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A ONE AND ONE-HALF DIMENSIONAL MODEL
OF THE EBT REACTOR

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I. INTRODUCTION

The Elmo Bumpy Torus (EBT) device has been successful in exhibiting stable, steady-state points of operation.¹ The energy confinement time in the experiment is consistent with that predicted by neoclassical theory, and a favorable increase in confinement time with temperature is indicated by the data. In terms of dimensionless plasma parameters (with the exception of β), the EBT-1 device is operating in a regime which, if attained in a reactor, makes EBT an attractive ignition device.

Other features which make EBT attractive as a reactor are its steady-state operation, its potential for high beta and power density, its large aspect ratio for easy accessibility, and its weak field curvature implying a reduced structural loading. Furthermore, the EBT concept is economically competitive with other reactor proposals.

In addition to the confinement time, the temperatures, density, and the sign and magnitude of the ambipolar potential in EBT-1 are all consistent with neoclassical transport theory. The fact that EBT-1 can be explained by neoclassical theory gives credence to the scaling of EBT to the reactor regime.

Previous attempts at modeling the EBT reactor (EBTR) have been with zero dimensional models (Ref. 2 and references therein). These codes generally use the minor radius as a gradient scale length, and while sufficient to give the gross features of the system, they cannot give useful profile information such as the effect of the refueling on the density profile, or the effect of the high temperature plasma digging a well in the magnetic field.

To give a more quantitative answer to the question of whether a device can be scaled to a reactor from the present EBT-1 and future EBT-P devices, time and space dependent models of plasma and alpha transport must be formulated. The effect on the background magnetic field of the high temperature and density reactor plasma must also be included in the model, since the magnetic field affects the transport³ and the overall MHD stability of the system.⁴ This requires a self-consistent calculation of the magnetic field in conjunction with the plasma transport calculation.

In this paper we describe a one-dimensional, time-dependent model of plasma particle and energy transport and alpha particle transport coupled with

magnetic field evolution in a geometry appropriate to EBT. The transport equations used are derived from exact moments of the Boltzmann equation,⁵ and the magnetic field is calculated from Faraday's and Ampere's laws. The set of transport equations is closed by incorporating into them transport coefficients derived from the appropriate kinetic equation. Also included in the model is a Fokker-Planck calculation of the alpha particle slowing down and resultant plasma heating.

Other attempts at modeling EBT in one dimension consist of treating the plasma transport in cylindrical geometry while maintaining a constant magnetic field profile.^{6,7} A novel feature of the present code is the calculation of the transport in a geometry appropriate to EBT.

This geometrical effect is accounted for in the model by employing the magnetic field lines as the basis of the coordinate system. Transport of mass and heat in a plasma is much faster along a field line than across it. Thus, on the diffusion time scale, in which we are interested, the plasma density and temperature can be considered constant along a field line. Surfaces of constant pressure, or equivalently surfaces of constant magnetic flux, can then be mapped out, and variation of the plasma quantities is considered only normal to these surfaces.

A multi-dimensional equilibrium calculation is required to determine the shapes and positions of the flux (pressure) surfaces, and thus the geometry, at each instant of time. Once this family of surfaces is determined, the one-dimensional calculation of plasma and field diffusion across these surfaces can be performed. Thus, the fully self-consistent treatment requires a calculation of flux surface shapes changing in response to plasma diffusion which is influenced by the instantaneous shapes. This technique has been successfully employed in the modeling of tokamaks (Ref. 8 and references therein).

An equilibrium calculation determines the shapes of the surfaces and not the value of the flux on each surface. If a knowledge of the shapes of the family of surfaces is assumed a priori and these shapes are taken to remain constant in time, the description is reduced to a one-dimensional one. This is the basis of the one-dimensional version of the code that we will use for the modeling studies presented further on.

As a final note, the model described in the following sections pertains only to plasma transport interior to the annulus, and the effect of the annulus on the field and plasma heating is not included.

