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**KELVIN-HELMHOLTZ-LIKE INSTABILITY OF A SHEAR
LAYER SUBJECT TO FREE BOUNDARY CONDITIONS**

by

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ABSTRACT

For free boundary conditions a shear layer with linear velocity profile supports irrotational disturbances, one mode being unstable when its wavelength $\lambda > 2.619$ times the thickness h of the shear layer. For long wavelengths the dispersion relation of the mode approaches Kelvin-Helmholtz form.

The Kelvin-Helmholtz instability involves two counterstreaming fluids separated by a shear layer which is sometimes idealized as a vortex sheet.^{1,2} Although this instability is basically a surface wave, both fluids are actively involved in its evolution. In the linear regime the unstable mode extends into both fluids. The response to an initial disturbance is thus a coherent response involving both fluids as well as the shear layer. In the nonlinear regime, for example, in the turbulent mixing layer, the basic dynamics of vortex roll up, vortex amalgamation, and entrainment of passive fluid - which together set the spreading rate of the mixing layer³ - takes place almost entirely in the two counterstreaming fluids. While most canonical shear layer velocity profiles yield similar instability evolutions, the mathematical description of the instability is simplest for the linear velocity profiles dealt with here. For these profiles the instability is an irrotational disturbance.

It is also possible to obtain experimental realizations of the shear layer alone, without the surrounding counterstreaming fluids. Most commonly this is done by putting the shear layer in a box, for example, in laboratory attempts to simulate homogeneous turbulence in the presence of shear.⁴ In such cases the rigid wall conditions prevent the fluid from supporting irrotational disturbances. This is not to say that the fluid configuration is stable - after all, the fluids are often turbulent - but only that three dimensional, rotational motions are the unstable ones. In particular, numerical simulation has shown⁵ that stagnation points corresponding to the 'right' direction of energy transfer among the components of fluctuating flow velocities are unstable structures, while stagnation points corresponding to the 'wrong' direction of energy transfer are damped. One may

show that a stagnation point in the fluid is also a source of secondary instabilities of the vortex stretching type.

But it is also possible to obtain flow configurations that may be idealized as a shear layer of incompressible fluid satisfying free boundary conditions. Such flows are of frequent occurrence in inertial confinement fusion schemes. One often has a shell of heavy fluid surrounding light fluid - the DT - and being driven by a 'light fluid', e.g., a magnetic field, laser beams, or electron beams. Furthermore, tangential flows may occur in the heavy fluid as the result of geometric misalignments, deviations of the shell from ideal symmetry, and azimuthal pressure variations, or such flows may be deliberately induced, as in the rotational stabilization of the metal liner implosion.⁶ Such tangential flows may usually be idealized as shear layers, but even when there is no initial shear⁶, shear usually develops during the course of the implosion as the result of angular momentum conservation. Of course, convergence effects and radial pressure gradients are important to the full evaluation of such flows. But the heavy fluid is often not greatly compressed during much of the implosion. Frequently, too, the pressure exerted on the heavy fluid by the light fluids may be assumed not to vary when the surfaces of the heavy fluid are given volume preserving perturbations. Thus, for the purpose of studying the stability of tangential flows, a reasonable extreme idealization of the configuration is that of a plane shear layer of incompressible fluid satisfying constant (and equal) pressure boundary conditions, i.e., free boundary conditions. For mathematical simplicity the velocity profile of the shear layer is taken to be linear. Obviously, a less highly idealized configuration would display Rayleigh-Taylor instability as well as significant convergence effects on the develop-

most of all modes.

We consider the stability of the two-dimensional equilibrium shear layer configuration of an incompressible fluid

$$\mathbf{u}_0(x, t) = (Sy, 0, 0) \quad (1)$$

and

$$p_0(x, t) = p_0, \quad \text{for } -\frac{h}{2} < y < \frac{h}{2} \quad (2)$$

and x and z unlimited. The whole configuration is required to satisfy the free boundary conditions

$$p = p_0 \quad \text{at } y = \pm \frac{h}{2} \quad (3)$$

We presume that the system undergoes a two dimensional disturbance

$$\delta \mathbf{u} = (\delta u, \delta v, 0) \quad (4)$$

and note that the linearized Euler equations for the disturbance take the form

$$\left(\frac{\partial}{\partial t} + Sy \frac{\partial}{\partial x} \right) \delta u + S \delta v = - \frac{\partial}{\partial x} \delta \phi \quad (5a)$$

and

$$\left(\frac{\partial}{\partial t} + Sy \frac{\partial}{\partial x} \right) \delta v = - \frac{\partial}{\partial y} \delta \phi \quad (5b)$$

where

$$\delta \phi = \frac{1}{\rho} \delta p \quad (5c)$$

The disturbance must also satisfy the continuity equation and the free boundary conditions

$$\delta \phi = 0 \quad \text{at } y = \pm \frac{h}{2} \quad (6)$$

irrotational disturbance modes exist for this problem, and we consider a single such mode, namely

