CONF-9207109-1

Imaging Algorithms for Geophysical Applications of Impedance Tomography

CONF-9207109--1

DE92 016784

Alan J. Witten
Oak Ridge National Laboratory
P.O. Box 2008
Oak Ridge, TN 37831-6200

and

John E. Molyneux
Widener University
Department of Mechanical Engineering
Chester, PA 19013

"The submitted manuscript has been authored by a contractor of the U.S. Government under contract No. DE-AC05-84OR21400. Accordingly, the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so for U.S. Government purposes."

Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37831
managed by
MARTIN MARIETTA ENERGY SYSTEMS, INC.
for the
U.S. DEPARTMENT OF ENERGY
under Contract No. DE-AC05-84OR21400

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED *EP*

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Imaging algorithms for geophysical applications of impedance tomography

Alan J. Witten

Oak Ridge National Laboratory
P.O. Box 2008, Oak Ridge, TN 37831-6002

John E. Molyneux

Widener University, Department of Mechanical Engineering
Chester, PA 19013

June 2, 1992

ABSTRACT

The methods of impedance tomography may be employed to obtain images of subsurface electrical conductivity variations. For practical reasons, voltages and currents are usually applied at locations on the ground surface or down a limited number of boreholes, but almost never over the entire surface of the region being investigated. The geophysical inversion process can be facilitated by constructing algorithms adopted to these particular geometries and to the lack of complete surface data. In this paper we assume that the fluctuations in conductivity are small compared to the background value. The imaging of these fluctuations is carried out exactly within the constraints imposed by the problem geometry. Several possible arrangements of injection and monitoring electrodes are considered. In two dimensions these include: Cross-line geometry, current input along one line (borehole) and measurements along a separate parallel line. Single-line geometry, injection and monitoring using the same borehole. Surface reflection geometry, all input and measurement along the ground surface. Theoretical and practical limitations on the image quality produced by the algorithms are discussed. They are applied to several sets of simulated data, and the images produced are displayed and analyzed.

1. INTRODUCTION

Since the inversion procedures we consider are intended for application in geophysical field work, we have adopted two constraints in their formulation: First, we have chosen geometries that are, at least approximately, similar to those normally met in geophysical applications. Second, we have structured the algorithms so that they may be implemented using computing power readily available in the field -- essentially a personal computer or equivalent. To meet these constraints, we have found it necessary to assume that the fluctuations in conductivity are sufficiently small for perturbation methods to be used, and that lowest order results give sufficient accuracy (see [5, 9, 6] for numerical approaches to the exact (nonperturbative) problem in the case where data is gathered over the entire surface of the region, and [8] for a rigorous discussion of the mathematical aspects of the geophysical problem).

In section 2. we specify the injection and monitoring electrode placement geometries for which we have obtained complete or partial inversion formulas. Only two-dimensional problems are considered although similar results can be derived in the three-dimensional case. We formulate the equations which relate the fluctuations in subsurface voltage to the fluctuations in conductivity, and linearize these using small perturbation theory. The result of these manipulations is an integral equation of the first kind which relates the measured voltage (as a function of monitor and injection electrode position) to the subsurface conductivity fluctuations. Since equations of this type are known to be generally ill posed (in the sense that small errors in the data can lead to large variations in the computed result), some care must be exercised in the inversion process.

In section 3. we use transform methods to convert the integral equation just described to an algebraic relationship that determines the Fourier-Laplace transform of conductivity in terms of the Fourier transform of the measured voltage. To complete the solution, we then develop the required transform inversion procedure. Fourier inversion is well behaved and presents no problems, but the Laplace inversion process is (as above) ill posed. Instead of directly employing an existing routine, we have developed a Tikhonov regularization method especially adapted to the current problem. This routine takes advantage of the assumed smoothness of the conductivity function and the knowledge we have as to its support. The details of the resulting numerical method are briefly discussed. We also show in this section that each of the various measurement geometries yields a different amount of wave number information. In effect, each gives the transform of the conductivity multiplied by a certain (low pass) filter or window function. We find that, in general, only the so-called full coverage case is capable of producing a (theoretically) complete, unfiltered image.

