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James W-K. Mark

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A THEORY OF BENDING WAVES WITH APPLICATIONS TO DISK GALAXIES*

James W-K. Mark

Lawrence Livermore National Laboratory

Livermore, CA, 94550, U.S.A.

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Abstract: A theory of bending waves is surveyed which provides an explanation for the required amplification of the warp in the Milky Way. It also provides for self-generated warps in isolated external galaxies. The shape of observed warps and partly their existence in isolated galaxies are indicative of substantial spheroidal components. The theory also provides a plausible explanation for the bending of the inner disk (< 2 kpc) of the Milky Way.

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I. INTRODUCTION

The bending of the disk of the Milky Way has been observed earlier by Burke (1957), Kerr (1957) and Westerhout (1957), and has generated considerable interest for theoretical explanation (e.g. Kahn and Woltjer 1959, Elwert and Hablick 1965, Lynden-Bell 1965, Avner and King 1967, Hunter and Toomre 1969). The work of Hunter and Toomre and that of Fujimoto and Sofue (1976, 1977), Spight and Grayzeck (1977) all suggested that the most attractive reason for the bending was due to tidal distortion by the LMC. But quantitative comparison was satisfactory only if the tidally generated warps could somehow be amplified "by a factor approaching 3" (cf. p. 773 of Hunter and Toomre 1969).

Toomre (1966), briefly with moment equations, and Kulsrud et al. (1971) and Mark (1971) with detailed stellar dynamics, reported on the possibility of a "fire-hose" type instability in the stellar disk which could provide for self-generation and/or amplification of bending waves. A small warp in the more massive stellar disk could drive larger amplitude warps in the outer gaseous disk. This mechanism appeared useful for providing wave amplification in the Milky Way. It became more interesting in the light of subsequent observations of warps in rather isolated galaxies (Sancisi 1976, Bosma 1978; Rogstad et al. 1974 found it difficult to conclude that the warp of M83 was tidal). Also some stellar disks showed warps (Tsikoudi 1977, 1980; van der Kruit 1979). However this mechanism

depended upon galaxy disks possessing sufficiently large stellar velocity dispersions in the plane relative to that out of the plane. The only obvious observational test based on stellar dispersive speeds in the solar neighborhood is suggestive of stability or near marginal stability, making the question of the allowed amount of residual wave amplification more difficult to ascertain (we recall that only a factor of 3 amplification was needed in the Milky Way). Although difficult to test, the possibility exists that such a mechanism plays a role in some presently observed galaxy warp or in the warps at earlier states of galaxy evolution. For example, it might represent one of the possible constraints which caused disk galaxies to maintain a certain ratio of stellar velocity dispersion in their disk. Perhaps dispersions in the plane were needed to stabilize Jeans instability while the vertical dispersion had to maintain a proper ratio to these latter in order to stabilize the "fire-hose" instability.

More recent theoretical studies of bending waves in disk-like systems have been motivated firstly by the observational discovery of warps in numerous external galaxies (Roberts 1966, Rogstad et al. 1974, 1976, Roberts and Whitehurst 1975, Sancisi 1976, Bosma 1978 and Weliachew et al. 1978) showing that the phenomena is rather widespread as well as occurring also in isolated galaxies. Interest has also been generated by the conjecture that the phenomena of warps might be more easily understood because of the occurrence of extended spheroidal "halo" matter (Polyachenko 1977, Binney 1978, Saar 1978,

Sanders 1978, Polyachenko and Shukhman 1979, Tubbs and Sanders 1979). Also, other possibly bent disks in or around galaxies require dynamical explanation, for example that in the inner few kiloparsecs of the Milky Way (Burton and Liszt 1978; Liszt and Burton 1978, 1980; Sinha 1979). Warps in isolated galaxies (cf. Sancisi 1976, Bosma 1978) are indicative of the need for internal excitation mechanisms. These recent theoretical descriptions of warped structures either lack detailed dynamical justification (particularly discussions of properties of such bending waves), or, they assume a priori a very dominant "halo" (sometimes even triaxial in shape, Binney 1978), while the very question of the mass and distribution of spheroidal matter is a subject for detailed observational determination.

