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NEOCLASSICAL EFFECTS ON RF CURRENT DRIVE

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ABSTRACT

A functional form is derived for the efficiency of current drive in toroidal geometry with low frequency waves (frequencies below the electron cyclotron frequency). By fitting constants in order to duplicate numerical results for the efficiency we obtain an accurate function which will be useful for computer calculations of current drive.

1. INTRODUCTION

In an axisymmetric tokamak in steady-state equilibrium, assuming there is no inductive electric field, the current density parallel to the magnetic field may be expressed [1] as $j_{\parallel} = j_{\parallel}^{\text{PS}} + B(G + H)$, where $j_{\parallel}^{\text{PS}}$ is the well-known Pfirsch-Schlüter current, which is proportional to the pressure gradient, and B is the magnetic field strength. A goal of steady-state current drive is to arrange for the bootstrap current, H , and noninductive current, G , contributions such that the resulting MHD equilibrium has desirable stability properties with a minimum amount of external driving power. Whereas the bootstrap current owes its existence to the neoclassical orbit effects associated with finite aspect ratio, [2] it is clear that magnetic trapping will degrade the efficiency of generating current with radio waves (rf). In this note an analytic expression is derived in order to provide a convenient function for

accurate ray tracing or full wave calculations of rf current drive in realistic toroidal geometries.

The analysis of current drive presented here hinges on the formulation of the problem in terms of a response function χ [3]. This response function is the solution to the perturbed electron distribution in the presence of collisions and a weak electric field. Given χ , the current driven by a particular rf-induced flux of electrons in velocity space is given by a simple velocity-space integral. Our work consists of finding a simple analytical expression for the current drive efficiency based on an approximate expression for χ and on certain simplifications in carrying out the velocity-space integrals over χ . This is checked against the exact results found by calculating χ and the velocity-space integrals numerically.

Our analytic approach demonstrates that there are two principal neoclassical effects which act to diminish rf current drive efficiency. For current drive at high phase speeds, typically achieved by Landau damping of lower hybrid slow waves resonant at parallel velocities above the electron thermal speed ($v_e < \omega/k_{\parallel}$), wave energy is added to electrons far from the trapped region of velocity space, but these current carriers will eventually diffuse via collisions until they become magnetically trapped and cease to provide a parallel electric current. On flux surfaces with a local inverse aspect ratio $\epsilon \lesssim 0.1$ the current drive efficiency is moderately reduced [4]. On the other hand, some waves, such as the low-frequency fast (compressional Alfvén) wave, may resonate with low phase speed electrons via combined Landau and transit time magnetic damping. Since these waves mostly add energy at low parallel velocities, $\omega/k_{\parallel} \lesssim v_e$, and over a range of large perpendicular velocities, $v_e \lesssim v_{\perp}$, they directly interact to a great extent with magnetically trapped electrons. As the wave's phase speed $\omega/k_{\parallel} \rightarrow 0$, we expect the current drive efficiency to vanish. Both of these neoclassical effects discussed above are qualitatively derived in our analytic work; however this approach, which uses Giruzzi's response function [4], cannot be quantitatively accurate due to the high energy approximation assumed for the electron collision operator.

Therefore, we additionally solved for the current drive efficiency by integrating the numerically derived response function computed from the bounce-averaged adjoint equation in toroidal geometry [5]. These calculations

are done for several choices of ion charge (Z), inverse aspect ratio, poloidal location where the wave heating occurs (θ), a large range of phase speeds ($w \equiv \omega/k_{\parallel} v_e$), and both Landau damping (LD) and Alfvén wave (AW) types of resonance. (Relativistic corrections are small, $O[T_e/m_e c^2]$, for plasma electron temperatures $T_e \lesssim 50$ keV for the waves we consider, so only the nonrelativistic results are presented here.) On a given poloidal flux surface, ψ , the rf-driven current density is expressed as [1,5]

$$\langle j_{\parallel}^{rf} B \rangle / \langle B^2 \rangle \equiv G(\psi) = \frac{V'}{L} \eta \langle p^{rf} \rangle ,$$