Tests of the inversion formulas presented here have, to date, been performed using only

simulated data, and several examples are presented in section 4.. The image quality that can be achieved using the various geometries is discussed and analyzed using simulated targets. Finally, section 5. presents some practical considerations which may effect the applicability of our work to certain geophysical problems, and summarizes the algorithms obtained.

2. FORMULATION OF THE PROBLEM

Our algorithm development will concentrate entirely on two-dimensional geometries. Most of our results, however, can be directly extended to the three-dimensional case. We also restrict ourselves to the case in which the conductivity of the subsurface region (suitably normalized) is of the form $\sigma = 1 + \epsilon\sigma'(x, y)$, where $0 \leq \epsilon \ll 1$, the support of σ' is a bounded region, and $\max|\sigma'|$ is of order one. This may represent a fairly stringent limitation for some problems of geophysical interest, and we will return to a further discussion of this point in the final section. The potential in the subsurface region satisfies the equation

$$-\nabla \cdot \sigma(x, y) \nabla \phi = c(x, y), \quad (1)$$

where $c(x, y)$ is the prescribed current injection density.

As will become apparent, the ease with which the problem of inversion can be solved is strongly dependent on geometry. We will concentrate on the following two basic configurations.

- i. First, we suppose that the inhomogeneities to be imaged are known to lie far from the surface, and we make the assumption that all boundary effects can be neglected. Such 'deep' objects are thus effectively situated in an infinite surrounding medium. We suppose that the support of σ' is known *a priori* to lie in the semi-infinite slab $0 < x < b$. We probe this region by placing the injection electrodes in a vertical borehole which coincides with the line $x = 0$ (i.e. $c(x, y) = \delta(x)c(y)$), and using one of the following strategies:
 - (a) Measuring the voltage along the same line, $x = 0$, on which the current is injected. In this *single-line* case, we will show that a partial reconstruction of σ' can be obtained.
 - (b) Measuring the voltage at points along a second borehole which coincides with the vertical line $x = b$. We call this the *cross-line* strategy and will show that it also leads to partial determination of σ' .

- (c) Finally, again using two boreholes, but now measuring the voltage on both lines $x = 0$, and $x = b$. As we will see, this *full coverage* case leads to a complete reconstruction of the conductivity.
- ii. Second, we consider a geometry in which the ground surface coincides with the line $x = 0$, and the inhomogeneities are sufficiently shallow that the effects of this surface cannot be ignored. In this *surface reflection* case, we assume all current injection and monitoring electrodes are located along the surface, and we again obtain a partial reconstruction of σ' .

We will first present the analysis of the infinite medium cases described above, and will then describe the modifications necessary for study of the semi infinite model. To complete the specification of the direct problem for the potential (in which σ' and c are both taken as known functions) we require that it satisfy Eq.(1) for $\mathbf{x} = (x, y) \in R^2$, and $c(x, y) = \sum_{j=1}^n I_j \delta(x) \delta(y - q_j)$, where I_j is the current injected or withdrawn at the j -th source point $\mathbf{p}_j = (0, q_j)$. As $|\mathbf{x}| \rightarrow \infty$, we require that $\phi \rightarrow \sum_{j=1}^n I_j g(|\mathbf{x}|)$ uniformly in $\mathbf{x}/|\mathbf{x}|$, where for $t > 0$, $g(t) = \ln(1/t)/(2\pi)$. By linearity, the multiple source problem can, of course, be solved by a superposition of single source solutions. Thus, we can develop the case $s = 1$ with a unit source at $\mathbf{p} = (0, q)$. The potential in this case depends on the parameters q and ϵ . In particular, when $\epsilon = 0$, the solution, ϕ_0 , is well known

