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UNDER BLANKET RELEVANT CONDITIONS***

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ABSTRACT

A flow instability has been observed in a rectangular thin conducting wall duct in MHD experiments at ANL's ALEX facility. It developed in a side layer jet attached to a side wall parallel to the applied B-field. The nature of the instability resembles very strongly that of a classic laminar instability in ordinary fluid mechanics. The phenomenon is distinguished from ordinary turbulence by its strong periodicity and its lack of small scale structure. A preliminary identification of the factors which may influence the instability has been made. For the conditions and test section geometry reported here, the instability appears at an approximately constant critical Reynolds number $2650 < Re_{cr} < 5100$, independent of Hartmann number in the range $2700 < M < 5400$.

I. INTRODUCTION

To improve the level of understanding of liquid-metal MHD flows as they apply to blankets of magnetically confined fusion reactors, an experimental and code development effort is under way at Argonne National Laboratory (ANL). To date, both experiments, carried out at the ALEX (Argonne's Liquid Metal Experiment) facility, and modeling work have been focused on laminar, rectilinear MHD flows in both uniform and spatially varying magnetic fields near fusion reactor relevant conditions. The question of the existence of turbulence or flow instabilities in the ALEX experiments has been addressed only briefly previously. In that case, the test section was a round pipe in the fringing and uniform regions of a transverse magnetic field. For the case of Hartmann number and interaction parameter of 6000 and 10,000, respectively (close to reactor relevant conditions), no evidence of turbulence or indeed any flow instabilities was found.

The existence of the instability reported here was first noticed as unexpected and quite erratic fluctuations in oscilloscope traces of the output of the velocity measuring probe when it was placed very near the wall of a rectangular duct. Other results, such as measurements of mean velocity distribution in the flow direction, were consulted for corroborating evidence of erratic velocities near the wall. When this was found, a systematic effort was

- interaction parameter, N ,
- Hartmann number, M ,
- lateral coordinate, z , and
- axial coordinate, x ,

at which the instability first appears, and the structure of the instability as a function of these parameters.

If the subject instability occurs under fusion reactor relevant conditions, and it can be properly engineered into the design, its presence could enhance heat transfer and provide increased design margin for liquid-metal cooled blankets. It is possible that the combined effects of flow instabilities and flow tailoring can produce a very high performance liquid-metal cooled blanket.

II. LAMINAR WALL JETS IN MHD FLOWS

Before describing the facility, the test section, and results, the following brief review of laminar sidewall jets in MHD flows is presented because a simple description of the features of the jet for the laminar case can provide a common basis from which to begin understanding the physics of the instabilities.

We speak here of MHD flows in ducts with thin conducting walls under strong transverse magnetic fields and the accompanying boundary layer wall jets associated with the walls which are parallel to the magnetic field (sidewalls). These wall jets have a thickness of order $LM^{-1/2}$ and, in general, carry a sizable fraction of the total flow. Here, L is the characteristic transverse dimension of the duct (usually one-half of the duct dimension along the magnetic field lines), see Fig. 1, and $M = BL/\sigma\mu$ (B is the magnetic flux density and σ and μ are the electrical conductivity and viscosity of the fluid) is the Hartmann number. For liquid metals and strong magnetic fields, M is of order of 10^3 and higher. In contrast, the boundary layers on walls which have a non-zero normal component of the magnetic field, B_n , have a thickness of order $LM_n^{-1/2}$ (M_n is based on B_n) and carry a negligible fraction of the total flow. Detailed solutions for the sidewall jets in both constant and variable area rectangular ducts in a uniform transverse magnetic field have been obtained by Walker.² For fully developed flow in a constant area duct, a much simpler treatment can be used to obtain averaged parameters of the sidewall jets and to bring out the physi-

Although the treatment is limited to fully developed flows in uniform magnetic fields, the basic mechanisms that leads to the formation of the jets has a much broader validity.