where $V' = dV/d\psi$ is the derivative of the toroidal volume, p^{rf} is the rf heating power density, and $\langle \rangle$ denotes the flux surface average [6]. The quantity L is the connection length of a field line, from one intersection of the midplane to the next ($L = 2\pi R_0 q(\psi)$, where R_0 is the major radius and q is the safety factor). In our normalization the current drive efficiency is related to a dimensionless function $\tilde{\eta}$ as $\eta = \frac{38.4 \times 10^{18}}{\ln A} \frac{T_e}{n_e} \tilde{\eta}$, where $\ln A$ is the Coulomb logarithm, n_e is the local electron density, and all units are SI and keV. By comparing the numerically determined $\tilde{\eta}$ with our analytic expression it proves possible to select constants in the $\tilde{\eta}$ expression in order to fit closely the numerical values over the range studied. The final results are functions $\tilde{\eta}(Z, \epsilon, \theta, w)$ of four variables, for both wave types.

In addition to Giruzzi's work [4] there have been other previous studies relevant to our problem. For current drive by Landau damping an early account of trapping effects was provided by Cordey, Eddington, and Start [7], and more accurate results have been recently provided by the CQL computer code [8]; in both references, however, the bounce-averaged Fokker-Planck equation essentially assumes uniform flux surface illumination by the rf, so no explicit θ dependence appears. For Alfvén wave heating even the published $\epsilon=0$ results are inaccurate, i.e., the Fokker-Planck calculation of Fisch and Karney [9] resulted in $\tilde{\eta}$ typically 20% smaller than the adjoint solution found here; and results for $\epsilon=0.1$ in Ref. 10 are quantitatively correct only for $w \gg 1$ due to the neglect of electron-electron collisions.

2. ANALYTIC MODEL

In the semi-analytic calculation of $\tilde{n}$ we select velocity space variables which are local to the poloidal position θ where the rf heating occurs on a flux surface. An electron's parallel and perpendicular velocity components are normalized to the thermal speed, $v_e \equiv \sqrt{T_e/m_e}$, as $w = v_{\perp}/v_e$ and $x = v_{\parallel}/v_e$; and its normalized speed is $u = \sqrt{x^2 + w^2}$. If the maximum field amplitude on a flux surface is $B_M \equiv B(\psi, \pi)$ and the local field is $B = B(\psi, \theta)$, then electrons with $x < x_t \equiv w (B/B_M)^{1/2} [1 - (B/B_M)]^{-1/2}$ are free to circulate along the field line, while electrons with $x_t < x$ are trapped and cannot contribute to $j_{\parallel}^{rf}$ in a time-averaged sense. It is convenient to also define a pitch angle $\lambda \equiv w/u$; trapped electrons have $0 \leq \lambda < \lambda_t$, where

$$\lambda_t^2 = 1 - (B/B_M) \quad . \quad (1)$$

Note that for circular flux surfaces at low beta and large aspect ratio the field is given by $B \approx 1/(1 + \epsilon \cos \theta)$, so

$$\lambda_t^2 \approx 1 - [(1 - \epsilon \cos \theta)/(1 + \epsilon)] \quad ,$$

where $\epsilon = r/R_0$.

If the response function $\chi(w, x)$ is known at θ then the rf current density is found [3] by an integration over circulating particle velocity space:

$$\tilde{j} = 2\pi \int_{-\infty}^{\infty} dw \int_0^{x_t} dx \, x \, \underline{S} \cdot \frac{\partial \chi}{\partial \underline{u}} \quad , \quad (2)$$

where

$$\underline{S} \cdot \frac{\partial \chi}{\partial \underline{u}} = (w D^{rf} f) \frac{\partial \chi}{\partial w} \quad , \quad (3)$$

assuming linear damping by waves which push electrons along the field lines, so the electron distribution remains nearly Maxwellian -- $f = Ne^{-u^2/2}$. The perpendicular velocity dependence of rf diffusion differs for Landau damping and Alfvén waves [9,11]:

$$D^{rf} = D_0 \delta\left(\omega - \frac{\omega}{k_{\parallel} v_e}\right) \times \frac{1}{(2 - x^2)^2} \quad ; \quad \begin{matrix} \text{LD} \\ \text{AW} \end{matrix} \quad (4)$$

where D_0 is a constant and a narrow $k_{\parallel}$ spectrum is assumed. Wave heating occurs for trapped as well as circulating electrons and is given by:

$$\tilde{p} = 4\pi \int_{-\infty}^{\infty} d\omega \int_0^{\infty} dx \, x \, \underline{S} \cdot \frac{\partial}{\partial \underline{u}} \left(\frac{u^2}{2} \right) \quad , \quad (5)$$

where we have

$$\underline{S} \cdot \frac{\partial}{\partial \underline{u}} \left(\frac{u^2}{2} \right) = \omega^2 D^{rf} f \quad . \quad (6)$$