$$\delta u = \nabla w \quad (7a)$$

and

$$w = \left[A(t)e^{ky} + B(t)e^{-ky} \right] e^{ikx} . \quad (7b)$$

The linear velocity profile of Eq. (1) permits the two determinations of the perturbed pressure, from Eqs. (5a) and (5b) respectively, to be consistent. The solution is completed by expressing the perturbed pressure in terms of A and B, through use of Eq. (8), and imposing the two boundary conditions of Eq. (6). This yields evolution equations for A and B, to be solved subject to initial conditions. Specifically, if we define

$$\tau = iSt \quad (8)$$

the evolution equations become

$$\frac{d}{d\tau} A = [f(q)A - qB]/(e^q - e^{-q}) \quad (9a)$$

and

$$\frac{d}{d\tau} B = [qA - f(q)B]/(e^q - e^{-q}) , \quad (9b)$$

where

$$q = kh \quad (9c)$$

and

$$f(q) = \left(1 - \frac{1}{2}q\right)e^q - \left(1 + \frac{1}{2}q\right)e^{-q} . \quad (9d)$$

The parameter q measures the shear layer thickness in disturbance wavelengths.

If we assume that A and B are proportional to $e^{i\omega t}$ we obtain the disper-

tion relation

$$p^2 = - [q^2 - f^2(q)] / [(e^q - e^{-q})^2] \quad (10)$$

A negative imaginary part for p implies an unstable mode. For the dispersion relation of Eq. (10) the disturbances are oscillatory at short wavelengths, $q > q^*$, while one purely growing disturbance exists at long wavelengths, $q < q^*$. The stability boundary corresponds to a critical wavelength

$$\lambda^* = 2.619 h. \quad (11)$$

In the long wavelength limit the dispersion relation becomes

$$p^2 = - \frac{1}{12} q^2, \quad (12)$$

which corresponds to an instability growth rate $\exp\left(\frac{1}{2\sqrt{3}} kUt\right)$, where $U = Sh$ is the velocity jump across the interface. This growth rate satisfies a similar dispersion law to that of the natural Kelvin-Helmholtz instability associated with this shear layer configuration, in which counterstreaming fluids of the same density as the shear layer flow with the velocity of the edges of the shear layer, except that here the growth rate is reduced by a factor $(1/3)^{1/2}$.

Alternatively, the growth rate is precisely that of the natural Kelvin-Helmholtz instability associated with a shear layer with the same velocity shear but of thickness $(3)^{-1/2}h$. Note that in the present configuration there are no external fluids to help support the instability. Nevertheless, the long wavelength behavior of the mode is very similar to that of the Kelvin-Helmholtz instability, as is the short wavelength cutoff of the instability, although the present mode persists to shorter wavelengths than does the K-H mode, which cuts off at $\lambda^* = 4.915 h$. The instability is also present if the shear S is time dependent.

While definitive judgments on the role of the Kelvin-Helmholtz-like mode discussed here to the rotationally stabilized metal liner program⁶ must await more realistic calculations in cylindrical geometry, it is of interest to note that this mode may be relevant. In the rotationally stabilized liner the inner radius R is stabilized to Rayleigh-Taylor (R-T) modes by keeping the effective acceleration $\ddot{R} - v^2/R$ negative until turnaround. This is not the whole story for these modes, for $\dot{R}$ also influences the stability of an implosion, but, in addition, the liner develops shear during the implosion. While the liner is initially in solid body rotation, $v(r,0) = \Omega r$, angular momentum conservation implies that shear develops via

$$v(r,t) = \Omega(r + [R^2(0) - R^2(t)]/r), \quad (13)$$

the cylindrical analog of the linear shear layer discussed here. The present results would suggest that a metal liner, driven by a magnetic field and imploded onto a magnetic field or a low density plasma, may be subject to low n (long wavelength) instabilities of the type discussed here.

However, rotationally stabilized liners are usually realized experimentally by driving the outer liner surface with a mechanical pusher, which is, in turn, driven magnetically. Such a liner configuration is idealized in the present context as a plane shear layer with one rigid wall and one free boundary. For such a configuration the mode discussed here exists, but it becomes a stable oscillation. Thus it may be that the mechanical pusher stabilizes the present mode, as well as stabilizing the outer surface of the liner to R-T modes.

REFERENCES

1. Lord Kelvin, Mathematical and Physical Papers, V.4. Hydrodynamics and General Dynamics (Cambridge, England, 1910), p. 76.
2. S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability (Oxford, England, 1961).
3. G. L. Brown and A. Roshko, J. Fluid Mech. 64, 775 (1974).
4. P. H. Champagne, V. G. Harris and S. Corrsin, J. Fluid Mech. 81, 81 (1970).
5. S. Corrsin, Turbulence in Internal Flows, S. N. B. Murthy, Ed. (Hemisphere, Washington, 1977), p.11.
6. D. L. Book and N. K. Winsor, Phys. Fluids 17, 662 (1974).