Consider fully developed flow in the rectangular duct shown in Fig. 1. The aspect ratio of the duct is b ; the sides have, in general, unequal thicknesses and electrical conductivities whereas the top and bottom walls are identical. In the core of the flow, outside the side layers with thickness of order $LM^{-1/2}$ and the Hartmann layers with thickness of order LM^{-1} , the flow is inviscid. The governing equations are reduced to a balance of the pressure gradient $-\nabla p$ by the ponderomotive force $j \times B$, i.e.,

$$-\nabla p + j \times B = 0 \quad (1)$$

The additional equations to be satisfied are the conservation of mass, electric charge, and Ohm's law, namely,

$$\nabla \cdot u = 0, \quad \nabla \cdot j = 0, \quad j = \sigma(-\nabla \phi + u \times B) \quad (2)-(4)$$

Here, u is the velocity, ϕ is the electric potential, and j is the electric current density.

In fully developed flow, $-\nabla p = K$, where K is a constant. It follows from Eqs. (1) and (3) and the symmetry about the $y=0$ axis that the only non-zero component of the current density is $j_z = K/B$. Equation (4) requires that the same be true for the gradient of the electric potential, i.e.,

$$\frac{\partial \phi}{\partial x} = 0, \quad \frac{\partial \phi}{\partial y} = 0, \quad \text{and} \quad \frac{\partial \phi}{\partial z} = -\frac{j_z}{\sigma} + uB \quad (5)$$

The uniform current density crosses the core/side layer interface and flows across the side layer and into the side wall at $z = bL$ virtually unchanged, provided the side layer offers much higher electrical resistance to current flow than does the side wall. The condition for this to occur is $M^{-1/2} \ll \sigma_1 t_1 / \sigma L$, ($i = 2, 3$).

For $y > 0$, the current that has entered the side wall flows upward. The sidewall electric current per unit axial length increases linearly from $I = 0$ at $y = 0$ (by virtue of symmetry) to $I = Lj_z$ at $y = L$. From there, the current flows unchanged across the top wall, flows down the side wall at $z = -bL$ decreasing linearly, crosses the side layer there and enters the core of the flow, thus completing the circuit.

Ohm's law applied at the side wall gives a linear variation of the voltage gradient corresponding to the linear distribution of the electric current. It follows that the voltage along the side wall has a parabolic distribution

$$\phi_2 = \phi_0 + \frac{j_z}{2\sigma_2 t_2} (L^2 - y^2) \quad (6)$$

where ϕ_0 is the potential at the corner $y = L$, $z = bL$. Because the side layer thickness, δ , is very small, $\delta = O(LM^{-1/2})$, ϕ_0 is also the potential at the top wall, at $z = bL - \delta$. The electric potential is continuous across a liquid/solid interface and across the Hartmann layer. It is also constant in the core along the magnetic field lines, since $\partial \phi / \partial y = 0$. It follows that at the interface between the core and the side layer, the potential is constant and equal to ϕ_0 . Consequently, in Eq. (6), the

second term represents a jump in electric potential across the side layer.

$$\Delta \phi_{sl} = \frac{j_z}{2\sigma_2 t_2} (L^2 - y^2) \quad (7)$$

Ohm's law, Eq. (4), dictates that a finite flow rate be associated with this jump. More specifically, integration of Ohm's law across the side layer yields

$$\int_0^\delta u_{sl} dz = \frac{1}{B} \left[\frac{j_z}{\sigma} \delta + \Delta \phi_{sl} \right]$$

where sl denotes side layer variables. The left hand side of the equation is the volumetric flow rate in the side layer per unit y . The second term in the parentheses of the right hand side is given by Eq. (7). The first term in the parentheses is of order $c_2 M^{-1/2}$ compared to the second one and can be neglected. Therefore, the flow per unit y in the side layer, Q_2 , varies parabolically in y and is given by

$$Q_2 = \frac{j_z}{2\sigma_2 t_2 B} (L^2 - y^2) \quad (8)$$

whereas the total flow in the side layer, Q_2 , is given by

$$Q_2 = \frac{2}{3} \frac{j_z L^3}{\sigma_2 t_2 B} \quad (9)$$

Next, to provide a relationship between the current density j_z and the velocity, Ohm's law is applied to the top wall.

$$\frac{\partial \phi_w}{\partial z} = -\frac{j_w}{\sigma_1} = j_z \frac{L}{t_1} \frac{1}{\sigma_1} \quad (10)$$

As explained above, the voltage is continuous in the y direction and the voltage gradients given by Eqs. (5) and (10) are identical. It follows that

$$\frac{j_z}{\sigma} \left(1 + \frac{L\sigma}{t_{1\sigma_1}} \right) = uB \quad (11)$$

Note that because j_z is uniform over the cross section, so is the velocity u . The ratio $t_{1\sigma_1}/L\sigma$ is the wall conductance ratio of the top wall.