II. THE EBT REACTOR CODE

A. BASIC EQUATIONS

1. Transport Equations

The general transport equations describing the evolution of particle density and energy density may be obtained from either moments of the bounce averaged drift kinetic equation⁶ or from moments of the Boltzmann equation.⁵

The electron and alpha particle conservation equations are

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma_e = S_e \quad (1)$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot \Gamma_\alpha = S_\alpha \quad (2)$$

where Γ_e and Γ_α are the electron and alpha fluxes respectively, S_e is the source of electrons provided by ionization of neutrals and impurities, and S_α is the source of alphas provided by fusions of hydrogens. We assume quasi-neutrality for electrons, hydrogens, alphas and impurities, i.e., $n_e = n_h + 2n_\alpha + \sum_I Z_I n_I$, where Z_I and n_I are the charge and density of impurity specie I.

The hydrogen is described by an average DT particle and the impurity density in this model is a fixed fraction of the total ion density and does not diffuse independently.

The electron and ion energy conservation equations are

$$\frac{3}{2} \frac{\partial p_e}{\partial t} + \nabla \cdot Q_e + e\Gamma E_r + Q_\Delta + R = E \cdot J + P_{\mu e} + P_{\alpha e} \quad (3)$$

$$\begin{aligned} \frac{3}{2} \frac{\partial p_i}{\partial t} + \nabla \cdot Q_i + \frac{3}{2} n n_0 \langle \sigma v \rangle_x k (T_i - T_0) = \\ P_i + Q_\Delta + e\Gamma E_r + \frac{3}{2} n n_0 \langle \sigma v \rangle_i k T_0 + P_{\alpha i} \end{aligned} \quad (4)$$

Here $p = \frac{3}{2} n kT$ is the electron or ion energy density, and Q_e and Q_i are the total electron and ion energy fluxes which include both convective and conductive

losses; $E \cdot J$ is the Joule heating from any plasma currents and inductive electric fields; $P_{\mu e} = (\text{const}) \times n^\gamma$ is the electron heating rate due to microwave absorption, which we take to be proportional to density raised to some arbitrary power γ . The electron-ion energy exchange rate, Q_Δ , is given by

$$Q_\Delta = \frac{3n}{\tau_{ei}} \frac{m_e}{m_i} k (T_e - T_i)$$

where τ_{ei} is the electron-ion collision time

$$\tau_{ei} = \frac{3}{4} \sqrt{\frac{m_e}{2\pi}} (kT_e)^{3/2} / (e^4 n \ln \Lambda)$$

and $\ln \Lambda$ is the Coulomb logarithm. E_r is the radial ambipolar electric field determined self-consistently by equating $\Gamma_e = \Gamma_i + 2\Gamma_\alpha$. Energy losses from cyclotron radiation and line, recombination, and bremsstrahlung radiation arising from collisions between electrons and impurities is represented by R . Neutral ionization and charge exchange rates are given by the terms $n_0 \langle \sigma v \rangle$, where n_0 is the neutral density, and T_0 is the neutral temperature. P_i is any additional ion heating, e.g., from RF sources or neutral beams, and $P_{\alpha e}$ and $P_{\alpha i}$ are the electron and ion heating from alpha slowing down.

2. Magnetic Field Equations

As we have discussed, the magnetic field in the plasma depends on the plasma and alpha pressures, so that a self-consistent calculation is required for determining the field. Also, since the pressures change on a diffusion time scale, the change in magnetic field must also be computed on this time scale.

The magnetic field and the plasma are related by the equation of magnetohydrodynamic equilibrium and Ampere's law. The equilibrium equation

$$J \times B = c \nabla p \tag{5}$$

relates the currents, field, and total pressure, $p_e + p_i + p_\alpha$. Ampere's law relates the field to the currents

$$\nabla \times B = 4\pi \frac{J}{c} \tag{6}$$

Finally, conservation of magnetic flux is expressed by Faraday's law

$$\frac{\partial B}{\partial t} = - c \nabla \times E \quad (7)$$

where E is the inductive electric field.

B. ONE-DIMENSIONAL EQUATIONS

The above set of equations describe the evolution of an EBT plasma in three dimensions. As discussed in the introduction, a reduction in complexity in the description of EBT can be accomplished if the magnetic field geometry is used as the basis for the coordinate system. The diffusion of plasma quantities can be modeled with one-dimensional equations instead of three-dimensional ones.

