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Sterile Neutrinos as Dark Matter

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ABSTRACT

The simplest model that can accomodate a viable nonbaryonic dark matter candidate is the standard electroweak theory with the addition of right-handed or sterile neutrinos. This model has been studied extensively in the context of the hot dark matter scenario. We reexamine this model and find that hot, warm, and cold dark matter are all possibilities. We focus on the case where sterile neutrinos are the dark matter. Since their only direct coupling is to left-handed or active neutrinos, the most efficient production mechanism is via neutrino oscillations. If the production rate is always less than the expansion rate, then these neutrinos will never be in thermal equilibrium. However, they may still play a significant role in the dynamics of the Universe and possibly provide the missing mass necessary for closure. We consider a single generation of neutrino fields (ν_L, ν_R) with a Dirac mass, μ, and a Majorana mass for the right-handed components only, M. For $M \gg \mu$ we show that the number density of sterile neutrinos is proportional to μ^2/M so that the energy density today is *independent of* M. However M is crucial in determining the large scale structure of the Universe. In particular, $M \simeq 0.1 - 1.0$ keV leads to warm dark matter and a structure formation scenario that may have some advantages over both the standard hot and cold dark matter scenarios.

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The COBE DMR experiment's recent detection of large-scale anisotropy in the cosmic microwave background[1] has considerably strengthened the view that the large scale structures seen today evolved from very small primeval density inhomogeneities. Still, the two primary ingredients which dictate how structure forms, namely the nature of dark matter and the shape of the primeval fluctuation spectrum, remain unknown.

The best studied and perhaps most successful model for structure formation is known as the cold dark matter (CDM) theory[2]. In the standard CDM model, the Universe is assumed to be spatially flat ($\Omega = 1$) with $90 - 95\%$ of the mass density in dark matter and the balance in baryons ($5-10\%$) and photons and light neutrinos ($\ll 1\%$). Primeval fluctuations are generated during inflation and are Gaussian with a scale-invariant spectrum. CDM, with the additional assumption that galaxy formation is 'biased' to occur first at the highest peaks in the density fluctuation spectrum can successfully explain galaxy-galaxy and cluster-cluster correlation functions on scales of order $1 - 5$ Mpc and is at least consistent with the morphology of galaxies. However, CDM now appears to be inconsistent with various sets of observational data. Perhaps its greatest difficulties come with large scale structure data such as the APM galaxy survey[3], which suggest more power on large scales than standard CDM model predictions. On small scales, the observed pairwise velocity dispersion for galaxies appears to be significantly smaller than those predicted by CDM[4].

One alternative[5] which has recently received a fair bit of attention is cold + hot dark matter (C+HDM). HDM is usually taken to be a light neutrino with $m_\nu = \left(92\Omega_\nu h^2\right)$ eV where $H = 100h$ km/sec/Mpc is the Hubble parameter. In models with HDM alone, the processed fluctuation spectrum is characterized by the typical distance a neutrino travels over the history of the Universe, $\lambda_\nu \simeq 40\,(30\text{ eV}/m_\nu)$ Mpc. This in turn sets the mass scale below which damping occurs due to free-streaming, $M_{\rm FS} \equiv 4\pi\rho\left(\lambda_\nu/2\right)^3/3 \simeq 3\times 10^{15}\,(30\,{\rm eV}/m_\nu)^2\,\Omega_\nu^{-1}\,M_\odot$. In HDM models, the first structures to form are pancake-shaped objects of size λ_ν with smaller scale structures such as galaxies and clusters forming later via fragmentation. However, we know from the galaxy correlation function, that the scale which is just becoming nonlinear today is around $5h^{-1}$Mpc. Essentially, the problem with HDM alone is that λ_ν is too large: If galaxy formation occurs early enough to be consistent with high-redshift galaxies and quasars, then structures on $5h^{-1}$Mpc will overdevelop. The hope is that C+HDM will combine the successes of both models. In fact, a survey[6] of models with various amounts of hot dark matter, cold dark matter and baryons points to $\Omega_{\rm baryon} = 0.1$, $\Omega_{\rm CDM} = 0.6$, $\Omega_\nu = 0.3$ and a Hubble constant of $h = 0.5$ as the best fit model for microwave anisotropy data, large scale structure surveys, and measures of the bulk flow within a few hundred megaparsecs.