By virtue of Eq. (11), Eqs. (8) and (9) can be put in the more useful form

$$q_i = \frac{1}{2} \frac{c_i}{c_1} \frac{1}{1+c_i} \frac{u}{L} (L^2 - y^2) \quad (12)$$

and

$$Q_i = \frac{2}{3} \frac{c_i}{c_1} \frac{1}{1+c_i} uL^2 \quad (13)$$

$i = 2, 3$.

The flow rate in the core of the flow is given by $Q_{\text{core}} = 4uL^2bu$. Therefore,

$$Q_i = \frac{1}{6b} \frac{c_i}{c_1} \frac{1}{1+c_i} Q_{\text{core}}, \quad i = 2, 3$$

For a square duct with equal wall thicknesses $b = 1$, $c_1 = c_i$, each side layer carries one-sixth of the core flow or, alternatively, one-eighth of the total flow. Since each side layer carries a flow of the same order of magnitude as that in the core but has a cross sectional area which is smaller by a factor of M^2 , the velocities in the side layer are higher than the velocities in the core by the same factor, M^2 .

The existence of the side layers on walls parallel to the magnetic field is not confined to fully developed flows. As shown above, side layers must exist because the constant potential along the magnetic field lines (a fact that is valid in general as long as $c \ll 1$) does not match the non-uniform potential distribution in the side wall. The resulting potential difference can only be supported through a finite flow rate in the side layer.

The fact that side layer jets exist as a result of a potential jump across the side layers can be used to make general statements about the factors that affect the flow rate carried by the side layers. For example, increasing sidewall thickness and decreasing current density both make the potential distribution in the side layer more uniform. This reduces the potential jump, and thus reduces the flow rate in the side layer. The current density decreases with decreasing velocity or magnetic field, or conductance ratio, c . On the other hand, 3-D MHD effects increase the current density that enters the side wall and hence the flow rate in the side layers. Decreasing M

increases the side layer thickness, allowing more current to flow in the side layer along magnetic field lines and thus less current to enter the side wall. Consequently, the flow rate in the side layer decreases although the side layer thickness increases.

With this review of the basic mechanisms governing the structure of laminar wall jets as a backdrop, the present results describing instabilities within the wall jet can be more fully understood.

III. DESCRIPTION OF FACILITY AND TEST SECTION

The results presented here were acquired from a rectangular thin-walled stainless steel test section of uniform cross section, $2L \times 2L$ (see Fig. 1), which was installed in the ALEX facility. ALEX is a facility for collecting data on magnetohydrodynamic (MHD) effects in liquid metal cooled fusion reactor blankets. The facility, designed to extend significantly the existing data base on liquid metal MHD, employs eutectic NaK as the working fluid in a room-temperature closed loop. The instrumentation system is capable of collecting detailed data on pressure, voltage, and velocity distributions in the liquid metal at any position within the uniform and fringing field of a 2 Tesla conventional electromagnet. The magnet gap dimensions, for the uniform field of 2 T, are 20 cm high x 0.75 m wide x 1.83 m long. The experimental facility is capable of attaining maximum Hartmann numbers (M), and interaction parameters (N) in the range of 10^3 - 10^4 , which is close to the range of 10^4 - 10^5 that these parameters attain in the reactor blanket. Some steady state results from the same test section have been reported previously.³⁻⁵ Table 1 summarizes the dimensions of the test article and the range of test parameters covered. All four walls are of nominally equal thickness. The test section duct was fabricated by extrusion and, as a result, has rounded corners. The four inside corner radii vary within the limits given in Table 1.

Table 1. Summary of Square Test Section Parameters

Parameter	Value(s)
L , cm	4.39
r (corner radii), cm	0.5-0.9
c	0.07
B (max/min), T*	2.3/1.1
M (max/min)	5800./2900.
N (max/min)	1.26×10^5 /540
U (max/min) cm/s**	33.9/0.58

* B is the maximum field inside the magnet.

** U is the average velocity in the duct.

IV. DESCRIPTION OF INSTRUMENTATION

The results presented and discussed here are in the form of power spectral density (PSD) plots and time domain records of the voltages generated by a liquid-metal electromagnetic velocimeter (LEVI) velocity probe. The LEVI probe has been described in detail elsewhere.