Although the rf current density and heating power density vary along a field line, the flux-surface-averaged efficiency is given by the ratio of the local quantities in Eqs. (2) and (5),

$$\tilde{\eta} = \tilde{j} / \tilde{p} \quad . \quad (7)$$

Use of the approximate response function of Giruzzi [4], $x = [1 - (\lambda_t/\lambda)^{\alpha}]^{1/2} \omega u^3 / (5 + Z)$, results in the following partial derivative in Eq. (3):

$$\frac{\partial x}{\partial \omega} = \left[1 - \left(\frac{\lambda_t}{\lambda} \right)^{\alpha} \right] \frac{2(3 \omega u^2 + u^3)}{(5 + Z)} + \frac{2}{(1 + Z)} \left(\frac{\lambda_t}{\lambda} \right)^{\alpha} x^2 u \quad . \quad (8)$$

For a straight magnetic field ($\lambda_t \equiv 0$) this reduces to the well-known Fisch-Boozer result [12]. The derivation of this expression assumed that wave energy is transferred to high energy electrons, which is a good approximation at $w \gg 1$. At low phase speeds ($w \lesssim 1$) this condition can be mildly satisfied, e.g., for AW damping at $\epsilon=0$, since the rf diffusion of Alfvén waves is dominated by electrons at $x \gtrsim 1$. For $\epsilon \neq 0$, however, the assumption is violated at $w \ll 1$ since only electrons with $x < 1$ are free to carry current. As we shall see, Eq. (8) nevertheless results in the correct functional dependence of $\tilde{\eta}$ for $\epsilon \leq 0.1$ and $w \geq 0.2$. Our approach now is to evaluate $\tilde{\eta}$ in the two phase speed limits and then seek a uniform approximation over the full range of w .

At high phase speeds the exponential dependence $f = e^{-x^2/2}$ cuts off the integrals at large x so we can use the approximation $x \ll w$. Then $u = w$ in the first term of Eq. (8) and we can neglect the second term. The resulting efficiency at $w \gg 1$ is given by

$$\tilde{\eta} = \lim_{w \gg 1} \left[\frac{\partial x / \partial w}{w} \right] = \left[1 - \left(\frac{\lambda_t}{\lambda_r} \right)^\alpha \right] \frac{4 w^2}{(5+Z)} \quad (9)$$

We have evaluated the pitch angle at $\lambda_r \equiv w/u_r$ with $u_r \equiv \sqrt{x_r^2 + w^2}$, where x_r is a constant. Giruzzi suggests evaluating λ_t for these circulating electrons at an "averaged" value of θ , so $\lambda_t \sim \epsilon^{1/2}$, and he finds the exponent $\alpha = (5+Z)/(1+Z)$, which approaches unity as Z becomes large. In order to match our numerical results for $\tilde{\eta}$ we replace Eq. (9) with

$$\tilde{\eta} = R(\epsilon, w) \eta_0(Z, w)$$

where

$$R = \left[1 - \frac{\epsilon^n \sqrt{x_r^2 + w^2}}{\epsilon^n x_r + w} \right] \quad (10)$$

and, in the $w \gg 1$ limit,

$$\eta_0 \rightarrow \frac{4}{5} \frac{\omega^2}{2} \quad (11)$$

The function R is a generalization which qualitatively agrees with previous calculations [4,7,8] that show a reduction in $\tilde{\eta}$ for $c \neq 0$, at $\omega \gg 1$ (assuming x_p and ϵ^n are of order unity), but which also behaves as $R \rightarrow \omega/\epsilon^n x_p$ as $\omega \rightarrow 0$. (This small ω behavior of R , which has not been derived, will be required in order to reproduce the correct ω dependence of $\tilde{\eta}$ when $\omega \leq 1$ and $c \neq 0$.) The exponent n will be determined by comparison with the numerical results.