Based on the earlier studies of bending waves with self-gravity by Hunter and Toomre (1969), Kulsrud et. al. (1971), Mark (1971), and based on a new approach to the gravitational potential theory for waves in disks (cf. Mark 1976c), the recent study of Bertin and Mark (1980) further examines the effects of unstable, two-stream interactions with a spheroidal subsystem (the disk is like "a flag waving in the wind"). This subsystem could be a massive "halo" but the theory allows it also to be just the extensions of a "bulge" (less massive). We find that sensible explanations of self-excited warps or driven waves are possible without necessarily postulating "haloes" which are either excessively large or triaxial (or even flattened). A crude heuristic description of our detailed dynamical theory is included in Section II. The very fact that warps could

also be driven by small spheroidal subsystems or other mechanisms (e.g. "fire-hose") makes it at first more difficult to connect the properties of halo matter with that of warps in isolated galaxies.

On the other hand, by a novel use of a wave energy or angular momentum relation (cf. Eq. 1 below), Bertin and Mark (1980) showed that just the typically observed qualitative shapes of galaxy warps are indicative of extensive spheroidal matter within the radii of the observed warps (cf. Section III). Preliminary confirmation with a few details has been carried out by Lake and Mark (1980) for the Milky Way and some external galaxies; additional suggested examples are given in the paper by Bertin and Casertano (1982). Since our theory does not assume substantial haloes to begin with, it could provide independent evidence for halo sizes. As we will emphasize in Sections II and III, our arguments depend only on very few basic concepts of the theory and are thus largely independent of the detailed mathematical analyses. In fact they only depend on bending wave propagation properties in the disk and do not entail the details of whether self-generated warps interact with the halo or with fire-hose instabilities in the stellar disks.

Our theory of bending waves also seems to provide a plausible dynamical rationale (cf. Section IV and Blitz et. al. 1981) regarding the interpretation that the inner gaseous disk (< 2 kpc) of the Milky Way is bent out of the plane of the main galaxy disk (Burton and Liszt 1978, Liszt and Burton 1978, 1980, Sinha 1979). In this

case it is due to an analogous two-stream type instability between the inner disk and the bulge (or spheroidal matter with this smaller length-scale). Further theoretical work and comparisons with observations would be useful.

II. A HEURISTIC DISCUSSION OF THE BENDING WAVE THEORY

For details on the analytical derivations of the bending wave theory as used here, we refer the reader to Mark (1971), Bertin and Mark (1980). We will not attempt to summarize these extensive calculations here. Rather, we provide discussions of much simplified models as a guide to the basic physics involved.

If we artificially "turn-off" the self-gravity in the disk, then we are left with so-called "kinematic waves". Their nature is best illustrated in the case of a massless flat disk imbedded in a spherical halo (cf. Figure 1). Particles displaced from the disk will remain in orbits inclined from the original flat disk. There are no actual torques on the particle orbits so that the appearance of vertical oscillations is purely kinematical. If the individual particle displacements are systematic over the disk (such as due to initial external driving), they could form a warped disk. In such a kinematic wave, each particle's bending amplitude above the initial plane bears no particular universal relation to that of particles at other radii except where it relates to the initial external forcing.

Also lack of coherence of frequencies at different radii results in eventual "washing-out" of the warp due to phase-mixing.

Restoring of self-gravity to the disk changes the vertical oscillation frequency as well as allows horizontal propagation of wave information. This self-gravitating bending wave can be crudely illustrated by the model of Figure 2. Let us denote the height of bending $\Delta z = h(r, \theta, t) = \text{Re}\{\exp[i(\omega t - m\theta + jkdr)]\}$ in the usual cylindrical polar coordinate system (r, θ, z) where t is the time [$k(r)$ can be a complex-valued wavenumber containing both phase and amplitude information]. Clearly these waves would exhibit a dispersion relation $D(\omega, m, k) = 0$ and group velocity $c_g = -d\omega/dk$. Even without entering into details, we can surmise that conservation of wave energy implies that the flow of wave energy through a fixed radius $2\pi r c_g \mathcal{E} \approx \text{constant}$, where $\mathcal{E}$ is wave energy density. From general principles we expect $\mathcal{E} \propto \Sigma(r) h^2$, where $\Sigma(r)$ is the local surface mass density of the disk. Also $c_g \propto \Sigma$ and thus we expect

$$r \Sigma^2 h^2 \approx \text{constant}, \text{ or, } h \propto [r^{1/2} \Sigma(r)]^{-1}. \quad (1)$$