The LEVI is essentially a miniature local electromagnetic flowmeter; a device that can measure the local voltage gradient, which, for the negligible current densities of the present MHD experiment, is proportional to the applied magnetic field and the local velocity. The LEVI used here requires a strong applied magnetic field, flow in a thin walled or insulating duct, and, of course, a conducting fluid. The velocity is related to the measured quantities as follows:

$$u = \frac{\Delta\phi}{d \cdot B}$$

where $\Delta\phi$ is the local voltage difference measured by the two electrodes, d is the LEVI electrode separation, and B is the local applied magnetic field. Typical electrode separations were ~ 1.5 mm. The LEVI signals were amplified by a gain of 1000 before input to the oscilloscope and FFT analyzer.

The power spectral density (PSD) plots of the LEVI probe was acquired using the commercially available dual-channel fast Fourier transform (FFT) analyzer - Nicolet Scientific Corporation Model 660A. Ensemble-averaging features were used to statistically eliminate random noise contributions from the PSDs. The FFT analyzer employs standard digital signal processing circuitry to generate the PSD information.

Time domain records of the LEVI voltages were collected by the simple method of photographing the screen of a memory-oscilloscope. Such photographic records are admittedly subjective, but care was exercised in selecting representative oscilloscope traces. For this task, the memory feature of the oscilloscope was particularly helpful. In general, at least one companion photo was taken for each ensemble-averaged PSD acquired when the oscilloscope traces showed an interesting or unusual behavior.

To prepare the figures presented in this paper, it was necessary to digitize the hard copy plots produced by the FFT analyzer using a graphics pad and cross-hair device. For the purposes of the essentially qualitative analysis presented here, this technique is considered adequate. The oscilloscope photos of the LEVI velocity time histories were also digitized to enable the presentation of families of time histories on a single figure. Figure 2 presents a comparison of the raw photo and the resultant

digitized figure. The major difference between the two curves is the additional high frequency content of the oscilloscope photo. Since the digitized photos were used primarily for comparison purposes, and the subject high frequency components are considered to be significantly higher than the main frequencies of interest, this difference between the two curves in Fig. 2 is considered unimportant in the present context.

V. OBSERVATIONS OF INSTABILITIES IN THE NEAR WALL REGION

An oscilloscope trace similar to the one shown in Fig. 2 was the first evidence of an instability in the square duct. Following a preliminary evaluation of the behavior of the instability to determine the possible parameters that affect its existence, an extensive matrix of test conditions was carried out to fully characterize the nature of the instability. Approximately 1800 PSD plots and 750 oscilloscope photos were acquired for this investigation. Other data, such as mean velocity measurements and pressure distributions, showed evidence of the instability as well.

Figure 3 was constructed from seven axial velocity distributions acquired in the $y = 0$ plane, each at a different value of Z/L . X/L is the non-dimensional x coordinate along the axis of the duct. The origin of the x -coordinate axis, which is directed away from the uniform field region, is at the upstream corner of the magnet pole face. The normalized B -field distribution with respect to X/L is shown near the top of the figure. This distribution is uniform in the z -direction. Figure 3 is a surface of the mean velocity extending from near the inlet of the magnet ($X/L = +3$) to near the exit of the flow from the magnetic field ($X/L = -35$) and for $0.95 > Z/L > 0.60$. For values of $Z/L < 0.60$, the velocity is uniform at $u/U = 0.75$. The irregularities in the mean velocity surface are a direct reflection of the presence of the instabilities in the flow. The data acquisition system used to acquire the mean velocity measurements has low-pass input filters to reduce the effects of high frequency disturbances on the mean measurements, but the frequency content of the instability was such that a steady mean velocity measurement could not be obtained for these conditions. Hence, Fig. 3 shows, in a qualitative way, the physical extent of the instability and its relative position with respect to the magnetic field distribution, for the specific case of $M = 5300$ and $N = 2900$. From Fig. 3, one can see that at $Z/L = 0.95$ the instability forms in the following rather small region of the duct: $-10 \geq X/L \geq -15$. One can further see that, once initiated, the instability quickly expands to occupy a region of the flow which extends more than 20 characteristic lengths of the duct in the X/L direction within the uniform region of the field, and whose width

is approximately constant in the z-direction at $0.70 \leq Z/L < 1.0$.