In this section we derive a set of one-dimensional equations suitable for modeling EBT.

The magnetic field can be written in terms of the scalar variables β and ψ , the toroidal flux, such that

$$B = \nabla \beta \times \nabla \psi \quad (8)$$

where B , $\nabla \beta$, and $\nabla \psi$ are mutually orthogonal. We adopt a third coordinate variable, χ , which lies along B . This system is illustrated in Figure 1. Assuming that no current flows in the "radial" or ψ direction, we see from Ampere's law, Equation (6), that the current lies in the direction of $\nabla \beta$

$$\frac{4\pi}{c} J = \frac{4\pi}{c} J_\beta = \frac{1}{h_\chi h_\psi} \frac{\partial}{\partial \psi} \frac{h_\chi}{h_\beta h_\psi} \quad (9)$$

where the h are metric coefficients, i.e., $\nabla i = \frac{1}{h_i}$. It then follows from Equation (5) that the pressure is constant on a surface of constant ψ ,

$$(\nabla p)_\psi = J_\beta B_\chi \quad (10)$$

Substituting from Equation (9),

$$4\pi \frac{\partial p}{\partial \psi} + \frac{1}{h_\psi h_\chi h_\beta} \frac{\partial}{\partial \psi} \frac{h_\chi}{h_\beta h_\psi} = 0 \quad (11)$$

we arrive at the Grad-Shafranov equation^{9,10} expressed in the general geometry coordinate system.

The time rate of change of the flux, from Equation (7), is given by

$$\dot{\psi} = h_\beta c E_\beta \quad (12)$$

We make the assumption that heat flow is infinitely fast along a field line, and thus the temperatures are constant on the constant pressure surfaces. This, then, implies that the density is also a constant since $n = \frac{p}{kT}$. Therefore, by averaging Equations (1)-(4) over a small volume enclosing a flux surface, all reference to χ and β can be eliminated. We introduce the additional surface label, V , where V is the volume contained within a ψ surface. The flux surface average of a quantity A is defined by

$$\langle A \rangle = \frac{\partial}{\partial V} \int_V d^3x A \quad (13)$$

where the integral is over the volume contained within the surface. From Gauss' theorem

$$\nabla \cdot A = \frac{\partial}{\partial V} \langle A \cdot \nabla V \rangle \quad (14)$$

for any vector A .

With the notation $\langle n \rangle = n$, $\langle p \rangle = p$ and $\frac{\partial}{\partial t} = \frac{\partial}{\partial t} \Big|_V$, the conservation equations, Equations (1)-(3), become

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial V} \langle \Gamma_e \cdot \nabla V \rangle = \langle S_e \rangle \quad (15)$$

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial V} \langle \Gamma_\alpha \cdot \nabla V \rangle = \langle S_\alpha \rangle \quad (16)$$

$$\frac{3}{2} \frac{\partial p_e}{\partial t} + \frac{\partial}{\partial V} \langle Q_e \cdot \nabla V \rangle = \langle H_e \rangle \quad (17)$$

$$\frac{3}{2} \frac{\partial p_i}{\partial t} + \frac{\partial}{\partial V} \langle Q_i \cdot \nabla V \rangle = \langle H_i \rangle \quad (18)$$

where the S's and H's are the remaining terms in Equations (1)-(4). The equations determining the evolution of the magnetic field, Equations (12) and (11), are

$$\langle \dot{\psi} \rangle = \langle h_{\beta} c E_{\beta} \rangle \quad (19)$$

$$4\pi \frac{\partial p}{\partial V} + \frac{\partial \psi}{\partial V} \frac{\partial}{\partial V} K \frac{\partial \psi}{\partial V} = 0 \quad (20)$$

where K is a purely geometrical function and depends only on the shapes of the flux surfaces,^{11,12}

$$K = (\oint B dl) (\oint dl/B) \quad (21)$$

The one dimensional equations (15)-(19) are the conservation equations of mass, energy, and flux. The equilibrium equation, Equation (20), is a constraint equation on the field and pressure. These equations are exact in the sense that no approximations have been made in reducing their dimensionality.