As appealing as C+HDM may be for large scale structure phenomenology, it is somewhat unpalatable from the point of view of particle physics. Since there are no stable, neutral, massive particles in the 'standard model' for electroweak interactions, the existence of nonbaryonic dark matter implies new physics. Given that the existence of the baryon-antibaryon asymmetry also requires new (and probably distinct) physics, it seems already a great coincidence that $\Omega_{\rm DM}$ and $\Omega_{\rm baryon}$ be as close as they are[7]. Two types of dark matter imply further additions to the standard model with yet another coincidence in order to have $\Omega_{\rm HDM}$, $\Omega_{\rm CDM}$, and $\Omega_{\rm baryon}$ all within one or two orders of magnitude of each other[8].

By far the simplest dark matter candidate, at least from the point of view of particle physics is the neutrino. Massive neutrinos require only the addition of right-handed or sterile neutrino fields to the standard model. In fact, it is the *absence* of right-handed neutrinos that seems contrived in light of the fact that all other fermions in the standard model have both left and right-handed components.

Here we focus on the possibility that sterile neutrinos are the dark matter and that they are somewhat heavier but less abundant than the usual HDM neutrino. Such a 'warm' dark matter particle may have advantages for structure formation over both hot and cold dark matter scenarios. Our work is similar in some respects to that of Bond, Szalay, and Turner[9] who consider a particle that is in thermodynamic equilibrium at early times but decouples before ordinary neutrinos do so that g_*, the number of effectively massless degrees of freedom, is relatively high ($g_* \gtrsim 100$). Warm dark matter has been for the most part been ignored, to a large extent because there have been no compelling candidates proposed thusfar. In part, the motivation for this work is to propose a 'realistic' warm dark matter candidate.

For simplicity, we consider only one generation of neutrinos. The mass terms for the neutrinos are then[10]:

$$\mathcal{L} = \mu\left(\frac{\phi}{v}\right)\bar{\nu}_L\nu_R + M\nu_R\nu_R + \text{ h.c.} \qquad (1)$$

where ϕ is the standard model Higgs field with $\langle\phi\rangle = v$. The usual HDM case, wherein the active neutrinos constitute the dark matter, corresponds to $\{\mu = 92h^2\text{eV},\ M \ll \mu\}$ or $\{\mu^2/M = 92h^2\text{eV},\ M \gg \mu\}$. When sterile neutrinos are the dark matter, the relevant mass is M. At tree-level, ν_R couples only to ν_L and therefore the most efficient way to produce sterile neutrinos[11,12,13] is via oscillations $\nu_L \to \nu_R$. The probability of observing a right-handed neutrino after a time t given that one starts with a pure monoenergetic left-handed neutrino is $\sin^2 2\theta_M \sin^2 vt/L$ where θ_M is the 'mixing angle', L is the oscillation length, and v is the velocity of the neutrinos. In vacuum, and with $\mu \ll M$ (see-saw model) $\theta_M = \mu/M$ and $L = 4E/\left(M^2 - \mu^2\right)$ where E is the energy of the neutrinos. In the early Universe, the observation time t is replaced by the interaction time for the left-handed neutrinos. Recent work[14,15,16] has fine-tuned this picture taking into account the effect of finite density and temperature on the mixing angle.

Here we are interested in the case where the right-handed neutrinos are produced at temperatures of order 100 MeV though the production rate is never so fast that they equilibrate. We begin with the Boltzmann equation for the sterile neutrinos:

$$\left(\frac{\partial}{\partial t} - HE\frac{\partial}{\partial E}\right) f_S(E,t) = \left[\frac{1}{2}\sin^2(2\theta_M(E,t))\,\Gamma(E,t)\right] f_A(E,t) \qquad (2)$$

where f_S and f_A are the distribution functions of the sterile and active neutrinos. In the epoch under consideration ($T \gg 1$ MeV) the left-handed neutrinos are in thermal equilibrium so that $f_A = \left(e^{E/T}+1\right)^{-1} \simeq \left(e^{p/T}+1\right)^{-1}$. The quantity in square brackets is the probability per time of an active neutrino converting into a sterile one[16] where we have used the fact that for parameters of interest, the collision time is always much greater than the oscillation time (i.e. $\sin^2 vt/L$

averages to 1/2). The mixing angle and the collision rate are[17]

$$\sin^2(2\theta_M) = \frac{\mu^2}{\mu^2 + [(c\Gamma E/M) + (M/2)]^2} \quad ; \quad \Gamma \simeq \frac{7\pi}{24} G_{\rm Fermi}^2 T^4 E \tag{3}$$

where $c \simeq 4\sin^2(2\theta_W)/15\alpha \simeq 26$.

