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SELF AND TRANSFER IMPEDANCES OF COAXIAL COILS
FOR INTERROGATING A BOREHOLE FORMATION

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ABSTRACT for "Self and Transfer Impedance of Coaxial Coils for Interrogating a Borehole Formation" by Jiunn S. Yu (505 -844-6727), Division 4733, Sandia National Laboratories, Albuquerque, NM 87185

The driving-port and receiving-port impedances of conducting coils are measurable quantities that vary with the electro-magnetic (EM) properties of surrounding matter. Interrogations using coaxial coils in a borehole can thus permit the evaluation of certain compositional and structural features of earth formations. Some correlations between the EM properties and the production parameters (PP) of hydrocarbon reservoirs are already in use, e.g., the Archie's rule and the parameterized data by Poley, et. al. In anticipation of more refined correlations for tight or shaly sands where the PP are more difficult to discern, the need to reduce the ambiguity in formation evaluations has motivated borehole interrogations using multiple frequencies. Frequencies from 10 to 100 MHz seem an optimal range for operating coaxial coils. The coils' impedances below 10 MHz become less sensitive to the dielectric properties of formation, and higher-order-mode couplings above 100 MHz become too complex to invert measurable impedances. Only the dipole and quadrupole couplings are retained for typical coil diameters (5 to 20 cm) to reduce coil-design and data-inversion inaccuracies. Calibration requirements consistent with theoretical assumptions provide means of validating the operation ranges of actually designed coils. Equivalent circuits between coaxial-coil ports are formulated to visualize the coupling mechanisms involved. Various measurements on the self and transfer impedances enable the self-consistency checks on whether an interrogated formation is indeed homogeneous and isotropic. Perhaps more importantly the impedance measurements also provide a data base for parameter-optimization techniques to quantify the often-heterogeneous/anisotropic formations along a borehole.

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We consider two conducting coils aligned coaxially to a borehole axis. One is connected to a transmitter of constant-voltage source and the other to a receiver of infinitely large impedance. As formation properties along the borehole vary, the self impedance of transmitting coil and the transfer impedance at receiving coil will also vary. The purpose of borehole interrogation is to evaluate formation properties from the two measurable impedances.

The transfer impedance between two small coils have already served to evaluate certain formation parameters. For examples, earth conductivity [Moran & Kunz] and electric permittivity [Fuller & Wait] have been evaluated including the effects of borehole sizes and fluids. Field tests in reservoir rocks have also been demonstrated [Meador & Cox, and Berry, II, et al] to estimate hydrocarbon saturations.

All these previous works and the present paper construct analytical models to invert measurable responses for desired parameters by assuming that the formation is ideally linear, isotropic and piecewise homogeneous. However, borehole formations are often heterogeneous and anisotropic and cause parameter evaluations to be in error. A quick estimate on the degree of heterogeneity/anisotropy can thus be of great value in practice to prevent undue error that may otherwise arise. This paper emphasizes the use of self and transfer impedances simultaneously to allow a self-consistent check [Yu] on the inverse process of evaluating desired parameters from measurable responses.

Another assumption made in the previous works was that, in their respective frequencies of interrogation, the coaxial-coil coupling is due to magnetic dipole alone. Significant error may result from this assumption if nonuniform components of transmitting-coil current become substantial. For example, a formation with a dielectric constant 9 at 64 MHz would be evaluated to have a dielectric constant 6 if use were made of the transfer impedance between two coils 0.1 m in diameter and 1.0 m in separation. This paper introduces

magnetic-quadrupole coupling to reduce and prevent this type of error from taking place when source frequencies from 10 to 100 MHz are used. In addition, a highly desirable feature of operating multiple frequencies simultaneously would help to make formation evaluations more unique by exploiting the dispersive characteristics of reservoir rocks [Poley, et al].

In log data analyses it has been common to assume that the magnetic permeability of earth formations is the same as that of an empty space. This assumption is always in doubt until verified, particularly for lossy formations. An in-situ verification can be very useful in differentiating several possibilities in log interpretation. This paper proposes to facilitate this verification by making use of measurable impedances at both coils with lower frequencies.

Another problem is often encountered in interrogating reservoir rocks that are partially saturated by saline water. The formation could become so lossy that no detectable signal would reach the receiving coil to permit an accurate measurement of transfer impedance. The emphasis on self-impedance measurement here is to also permit an accurate evaluation of reservoir rocks that may be partially saturated by highly saline water.

