CLOSED ORBIT PRODUCED BY AN IMPERFECT BUMP*

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J. M. Peterson
Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720
and

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

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Please note the following changes:

For Introduction "Paragraph 1" read A "bump" in the closed orbit of a storage ring or other type of circular accelerator is a deliberate local variation of the closed orbit produced by two or more magnets.

"Paragraph 1" read In this case the net effect of all the bump magnets is zero outside the region of the bump.

"Paragraph 1" read A particle that enters the bump on the central closed orbit is moved off the normal closed orbit while in the bump but is restored to that orbit when leaving the bump region.

"Paragraph 2" read We shall develop an expression for the perturbed closed orbit in terms of the bump's imperfections.

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J. M. Peterson
Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

Summary

An imperfect orbit bump perturbs the closed orbit throughout the storage ring. The magnitude and phase of the perturbation is evaluated in terms of the imperfections of the bump.

Introduction

A "bump" in the closed orbit of a storage ring or other type of circular accelerator is a deliberate local variation of the closed orbit produced by two or more magnets. In the PEP storage ring, for example, each injection system uses two bumps; one is a d.c. bump, which is produced by four d.c. magnets, and the other is a "fast" bump, which is produced by three kicker magnets. If each set of magnets is correctly positioned and properly energized, each bump is a purely local distortion of the closed orbit and does not affect the closed orbit in the rest of the machine. In this case the net effect of all the bump magnets is zero; outside the region of the bump a particle that enters the bump on the central closed orbit is moved off the normal closed orbit while the bump but is restored to that orbit when leaving the bump region.

However, if the bump magnets are not correctly positioned or not properly energized, the net effect of all the bump magnets is not zero, and the closed orbit in the rest of the machine is perturbed. We shall develop an expression for the perturbed closed orbit in terms of the bumps imperfections.

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We shall consider the case where the whole system of bump magnets is in a region of the lattice with no intervening quadrupole or other type of focusing magnet. In this case the analysis is simplified because the betatron function $\beta(s)$ is of the relatively simple form $\beta(s) = \beta_0 + (s-s_0)^2/s_0$ throughout the region of the bump. β_0 is the β -value at the minimum of the parabola, and s_0 is the position of the minimum. This situation holds for the injection bumps in the PEP ring, where each bump-magnet system is within a long straight section.

The Effect of a Single Magnet

The perturbation of the closed orbit in a linear lattice that is produced by one dipole magnet that deflects the beam by the angle θ can be readily derived. In place of the betatron amplitude $x(s)$ and slope $x'(s)$, it is convenient to transform to the variables $\eta(\phi)$ and $\eta'(\phi)$ ($\eta' \equiv d\eta'/d\phi = \beta d\eta/ds$):

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} 1/\sqrt{\beta(s)} & 0 \\ \alpha(s)/\sqrt{\beta(s)} & \sqrt{\beta(s)} \end{pmatrix} \begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} x/\sqrt{\beta} \\ \alpha x/\sqrt{\beta} + x'\sqrt{\beta} \end{pmatrix} \quad (1)$$

The advantage of the η, η' variables is that a betatron oscillation is represented in the η, η' phase plane by a point on a constant-radius circle, the angular position being given by the betatron phase angle $\phi = \int ds/\beta$. The radius-squared of the circle is its emittance:

$$\eta^2 + \eta'^2 = \varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2 \quad (2)$$

which is the same as for the corresponding ellipse in the x, x' phase plane at any point s .

The closed orbit produced by the deflection angle θ_1 of the magnet under consideration is the unique orbit that closes on itself after one revolution. Let the lattice tune be ν . The following diagram in η, η' space makes it clear that amplitude of the closed orbit perturbation at $\phi \geq \phi_1$ is

described by:

$$\eta(s) = \frac{\sqrt{\beta_1} \theta_1}{2 \sin \pi \nu} \cos(\phi - \phi_1 - \pi \nu)$$

$$\phi_1 \leq \phi \leq \phi_1 + 2\pi \nu$$

$$(3)$$

where $\phi - \phi_1$ is the betatron phase relative to the position s_1 of the perturbing magnet.