$$\phi_0(\mathbf{x}, \mathbf{p}) = G(\mathbf{x}, \mathbf{p}), \quad (2)$$

with $G(\mathbf{x}, \mathbf{x}') = g(|\mathbf{x} - \mathbf{x}'|)$. When $\epsilon > 0$, Eq.(1) is formally equivalent to the integral equation

$$\phi(\mathbf{x}) = \phi_0(\mathbf{x}) + \epsilon \int_S G(\mathbf{x}, \xi) \nabla \cdot \sigma' \nabla \phi d\xi, \quad (3)$$

where $S \in (0, b) \times (-\infty, \infty)$ is the support of σ' . A closed form solution of this equation (assuming σ' to be known) is, in general, no longer possible, but when $\epsilon \ll 1$ an approximate solution may be obtained using perturbation methods. To lowest order, i.e. correct to $O(\epsilon^2)$, the potential is given by Eq.(3) with ϕ in the integral on the right hand side replaced by ϕ_0 . Making this approximation, letting $\phi_1 = (\phi - \phi_0)/\epsilon$, substituting from Eq.(2), and using the divergence theorem to integrate by parts, we obtain

$$\phi_1(\mathbf{x}, \mathbf{p}) = - \int_S \nabla_2 G(\mathbf{x}, \xi) \cdot \nabla_1 G(\xi, \mathbf{p}) \sigma'(\xi) d\xi, \quad (4)$$

where ∇_k operates on the k -th argument of G .

The right hand side of Eq.(4) can be determined by voltage measurements so we may attempt to determine σ' by inverting this equation under the assumption that ϕ_1 is known on some set in $L \in R^2$. Inversion by integral transformation is suggested, since Eq.(4) is similar in form to a convolutional integral equation. If $f(x, y)$ is a function

of two variables, we define its Fourier transform with respect to the second variable by $(\mathcal{F}_2 f)(x, k) = \int_{-\infty}^{\infty} f(x, y) \exp(-iky) dy$. It is easy to show that the transform of the vector $g(x, y) = \nabla g(|(x, y)|)$ is (for $x, k \neq 0$)

$$\mathcal{F}_2 g(x, k) = -\frac{1}{2} \exp(-|kx|)(\operatorname{sgn}(x), i \operatorname{sgn}(k)). \quad (5)$$

This result together with the properties of the Fourier transform lead to the representation

$$\begin{aligned} \nabla_1 G(\xi, x) \cdot \nabla_1 G(\xi, p) &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(ik(\eta - y) + ik'(\eta - q)) \\ &\quad \exp(-|k(\xi - x)| - |k'\xi|) \\ &\quad \frac{1}{4} (\operatorname{sgn}(\xi_1 - x) \operatorname{sgn}(\xi) - \operatorname{sgn}(k) \operatorname{sgn}(k')) dk dk'. \end{aligned}$$

If we substitute this expression into Eq.(4) and recall that $\xi \in S \Rightarrow 0 < \xi < b$, we see immediately that for x restricted to the two values 0, b it is possible to relate the Fourier transform of $\phi_1(x, y, q)$ with respect to the variables y and q to the Fourier-Laplace transform of $\sigma'(x, y)$. After some algebra we obtain¹

$$\begin{aligned} \tilde{\phi}_1(b\delta_{x,b}, k, k') &\equiv \mathcal{F}_2 \mathcal{F}_3 \phi_1(b\delta_{x,b}, k, k') \\ &= \frac{1}{4} (1 - 2\delta_{x,b} - \operatorname{sgn}(k) \operatorname{sgn}(k')) \exp(-b\delta_{x,b}|k|) \\ &\quad \cdot (\mathcal{L}_1 \mathcal{F}_2 \sigma')(1 - 2\delta_{x,b}|k| + |k'|, k + k'). \end{aligned} \quad (6)$$

(7)