Increasingly log tools of better accuracy and sensitivity are needed to help making hydrocarbon production more economical for reservoirs of moderate to low qualities. The following sections of this paper offer a set of analytical tools to probe designers and log analysts who are in need of multiple-frequency interrogations to enhance the uniqueness of their formation evaluations. Operating frequencies are, for the time being, limited between 10 to 100 MHz for coaxial-coil couplings. The low limit is set to avoid the delicate and often-unstable nullings needed to suppress the dominant responses carrying the magnetic-permeability-only information. The high limit is set to avoid the need of including magnetic-octupole couplings in the inverse process of evaluating the desirable parameters from the measurable responses.

II.

HARMONIC CURRENTS ON TRANSMITTING COIL

There is little exaggeration to say that the accuracy of inverse process hinges on how accurate the current distribution can be measured on the transmitting coil. We consider a single-turn circular coil driven by a constant voltage V across an infinitesimal gap at $\phi = 0$ of the cylindrical coordinates (r, ϕ, z) . All the geometric and electromagnetic parameters, as well as measurable responses, of interest in this paper are defined in the table provided for convenient reference. King, Wu, and Storer have investigated the current distribution of such a coil by a Fourier-series expansion as

$$I(\phi) = \sum_{n=0}^{\infty} I_n \cos(n\phi) = V \sum_{n=0}^{\infty} Y_n \cos(n\phi) \\ \approx V[Y_0 + Y_1 \cos(\phi)] \quad , \quad \text{if } |ka| < 1; \text{ where} \quad (1a)$$

$$Y_0^{-1} = Z_0 \approx \omega \mu a [(\pi/6)(ka)^3 + jq(a,b)] \quad , \text{ and} \quad (1b)$$

$$Y_1^{-1} = Z_1 \approx (2\omega \epsilon a)^{-1} [(\pi/3)(ka)^3 - jq(a,b)] \quad , \text{ with} \quad (1c)$$

$$q(a,b) = \ln(8a/b) - 2 \approx 3 \text{ for } a/b = 18.5. \quad (1d)$$

If the coil radius a is 6 or more times larger than the wire radius b , the shunt impedances Z_0 and Z_1 would be purely reactive for all magnitudes of ka 0.3 or less. For these cases the uniform current I_0 would be dependent only on the magnetic permeability of surrounding medium. As a result, no useful information about the medium would be obtained from measuring I_0 , if the medium is known to have the same magnetic permeability as that of empty space.

Geometric Parameters, Electromagnetic Parameters and Measurable Responses of Coaxial-Coil Couplings

$(a, b) \equiv$ radii of (coil, wire) for transmitting and receiving coils

$(r, \phi, z) \equiv$ the circular-cylindrical coordinates referenced to transmitting coil center

$s = \sqrt{a^2 + z^2}$ in meters $\equiv$ distance from the center of transmitting coil to the wire-center of receiving coil

$(\mu_0, \epsilon_0) \equiv$ (magnetic permeability, electric permittivity) of empty space

$$\approx (4\pi \times 10^{-7} \text{ henry/meter}, 10^{-9} / 36\pi \text{ farad/meter})$$

$(\mu, \epsilon) \equiv$ (magnetic permeability, electric permittivity) of matter

$$= (\mu_0 \mu_r = \mu_0 (\mu'_r - j\mu''_r), \epsilon_0 \epsilon_r = \epsilon_0 (\epsilon'_r - j\epsilon''_r))$$

$j = \sqrt{-1}$ for the time-harmonic convention $\exp(+j\omega t) = \exp(+j2\pi f t)$

$$(k_0, \eta_0) = (\omega \sqrt{\mu_0 \epsilon_0} = \omega/c = 2\pi/\lambda_0, \sqrt{\mu_0/\epsilon_0} \approx 120\pi \text{ ohms})$$

$$\begin{aligned} (k, \eta) &= (\omega \sqrt{\mu \epsilon} = k_0 \sqrt{\mu_r \epsilon_r} = k_0 v_r, \sqrt{\mu/\epsilon} = \eta_0 \sqrt{\mu_r/\epsilon_r}) \\ &= \eta_0 \eta_r \text{ where } (v_r, \eta_r) \text{ are complex (refractive index, relative intrinsic impedance)} \end{aligned}$$

$V \equiv$ applied voltage across $\phi = 0$ of the transmitting coil

$V_{oc} \equiv$ open-circuited voltage across $\phi = 0$ of the receiving coil

$I(\phi) \equiv$ current distribution around the transmitting coil

$Z_{11} \equiv$ self or driving-port (across $\phi = 0$) impedance of transmitting coil

$$= V/I (\phi=0)$$

$Z_{21} \equiv$ transfer or receiving-port (across $\phi = 0$) impedance at receiving coil

$$= V_{oc}/I(\phi=0)$$

The cosinusoidal current I_1 , if made measurably large, would depend on the electric permittivity of surrounding medium. The condition for I_1 to be larger than -34 dB relative to I_0 is that the magnitude of ka becomes larger than 0.1. Otherwise the coil will support predominantly I_0 and can be approximated as a simple magnetic dipole.