For the M and N conditions of Fig. 3, LEVI PSD plots were acquired at $Z/L = 0.95$ as a function of X/L . Figure 4 shows only a representative sample, for clarity, of the evolution of the PSD in the downstream direction. Note that, in this and all subsequent figures, the flow is moving from positive to negative values of X/L . A careful study of the more than 30 PSD plots from which those in Fig. 4 were selected revealed that the instability first appears near $X/L = -10$ to -12 . The two spectra in Fig. 4 nearest to $X/L = -10$ show its evolution in the earliest stages. According to the PSD plots and the accompanying oscilloscope traces, the instability begins as a harmonic oscillation in the mean velocity which gives rise to a triangular-shaped region on the PSD plot. The center frequency of the triangular-shaped region can be designated as the characteristic frequency of the instability. Its value in this case is about 140 Hz. The fact that the PSD peak is triangular in shape indicates that a number of frequencies are contributing to the instability; here the approximate bandwidth of the triangular region is 130 Hz.

With the aid of PSD plots such as those shown in Fig. 4, the formation and evolution of the instability can be assessed more precisely than from information such as that contained in Fig. 3. For example, Fig. 4 (for $Z/L = 0.95$) shows a formation region for the instability which extends over the range $-10 \geq X/L \geq -13$. From $X/L = -13$ to -15 , the shape of the PSD changes little from that at $X/L = -13$. From $X/L = -15$ onward to near the exit from the magnetic field ($X/L = -35$), the low frequency content of the PSD ($f \leq 10$ Hz) drops by 1-2 orders of magnitude; this behavior can be seen in Fig. 4. Hence, even though the physical size of the region occupied by the instability is constant as one moves downstream, Fig. 4 reveals that there is movement of energy out of the low frequencies ($f \leq 10$ Hz) as the flow proceeds downstream from $X/L = -15$ to $X/L = -35$.

Figure 5 presents the evolution of oscilloscope traces with X/L , for a fixed value of $Z/L = 0.95$. Each trace in the figure is a plot of velocity vs. time, similar to Fig. 2, and is representative of the X/L ranges over which the PSD's given in Fig. 4 have the characteristic shapes described above. The velocity trace at $X/L = -11$ is representative of the LEVI signals observed during the formation of the instability ($-10 \geq X/L \geq -13$). The trace at $X/L = -15$ is indicative of the LEVI signals throughout the region $-13 \geq X/L \geq -15$. Finally, the traces at $X/L = -22$ and $X/L = -36$ accompany the corresponding two PSD's in Fig. 4.

There are two very striking features of the traces in Fig. 5. First, the amplitude of

the instability oscillations, with respect to the mean velocity, is quite large; reaching peak values of the order of the mean velocity. Second, the velocity signals develop a strongly periodic character as the flow proceeds downstream. This periodicity is obvious in the velocity trace of Fig. 5 at $X/L = -36$. Based on the velocity traces, the instability appears to have a well organized structure, which becomes more and more coherent as the flow moves in the downstream direction. The PSD for this position in Fig. 4 shows a well defined peak at a frequency range corresponding to the period of the oscillations.

Figures 6 and 7 show, for the same values of M and N as Figs. 4 and 5, the variation of PSD and velocity traces with Z/L for $X/L = -12$. In Fig. 6, one can see at $Z/L = 0.95$ the same PSD curve with the triangular-shaped region that is shown in Fig. 4. Figure 6 shows that the prominence of the peak decreases rapidly with distance from the near-wall region of the duct. In fact at $Z/L = 0.75$, it is difficult to discern the existence of any peak. It should be pointed out that the whole spectrum, including the low frequencies (not just the triangular-shaped region), increases in energy content as the wall is approached. The increase is roughly an order of magnitude from $Z/L = 0.85$ to $Z/L = 0.95$. The velocity time histories shown in Fig. 7 do not exhibit any unusual behavior as the wall is approached. This is consistent with the information in the PSD plots which shows that the energy in the frequencies associated with the instability ($70 \leq f \leq 200$) is essentially equal to that in the low frequency range, below about 20 Hz. Hence, though the instability associated with the triangular part of the spectrum is present and growing as one approaches the wall, it is not growing significantly faster than the low frequency, broadband part of the spectrum.

