameras and videotape recorders are convenient and sensitive data recording media.

We have constructed one such line of sight (Fig. 7) using a Hadland Imacon 700 streak camera and a Cohu 4400 low-light-level SIT TV camera, both provided by the Plasma Diagnostics Group (CTR-8) of LASL's CTR Division. An IVC videotape recorder, a videodisk unit, and two small TV monitors

Fig. 6.

A simple scheme for detecting the existence of limb darkening in a planar plasma.

were provided by the Atmospheric Sciences and Optical Physics Group (J-10) of LASL's J Division.

The Imacon is operated at a streak rate of 1 ns/mm. Its photocathode has an S-20 response. It is triggered by a portion of the laser oscillator pulse that is switched out before passing through the main laser amplifiers. The trigger pulse passes through a single small YAG amplifier and then to a trigger photodiode that detects it and generates an electrical pulse, which finally triggers the Imacon.

The TV camera has been modified by EG&G, Inc., to allow manual gain adjustment; we operate it at maximum gain. The TV camera is about ten times more sensitive than Polaroid ASA 10000 film, which we have also tried as a recording medium using a Hadland film back with lens coupling. The TV camera is allowed to free-run, without synchronization between the laser pulse and the read-out of TV frames. The pulse may sometimes occur in the middle of a read-out. This is inconvenient but not

Fig. 7.

Experimental layout for observing back side of laser target. See text for discussion.

Fig. 8.
TV monitor photograph of a TV frame of videotape data.

critical; in the future we shall provide for externally blanking the read-out until after the pulse.

The system is sensitive enough to magnify the image of the backside plasma by a factor of 10 and still record a usable signal. If the plasma is $150\text{ }\mu\text{m}$ across, its image is about 1.5 mm across at the Imacon photocathode. The dynamic spatial resolution of the Imacon is probably from 3 to 5 line pairs/mm,¹⁵ so we have from 4 to 8 resolution elements across the plasma. This allows us to measure the center-to-edge brightness variation due to laser beam nonuniformity.

Magnification is achieved by placing a small $f/0.9$ lens of approximately 3-cm focal length inside the vacuum chamber so that the plasma is at its focus.

The collimated light exits the chamber and is refocused onto the Imacon photocathode by a large lens of 30.7-cm focal length. Extensive light baffling

Fig. 9.
Contour plot of the TV frame shown in Fig. 8. Contours of constant video amplitude are shown.

covers the entire line of sight to exclude scattered front-side light. The exterior mirrors can be easily removed to give us a 15° viewing angle instead of the 30° angle shown.

The videotape recording of the streak camera data is digitized at the EG&G Video Editor facility and then written out on magnetic binary tape for further reduction. Figure 8 shows a TV monitor photograph of a typical streak camera record. The time axis increases downward. The spatial axis runs horizontally. The rectangle at the top of the frame is the static image of the photocathode recorded before the Imacon is triggered.

The streak is characterized by a bright flash at its origin followed by a fainter tail. The variation in brightness across the streak indicates nonuniform heating of the plasma. The laser shot contained 26 J, and the target was $12.5\text{-}\mu\text{m}$ -thick aluminum foil.

The viewing angle was 30° . The exposure was made in white light; the observational band is determined by the S-20 photocathode response and the KG-3 laser-light filters protecting the Imacon. The streak is about $200\text{ }\mu\text{m}$ across and runs for about 10 ns.

More detail is apparent in the contour plot of the same TV frame shown in Fig. 9. This was produced from the digitized data using the TV reduction codes developed by Gow.¹⁸ The contours show levels of constant video amplitude. The initial flash has a roughly circular core about $100\text{ }\mu\text{m}$ across. The shape of the core is apparent because we did not use a slit to reduce spatial smearing along the time axis. Thus, both time and space information appear along the vertical axis, as indicated by the vertical arrow showing the time and space scale. The total field of view is about $600\text{ }\mu\text{m}$ by $800\text{ }\mu\text{m}$. In future shots we may use a slit to reduce the field of view in one dimension to permit higher time resolution. Alternatively, damagnifying the plasma image also permits higher time resolution at the expense of space resolution.

To observe limb darkening, we need a relative intensity calibration of the observing system, which can be done using a xenon flashlamp as a source and introducing various degrees of neutral-density attenuation. To measure $T(\tau)$ we need an absolute calibration, for which the J-10 calibrated xenon flasher may be useful.

To begin measuring limb darkening, one might assemble a second identical observing system as shown in Fig. 6. A more elegant procedure is to construct an optical device that allows several different views of the plasma to be recorded on a single streak camera.

A schematic diagram of such an arrangement is shown in Fig. 10. This arrangement permits viewing the target simultaneously along the target normal and at an angle of 30° to the normal. Note that both views make the same angle with the laser beam and therefore will not detect intensity variations owing to an angular correlation between the laser beam and the emitted radiation. The two optical paths are identical. The light acceptance cone is about $f/2$, which will give sufficient resolution of the direction of the emitted radiation and will collect enough light to measure with the Imacon/TV system.