In the low phase speed limit the evaluation of $\tilde{\eta}$ requires explicit integrations over x for $\tilde{p}$ and $\tilde{j}$; the integral for $\tilde{p}$ is straightforward, while the endpoint, x_t , of integration plays a dominant role in $\tilde{j}$. Since $D^{rf}(x)$ differs for LD and AW heating the two wave types are treated separately. Considering first LD, we can set $u = x$ and $\lambda = \omega/x$ when $\omega \ll 1$. Doing this, we insert both terms of Eq. (8) into Eq. (2), which results in an expression for $\tilde{j}$ which involves functions of the form $I(p, x_t) = \int_0^{x_t} dx x^{p+1} e^{-x^2/2}$. To simplify matters we replace $I(p, x_t)$ with a monotonic function of x_t which preserves the correct values in both the large and small x_t limits:

$$I(p, x_t) \approx 2^{p/2} \Gamma\left(\frac{p+2}{2}\right) \{1 - \exp[-b(p)(y_t/2)^{(p+2)/2}]\} \quad ,$$

where

$$y_t \equiv x_t^2 = \frac{(1 - \lambda_t^2) \omega^2}{\lambda_t^2} \quad , \quad (12)$$

$b(p)^{-1} \equiv [(p+2)/2] \Gamma(\frac{p+2}{2})$, and $\Gamma(z)$ is the gamma function. Comparing the various contributions to $\tilde{j}$ we find that terms proportional to ω^2 are generally negligible; and an adequate approximation to the result is

$$\tilde{j} = \frac{6 \sqrt{2} \pi^{3/2}}{(5 + Z)} D_0 N w e^{-w^2/2} C(\epsilon, \theta, w) M(\epsilon, \theta, w)$$

where

$$C = \{1 - \exp[-(cy_t)^m]\} \quad (13)$$

and

$$M = [1 + A (\lambda_t/w)^k] \quad (14)$$

Dividing this by $\tilde{p}$ we obtain the low phase speed limit $\tilde{\eta} = \eta_0(Z, w) C(\epsilon, \theta, w) M(\epsilon, \theta, w)$

where, for $w \ll 1$

$$\eta_0 \rightarrow K/w. \quad (15)$$

Our theoretical value of K is $3\sqrt{\pi/2} / (5 + Z)$, which yields the correct asymptotic LD form for η_0 only in the Lorentz gas limit ($Z \gg 1$); the actual value should be [7] $K = 3\sqrt{\pi/2} / Z$. When $\epsilon \neq 0$ and $\theta \neq \pi$, the factor C severely cuts off the current drive efficiency at small w , with our LD calculation having $m=2.5$ and $c=0.31$. In the factor M , which is expected to be of order unity for $w \geq 1$, both A and k ($= \alpha$) are functions of Z , but we will suppress this Z variation in our expression for $\tilde{\eta}$. All the constants in Eqs. (13)-(15) will be adjusted to fit the numerical results for $\tilde{\eta}$ at $Z=1.0$ and 2.0 .

The derivation of $\tilde{\eta}$ for AW damping in the $w \ll 1$ limit follows a similar procedure with the appropriate $D^{rf}(x)$ from Eq. (4) in both the $\tilde{p}$ and $\tilde{j}$ integrals. The functions for $\tilde{\eta}$ are identical to Eqs. (13)-(15) except the constants differ from the LD values. We calculate, for example, $K=32.9/(5+Z)$ [which differs from the correct AW asymptotic result [11] which is roughly $K = 15/(0.678 + Z)$]. K is of course larger for the AW than for LD since less-collisional electrons at larger x are carrying the rf current. By the same token the detrimental effects of trapping set in more quickly for the AW as w

is reduced; we calculate a larger $m=4.5$, with $c=0.21$. In the factor M the exponent k should be the same as for LD, but the coefficient A is calculated to be larger for AW current drive.

Since C and M approach unity for large w we attempt a uniform approximation to the efficiency as

$$\tilde{\eta} = C M \eta_0 R; \quad (16)$$

the straight-field efficiency, η_0 , must agree with the limiting forms in Eq. (11) and (15), so we adopt the following function

$$\eta_0 = \left\{ \frac{K}{w} + C + \frac{4 w^2}{(5 + Z)} \right\} \quad (17)$$

As with $K(Z)$ we expect the value of C will depend on Z . Although an analytic expression for $C(Z)$ is suggested in Ref. 11, our method is to select both $K(Z)$ and $C(Z)$ by comparison with the numerical results computed for the cases with $\epsilon=0$.