Moving further downstream to $X/L = -15$, Figs. 8 and 9 present the same kind of information as do Figs. 6 and 7. These figures are typical of the behavior of the instability for $-13 \geq X/L \geq -15$. Figure 8 shows an essentially smooth and monotonic growth of spectral energy content as one approaches the wall region. No narrow peaks or triangular-shaped regions, such as those in Fig. 6 (which we associate with the presence of a harmonic instability), are present here. The main change, of course, is the dramatic increase in the amplitude of the PSD (by about three orders of magnitude) as one moves from $Z/L = 0.55$ to $Z/L = 0.95$. To first order, one can say that, at $X/L = -15$, the spectra shift upward, in all frequencies, as the wall is approached. Further examination of the energy spectra in Fig. 8 at the maximum frequency plotted, in this case 200 Hz, reveals that as one approaches the wall, the frequency content of the spectra includes higher and higher frequencies. Figure 8 indicates that

only frequencies up to 200 Hz are present for $Z/L \leq 0.75$ and that frequencies beyond 200 Hz are present for $Z/L \geq 0.75$.

Figure 9 shows a considerable gradient in the "activity" or intensity of the fluid motion as one travels from $Z/L = 0.55$ to $X/L = 0.95$. One can even find an occasional large drop in the velocity time history corresponding to a negative velocity. Each LEVI time trace is matched with its respective PSD plot in Fig. 8. The LEVI time history at $Z/L = 0.95$ is the same one that appears in Fig. 5 at $X/L = -15$. Thus a picture of both the axial and transverse distributions of the features of the instability can be composed.

Further downstream, at $X/L = -36$, Figs. 10 and 11 give the variations of PSD and velocity time histories with Z/L . The downstream position of $X/L = -36$ is still within the uniform region of the field. The data indicate that this position is far enough upstream of the fringing region at the outlet end of the magnet to be unaffected by it. Figures 10 and 11 present a picture of the final state to which the instability has rather slowly evolved from $X/L = -15$ to $X/L = -36$. Figure 10 paints the following picture as one follows the distribution of the PSD from $Z/L = 0.5$ to the wall; an instability is present in a band of frequencies ranging from about 10 Hz to 50 Hz having a central peak at approximately 20 Hz. The amplitude of the spectra increase with no preferential growth or decay of energy within any given frequency bandwidth as the wall is approached, up to $Z/L = 0.85$. However, from $Z/L = 0.85$ to 0.95, there is a significant movement of energy out of the low frequencies ($f \leq 10$ Hz). The high degree of coherence attained by the instability is most pronounced at this axial position and at $Z/L = 0.95$. The highly organized trace of the LEVI velocity at $Z/L = 0.95$ in Fig. 11 and the disappearance of the low frequency content of its companion PSD in Fig. 10 seem to be strongly linked.

The spectra and velocity traces discussed until now were taken at $M = 5300$ and $N = 2900$. Additional measurements at higher interaction parameters indicated that the instability was progressively suppressed as the interaction parameter was increased. Figures 12 and 13 show PSD and velocity time histories, respectively, at $Z/L = 0.95$, for the same Hartmann number as Figs. 2-11 but for $N = 11,000$. Figures 12 and 13 should be compared to Figs. 3 and 4. For the higher interaction parameter the instability is still present, but has a greatly reduced impact on the nature of the flow. The PSDs in Fig. 12 show the origin of the instability at $Z/L = 0.95$ to lie near $X/L = -16$ in this case, and to have a characteristic frequency near 35 Hz. In contrast to the lower interaction parameter case covered in Figs. 2-11, only the spectrum near this frequency grows in the downstream direction

over the range $-16 \geq X/L \geq -22$; the energy in all other frequencies is invariant in this range. Further downstream, for $-22 \geq X/L \geq -35$, the main change in the PSDs is a growth of energy in the frequencies just below the instability related frequencies ($3 \leq f \leq 30$). In fact, for all the PSDs shown in Fig. 12, all spectral evolution is confined to the frequency range $3 \leq f \leq 50$ Hz; outside this range the spectra are invariant. Figure 13 shows the companion velocity time history plots for Fig. 12. These plots suggest a negligible increase in the intensity of the fluctuations as one moves downstream. Accordingly, the PSD plots show only small increases compared to those of Fig. 3.