III. SELF IMPEDANCE FOR PROBE CALIBRATION

Making use of (1b) to (1d), the driving-port impedance of an idealized small coil has an equivalent circuit consisting of Z_0 and Z_1 in parallel:

$$Z_{11} \approx (Y_0 + Y_1)^{-1} = V / (I_0 + I_1) \quad (2a)$$

$$= \frac{j\eta(ka) [q^2(a,b) + j(\pi/6)(ka)^3 q(a,b) + (\pi^2/18)(ka)^6]}{(1-2k^2 a^2) q(a,b) + j(\pi/3)(ka)^3 (1+k^2 a^2)}$$

$$\approx j3\eta(ka)/(1-2k^2 a^2) \text{ for } |ka| < \pi/10 \text{ and } a/b = 18.5 \quad (2b)$$

$$\approx j3\omega\mu a \text{ for } |ka| < 0.1/\sqrt{2}.$$

The condition for the last expression is set by assuming the noise level in practice to be about -40 dB below the predominant current I_0 .

A real transmitting coil will have a finite feed-gap, a feeder cable from transmitter, and a dielectric mandrel of finite thickness. Thus a well-designed calibration procedure is essential to check whether the analytical model in (1) or (2) can adequately describe the coil operations from 10 to 100 MHz. As a minimum the coil has to be calibrated in the air to determine an effective dielectric constant of slightly larger than unity for the coil assembly. Additional calibrations in a medium of high dielectric constant with negligible loss is desirable. Perhaps a conducting cylinder filled with distilled water is a good choice, although it would necessitate the solution of boundary-value problems associated with the

metallic tank.

Suppose that a transmitting coil had been satisfactorily calibrated. We could then proceed with confidence in knowing that (1) and (2) are correctly modeling the coil performance. It is tempting to evaluate the medium parameters from measurements on Z_{11} , with operating frequencies that make I_1 measurable. This inverse process can be developed using the ratio of Z_{11} measured in an undetermined medium to Z_{11} measured in the air:

$$S(\mu_r, \epsilon_r) = Z_{11}(\mu, \epsilon) / Z_{11}(\mu_0, \epsilon_0) = S' + jS'' \quad (3)$$

The ratio S is in general a complex number at a given frequency. If we assume $\mu_r = 1$, then complex values of ϵ_r across 10-100 MHz could be obtained from (3) as an inverse process.

IV. TRANSFER IMPEDANCE FOR SONDE CALIBRATION

Having assured that (1) or (2) is accurate after calibrations, the source currents can then be used to formulate the coupling mechanisms between two coaxial coils in a sonde. We use the multipole expansion technique [Stratton, Jackson] and obtain the multipole coefficients by integrating the source contributions of transmitting coil. The current I_0 contributes to magnetic dipole only, while I_1 contributes to both electric dipole and magnetic quadrupole. These multipoles constitute the field vectors produced by the transmitting-coil currents.

Only the ϕ -component E_ϕ of the electric field vector is of interest for our coaxial-coil coupling. Our transmitting coil is placed at $z = 0$ and the receiving coil is placed at an arbitrary distance z along the borehole axis. With the absence of receiving coil, the E_ϕ at z based on the multipole-expansion technique is

$$E_{\phi} \approx \frac{\eta}{4s} (ka) \left[\left(\frac{3a}{s} \right) \frac{\sin(ka) - (ka) \cos(ka)}{k^2 a^2} (I_0) \left(1 - \frac{j}{ks} \right) - \right. \\ \left. - j \frac{\sin(ka)}{ka} (I_1) \left(1 - \frac{j}{ks} - \frac{1}{k^2 s^2} \right) \cos \phi \right] \exp(-jks), \text{ where} \quad (4a)$$

$$s = (a^2 + z^2)^{\frac{1}{2}}. \quad (4b)$$

When this field is reduced to the radiation zone ($z/a \rightarrow \infty$) it is in agreement with the same field obtained by King. Moreover, the magnetic-dipole coefficient associated with I_0 is also in agreement with the point-source approximation ($z/a \gg 1$) from the closed-form solution of a constant-current loop [Yu].