3. NUMERICAL RESULTS

The response function χ was calculated numerically in Ref. 5. (There is one departure in the results presented here from those of Ref. 5: we treat the linearized electron-electron collision term exactly rather than truncating the Legendre expansion of the collision term after the first term. In fact, this changes the results only slightly.) The important approximations are: the collision time is much larger than the transit time or bounce time for the electron (the banana limit); the drifts of electrons across the field lines is neglected; the ion collision term is calculated in the Lorentz limit ($m_i \rightarrow \infty$). In addition, by expressing the current drive in terms of a response function, we are assuming that the collision term may be linearized about a Maxwellian. The results quoted here are all nonrelativistic; i.e., the Landau collision operator is used and relativistic effects are not included in the wave-particle interaction. The magnetic surfaces are assumed to be circular with

$$\begin{aligned}
R &= R_0 (1 + \epsilon \cos \theta), \\
B_{\text{tor}} &= B_{\text{tor},0} / (1 + \epsilon \cos \theta), \\
B_{\text{pol}} &= B_{\text{pol},0} / (1 + \epsilon \cos \theta).
\end{aligned}$$

The current-drive efficiency $\tilde{\eta}$ is then given by Eqs. (7), (2), and (5) with $\underline{S}$ given by Eqs. (3) and (4). The analytic function was fitted to the numerical results with the following constants:

$$\begin{aligned}
K &= \begin{cases} 3.0/Z \\ 11.91/(0.678 + Z) \end{cases} \begin{matrix} , \text{ LD} \\ , \text{ AW} \end{matrix} \\
C &= \begin{cases} 3.83/(Z^{0.707}) \\ 4.13/(Z^{0.707}) \end{cases} \begin{matrix} , \text{ LD} \\ , \text{ AW} \end{matrix} \\
n &= 0.77 \qquad \qquad \qquad x_r = 3.5 \\
m &= \begin{cases} 1.38 \\ 2.48 \end{cases} \begin{matrix} , \text{ LD} \\ , \text{ AW} \end{matrix} \\
c &= \begin{cases} 0.389 \\ 0.0987 \end{cases} \begin{matrix} , \text{ LD} \\ , \text{ AW} \end{matrix} \\
k &= 3.0 \\
A &= \begin{cases} 0.6 \\ 12.3 \end{cases} \begin{matrix} , \text{ LD} \\ , \text{ AW} \end{matrix}
\end{aligned}$$

The analytic function agrees within $\pm 12\%$ of the numerical results over the range of interest -- $w > 0.5$ and for non-negligible $\tilde{\eta}$ (> 3), for $0 \leq \epsilon \leq 0.25$, for $Z=1$ and 2 , and for both wave polarizations, and for most θ . The exception is for $\theta = 3\pi/4$, $\epsilon = .01$ for the AW at $w = 0.5$, where the error is in excess of 40% ; this inaccuracy occurs where $\tilde{\eta}$ is a fast-varying function of θ and w and is acceptable for ray tracing and full wave calculations in which the actual current generation is due to a broad range of w and θ values.

The analytic function (curves) is compared with the numerical results (points) in Figs. 1 and 2 for LD. Since LD is only important at $w \geq 1$ (wave damping typically occurs before $w \leq 1$ is achieved) the agreement is seen to be quite satisfactory. Note that $\tilde{\eta}$ is essentially independent of θ for $w \geq 1$.

Figure 1. Current drive efficiency vs. parallel phase speed for Landau damping and $Z = 1.0$. Top curve is $\epsilon = 0$; dashed is $\epsilon = 0.03$ and $\theta = \pi$; bottom is $\epsilon = 0.25$; and solid curves are $\epsilon = 0.10$ with wave damping at four different poloidal angles. Curves are analytical formula and points are numerically derived.

Figure 2. Current drive efficiency, Landau damping and $Z = 1.0$, for $\epsilon = 0.03$ and four poloidal angles, as well as for $\epsilon = 0$ (top curve).