Figures 14 and 15 show the distributions of PSD and velocity time history as a function of Z/L far downstream at $X/L = -34$ for $N = 11,000$ and $M = 5300$. Here the effects of the instability are more closely confined to the near wall region than in the lower interaction parameter case. As one approaches the wall starting from $Z/L = 0.75$ in Fig. 14, the energy content of the spectra increases not only at the characteristic frequency of the instability (about 30 Hz), but at all lower frequencies as well. For values of $Z/L < 0.75$, there is virtually no presence of the instability at 30 Hz in the PSD plots, and they are approximately invariant. Further, in both Figs. 12 and 14, all spectra are essentially invariant for frequencies greater than 50 Hz.

The effect of the interaction parameter, N , on the PSD at $X/L = -31$, and $Z/L = 0.95$, for a constant $M = 5300$, can be seen in Fig. 16. One can easily spot the triangular-shaped instability region at $N = 11,000$ and can follow the evolution of this region to either lower or higher interaction parameter values. The most notable feature of the PSDs in Fig. 16 is a reversal in the following general trend: for all N , the amplitude of the PSD at any given frequency increases as N decreases (the rate at which it increases is, of course, a function of the frequency and many other factors). For the frequency range $f \leq 20$ Hz, this general trend reverses in going from $N = 4700$ to the lowest N of $N = 2900$. One can see a significant reduction in the energy content of the low frequency range ($f \leq 20$ Hz) for the $N = 2900$ PSD when compared to the $N = 4700$ PSD. In going from the $N = 11,000$ PSD to the two PSDs at higher values of N , a very rapid suppression of the instability is evident so that at the highest value of $N = 48,000$ only a small harmonic peak is present having an amplitude comparable to some of the background noise signals.

The effect of Hartmann number, M , on the instability is shown in Fig. 17 for $X/L = -31$, $Z/L = 0.95$, for a constant Reynolds number (based on the duct width, $2L$) of 5100. Note that, as defined here, $Re = 2M^2/N$. It is seen that, for conditions in which the flow rate is

held constant, the Hartmann number has a weak influence on both the magnitude and distribution of the energy spectra. Since the subject instabilities are inertially driven phenomena, they may scale more properly with the interaction parameter, N . One may argue that the characteristic frequency of the instability is associated with the mean velocity u and a characteristic length scale l . The thickness of an inertial side layer is of the order of $L/N^{1/3}$.⁶ Hence, taking the inertial side layer thickness to be the characteristic length scale suggests that, for constant velocity in the flow, the spectral frequencies scale with $N^{-1/3}$. Based on this idea, Fig. 18 was plotted; it presents the PSDs from Fig. 17 for frequency scaled by $N^{-1/3}$. The three spectra, for Hartmann numbers varying by as much as a factor of two, seem to collapse on each other, to first order. This surprising result demonstrates a marginal importance of the Hartmann number (as long as it is large enough) on the onset of instability.

The narrow triangular spectra that are shown in Fig. 17 are characteristic of the early stages of the instability involving low amplitude harmonic oscillations. Therefore, Fig. 17 suggests that the conditions corresponding to the three spectra shown are all close to the onset of instability. These conditions have Hartmann number ranging by a factor of two and interaction parameter ranging by a factor of four. Yet, they all have the same Reynolds number, $Re = 5100$. It appears that, over the ranges of M and N examined, the onset of instability depends primarily on the Reynolds number. To examine this conclusion further, spectra at $Re = 2550$ were taken for the same Hartmann numbers as in Fig. 17. The spectra show no evidence of the instability. Because Reynolds numbers between 2550 and 5100 were not studied, we can reach the conclusion that the critical Reynolds number lies between 2650 and 5100 and is independent of the Hartmann number in the range $2700 < M < 5400$. This insensitivity of the onset of the instability on M suggests that the stability of the wall jet is not, to a first order, affected by the magnitude of the magnetic field. The magnetic field causes the wall jets but, because the jet velocities are of order $U \cdot M^{1/2}$ and the width of the jets is of order $L \cdot M^{-1/2}$, the Reynolds number based on the maximum jet velocity and half jet width is independent of M and is of the same order as the Reynolds number based on the average velocity and duct width. It should be pointed out that although the data suggest that the onset of the instability is insensitive to the Hartmann number, the data also indicate that the evolution and final state of the instability and therefore its heat and mass transfer characteristics depend strongly on M .