A two or three-dimensional equilibrium calculation is required to determine the shapes and positions of the flux surfaces and then the geometry coefficients for insertion into the one-dimensional set. The flux (or B) is determined from Faraday's law, Equation (19), with an electric field consistent with the transport.

The difference equations representing Equations (15)-(19) are written such that they preserve the conservation properties of these equations, so that only physical processes create or destroy particles, energy and magnetic flux. The method of obtaining the simultaneous solution of the nonlinear set of one-dimensional equations, Equations (15)-(20), is discussed in Reference 13.

A self-consistent solution requires alternating between the 1-D set and the 2 or 3-D equilibrium equation.^{8,11,12} If the geometry changes are expected to be relatively minor, then the geometry can be specified a priori and the code run in a 1-D mode. The 1-D mode is used for the model calculations presented in this paper.

C. EQUILIBRIUM AND GEOMETRY

The role of the equilibrium calculation is to determine the geometrical coefficients for insertion into the 1-D equation set. For computational ease in calculating equilibria, we approximate the EBT device by its infinite aspect ratio limit, the axisymmetric bumpy cylinder, Figure 2. Effects of toroidal curvature are included through the transport coefficients. With this approximation the geometry function, K, given by Equation (21) reduces to $K = \langle |\nabla V|^2 / r^2 \rangle$.

A background magnetic field from given external currents determines the initial grid shape. In a fully self-consistent calculation, the grid is updated periodically in the following manner:

Given profiles of $p(V)$ from the 1-D set of equations, the grid shape is altered while preserving the volume inside each flux surface until the solution for ψ from Equation (11) agrees with that obtained from the 1-D set. Standard techniques for solving elliptic partial differential equations are employed for the solution of Equation (11). The iteration technique for solving for the grid shape is described in more detail in Reference 8. Since the shape changes occur on a much slower time scale than the plasma diffusion, very few iterations are required for convergence.

D. TRANSPORT

1. Collisional Transport

In order to close the system of equations, Equations (15)-(20), transport coefficients in the particle and energy fluxes, must be specified. Transport in EBT is the subject of intense investigation, and a review of its status can be found in Reference 3.

The fluxes can be expressed in terms of integrals over the solution of a suitable bounce averaged drift kinetic equation

$$\Omega \frac{\partial f}{\partial \beta} + V_D \frac{\partial f}{\partial \psi} = Cf \quad (22)$$

where β is again the poloidal angle variable and ψ designates constant pressure surfaces, Ω is poloidal drift frequency, $\Omega = -\frac{\partial J}{\partial \psi}$, V_D is the vertical drift, $V_D = \frac{\partial J}{\partial \beta}$, J is the adiabatic invariant, $J = \int ds v_{||}$, and Cf is the bounce

averaged collision operator. Bounce averaging is justified in forming Equation (22) since the bounce frequency is much greater than the poloidal drift and collision frequencies.

The particle and energy fluxes normal to a pressure surface are determined from the solutions of Equation (22),

$$\Gamma_{\psi} = \int \frac{d\beta}{2\pi} \int d^3v V_{D\psi} f \quad (23)$$

$$Q_{\psi} = \int \frac{d\beta}{2\pi} \int d^3v V_{D\psi} \frac{mv^2}{2} f \quad (24)$$

Results of these calculations give the fluxes in terms of diffusion coefficients and gradients

$$\Gamma_j = - D_{nj} \frac{\partial n}{\partial r} - D_{Tj} \frac{\partial T_j}{\partial r} + \frac{ne_j}{kT_j} D_{nj} E_r \quad (25)$$

$$Q_j = - K_{nj} \frac{\partial n}{\partial r} - K_{Tj} \frac{\partial T_j}{\partial r} + \frac{ne_j}{kT_j} K_{nj} E_r \quad (26)$$

where the subscript j refers to particle specie, and the "radial" derivative refers to the derivative across the pressure surface and is given, for example, by $\frac{\partial n}{\partial r} = \frac{\partial n}{\partial V} \nabla V$, where V is the volume enclosed by the surface. The quantity E_r in these equations is the radial ambipolar electric field. This field is determined self-consistently in the code by solving the nonlinear equation $\Gamma_e = \Gamma_i + 2\Gamma_{\alpha}$ for E_r .