Figure 3. Alfvén wave current drive, $Z = 1.0$: $\epsilon = 0$; $\epsilon = 0.03$, $\theta = \pi$; and $\epsilon = 0.10$, $\theta = \pi$, $3\pi/4$, $\pi/2$ and 0 .

Figure 4. Alfvén wave current drive, $Z = 1.0$: $\epsilon = 0$; and $\epsilon = 0.03$, $\theta = \pi$, $3\pi/4$, $\pi/2$, and 0 .

Figure 5. Alfvén wave current drive, $Z = 2.0$: $\epsilon = 0$; $\epsilon = 0.03$, $\theta = \pi$; and $\epsilon = 0.10$, $\theta = \pi$, $3\pi/4$, $\pi/2$, and 0 .

Figure 6. Alfvén wave current drive, $Z = 2.0$; $\epsilon = 0$; and $\epsilon = 0.03$, $\theta = \pi$, $3\pi/4$, $\pi/2$ and 0 .

The reduction of $\tilde{\eta}$ with increasing ϵ (including the calculated values at $\epsilon = 0.25$ from the CQL code [8]) is substantial.

Figures 3 and 4 show for the same $Z = 1.0$ that the AW has a larger $\tilde{\eta}$ than LD when $\epsilon = 0$ or $\theta = \pi/2$. In realistic situations ($\epsilon \neq 0$, $\theta < \pi/2$) the AW result is, however, smaller than the LD value when $w \leq 1$. Our experience with ray tracing shows that AW current drive is dominated by $w \leq 2$ so it is important to select spectra and launch positions such that $w \geq 1$ for fast-Alfven wave current drive. Finally, Figs. 5 and 6 show a significant reduction of efficiency with Z , underscoring the desirability of maintaining low impurity content. Overall the analytic and numerical AW results are seen to be in good agreement over the important range $1.0 \leq w \leq 2.0$.

4. CONCLUSIONS

We have derived a function of multiple variables which is the efficiency for driving current in toroidal geometry with radio waves via either Landau damping or the Alfven wave-mechanism. The function, given by Eqs. (10), (13), (14), (16), and (17), with the constants given in Sec. 3, agrees well with the numerically determined current drive efficiency over the range of variables of practical interest to large tokamaks. This function should facilitate numerical (ray tracing or full wave) treatments of tokamak current drive.

It is noteworthy that our $w \ll 1$ results may be unduly pessimistic. Some recent calculations have suggested that energy dissipated on trapped electrons will also result in significant current drive [13], and further work along these lines should be encouraged.

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REFERENCES

1. D.A. Ehst, Nucl. Fusion 25 (1985) 629.
2. S.P. Hirshman, Phys. Fluids 31 (1988) 3150.
3. T.M. Antonsen, Jr. and K.R. Chu, Phys. Fluids 25 (1982) 1295.

4. G. Giruzzi, Nucl. Fusion 27 (1987) 1934.
5. C.F.F. Karney, N.J. Fisch, and A.H. Reiman, in Radio-frequency Power in Plasmas, 8th Top. Conf., Irvine, CA, 1989, edited by R. McWilliams (American Inst. of Physics, New York, 1989), p. 430.
6. D.A. Ehst, K. Evans, Jr., and D.W. Ignat, Nucl. Fusion 26 (1986) 461.
7. J.G. Cordey, T. Eddlington, and D.F.H. Start, Plasma Physics 24 (1982) 73.
8. S.C. Chiu, V.S. Chan, R.W. Harvey, and M. Porkolab, Nucl. Fusion 29 (1989) 2175.
9. N.J. Fisch and C.F.F. Karney, Phys. Fluids 24 (1981) 27.
10. K. Yoshioka and T.M. Antonsen, Jr., Nucl. Fusion 26 (1986) 839.
11. C.F.F. Karney and N.J. Fisch, Phys. Fluids 28 (1985) 116.
12. N.J. Fisch and A.H. Boozer, Phys. Rev. Lett. 45 (1980) 720.
13. S. Puri and R. Wilhelm, in Radio-frequency Power in Plasmas, op. cit., p. 458; A.G. Elfimov and S. Puri, Max-Planck-Institut für Plasmaphysik Rep. IPP4/240 (1989).