This approximate relation depends only on self-gravity and very little on the details of the calculations (cf. Bertin and Mark 1980). The fact that this relation seems to be obeyed in a number of applications (cf. Sections III-IV) to warped and bent disks is an indication of the importance of disk self-gravity. If we recall from electrostatics that self-forces of disk charges are important relative to distributed volume charges, we are then not too surprised that self-gravity should

be important unless this disk mass is rather miniscule. In fact even in the extreme case of Saturn's rings relative to the mass of Saturn, preliminary evidence suggests bending waves where self-gravity affects the wave amplitude relation (Shu et al. 1982).

A somewhat more detailed (but still heuristic) calculation could equate the vertical accelerations

$$\left[\frac{\partial}{\partial t} + \Omega(r) \frac{\partial}{\partial \theta} \right]^2 h = v_z^2(r) h + g_z \quad (2)$$

where $\Omega(r)$ is the angular frequency of rotation of the disk, and $v_z^2(r)$ represents the kinematic oscillation frequency of Figure 1. The retarding self-gravity can be estimated by using the physical picture at the left frames of Figure 3. First we might analyze the wavy bent disk into two planes with densities $\Sigma/2$ and attraction $g_z \sim G\Sigma$ (G = gravitational constant). Clearly the real situation has an additional angular projection factor ($h/\text{wavelength}$) or kh . Thus $g_z \sim -G\Sigma|k|h$ for the warped disk. Equating to equation (2) gives a dispersion relation

$$[\omega - m\Omega(r)]^2 = v_z^2(r) + 2\pi G\Sigma(r)|k(r)| \quad (3)$$

where the extra 2π factor can be known only by detailed calculations (cf. Mark 1971, Bertin and Mark 1980). The right hand frame of Figure 3 illustrates this dispersion relation for $v_z = 0$. Additional effects such as disk thickness, response of stellar disk including vertical mode structure and resonances were discussed by Mark (1971). This paper also gives some details of the "fire-hose"

instability mentioned already in the introduction. Bertin and Mark (1980, particularly App. D.) summarizes recent applications where response of gaseous disk is emphasized. From the above, it is clear that the difference in amplitude of response of gas and star disks is due mainly to their relative inertias; but in addition, smaller effects due to dispersive velocities enter. The theory does allow occasional observable warping of stellar disks also, and these appear to have been detected in some galaxies (Tsikoudi 1977, 1980, van der Kruit 1979).

In the above discussion the halo, if present, provides an inactive basic gravitational field. On the other hand, the warped disk has an azimuthal component of the gravitational force which exerts a torque on the particles of the halo that pass close enough to the disk (cf. Figure 4a). This interaction allows a two stream instability (cf. Bertin and Mark 1980) which amplifies wave disturbances in the disk. For example, a driven wave excited by passage of a companion could be so amplified as it propagates inwards and as it is reflected and becomes outward propagating (Figure 4b). According to Bertin and Mark (1980), this could be sufficient to explain the amplitude discrepancy (Hunter and Toomre 1969) in driving of our galaxy's major warp by the Large Magellanic cloud. The amount of wave amplification per wave crossing time is sufficiently small that equation (1) remains roughly valid. Bertin and Casertano (1982) performed further theoretical confirmations of the disk-halo interactions by means of a gas-dynamical model (cf. also Yoshii and Fujimoto 1981 who specified an actual gaseous halo rather than using

gas-dynamics as a model; and Nelson 1976 whose discussion is relevant to the gas disk at intermediate radii).