There are three regimes of neoclassical transport in EBT, and the diffusion coefficients appropriate to each regime have been calculated at least in some approximation.

The collisional regime is characterized by the parameter $\nu/\Omega_0 > 1$, where ν is the collision frequency and Ω_0 is the poloidal precession frequency of a thermal particle in a mirror field with radius of curvature, R_c , $\Omega_0 = \frac{ckT}{eBrR_c}$. This regime was first treated by Kovrizhnykh¹⁴ in the large electric field limit, $\frac{e\phi}{kT} \gg 1$, where ϕ is the potential associated with the ambipolar field. Even for smaller fields, the results give a good approximation to the transport.¹⁵

The coefficients for this regime are

$$\begin{aligned}
 D_{nj} &= \frac{V_{oj}^2}{24} \frac{v_j}{v_j^2 + \Omega_j^2} \\
 D_{Tj} &= \frac{2n_j}{T_j} D_{nj} \\
 K_{nj} &= \frac{7}{2} kT_j D_{nj} \\
 K_{Tj} &= \frac{101}{2} kn_j D_{nj}
 \end{aligned} \tag{27}$$

where

$$V_{oj} = 2kT_j / (m \omega_{cj} R_T)$$

is the average vertical drift speed due to the toroidal curvature, R_T is the major radius of the torus, ω_c is the cyclotron frequency, v is the collision frequency, and Ω is the total poloidal bounce averaged precession frequency,

$$\Omega = \frac{ckT}{eBrR_c} + \frac{cE_r}{Br} \tag{28}$$

where R_c represents the effect of the mirror radius of curvature and the magnetic field gradient. In these equations all quantities refer to their mid-plane values.

In the more collisionless regimes of operation where $v/\Omega_0 < 1$, there are classes of particles for which the electric and magnetic drifts cancel (ions for radially inward pointing electric fields), in which case the flux can be enhanced.

Hazeltine, et al.¹⁵ have given expressions in integral form for the fluxes in the regime $\delta^{3/2} < v/\Omega_0 < 1$, where $\delta = a/R_T$ is the inverse aspect ratio. They find that the fluxes are independent of collision frequency, hence the name "plateau." Jaeger, et al.¹⁶ have evaluated the integrals using a model magnetic field and an ad hoc value of the poloidal component of the ambipolar field so that equipotential surfaces and contours of constant B are aligned in the midplane. These results are convenient for use in modeling and can be written as follows:

$$\begin{aligned}
D_n &= \frac{3\pi}{8\sqrt{a}} \left(1 + \frac{1}{2a}\right)^2 \frac{r}{R_T^2} \frac{ckTR_c}{eB} e^{-\hat{E}} \\
D_T &= \frac{n}{T} D_n (1 + \hat{E}) \\
K_n &= kT D_n \left(\frac{5}{2} + \hat{E}\right) \\
K_T &= kn D_n \left(5 + \frac{7}{2} \hat{E} + \hat{E}^2\right)
\end{aligned} \tag{29}$$

where $a = 2.1$, and $\hat{E} \equiv -e R_c E/kT > 0$. For radially inward electric fields, the Kovrizhnykh transport coefficients of Equation (27) give a good approximation to the electron transport even for $v/\Omega_0 < 1$ in this regime, since the poloidal magnetic and electric field drifts are in the same direction.

In the plateau regime the particles with $\Omega \approx 0$ suffer a collision before they complete a closed orbit. In the very collisionless operating regime such that $v/\Omega_0 < \delta^{3/2}$, the orbits are closed and are shaped like crescents or bananas. Here the spatial drifts of particles with $\Omega \approx 0$ become particularly important in the calculation of the diffusion coefficients. Calculations are in progress using Fokker-Planck collision integrals and preliminary results have been reported.¹⁶⁻¹⁸ However, due to the small value of the inverse aspect ratio, δ , the banana regime may be difficult to access. In fact, recent point model calculations of the EBT reactor² indicate favorable steady state burn points in the more collisional regimes of operation.

