Beta optimization in the context of reactor relevant tokamaks.

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Abstract

In a reactor relevant tokamak the appropriate definition of β , the ratio of the particle and magnetic field pressures, is $\beta^* \equiv [2 \langle p^2 \rangle / B^2]$, which exceeds the conventional definition by a factor dependent on the pressure peaking factor, PPF . A simple scaling is obtained which relates the two definitions, $\beta^* / \beta \simeq 0.9 + 0.15 PPF$. Stability properties are determined in terms of β^* in a circular and dec-shaped tokamak.

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I. Introduction.

Beta optimization studies for tokamaks and comparison with experimental results have resulted in a widely accepted, simple expression for the maximum achievable beta value stable to ideal MHD instabilities.^{1,2} This is commonly referred to as the Troyon-Gruber-Sykes (TGS) beta limit and is given by $\beta_{\max} = gI/aB$, where $\beta \equiv 2 \langle p \rangle / B^2$ and is expressed as a percentage value; $\langle \cdot \rangle$ implies a volume average, I is the plasma current in megamps, a is the minor radius in meters and B is the vacuum toroidal field at the geometric center measured in Tesla. The dimensionless coefficient g ranges from 2.5 to 3.5 depending on the assumptions of the equilibrium model and the relative importance given to different ideal MHD instabilities. Extensive studies³ have since shown that these limits are not absolute in the theoretical sense, as better profile optimization schemes permit stable equilibria with higher values of β . However, these optimized profiles have not yet been achieved experimentally and the empirical TGS limit is, in fact, obeyed. As a consequence, design of reactor relevant tokamaks is based on this simple formula, with the value of g ranging from 2 to 3, depending on the conservatism of the design team. However, the relevant definition for β in a reactor context is⁴ $\beta^* \equiv 2 \langle p^2 \rangle^{1/2} / B^2$, which is a better measure of the neutron production rate. The ratio, β^*/β is greater than unity and depends on the 'peakedness' of the pressure profile. In this note we address the enhancement of $g^* (\equiv \beta^*/(I/aB))$ over g for different plasma profiles and geometries.

The following procedure was used in this study. The plasma geometry is defined through the aspect ratio, R/a , and the shape of the plasma cross-section. Pressure and safety factor, q , profiles are chosen. Several flux-conserving equilibria are generated with increasing values of pressure and examined for stability to high- n ballooning modes as well as to the $n = 1$ external kink mode with the conducting wall set at infinity. The equilibrium with the highest pressure which is stable to both instabilities is determined. Its β and β^* , and consequently g and g^* , are obtained. This procedure is repeated for different profiles, and two different plasma shapes.

II. Equilibrium model.

It is important to note that rather than optimize the profiles to achieve the highest β , we have chosen profiles which are representative of those considered to be experimentally achievable. In doing this we are guided by the observation that experimental data analysis using transport codes indicates that the density profile has the form, $n = n_0(1 - r^2/a^2)^{\alpha_n}$ where r is an equivalent cylindrical radius and a is the average plasma minor radius. The exponent α_n has the value, $0 \leq \alpha_n \leq 0.75$ for discharges with broad profiles and $1.5 \leq \alpha_n \leq 3.0$ for those with peaked profiles. Correspondingly, the temperature profile is given by $T = T_0(1 - r^2/a^2)^{\alpha_T}$, with $0.75 \leq \alpha_T \leq 1.75$. Combining these two, we define the pressure profile as $p = p_0(1 - \Psi)^{\alpha_p}$, where we have replaced r^2/a^2 by a poloidal flux label, Ψ , normalized to be zero at the plasma axis and unity at the plasma edge and introduced $\alpha_p = \alpha_n + \alpha_T$. We can then define a broad pressure profile to have $0.75 \leq \alpha_p \leq 2.5$ and a peaked profile to have $2.25 \leq \alpha_p \leq 4.75$. This represents a rather wide range of profiles and there is clearly a region of overlap. We also introduce a new parameter describing the equilibrium pressure profile, the pressure peaking factor PPF , as the ratio of the central β to the volume averaged β ; $PPF = \beta_0 / \langle \beta \rangle$. We believe that the PPF is a general parameter which is best suited to describe the pressure profile since it is not associated with a specific functional form of $p(\Psi)$, and is readily available from both experiment and theoretical simulations. It should be noted that fixing the functional form of the pressure profile, for example by fixing α_p , does not uniquely determine the PPF , which also depends on the geometry of the poloidal flux surfaces, which in turn is dependent on the plasma shape as well as the details of the current distribution. In order to complete the specification of the plasma profiles we must specify a q -profile or g -profile [$B_\phi \equiv B_0 R g(\Psi)/r$]. We have chosen to initially specify $g(\Psi)$ and generate a zero- β equilibrium with a desired q_{axis} and q_{edge} by adjusting the parameters which define the function $g(\Psi)$. The resulting q -profile is then used to generate a sequence of flux-conserving equilibria with increasing values of β . This procedure has the advantage of controlling the q -profile while ensuring a vanishing edge current density, at least at low β .