For a general forcing by point current sources, the solution will be a sum of single source solutions, $\phi_1(x, y, q_1, \dots, q_s) = \sum I_j \phi_1(x, y, q_j)$. A representation of this function may be obtained from Eq.(6) as above, and will contain $\sum I_j \exp(-ik'q_j)$. It is clear that keeping the relative positions, say $\Delta q_j = q_j - q_1$, $j = 1, \dots, s$ of the points fixed and transforming with respect to q_1 will yield a result similar to Eq.(7). In fact, it follows easily that

$$\tilde{\phi}_1(b\delta_{x,b}, k, k', q_2, \dots, q_s) = \tilde{I}(k', q_2, \dots, q_s) \tilde{\phi}_1(b\delta_{x,b}, k, k'), \quad (8)$$

where $\tilde{I} = \sum_{j=1}^s I_j \exp(-ik' \Delta q_j)$.

Equation (8) is the result we will use for the single-line, cross-line and full coverage geometries, and we now turn to a short discussion of the semi-infinite medium case. In this case, we take $c = 0$ in Eq.(1) and specify instead the input current normal to the ground surface, $x = 0$

$$\partial_1 \phi(0, y) = \frac{1}{2} \sum_{j=1}^{s'} I'_j \delta(x - q'_j). \quad (9)$$

¹ If $f(x, y)$ is a function of two variables, we let $\mathcal{L}_1 f(\nu, y) = \int_0^{\infty} \exp(-\nu x) f(x, y) dx$ be the Laplace transform with respect to the first variable, and note that when transforming σ' the upper limit can be taken as b .

As $|(x, y)| \rightarrow \infty$ in $x > 0$ we require that $\phi \rightarrow \sum_{j=1}^n I'_j g(|(x, y)|)$. By linearity we can again consider a single half-unit source, $\partial_1 \phi(0, y) = \frac{1}{2} \delta(y - q)$ on the surface. The zeroth order solution is still given by Eq.(2). The first order solution is determined by the integral

$$\phi_1(x, p) = - \int_S \nabla_2 H(x, \xi) \cdot \nabla_1 G(\xi, p) \sigma'(\xi) d\xi, \quad (10)$$

where $H = G(x, \xi) + G(x, \xi')$ and $\xi' = (-\xi, \eta)$ is the reflection of ξ in the y -axis. The Fourier transform of $\nabla g(|(x, y)|)$ can be used, as before, to obtain a representation of $\nabla_2 H \cdot \nabla_1 G$, and it is easy to see that along the line $x = 0$ this is equal to twice the result already obtained for $\nabla_2 G \cdot \nabla_1 G$. Thus, for $x = 0$ we have $\mathcal{F}^2 \phi_1(0, k, k') = 2\tilde{\phi}_1(0, k, k')$, where $\tilde{\phi}_1$ is defined in Eq.(7). The result for an array of current sources on $x = 0$ can now be read directly from Eq.(8).

3. INVERSION AND IMAGING

Equation (8) relates the Fourier-Laplace transform of the fluctuating portion of the conductivity to the double Fourier transform of the measured voltage data, and we must decide to what extent σ' is determined by this relationship. Since a real conductivity was assumed, it is sufficient to know $\tilde{\sigma}'(u, v) = \mathcal{L}_1 \mathcal{F}_2 \sigma'(u, v)$ for $v > 0$. The Laplace inversion technique we employ also requires data for only positive values of u . Thus, complete inversion requires $\tilde{\sigma}'(u, v)$ in the first quadrant of R^2 . From Eq.(7) we see that $\tilde{\phi}_1(b\delta_{x,b}, k, k')$ gives $\tilde{\sigma}'(u, v)$ at points $(u, v) = ((1 - 2\delta_{x,b})|k| + |k'|, k + k')$, and we are thus led to consider the transformation $(u, v) = T(k, k') = (k' - k, k' + k)$. T maps the sets $Q_{\pm} = \{(k, k') : \pm k < 0, k' > 0, k' \pm k > 0\}$ onto $Q'_{\pm} = \{(u, v) : v > 0, \pm(u - v) > 0\}$, and for all $(k, k') \in Q_{\pm}$ we have

$$\tilde{\phi}_1(b\delta_{x,b}, k, k', q_j) = \tilde{I} \tilde{\sigma}'(u, v), \quad (11)$$

and since $Q'_- \cup Q'_+$ is the first quadrant of R^2 , σ' is completely determined in the full coverage case. On the other hand, it is only determined in Q'_+ in the single-line and surface reflection cases, and in the cross-line geometry only in Q'_- .