In the modeling calculations discussed in succeeding sections, the neo-classical transport package that we use in the code encompasses the collisional and plateau regimes. Specifically, we use the transport coefficients given in Equations (27) and (29).

2. Inductive Flow

All of the calculations of neoclassical transport are for collisional diffusion relative to a constant ψ surface. However, the equilibrium equation allows for the rearrangement of flux inside of the system. This change of flux gives rise to an inductive electric field, Equation (19), and the resulting $E \times B$ drift of plasma should be included in the calculation of transport for

self-consistency. For reactors with plasma β 's of 30%, the inductive term can approach the collisional diffusion terms. The total flux is then given by

$$\begin{aligned}\langle \Gamma \cdot \nabla V \rangle &= \langle \Gamma_c \cdot \nabla V \rangle + \langle nc \frac{E \times B}{B^2} \cdot \nabla V \rangle \\ &= \langle \Gamma_c \cdot \nabla V \rangle + n \langle v_\psi \cdot \nabla V \rangle\end{aligned}\quad (30)$$

where Γ_c is the collisional flux given by Equation (23). Furthermore, the term $5/2 nT v_\psi$ is added to the electron and ion energy fluxes.

E. REFUELING MODEL

In the present EBT-1 device, plasma refueling is accomplished by feeding in cold neutral gas at the plasma boundary. The low density of this device ($n \approx 2 \times 10^{18} \text{ m}^{-3}$) permits the penetration of these neutrals to maintain a steady-state plasma density. Neutral penetration is also necessary for thermal stability of EBT-1.¹⁹

In a reactor device with density $> 10^{20} \text{ m}^{-3}$ and a minor radius $\approx 1 \text{ m}$, penetration by the neutrals to the plasma core becomes impossible. Calculations performed with a neutral transport model for reactor parameters indicate that the neutrals are ionized within a few centimeters of the wall. Consequently, owing to hydrogen depletion from fusion reactions and diffusion losses, the hydrogen profiles become inverted, if refueling is attempted by neutral influx from the wall.

Thus the EBTR plasma must be fueled by pellet injection. As a crude model of this process, we assume in the code a constant hydrogen source rate across the plasma volume sufficient to maintain a constant total number of hydrogen particles.

F. ALPHA HEATING MODEL

The alpha particles, born with energy 3.5 MeV, thermalize from collisions with the colder, denser plasma. The resultant electron and ion heating can maintain their temperatures to ensure a sustained ignition. The manner in which we calculate the plasma heating is as follows:

The alpha density is calculated from Equation (2). A routine, employing volume averaged alpha density and plasma densities and temperatures, solves a Fokker-Planck equation for a volume averaged alpha particle distribution function. Included in the calculation are terms accounting for the production of high energy alphas and the diffusion loss of thermal alphas. From the solution of the Fokker-Planck equation can be obtained volume average electron and ion heating rates and an average alpha energy. The heating is apportioned spatially to the electrons according to the product of electron and alpha densities, while conserving the total energy. The heating is apportioned to the ions in a similar manner.

G. BOUNDARY CONDITIONS

The one-dimensional equations, (15)-(20), and the two-dimensional equation, (11), require boundary conditions for their solution. The code requires either the specification of the value of a variable or its flux at the boundary; e.g., either n or Γ , T_e or Q_e , T_i or Q_i . In the case of the equilibrium equation, (20), $K \frac{\partial \psi}{\partial V}$ is $\frac{4\pi}{c}$ times the current enclosed within a flux surface. Thus Equation (20) requires as a boundary condition either the specification of the total current in the system or the amount of magnetic flux being introduced into the system. (The flux in the system would change if the currents in the external coils were to change. The normal boundary condition is $\dot{\psi} = 0$.) The outermost surface is assumed to be fixed in these calculations and is not a free boundary. Thus the total volume of the system remains constant.