The Effect of Several Magnets

Now consider the effect of n dipole magnets, whose deflecting angles are $\theta_1, \theta_2, \dots, \theta_n$, situated at $s_1, s_2, \dots, s_n$. The total perturbation η_T outside the region $(s_1 \dots s_n)$ is given in a linear system by the sum of the individual perturbations:

$$\begin{aligned} \eta_T &= \sum_{i=1}^n \eta_i \\ &= \sum_{i=1}^n \frac{\sqrt{\beta_i} \theta_i}{2 \sin \pi \nu} \cos(\phi - \phi_i - \pi \nu) \\ &= \frac{1}{2 \sin \pi \nu} \left\{ \cos(\phi - \pi \nu) \sum_{i=1}^n \sqrt{\beta_i} \theta_i \cos \phi_i + \sin(\phi - \pi \nu) \sum_{i=1}^n \sqrt{\beta_i} \theta_i \sin \phi_i \right\} \\ &= \sqrt{\epsilon_T} \cos(\phi - \phi_T - \pi \nu) \end{aligned} \quad (4)$$

$$\text{where } \epsilon_T = (2 \sin \pi \nu)^{-2} \left\{ \left(\sum_{i=1}^n \sqrt{\beta_i} \theta_i \sin \phi_i \right)^2 + \left(\sum_{i=1}^n \sqrt{\beta_i} \theta_i \cos \phi_i \right)^2 \right\} \quad (5)$$

is the emittance corresponding to the total perturbation in the region outside the n magnets, and

$$\phi_T \equiv \tan^{-1} \left[\frac{\sum_{i=1}^n \sqrt{\beta_i} \theta_i \sin \phi_i}{\sum_{i=1}^n \sqrt{\beta_i} \theta_i \cos \phi_i} \right] \quad (6)$$

The expression (5) can be simplified if the n magnets are all in a region with no intervening focusing elements, in which case.

$$\left. \begin{aligned} \beta_i &= \beta_o + (s_i - s_o)^2 / \beta_o \\ \phi_i &\equiv \int_{s_o}^{s_i} ds / \beta = \tan^{-1} \left(\frac{s_i - s_o}{\beta_o} \right) \\ &= \text{betatron phase relative to } s_o \\ \cos \phi_i &= (\beta_o / \beta_i)^{1/2} \\ \sin \phi_i &= (s_i - s_o) / (\beta_o \beta_i)^{1/2} \end{aligned} \right\} \quad (7)$$

Equations (5) and (7) then yield:

$$\epsilon_T = (2 \sin \pi \nu)^{-2} \left\{ \beta_o \left(\sum_1^n \theta_i \right)^2 + \frac{1}{\beta_o} \left(\sum_1^n (s_i - s_o) \theta_i \right)^2 \right\} \quad (8)$$

The first sum $\sum_1^n \theta_i$ is the net angular deflection ($\Delta\theta_b$) produced by the bump and is zero for a perfect bump. The second sum $\sum_1^n (s_i - s_o) \theta_i$ is the negative of the net deflection ($-\Delta x_o$) produced at the point s_o by the bump, which also is zero for a perfect bump.

The curly bracket in (8) is recognized as the Courant-Snyder invariant emittance ϵ_b corresponding to the lack-of-closure points $\Delta\theta_b$, Δx_o referred to s_o . It can equally well be referred to the last (the n th) bump magnet, where the lack-of-closure values are $\Delta\theta_b$, Δx_n . I.e., (since $\alpha_o \equiv \alpha(s_o) = 0$, and $\gamma = 1/\beta_o$ everywhere in a drift region)

$$\begin{aligned} \epsilon_b &\equiv \left\{ \frac{1}{\beta_o} \left(\sum_1^n (s_i - s_o) \theta_i \right)^2 + \beta_o \left(\sum_1^n \theta_i \right)^2 \right\} \\ &= \gamma_o (\Delta x_o)^2 + 2 \alpha_o (\Delta x_o) (\Delta\theta_b) + \beta_o (\Delta\theta_b)^2 \\ &= \gamma_n (\Delta x_n)^2 + 2 \alpha_n (\Delta x_n) (\Delta\theta_b) + \beta_n (\Delta\theta_b)^2 \end{aligned} \quad (9)$$

where $\Delta x_n \equiv \sum_1^n (s_n - s_i) \theta_i$.

Thus, the net perturbed orbit $x_p(s)$ everywhere outside the bump can be written as

$$\begin{aligned}
 x_p(s) &= \sqrt{\beta(s)} \eta_T \\
 &= \frac{\sqrt{\beta(s)} \epsilon_p}{2 \sin \pi \nu} \cos (\phi - \phi_T - \pi \nu)
 \end{aligned} \tag{10}$$

where $\phi_T = -\tan^{-1}(\Delta x_0 / \beta_0 \Delta \theta_b)$. Note that the phase angle ϕ_T is defined with respect to s_0 , the position of minimum of the beta function in the region of the bump. It is useful to realize that ϕ_T is the betatron phase at the point s_1 where the non-closure trajectory intercepts the normal closed orbit. I.e.,

$$\tan \phi_T = - (s_0 - s_1) / \beta_0$$

$$\text{where } s_0 - s_1 \equiv \Delta x_0 / \Delta \theta_b$$

Thus, an imperfect bump with the non-closure parameters $\Delta x_n, \Delta \theta_b$ at the n th magnet produces a perturbed closed orbit outside the bump region equal to that produced by a single magnet having a deflection angle equal to $\Delta \theta_b$ and situated at the point s_1 .