The above remarks apply to the ideal case in which voltage data can be obtained for all values of wave number. In practice, however, the number of monitoring and injection electrodes will be limited and they will be spaced a finite distance apart. For the sake of simplicity, we assume that values of $\tilde{\phi}_1$ are available for wave numbers $|k|, |k'| < K_{max}$. This yields information on $\tilde{\sigma}'$ in the following subregions of the right half plane

- i. Full coverage case: $D_1 \equiv \{(u, v) : 0 < u < 2K_{max}, 0 < |v| < 2K_{max} - u\}$.

- ii. Single-line and surface reflection geometries: $D_2 \equiv \{(u, v) : 0 < u < K_{max}, u < |v| < 2K_{max} - u\}$.
- iii. Cross-line case: $D_3 \equiv \{(u, v) : 0 < u < 2K_{max}, 0 < |v| < \min(u, 2K_{max} - u)\}$

In the next section we shall consider some examples which illustrate the effects these limitations in wave number coverage have on image recovery from simulated data.

Having analyzed the regions over which $\partial'(u, v)$ data is available, let us now turn to a brief discussion of the problem of inverting this data to obtain $\sigma'(x, y)$. For simplicity, we let $\tilde{\tau}(u, v) = W(u, v)\partial'(u, v)$, where W is a window or filter function defined as the characteristic function of one of the sets D_j defined above, assume that $\tilde{\tau}$ is given for all u, v in the right half plane, and seek the inverse Fourier-Laplace transform, $\tau(x, y)$. Formally, of course, we have

$$\tau(x, y) = (\mathcal{L}_1^{-1} \mathcal{F}_2^{-1} \tilde{\tau})(x, y) \quad (12)$$

The inverse Fourier transform portion of the above formula can clearly be implemented via an FFT. The Laplace inverse, on the other hand, is well known to be ill-posed[4]. The technique we have chosen for this inversion is based on a combination of Tikhonov regularization and the Galerkin method (see [7]). Ang, Lund, and Stenger[1] have recently developed some quite general methods for Laplace inversion based on regularization and assuming only that the unknown function is in $L^2(0, \infty)$. We have already specified that $\sigma'(x, y)$ should vanish for $x \notin [0, b]$ and implicitly assumed that it is smooth. With these additional restrictions, we can consider inverting the mapping $\mathcal{L}f = g$, with $(\mathcal{L}f)(t) = \int_0^b \exp(-tx)f(x) dx$, and subject to the conditions: The unknown function f and its first derivative f' are square integrable and f vanishes at $0, b$ (i.e. f belongs to the Sobolev space $H = H_0^2(0, b)$). The given function g is an element of $L' = L^2(0, b')$ for some $b' > 0$. An inversion scheme based on these conditions is certainly much less general than those proposed by Ang, et al, but is adequate for our purposes.