III. Results.

The main object of this paper is to demonstrate the difference between β and β^* for a variety of profiles and to make a preliminary assessment of the impact of using β^* in defining the beta limit. Therefore, in the interest of brevity we select three values of α_p for this study, i.e., $\alpha_p = 1.4, 2$, and 3 . These represent a clearly broad, intermediate and peaked pressure profile, respectively. Three q -profiles are chosen with $q_{axis} = 1.03$ and $q_{edge} = 2.1, 5.1$, and 8.3 . We consider these nine sets of profiles with two different plasma shapes. These are chosen to have the same aspect-ratio, $R/a = 5$, with a circular and dee-shaped cross-section. The latter has an elongation of 1.6 and triangularity of 0.4 . A representative set of pressure and toroidal current density profiles for a dee-shaped cross section is shown in Fig. 1. Figure 2 shows the relationship between β and β^* . Here we plot the ratio β^*/β as a function of PPF for all of the profile choices for the two plasma shapes. It is interesting to note that there is a fairly good linear relationship between this ratio and PPF , which is approximately described by $\beta^*/\beta \simeq 0.9 + 0.15PPF$. We caution that this does not include an aspect-ratio effect. A quick assessment based on limited data indicates that the enhancement of β^* over β is slightly less at lower aspect-ratios. We note that even with modestly peaked profiles, $PPF \simeq 3$, we obtain a 30% enhancement in β^* over β . Profiles characteristic of the TFTR supershot regime, which have a $PPF \simeq 4.5$, yield a 50% enhancement.

The β^* -limits, defined as the maximum β^* value in % which is stable to all ideal MHD instabilities, are presented as a function of the pressure peaking factor, PPF , in Figs. 3 and 4, for the circular and dee-shaped cross-section, respectively. We have connected the points with constant q_{edge} to differentiate the current effects from the PPF effects. In both geometries we observe that the highest values of β^* are generally obtained at the lowest values of PPF , corresponding to the broadest profiles. In addition, we note that the β^* values for the dee-shape are generally larger than the values for the circle, because of its larger current carrying capacity. It is interesting to note that in the circular case there appears to be an optimal PPF which is not the lowest value. This is particularly clear in the high q_{edge} case.

The results can also be viewed from the perspective of the normalized current, I/aB . Figures 5 and 6 show this information for the circle and dee cross sections, respectively. Plotted are the maximum stable β^* values in % for differ-

ent values of I/aB . Even though these profiles were not optimized, the results reflect the ease with which the Troyon limit is achieved at higher currents for the dee-shape, while lower currents are favored in the circular cross-section. Thus, for $I/aB \sim 0.5$ the dee has a significantly higher $g^* \sim 4$ than the circle which has $g^* \sim 3$. In addition, the circular case has $q_{edge} \sim 2$ which is close to the hard kink limit while the dee-shape has $q_{edge} \sim 5$, which is more stable to kinks. These features are similar to those observed in scatter plots from experimental data.

IV. Discussion.

In this note we have introduced a fusion relevant definition of the Troyon-Gruber-Sykes scaling parameter, g^* , and correlated it with the peakedness of the pressure profile, PPF. We observe that g^* can be enhanced significantly over g and reach values up to 5 quite easily. A simple expression relating β^* and β is given by, $\beta^*/\beta \simeq 0.9 + 0.15PPF$. In terms of the plasma current we have noted that when q_{edge} is high (~ 8), the circular cross-section cases have the higher g^* (3.5 - 4.5). When the q_{edge} is low (~ 2), the dee-shaped cross-section, has the higher g^* (3.5 - 5.5). In terms of the pressure profile, we observe that for the circle the maximum stable β remains approximately constant up to $PPF \sim 5$ and then drops off rapidly. This, coincidentally, reflects the 'supershot' regime of TFTR. In the dee-shape, as the PPF is increased the maximum β decreases uniformly. We conclude with the observation that we can reasonably expect that the use of g^* will give a 30% enhancement of the TGS parameter, g , for moderately peaked profiles with $PPF \simeq 3$. This feature should be included in reactor design studies.

V. Acknowledgements

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Figure captions

Figure 1. Representative profiles used in this study. a) Pressure profiles across the mid-plane cross section with $q_{edge} = 5.1$, and b) J_ϕ for a fixed pressure with $\alpha_p = 2$, with $q_{edge} = 2.1, 5.1$, and 8.3 .

Figure 2. Relationship between β^* and β for all the cases studied.

Figure 3. The maximum stable β^* , in %, against all ideal modes in a circular cross section as a function of the pressure peaking factor PPF. The solid lines connect the points with constant q_{edge} .

Figure 4. The maximum stable β^* in %, against all ideal modes in a dee-shape cross section with $E = 1.6, \delta = 0.4$, as a function of the pressure peaking factor PPF. The solid lines connect the points with constant q_{edge} .

Figure 5. The maximum stable β^* in %, against all ideal modes in a circular cross-section as a function of the normalized current I/aB .

Figure 6. The maximum stable β^* in %, against all ideal modes in a dee-shape cross-section with $E = 1.6, \delta = 0.4$, as a function of the normalized current I/aB .

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6