On the other hand, if the boundary conditions are suitable, bending modes (Hunter and Toomre 1969, Kulsrud et al. 1971, Mark 1971, Bertin and Mark 1980) could be set up which are now self-generated because of this destabilizing interaction with the halo. Some such modes are conceptually illustrated in Figure 5. We emphasize that these bending modes are standing waves in the radial direction but propagate in the azimuthal direction with rotation frequency $\Omega_p = R_e(\omega)/m$. For standing waves, the range of allowable outer conditions have not all been delineated. But it is sufficient (Hunter and Toomre 1969) to have a rather sharp cutoff in the outer disk density; and some galaxies indeed have unusually sharp outer boundaries (van der Kruit and Searle 1981a,b) which are more than the minimum required for standing waves. It would be helpful to pursue further theoretical developments similar to that carried out for spiral structure (e.g., cf. Lau et al. 1976, Mark 1976 a-c, 1977, Bertin et al. 1977, Bertin and Mark 1978).

III. WARPS AND EXISTENCE OF SUBSTANTIAL HALOES IN GALAXIES

Even very simple considerations are suggestive that the theoretical picture of self-gravitating warps is consistent with substantial haloes inside the region where the disk warps exist as a diagnostic. This relationship follows (cf. Bertin and Mark 1980)

from the simple scaling law (1) relating height of bending $h(r)$ to the disk surface mass density $\Sigma(r)$. For the sake of argument, let us first presume that the observed flat rotation curves (cf. Rubin 1978) in galaxies represent somehow a mass distribution all restricted to the disk. Then $\Sigma(r) \sim 1/r$ in the outer parts where the rotation curves are near flat and where warps are observed. From equation (1), this implies a contradiction in that warps have a height relation $h(r) \propto r^{-3/2}$ which should look conceptually like Figure 6a. Of course observed warps have more typically the shape like Figure 6b which is indicative of a much more rapid fall off of disk mass according to equation (1). Thus, we find that the shape of warps and our theory together give an independent argument that flat rotation curves indeed are suggestive of substantial haloes (nb. our argument here is independent of detailed disk-halo interactions). The remainder of this section discusses this point further in regard to the Milky Way and one external galaxy, NGC 2841.

Figure 7a plots surface mass density against radius in the Milky Way galaxy as derived from equation (1) using the data of Henderson (1978). If we temporarily adopt a flat rotation curve with the circular velocity beyond the solar radius (R_0) equal to 250 km/s, we find the resulting points are well-fit by an exponential with a scale length of $3.4 \pm 0.2 (R_0/10 \text{ kpc}) \text{ kpc}$. This procedure is superior to determinations of the disk profile from fitting HI velocity profiles, because nothing is initially assumed about the mass distribution as a function of radius. (Of course, in both cases

a rotation curve must be assumed or measured so as to derive distances from the observed line-of-sight velocities.) Assuming the same flat rotation curve, Knapp et al. (1978) derived a scale-length of $3.2(R_0/10 \text{ kpc}) \text{ kpc}$ from such fitting of HI profiles with an assumed exponential mass distribution. Using the Oort (1960) limit to normalize the density at R_0 , we obtain a mass model close to that of Caldwell and Ostriker (cf. Ostriker 1982). If we use the most recent rotation curve of Blitz et al. (1979) and Blitz (1979), the scale length is $4.0 \pm 0.2 (R_0/10 \text{ kpc}) \text{ kpc}$, the disk mass integrated to 20 kpc is approximately $7.6 \times 10^{10} M_\odot$ (changing by $\sim 10\%$ depending on whether a central gap is included), and the combined mass of the halo and bulge out to 20 kpc is $3.1 \times 10^{11} M_\odot$. If this new rotation curve is used to recompute the halo component in the Caldwell-Ostriker model (leaving their bulge and disk parameters fixed), we find that the fraction of the gravitational force due to the halo at R_0 , $1.5 R_0$ and $2 R_0$ is 17%, 42% and 64%, respectively. These studies regarding the warp in the Milky Way should be refined in particular to take into account the new data of Henderson et al. (1982). The "scallop" outer edges of the warp might well be due to an additional superposition of higher modes with $m > 1$ (cf. Fig. 5).