VI. CONCLUSIONS

This paper describes the results of measurements acquired to characterize an instability observed in a rectangular thin-conducting-wall duct in the presence of a transverse magnetic field, under fusion reactor relevant conditions. For the conditions and test section geometry reported here, the instability developed at a critical Reynolds number $2650 < Re_{cr} < 5100$ independent of the Hartmann number. When conditions were such that the instability did form, it always formed within the uniform region of the applied field, downstream of the inlet fringing region of the magnet. Once initiated, the instability formed quite rapidly in space (within a few characteristic test section dimensions, L) to reach a somewhat stable size which changed only very slowly over many characteristic lengths in the axial direction. The presence of the instability affects the mean velocity distribution within the region occupied by the instability by apparently thickening the wall jet layer.

The origin of the instability is a laminar harmonic velocity oscillation, whose amplitude grows in absolute terms as one approaches the wall. Both the PSD and velocity time histories show the strength of the instability to increase as the wall is approached. Also, the high frequency content of the energy spectra increase as the wall is approached. The absence of any observed flow instabilities in the round duct link the flow instabilities observed here to the high-shear layers of the stationary wall jets because in a round pipe, there are no wall jets. Based on these observations, the instability appears to be born within the wall jet layer. For a fixed Reynolds number of 5100, the spectral frequencies of the flow energy seem to scale with $N^{1/3}$.

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Fig. 1. Coordinate System

Fig. 3. Two-dimensional Velocity Distribution Surface in a Thin-walled Rectangular Duct Showing Wall-jet Instability Region; $M=5300$, $N=2900$

Fig. 2. Comparison of Example Oscilloscope Photograph and Resultant Digitized LEVI Velocity Time History Plot for $X/L=-28$, $Z/L=0.85$, $M=5300$, $N=5300$

Fig. 4. Axial Distribution of PSD Profiles in a Thin-walled Rectangular Duct for; $Z/L=0.95$, $M=5300$, $N=2900$

Fig. 5. Axial Distribution of Velocity Time Histories in a Thin-walled Rectangular Duct for; $Z/L=0.95$, $M=5300$, $N=2900$

Fig. 7. Transverse Distribution of Velocity Time Histories in a Thin-walled Rectangular Duct for; $X/L=12$, $M=5300$, $N=2900$

Fig. 6. Transverse Distribution of PSD Profiles in a Thin-walled Rectangular Duct for; $X/L=12$, $M=5300$, $N=2900$

Fig. 8. Transverse Distribution of PSD Profiles in a Thin-walled Rectangular Duct for; $X/L=15$, $M=5300$, $N=2900$

Fig. 9. Transverse Distribution of Velocity Time Histories in a Thin-walled Rectangular Duct for; $X/L=15$, $M=5300$, $N=2900$

Fig. 11. Transverse Distribution of Velocity Time Histories in a Thin-walled Rectangular Duct for; $X/L=36$, $M=5300$, $N=2900$

Fig. 10. Transverse Distribution of PSD Profiles in a Thin-walled Rectangular Duct for; $X/L=36$, $M=5300$, $N=2900$

Fig. 12. Axial Distribution of PSD Profiles in a Thin-walled Rectangular Duct for; $Z/L=0.95$, $M=5300$, $N=11,600$

Fig. 13. Axial Distribution of Velocity Time Histories in a Thin-walled Rectangular Duct for; $Z/L=0.95$, $M=5300$, $N=11,000$

Fig. 15. Transverse Distribution of Velocity Time Histories in a Thin-walled Rectangular Duct for; $X/L=0.95$, $M=5300$, $N=11,000$

Fig. 14. Transverse Distribution of PSD Profiles in a Thin-walled Rectangular Duct for; $X/L=0.95$, $M=5300$, $N=11,000$

Fig. 16. The Effect of Interaction Parameter, N , on PSD Profiles in a Thin-walled Rectangular Duct for; $X/L=0.95$, $Z/L=0.95$, $M=5300$

Fig. 17. The Effect of Hartmann Number, M , on PSD for;
 $X/L=-31$, $Z/L=0.95$, $Re=5100$

Fig. 18. The Effect of Scaling Frequency $f'' = f/N^{1/3}$ on
 PSD for Various Hartmann Number, M , at
 $X/L=-31$, $Z/L=0.95$, $Re=5100$