Let a receiving coil of the same geometry as the transmitting coil be placed at an arbitrary distance z along the borehole. The open-circuited voltage across an infinitesimal gap at $\phi = 0$ of the receiving coil is related to the transfer impedance between the two coils as

$$V_{oc} = \frac{-1}{I(\phi=0)} \int_0^{2\pi} I(\phi) E_{\phi}(a, d\phi); \text{ and} \quad (5a)$$

$$Z_{21} = \frac{V_{oc}}{I(\phi=0)} \approx -\eta \frac{\pi a}{2s} (ka) Z_{11}^2 \left[\left(\frac{3a}{s} \right) \frac{\sin(ka) - (ka) \cos(ka)}{k^2 a^2 z_0^2} \cdot \left(1 - \frac{j}{ks} \right) - j \frac{\sin(ka)}{2ka z_1^2} \left(1 - \frac{j}{ks} - \frac{1}{k^2 s^2} \right) \right] \exp(-jks); \quad (5b)$$

$$\approx \frac{-\eta \pi (ka)^3}{2ks(1-2k^2 a^2)^2} \left[\left(\frac{a}{s} \right) \left(1 - \frac{j}{ks} \right) - j 2(ka)^3 \left(1 - \frac{j}{ks} - \frac{1}{k^2 s^2} \right) \right] \exp(-jks);$$

$$\text{for } |ka| < \pi/10 \text{ and } a/b = 18.5, \quad (5c)$$

$$\approx j\omega\mu \frac{(\pi a^2)^2}{2\pi s^3} = j\omega M \quad \text{for } |ks| \ll 1; \quad (5d)$$

where M is the mutual inductance between the coils under quasistatic approximation.

Again a well-designed calibration procedure is essential for an actually implemented sonde to check whether the analytical model in (4) and (5) provides an accurate description for 10-100 MHz operations. The calibration procedure suggested for Z_{11} is applicable also for Z_{21} .

V. A SELF-CONSISTENCY CHECK BY Z_{11} & Z_{21}

The solutions for Z_{11} and Z_{21} are obtained by assuming that the medium surrounding the coaxial coils is linear, isotropic and homogeneous. However, borehole formations are often to some extents heterogeneous, anisotropic, and even nonlinear. As is well known, the assumption made in analytical model has to be verified to assure the accuracy of inverse process by which desirable formation parameters are evaluated from measurable responses.

Self-consistency checks on the analytical model of sonde are usually done after the sonde is properly calibrated in a known formation. For coaxial-coil interrogations, a self-consistency check may require that the formation parameters (μ_r, ϵ_r) evaluated from Z_{11} and Z_{21} are in agreement to within calibrated inaccuracies and measurement imprecisions. Unless the formation parameters inverted from measurements at different ports are self-consistent in this sense, it is very likely that the formation properties do not behave as assumed.

For example, without assuming $\mu_r=1$, a sonde may consist of two receiving coils placed at $z=z_1$ and z_2 . Use of (2a) and (5b) can produce three sets of complex (μ_r, ϵ_r) by three sets of combination: $Z_{11}(z=0)$ and $Z_{21}(z=z_1)$, $Z_{11}(z=0)$ and $Z_{21}(z=z_2)$, and $Z_{21}(z=z_1)$ and $Z_{21}(z=z_2)$. If the three sets of inverted

parameters are not self-consistent to within acceptable inaccuracies/imprecisions, then subsequent estimates on the production parameters of reservoir rocks would have to be cross-checked with other log data. Undue errors can be more systematically reduced when self-consistency checks are made in conjunction with log-data interpretations.

VI. IS RELATIVE MAGNETIC PERMEABILITY UNITY?

This question for a borehole formation is always in doubt until verified, especially for lower-frequency interrogations where the evaluation of formation conductivity may be significantly affected by small differences in magnetic permeability. An in-situ verification to this question may be implemented by making use of (2c) or (5d) where the impedances are directly proportional to μ if the frequency is set around 10 MHz.

The use of (5d) is a proximity-coil coupling [Yu] where residual errors in the evaluation of μ have been quantitatively established for a coil spacing of $z=2a$.