III. MODELING OF THE EBT REACTOR

To assess the importance of profile effects on EBT modeling we compare results from one-dimensional calculations to previously published zero-dimensional ones.² For a baseline we chose the parameters for which the zero-dimensional code predicts ignition

$$\begin{aligned}R_T &= 60 \text{ m (major radius)} \\a &= 1 \text{ m (minor radius at midplane)} \\a &= .71 \text{ m (minor radius at throat)} \\N &= 24 \text{ (number of mirror sectors)} \\B_0 &= 1.8 \text{ T (midplane magnetic field on axis)}\end{aligned}$$

The total microwave power delivered to the electrons was 240 MW corresponding to an average of $.2 \text{ MW/m}^3$. The average radius of curvature was taken to be $R_c = R_{co}^2/r$ where $R_{co} = 2.7 \text{ m}$.

The zero-dimensional code uses Kovrizhnykh transport only, Equation (27), so for these calculations we did the same and turned off the plateau transport. As a further comment on the model, we specified a spatially uniform source of ions at each time step sufficient to maintain a constant (in time) number of hydrogen particles.

In the first set of runs we took the alpha diffusion to be governed by the thermal alpha population. And the alpha thermal energy was taken to be the ion thermal energy. Thus the temperature employed in the calculation of the alpha transport was the ion thermal energy. Also the microwave power was programmed to fall to zero linearly between 15 and 20 sec.

In Figure 3 we plot time histories of volume averaged electron, ion, alpha temperatures. In Figure 4 are plotted volume averaged alpha and ion densities, and in Figure 5 the alpha heating and the thermal (conduction and radiation) loss and β . As noted above, the microwave heating falls to zero at 20 sec. Thus for these parameters a steady state ignition condition is achieved with average electron and ion temperatures of $\approx 10 \text{ keV}$, alpha energy

of 24 keV, hydrogen density of $2.5 \times 10^{20} \text{ m}^{-3}$, alpha density of $2.5 \times 10^{19} \text{ m}^{-3}$. Other results from this calculation were: plasma particle confinement time, 3.3 sec; energy confinement time, .72 sec; alpha particle confinement time, 11 sec; neutron wall loading, 2.6 MW/m^3 .

In Figure 6 we plot radial profiles of the electron, hydrogen, and alpha particle densities at $t = 40$ sec. The hydrogen density is not much changed from its initial profile. The electron density equalled the ion density initially and charge neutrality necessitates the increase in electron density since the hydrogen is maintained constant and alphas are produced. In Figure 7 we plot radial profiles of the electron and ion temperatures and the ambipolar potential at $t = 40$ sec, at the same time. The temperature profiles are flat except at the edge. And the potential rises in response to this temperature gradient. Figure 5 shows the value of β reaching .55 at 40 sec. To see the effect which this produces on the magnetic field, we plot in Figure 8 the rms average magnetic field (averaged along a field line) at $t = 0$ and 40 sec versus the radius at the midplane. The plasma has dug a hole in the field by pushing it toward the outside. This high value of β implies a much higher ring β to stabilize the plasma against MHD modes.⁴

In this run, for the electrons $v/\Omega \approx 6$ across the profile, for the ions $v/\Omega \approx .3$, and for the alphas $v/\Omega \approx 2$. Thus the use of Kovrizhnykh transport is justified for the electrons and alphas, but banana orbit effects might be coming into effect for the ions.

These results are similar to the 0-D results and thus it appears that the 0-D code can give a reasonable overall picture of the behavior of the system. However, the 1-D code can give useful profile information such as the effect of the plasma on the background field and refueling on the density profiles.

To check the effect of the alpha confinement on the behavior of the system, we performed a number of runs where we varied the alpha particle flux from the previous case. In Figures 9-11 are shown results from a run where we divided the alpha particle flux by 100, in effect preventing the alphas from escaping. In Figure 9 are plotted average temperatures and in Figure 10 are plotted average densities. In Figure 11 are plotted the alpha power, the thermal losses, and β vs. time. Except for the increase in alpha density, $\langle n_\alpha \rangle \approx 8.5 \times 10^{19} \text{ m}^{-3}$ and $\tau_\alpha \approx 4000$ sec, with an associated increase in

bremsstrahlung radiation from the previous case, .47 MW/m to 1.3 MW/m, the results are similar. The small increase in temperature has caused the conduction loss to decrease such that the total thermal loss is approximately the same. We note from Figure 11 that beta in this run is .85.