To get a feel for when and how many sterile neutrinos are produced, we derive the equation for $r \equiv n_S/n_A$ where $n_i \equiv 2\int d^3p f_i/(2\pi)^3$ is the number density of sterile (active) neutrinos with $i = S$ ($i = A$). Changing the time variable from t to a, the Robertson-Walker scale factor and integrating Eq. (2) over momenta, one finds that

$$\frac{dr}{d\ln a} = \frac{\gamma}{H} + r\frac{d\ln g_*}{d\ln a} \tag{4}$$

where

$$\gamma \equiv \frac{1}{n_A}\int \frac{d^3p}{(2\pi)^3} \sin^2 2\theta_M(p,T)\,\Gamma(p,T)\,\frac{1}{e^{p/T}+1}\ , \tag{5}$$

and we have used the fact that $g_* a^3 T^3$ = constant. For g_* constant, γ/H gives the number of sterile neutrinos, relative to the number of active neutrinos, that are produced in each log-interval of T. Substituting Eq. (3), using $H = 1.66 g_*^{1/2} T^2/m_{\rm Planck}$, and taking the limit $M \gg \mu$, we find that

$$\frac{\gamma}{H} = \frac{13}{g_*^{1/2}}\left(\frac{\mu}{\rm eV}\right)^2\left(\frac{\rm keV}{M}\right) x \int_0^\infty \frac{y^3 dy}{(e^y+1)(1+x^2y^2)^2} \tag{6}$$

where $x \equiv 78\,(T/{\rm GeV})^3\,({\rm keV}/M)$. Taking $g_* = 10.8$ and doing the integral numerically, we find that γ/H reaches a peak value of $1.9\,(\mu/{\rm eV})^2\,({\rm keV}/M)$ when $x \simeq 0.19$ or $T = T_{\rm max} \simeq 133\,(M/{\rm keV})^{1/3}$ MeV and falls off as T^3 for $T \ll T_{\rm max}$ and T^{-9} for $T \gg T_{\rm max}$. Evidently, the number density in sterile neutrinos is proportional to M^{-1} so that the energy density is *independent of* M. Note also that most of the neutrinos are produced when the Universe has a temperature $T \simeq T_{\rm max}$. As will be discussed below, our calculations simplify if we can assume that g_* is constant. Since g_* changes abruptly at $T \simeq 200$ MeV and varies slowly for 200 MeV $\gtrsim T \gtrsim$ 20 MeV, this assumption will be pretty good for $M \lesssim$ keV but breakdown for masses much larger than this.

Our interest is in the structures which form in a ν_R-dominated Universe and we therefore require the full sterile neutrino distribution function. Here, we make the assumption that g_* is constant. Using $\partial f_S/\partial t = -HT\partial f_S/\partial T$ and the identity

$$T\left(\frac{\partial f_S}{\partial T}\right)_E + E\left(\frac{\partial f_S}{\partial E}\right)_T = T\left(\frac{\partial f_S}{\partial T}\right)_{E/T} \tag{7}$$

and changing the integration variable from T to x one finds

$$\frac{f_S}{f_A} = \frac{7.7}{g_*^{1/2}}\left(\frac{\mu}{\text{eV}}\right)^2\left(\frac{\text{keV}}{M}\right)\; y\int_x^\infty \frac{dx'}{(1+y^2x'^2)^2} \tag{8}$$

where $y \equiv E/T$. In general, the right hand side of Eq. (8) is a complicated function of E and therefore will have a different energy dependence than f_A. There is no reason to expect otherwise: high energy and low energy neutrinos oscillate at different rates. Moreover, these rates change with temperature. HOWEVER, for $T \ll T_{\max}$ the lower limit of the integral can be set to zero and the right hand side of (8) becomes independent of E and T. In this limit, the integral is easily done and we find