With the above interpretation, the adjoint of $\mathcal{L}$ is a mapping $\mathcal{L}^*$ from L' to H , and the Tikhonov procedure replaces $\mathcal{L}f = g$ with $\mathcal{K}_\alpha f \equiv \alpha f + \mathcal{L}^* \mathcal{L}f = \mathcal{L}^* g \equiv g^*$, where α is a small parameter. Since the later integral equation is of the second kind it is well posed, and we can use the Galerkin method to solve it, i.e., we seek f in H such that² $(\mathcal{K}_\alpha f - g^*, h)_H = 0$ for all $h \in H$. It can be shown using the methods in Kress[7] that the problem simplifies to

$$\alpha((f, h)_L + (f', h')_L) + \int_0^b \int_0^b k(x, y)f(x)h(y) dx dy = (\mathcal{L}'g, h)_L, \quad (13)$$

² $(f, g)_H = (f, g)_L + (f', g')_L$ is the scalar product in H , where $(f, g)_L = \int_0^b fg dx$ is the scalar product in $L = L^2(0, b)$.

where $\mathcal{L}' : L' \rightarrow L$ is the adjoint of $\mathcal{L}$ as an operator from L to L' , and $k(x, y) = (1 - \exp(-\mathcal{V}(x + y)))/(x + y)$ is the kernel of the operator $\mathcal{L}'\mathcal{L}$. To solve Eq.(13) we use the standard finite element procedures: (i) Partition $[0, b]$ into n equal subintervals using the points $x_i = ib/n$, $i = 0, \dots, n$. (ii) Approximate f using linear interpolation with nodes x_i . (iii) Require Eq.(13) to hold for all piecewise linear h which vanish at $x = 0, b$. This process yields $n - 1$ equations for the unknown interior³ values of f . For a given g the solution will depend on both n and the parameter α . Numerical experiments with $b = b' = 1$ have shown that for n of order 10, $\alpha = 10^{-3}$ gives satisfactory results with L^2 errors of order 10^{-3} .

4. SIMULATION STUDIES

The quality of any image reconstructed using integral transform methods will be influenced by the extent of coverage in the transform (K) space. As noted in the previous section, the full coverage geometry provides the most complete K -space coverage and, hence, the best image quality. Ideally, for continuous measurements over infinite electrode lines, $K=(u, v)$ -space coverage is the half-plane $u > 0$. In practice, however, discrete measurements must be made over lines of finite extent and this fact leads to somewhat more limited K -space coverage of the triangular region $0 < |v| < 2K_{max} - u$, $0 < u < 2K_{max}$. Thus, for a particular value of u , the image of σ' is low-pass filtered in y with a cutoff spacial frequency of $v = \pm(2K_{max} - u)$. This filtering becomes more severe with increasing u , and shrinks to the point $v = 0$ at $u = 2K_{max}$.

The wave number space coverage in the full coverage geometry introduces an asymmetry whereby blurring of the image in the y direction increases with increasing u which, in turn, produces an apparent skewness of imaged features towards larger values of x . This is illustrated in Fig. 1 which shows plots of $\sigma'(x)$ along a horizontal and vertical line through the center of a square inclusion in a homogeneous background. In creating the reconstructions of σ' , the weak inhomogeneity approximation $\sigma' \ll 1$ was assumed, measurements were taken at 32 points along the lines $(0, y)$ and $(10, y)$ for $-15.5 \leq y < 0$. The inhomogeneity has dimensions of 2 units and is centered at $(5, -7.75)$. This figure exhibits the expected result that the center of the inclusion is shifted towards larger x , in this case from $x = 5$ to about $x = 6.5$, with considerable blurring in the y direction. A possible strategy to reduce the skewness of the image in the x direction and the blurring in the y direction is to introduce a window in u that limits the coverage in this direction to $0 < u < K_{cut}$ where $K_{cut} < 2K_{max}$. Figure 2 provides plots identical to those shown in Fig. 1 except that the coverage in u is truncated at $K_{cut} = K_{max}$. The result of this

³Recall that f is assumed to vanish at $0, b$.

ORNL-DWG 92M-10047

Figure 1: Plots of σ' versus x at $y = -7.75$ (dashed line) and σ' versus y at $x = 5$ (dotted line). The solid line indicates the extent of the assumed inclusion.

windowing is that the image is less skewed in the x direction (approximately one unit) and is less blurred in the y direction. The penalty of the windowing is a slight increase in blurring in the x direction. Reducing K_{cut} further is found to reduce the skewness with increased blurring in the x direction and no significant increase in sharpness in the y direction.