To calculate the effect of an increased alpha loss on the performance of the system, we used the average alpha energy as calculated by the Fokker-Planck routine in the evaluation of alpha diffusion coefficients. This energy can be over an order of magnitude greater than the ion thermal energy, and its use in the Kovrizhnykh transport coefficients implies a much larger particle flux.

In Figure 12 are plotted average ion, electron, and alpha temperatures versus time. In Figure 13 are plotted average ion and alpha densities, and in Figure 14 the alpha heating, the thermal loss rate and β . In this run the average ion and electron temperatures of 5.4 keV are reduced by roughly a factor of 2 from the baseline case and the alpha energy is increased from 24 keV to 95 keV. The increased alpha energy is due to the fact that the alphas escape before they can slow down. The alpha density is down from $2.5 \times 10^{19} \text{ m}^{-3}$ to $3.2 \times 10^{17} \text{ m}^{-3}$ with a decrease in confinement time from 11 sec to 6 sec. However, the system is ignited (alpha heating $\approx$ thermal loss) as illustrated in Figure 14. The lower plasma temperatures imply a lower conduction loss, and the lower alpha density implies a lower bremsstrahlung loss. The total thermal loss is thus reduced from the baseline case, and it equals the alpha heating.

In Figure 15 are plotted hydrogen density versus midplane radius at $t = 0$ and $t = 40$ sec, and in Figure 16 are plotted radial profiles of electron and temperatures and ambipolar potential at $t = 40$ sec. From Figure 15 the ion density has peaked in the center up from its initial value unlike the control case, but the total number of ions is constant. The temperature profiles from Figure 16 are essentially flat, and the ambipolar potential rises at the edge in response to temperature gradients.

It should be noted that the zero-dimensional code gave negative alpha particle confinement times for this calculation. This is due to the fact that the minor radius is used as a gradient scale length. The 1-D code did not have this difficulty, since the gradients can vary in space and time and adjust to the transport.

We were able to quench the ignition by a further increase in alpha particle flux. When alpha density was decreased to $n_{\alpha} \approx 10^{16} \text{ m}^{-3}$, the alpha heating was not sufficient to maintain the plasma temperatures, and the system did not ignite.

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Figure Captions

- Figure 1. Coordinate system for the EBT model equations.
- Figure 2. Bumpy cylinder model of EBT.
- Figure 3. Volume averaged electron, ion, and alpha energies vs. time. The ion thermal speed is used for the alpha transport calculation and the microwave power is decreased to 0 between 15 and 20 sec.
- Figure 4. Volume averaged ion and alpha densities vs. times for the case of Figure 3.
- Figure 5. Thermal losses, alpha heating and β vs. time for the case of Figure 3.
- Figure 6. Hydrogen, electron, and alpha densities at $t = 40$ sec vs. midplane radius for the case of Figure 3.
- Figure 7. Electron and ion temperatures and ambipolar potential at $t = 40$ sec vs. midplane radius for the case of Figure 3.
- Figure 8. RMS line averaged magnetic field at $t = 0$ and 40 sec vs. midplane radius for the case of Figure 3.
- Figure 9. Same as Figure 3 but with the alpha particle flux divided by 100.
- Figure 10. Same as Figure 4 for the case of Figure 9.
- Figure 11. Same as Figure 5 for the case of Figure 9.
- Figure 12. Same as Figure 3 but with the average alpha energy used in the calculation of the alpha transport.
- Figure 13. Same as Figure 4 for the case of Figure 12.
- Figure 14. Same as Figure 5 for the case of Figure 12.
- Figure 15. Same as Figure 6 for the case of Figure 12.
- Figure 16. Same as Figure 7 for the case of Figure 12.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16