$$f_S = \left(6.0/g_*^{1/2}\right)(\mu/\text{ eV})^2\,(\text{keV}/M)\; f_A. \tag{9}$$

f_S has the same functional form as f_A and therefore $\Omega_S/\Omega_\nu = (M/m_\nu)(f_S/f_A)$. From the relation $m_\nu/\Omega_\nu \simeq 92h^2$ eV we find that $\Omega_S = 1$ for $\mu = 0.22h$ eV where we have again set $g_* = 10.8$. Finally, we note that the contribution of sterile neutrinos to the energy density of the Universe at the time of primordial nucleosynthesis[18] must be $\lesssim 0.5$ times the contribution of a light neutrino species if standard big bang nucleosynthesis[19] is to be believed. This in turn implies that $M \gtrsim 200h^2$eV; that is, if sterile neutrinos are the dark matter then they are necessarily more massive than the standard HDM.

How do perturbations evolve when a sterile neutrino species is the dark matter? Several guiding principles help us understand the processed power spectrum. First, structure within the horizon grows only after the dominant component of matter becomes nonrelativistic and therefore the size of the horizon at matter-radiation equality $\lambda_H(a = a_{eq}) \equiv a_{eq} \int_0^{a_{eq}} dt'/a(t')$, defines a characteristic scale. Second, perturbations on scales smaller than the Jeans length $\lambda_J \equiv (\pi v_s^2 m_{\rm Planck}^2/\rho)^{1/2}$ (where v_s is the speed of sound) oscillate like pressure waves. Finally, for neutrinos, or any particle which is not completely non-relativistic, perturbations on scales smaller than the free streaming scale $\lambda_{FS} \equiv a \int_0^t dt' \langle (p/E)^2 \rangle^{1/2}/a(t')$ are exponentially damped. With the distribution function in Eq. (9), one can calculate these scales for sterile neutrinos. Figure 1 shows the relevant mass scales ($= 4\pi\rho(\lambda/2)^3/3$) as a function of the scale factor for the sterile neutrinos discussed here and for an ordinary light neutrino dark matter candidate. For light neutrinos, the damping scale and the horizon scale at equality are roughly equal [$\sim 10^{15} M_\odot$], of order supercluster size. This scale is the first to go non-linear. For sterile neutrinos, there is a large disparity between the two characteristic scales, so that perturbations with $10^{13} M_\odot \lesssim M \lesssim 10^{15} M_\odot$ are processed similarly; given an initial Harrison-Zel'dovich spectrum, they should all the the same final amplitude in linear theory. Power on scales smaller than this should be completely damped.

In conclusion, we have proposed a candidate for warm dark matter that exists in the simplest extension of the standard model. Warm dark matter has several advantanges over cold or hot dark matter, resulting from the fact that the power on scales of order $1 - 5$ Mpc is less than in CDM but greater than in HDM. In particular, the pairwise velocity dispersions in a WDM universe are likely to be smaller than in CDM and hence more in accord with observations. Since there is more power on small scales than in HDM, the epoch of galaxy formation is likely to be earlier and hence the observed high redshift quasars pose less of a problem for this model than for HDM. On large scales there is little difference among the three models: at present they all seem to be incompatible with the APM survey. Another advantage WDM has over HDM is that since the neutrino mass is higher, it is possible to fit more neutrinos into a given galaxy, thus evading Tremaine-Gunn limits[20]. Finally we point out a unique signature of WDM is an increase in the predicted primordial helium abundance; since a neutrino species that is in thermal equilibrium at the time of big bang nucleosynthesis adds $\Delta Y = 0.012$ to the primordial helium mass fraction, sterile neutrinos add

$$\Delta Y = .01 \left(\frac{100h^2\mathrm{eV}}{M} \right), \tag{10}$$

a potentially detectable deviation from the standard prediction.