VII. MAGNETIC-QUADRUPOLE REDUCES EVALUATION ERROR

An example may serve to illustrate the importance of including the magnetic-quadrupole coupling in using Z_{21} for parameter evaluation. We assume for simplicity that a formation is known to have $\mu_r=1$, and $\epsilon_r=9$ is the dielectric constant at 64 MHz to be inverted from measurements of Z_{21} .

Suppose we have two well-calibrated sondes (with $a/b=18.5$) by which measured Z_{21} are as good as that of (5b), then two complex ratios may be obtained using (5b) with two geometrical arrangements:

$$\frac{Z_{21}(a = 0.02\text{m}, z = 0.4\text{m}; \mu_r = 1, \epsilon_r = 9)}{Z_{21}(a = 0.02\text{m}, z = 0.4\text{m}; \mu_r = 1, \epsilon_r = 1)} \approx 1.5 - j0.9, \text{ and} \quad (6a)$$

$$\frac{Z_{21}(a = 0.05\text{m}, z = 1\text{m}; \mu_r = 1, \epsilon_r = 9)}{Z_{21}(a = 0.05\text{m}, z = 1\text{m}; \mu_r = 1, \epsilon_r = 1)} \approx -1.2 - j2.8 \quad (6b)$$

The first ratio corresponds to $ka \approx 0.2$, i.e., the magnetic-dipole-only assumption is quite good. If the complex number $(1.5 - j0.9)$ happened to result from actual measurements, a refractive-index chart [Yu] based solely on magnetic-dipole coupling could be used to evaluate the "unknown" ϵ_r . Indeed the chart inverts $(1.5 - j0.9)$ to the correct value of $\epsilon_r = 9$. The second ratio, on the other hand, corresponds to $ka \approx 0.5$ and the magnetic-dipole-only assumption is less valid. As a result, the same chart would invert $(-1.2 - j2.8)$ to a dielectric constant of 6, a 30% error due to assuming the magnetic-quadrupole coupling is negligible.

Equation (5b) is devised to reduce this type of errors when frequencies higher than 40 MHz are used in logging typical formations. An inverse process based (5b) is of course needed for the second ratio of example to yield the correct value of $\epsilon_r = 9$.

VIII. OPEN-CIRCUITED VOLTAGES CAN BE TOO NOISY

When certain coil spacings are fixed after a sonde is developed, there will be occasions when the induced voltages at receiving coils are too weak to be measured. These occasions would occur if formations are partially saturated by saline water. From (5b) we can see that, in addition to geometrical spreads due to a/s , the exponential decay could be very severe if the imaginary part of k becomes excessively large.

This problem becomes most severe at the lowest frequencies. For example, $\epsilon_r \approx 20 - j200$ at 10 MHz for a salt-water saturated sandstone of 14.5% porosity by volume and 4.3% salinity by weight [Poley, et al]. Fortunately when the V_{oc} becomes too noisy for Z_{21} measurements, the self impedance of transmitting coil becomes very sensitive to changes in formation parameters. Equation (2a) is thus given to emphasize that it can be used to develop an indispensable inverse process when coaxial coils are used to log reservoir rocks saturated partially by saline water.

IX. MULTIPLE-FREQUENCY LOGGING ENHANCES THE UNIQUENESS OF FORMATION EVALUATIONS

Reservoir rocks are highly dispersive for frequencies below 250 MHz, i.e., their electromagnetic parameters are very frequency-dependent, when fully or partially saturated by water of various salinity. Their dispersive characteristics have been demonstrated both empirically (e.g., Poley, et al) and analytically (e.g., Lysne). Two rocks of about the same porosity can have measurably different dielectric properties from 10 to 100 MHz. These different characteristics are attributable to poreshape distributions and/or baserock constituents. In either case it appears imperative to have multiple-frequency evaluations if the uniqueness of evaluating reservoir rocks is to be enhanced.

A reference to Figure 4 of Poley, et al may help to bring the nonuniqueness problem into focus. Two sandstone specimens of 12.7% and 12.9% porosity were saturated by fresh water. The real part of dielectric "constant" of the first specimen varied from 10.3 to 9.2, while the second specimen varied from 14.5 to 10.5, when frequencies were varied from 10 to 100 MHz. If these frequency-dependent properties were not adequately evaluated through multiple-frequency interrogations, it would be rather difficult to discriminate these two rocks uniquely.

This paper has improved the inverse-process algorithms for Z_{11} and Z_{21} so that coaxial-coil operations can cover 10-100 MHz, and allow multiple-frequency interrogations to take place simultaneously. This new capability seems particularly timely and important for evaluating reservoir rocks of moderate to low qualities where formation parameters are increasingly more difficult to discern.