This kind of arrangement, perhaps with an additional optical path to permit viewing the target at

Fig. 10.

A possible design of an optical arrangement to permit viewing two angles along a single line of sight.

an angle of $\sim 60^\circ$ from the normal, should demonstrate the existence of the limb-darkening effect. This effect occurs at angles considerably less than 90° from the normal, so there is no need to make observations at or near grazing angles, which would be more difficult.

We plan to construct such a device and use it to complete the feasibility study. Besides allowing more efficient data gathering, only one streak camera system requires calibration.

In summary, the feasibility study divides naturally into the three stages identified at the beginning of this section: (1) observation of limb darkening; (2) measurement of $T(\tau)$; and (3) determination of relative opacities. Each stage represents a significant milestone in its own right. We propose to report on each stage as it is completed.

ACKNOWLEDGMENTS

Without the contributions of a large number of people, our study would not have been possible. We would like to thank J. H. McNally, C. F. Keller, and D. D. Eilers for their interest and support; G. E. Barasch, S. N. Stone, M. G. Duran, and W. D. Gould for discussions, assistance, and the loan of

cameras, optics, and electronics; A. L. Merts, W. F. Huebner, D. Giovanielli, A. Englehardt, R. L. Carman, A. R. Larson, E. L. Lindman, and R. B. Schultz for discussions concerning plasmas and lasers; C. J. Stump, J. P. Jekowski, and C. E. Gow for discussions concerning TV systems and data reduction; F. C. Jahoda, K. A. Klare, and L. D. Caudill for the loan of and help with TV and streak cameras; B. Brixner, J. S. Ladish, and R. Carlson for providing optical and electronic apparatus; R. H. Robertson and I. V. Johnson for operating and maintaining the laser; and R. F. Martinez, H. V. Dehaven, K. E. Bartram, P. J. Kruse, and B. Carpenter for advice and assistance with instrumentation.

REFERENCES

1. A. R. Larson, G. R. Spillman, J. F. Barnes, J. F. Hafer, and W. Fickett, Los Alamos Scientific Laboratory, private communication, July 1976.
2. Ya. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena* (Academic Press, New York, 1967), Vol. 2, pp. 773-778.
3. A. K. Pierce and J. H. Waddell, "Analysis of Limb Darkening Observations," *R.A.S. Memoirs* 68, 89-112 (1961).
4. A. Zigler, H. Zmora, and J. L. Schwob, "The Origin of $K\alpha$ Radiation in Laser-Produced Aluminum Plasma," *Phys. Lett.* 63A, 275-278 (1977).
5. H. Boersch and G. Sauerbrey, "Transition Radiation, Bremsstrahlung, and Plasma Radiation," in *Optical Properties and Electronic Structure of Metals and Alloys*, F. Abelès, Ed. (North-Holland Publishing Co., Amsterdam, 1966), pp. 386-398.
6. W. Steinmann, "Optical Plasma Resonances in Solids," *Phys. Status Solidi* 28, 437-462 (1968).
7. Ya. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena* (Academic Press, New York, 1967), Vol. 2, p. 771.
8. L. R. Veaser and J. C. Solem, "Studies of Laser-Driven Shock Waves in Aluminum," *Phys. Rev. Lett.* 40, 1391-1394 (1978).
9. B. Bezzerides, D. F. Dubois, D. W. Forslund, J. M. Kindel, K. Lee, and E. L. Lindman, "Recent Developments in Understanding the Physics of Laser Produced Plasmas," *IAEA Symp. Plasma Physics and Controlled Nuclear Fusion Research*, Berchtesgaden, Germany, October 6-13, 1976.
10. G. Bekefi, *Radiation Processes in Plasmas* (John Wiley and Sons, Inc., New York, 1966), p. 36 ff.
11. V. Kourganoff, *Basic Methods in Transfer Problems* (Dover Publications, Inc., New York, 1963), pp. 72-85.
12. N. Hoffman, "Numerical Simulations of the Limb-Darkening Opacity Experiment," Los Alamos Scientific Laboratory report in preparation (1978).
13. D. H. Gill, R. C. Hyer, P. N. Mace, G. H. McCall, J. McLeod, J. E. Perry, Jr., and B. E. Watt, "Large Nd:Glass Laser System for Plasma Production," *Bull. Am. Phys. Soc.* 17, 1052 (1972).
14. D. H. Gill, R. C. Hyer, P. N. Mace, J. McLeod, J. E. Perry, C. H. Reed, R. H. Robertson, N. J. Terrell, and B. E. Watt, "A High-Power Nd:Glass Laser System for Fusion Research," *IEEE J. Quantum Electronics*, Vol. QE-10, 688-689 (1974).
15. G. E. Barasch and M. G. Duran, Los Alamos Scientific Laboratory, personal communication, January 1978.
16. C. E. Gow, EG&G, Inc., personal communication, January 1978.