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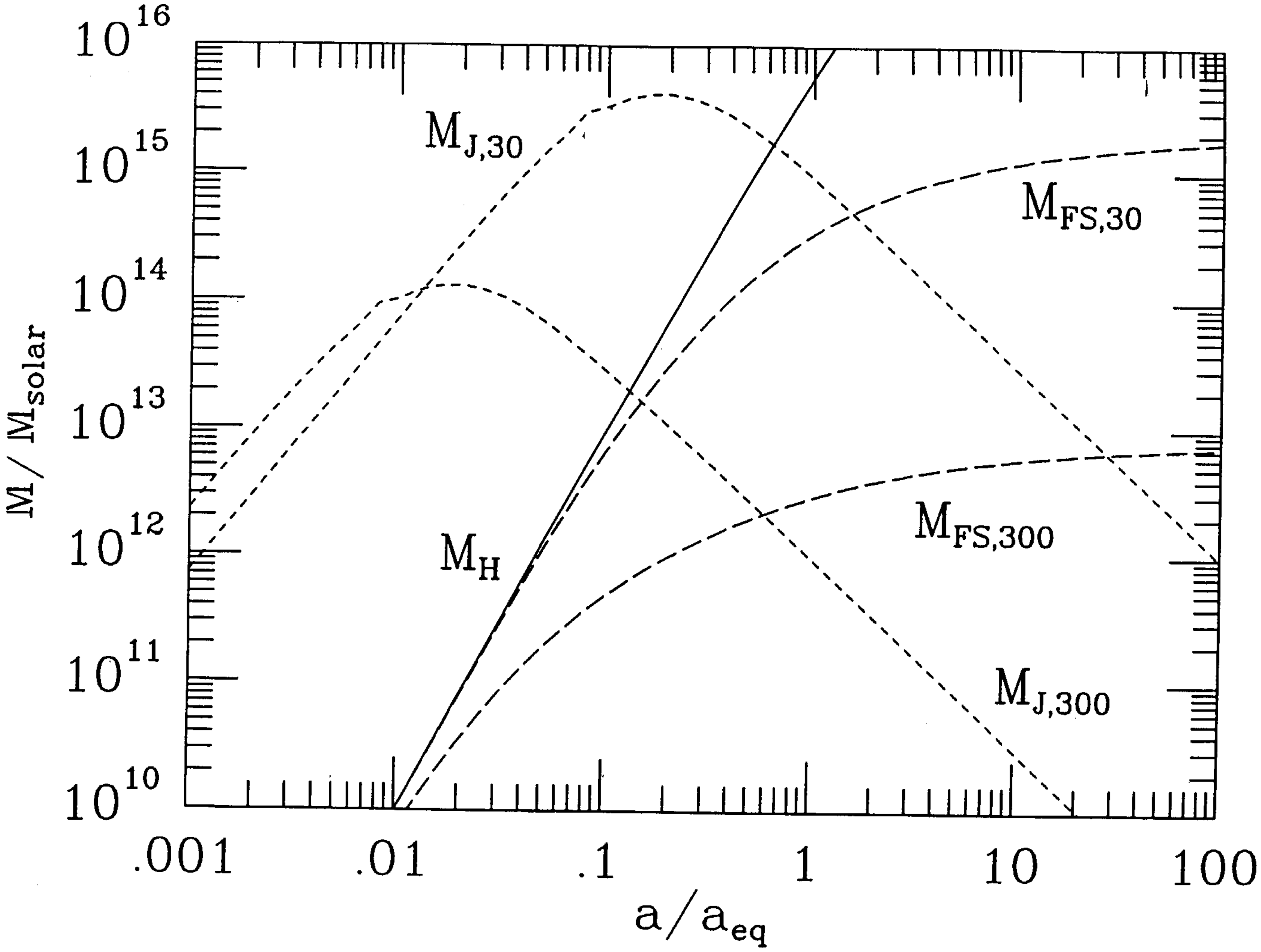
FIGURE CAPTIONS

1) Figure 1. Mass scales in hot dark matter and warm dark matter as a function of scale factor. M_H (solid line) gives the mass within the horizon. Long dashed lines give the free streaming mass for a 30 eV ($M_{FS,30}$) and 300 eV ($M_{FS,300}$) neutrino. Short dashed lines are the Jeans mass for a 30 eV ($M_{FS,30}$) and 300 eV ($M_{FS,300}$) neutrino.



M / M_solar
10^10
10^11
10^12
10^13
10^14
10^15
10^16
.001
.01
.1
1
10
100
a / a_eq
M_J,30
M_H
M_FS,30
M_FS,300
M_J,300