X. BOREHOLE EFFECTS MUST BE NEGLIGIBLY SMALL

Our models of Z_{11} and Z_{21} are formulated for coaxial coils without any supporting arrangements such as access cables, dielectric mandrel, sonde housing, borehole with mud, etc. The models are thus valid only when the effects of these supporting arrangements are negligibly small compared to those induced by the formation rocks of interest. This problem between theory and practice is by no means unique for coaxial-coil coupling. A closely related example is the measurement cares [Yu and Peters] needed to evaluate correctly the multipole coefficients of a specimen sphere which is analytically modeled with the absence of any supporting structure. Assuming a sonde has been accurately calibrated in a laboratory environment, then a borehole with mud would serve to support the sonde during downhole interrogation. Borehole effects contributing to Z_{11} and Z_{21} are thus required to be negligibly small if formation properties are to be accurately evaluated.

A most important condition to keep borehole effects negligible is to keep borehole diameter a small fraction of the cutoff wavelength when the borehole is treated as a cylindrical waveguide. To illustrate, we use Fig. 10 of Meador & Cox as a practical example. Relevant parameters and responses ("relative dipole" and "relative quadrupole" as used by Meador & Cox) are summarized in the following table.

The tabulated values are computed by specializing a generalized N-zone solution [Yu & Reardon] into a 3-zone solution with a_1 and a_2 as the radii of dividing surfaces. Only magnetic-dipole coupling is considered in the computations. For the coil radius $a_1 = 0.105$ cm, the coils are in effect treated as Hertzian dipoles totally submerged in borehole mud. For $a_1 = 5.25$ cm, the borehole mud occupies only the radial extent from 5.25 to 10.5 cm. The table shows that an error of less than 2% in magnitude would result if the sonde radius were varied from 5.25 to 0.105 cm.

Small Borehole Effects Caused by Changing the Radius of Coils

Formation ϵ_r in $r > a_2$	"Relative Dipole"		"Relative Quadrupole"	
	$a_1 = 5.25\text{cm}$	$a_1 = 0.105\text{cm}$	$a_1 = 5.25\text{cm}$	$a_1 = 0.105\text{cm}$
1-j1.76	0.973-j0.098	0.977-j0.100	1.04-j0.07	1.05-j0.07
20-j14	0.780-j0.995	0.784-j0.995	1.39-j1.50	1.41-j1.50
50-j112	-0.390-j0.231	-0.388-j0.238	-1.01-j0.53	-0.997-j0.554

$\epsilon_r = 1$ inside the sonde with a variable coil radius a_1 .

$\epsilon_r = 26 - j11.3$ for mud at 16 MHz and $26 - j6$ for mud at 30 MHz when "relative dipole" and "relative quadrupole" are evaluated within a borehole of radius $a_2 = 10.5\text{cm}$.

More extensive studies based on the N-zone solution can be made on the effects of borehole fluids and invaded-formation profiles. For the time being; we note that the analytical models of this paper are valid for log interpretations when operating conditions are designed to have negligible borehole effects and when formation rocks can be assumed uniform beyond the borehole wall.

We have improved the analytical models for coaxial-coil logging to permit a self-consistent and accurate evaluation of formation rocks using multiple frequencies within 10 to 100 MHz. For typical borehole diameters up to 20 cm, the inclusion of magnetic-quadrupole coupling allows the self and transfer impedances to be accurately calibrated during design, and self-consistently measured during downhole interrogation. Comprehensive processings of the measurable Z_{11} and Z_{21} are expected to enhance the uniqueness of evaluating the properties of reservoir rocks.

Like other logging tools, the application of our analytical models will be degraded when borehole effects become significant. As a rule, if the borehole diameter is twice the coil diameter of sonde, the magnitude of ka pertaining to borehole mud should be always less than $\pi/5$ to keep borehole effects sufficiently small. The frequency range of 10 to 100 MHz is allowable if a borehole diameter is less than 20 cm and a borehole mud is not water-based. However, economical considerations and practical operations often make it necessary to log boreholes filled with water-based mud. In these cases our analytical models are applicable only when the logging frequencies are lower than 50 MHz.

After the borehole effects are assured to be negligibly small, the magnitude of ka pertaining to formation rocks should also be restricted to less than $\pi/5$. If formation rocks are such that this restriction does not hold, then the coil diameter would have to be reduced, or the models as presented would have to be refined by including the magnetic-octupole coupling.

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