Although not shown, similar simulations have been performed for the other two measurement geometries considered. In these cases, images display artifacts anticipated from the associated K -space coverages. In the single line geometry, the v coverage is symmetric about $u = K_{max}$ with v coverage decreasing linearly away from this point. As a result, the image is symmetric, however, there is significant blurring at the lateral edges of the square inclusion. For the cross line geometry, v coverage is strongly biased towards small u . As a result, images become strongly skewed towards large x .

5. CONCLUSIONS

We have derived tomographic inversion formulas applicable to four arrangements of injection and monitoring electrodes. Although only one of these gives a theoretically complete image of the subsurface conductivity, all seem to perform fairly well on simulated

CRNL-DWG 92M-10046

Figure 2: Plots of σ' versus x at $y = -7.75$ (dashed line) and σ' versus y at $x = 5$ (dotted line) for $0 < u < K_{max}$. The solid line indicates the extent of the assumed inclusion.

data. Our assumption that the conductivity fluctuations are small may be somewhat limiting as regards the usefulness of our results. There is some evidence [2, 3], however, that perturbation results can at least give reasonably accurate position information even in cases where the small fluctuation assumptions are violated.

In the surface reflection geometry, it would probably be more realistic to assume a known stratified background conductivity, $\sigma_0 = \sigma_0(x)$, rather than use a constant average value $\sigma_0 = 1$ as we have done. In this case, a Fourier transform decomposition could still be employed in the y -direction, but the zeroth order potential solution and the Green's function for the unperturbed problem which were used extensively in our work would have to be obtained numerically or by some form of approximation (perhaps an additional perturbation series could be used for some problems). For the remaining geometries, the y -axis has been chosen in the direction of stratification, and the Fourier transform would no longer be applicable.

A practical limitation which would seem to be inherent in either the cross-line or full coverage geometries can be seen in Eq.(7). When $x = b$ so the monitoring and/or injection are done on separate parallel lines, the voltage data measured is the Fourier-Laplace transform of the conductivity multiplied by the exponential factor $\exp(-kb)$, where b is the borehole separation distance. Thus, for a fixed separation, usable wave number information will be limited approximately to the range $0 < k < 1/b$. Of course,

no such limitation exists for the surface reflection or single-line geometries.

6. REFERENCES

- [1] D. D. Ang, J. Lund, and F. Stenger. Complex variable and regularization methods of inversion of the Laplace transform. *Mathematics of Computation*, 53(188):598-608, October 1989.
- [2] M. Cheney, D. Isaacson, and F. I. Isaacson. Exact solutions to a linearized inverse boundary value problem. *Inverse Problems*, 6:923-934, 1990.
- [3] I. D. and E. I. Isaacson. Comment on Calderon's paper: "on an inverse boundary value problem". *Mathematics of Computation*, 52(186):553-559, 1989.
- [4] B. Davies and B. Martin. Numerical inversion of the Laplace transform: a survey and computational methods. *J. Comp. Phys.*, 33:1-32, 1979.
- [5] K. A. Dynes and R. J. Lytle. Analysis of electrical conductivity imaging. *Geophysics*, 46:1025-1036, 1981.
- [6] R. V. Kohn and A. McKenney. Numerical implementation of a variational method for electrical impedance tomography. *Inverse Problems*, 6:389-414, 1990.
- [7] R. Kress. *Linear Integral Equations*. Springer-Verlag, New York, 1989.
- [8] A. G. Ramm. Finding conductivity from surface measurements. *Computers Math. Applic.*, 21(8):85-91, 1991.
- [9] T. J. Yorkey, J. G. Webster, and J. T. Willis. Comparing reconstruction algorithms for electrical impedance tomography. *IEEE Trans. on Biomedical Engineering*, BME-34(11):843-852, 1987.