Next, we examine the galaxy NGC 2841, which has been observed by Bosma (1978). Figure 7b compares the surface density of neutral hydrogen with the surface density derived from the observations of the amplitude of the warp, normalized to the HI surface density in

the outer parts. (All distances in NGC 2841 assume $H_0 = 75$ km/s/Mpc.) If there is no systematic variation of the mass-to-light ratio with radius in the observed optical disk and the mass-to-light ratio is less than 6 (twice the values obtained from stellar population models, cf. Larson and Tinsley 1978), the surface photometry (van Houten et al. 1954) suggests that the density of the optical disk is negligible in the region from ~ 20 to 30 kpc; this allows the normalization of the derived density from the warp to the HI surface density in the outer parts. However, the optical disk may be the source of the extra mass at radii less than 20 kpc. In any case, the agreement in the range ~ 20 -35 kpc indicates that the HI surface density dominates the total surface density in this region. This is only $\sim 1\%$ of the local surface density needed to maintain the observed flat rotation curve (Bosma 1978), which implies that the mass density of the halo is inversely proportional to the square of the distance as may be derived from the rotation curve in this region.

Although these bending wave results were derived with the assumption of small inclinations in the warp, it is encouraging that the theory gives a fairly reasonable description of the large inclinations in galaxy M33 (cf. Lake and Mark 1980 for further discussions). Preliminary applications to several other galaxies can be found in Bertin and Casertano (1982).

For galaxies such as NGC 2841, there is the tantalizing possibility that with better data it may be possible to measure a mass-to-light ratio in the disks of external galaxies using the bending wave theory. This measurement makes possible a computation of the core radius of the halo component, an important parameter for constraining theories of its origin and composition (Peebles 1979). This is possible if the warp is measured over a range where it samples the mass distribution of the optical disk, and extends out to the region where the HI surface density dominates and provides the normalization. This project is ideally and uniquely suited for study with the VLA telescope, when accurate 21-cm line observations become possible.

IV. THE INNER DISK WITHIN 2 KPC OF GALACTIC CENTER

Independent analyses of the HI and CO within 2 kpc of the Galactic center have shown that the gas distribution is best described as a disk-like structure with a rotation axis inclined $20-25^\circ$ to that of the Galaxy (Burton and Liszt 1978, Liszt and Burton 1978, 1980, Sinha 1979, cf. our Fig. 8a). Detailed models of the gas as an expanding circular disk (Burton and Liszt 1978, Liszt and Burton 1978) or a thin elliptical bar (Liszt and Burton 1980) describe the observations equally well, but they are without any dynamical basis. We propose that the bending wave theory surveyed here can provide a plausible dynamical explanation for the tilt, and that the predictions of the theory are independent of the kinematic

model of the gas distribution. We show that the predicted relation between the disk surface density distribution and the height of the bending are satisfied for both the disk-like and bar-like models of the gas.

The original motivation for the theory was to provide an explanation for the warps in the outer parts of spiral galaxies (cf. Section I). The model shows that the warp arises as a self-excited bending wave which is driven unstable because of its motion through the slowly rotating halo. In this section we present some evidence that a similar model can provide a dynamical explanation for the tilt in the gas disk near the center of our Galaxy. In this case the gas disk moves through the more slowly rotating bulge and an analogous instability results in a "flapping" of this disk. Direct observations of the bulge component of the galactic nucleus using OH masers related to Mira variables (Baud 1978), and infrared radiation from population II stars (Maihara et al. 1978), indicates the presence of a slowly rotating spheroidal distribution of stars with a large scale height, comparable to what is seen in other galaxies. Liszt and Burton model their CO and HI observations of the central regions of the Galaxy by a massive disk or bar ($> 10^9 M_\odot$) with a gaussian scale height of 100 pc and typical rotational velocities of $200\text{--}300 \text{ km s}^{-1}$. Thus, the conditions of a thin rapidly rotating disk embedded within a larger, spheroidal mass distribution considered by the theory are applicable within the central two kiloparsecs of the Galaxy.

The bending wave theory (cf. Section II) predicts the scaling law given in equation (1). This scaling law arises because the bending wave propagates with a nearly constant rate of outflow of wave energy through circles of radius r . Although interactions with the bulge slowly amplify this rate of outflow, the amount of amplification per cycle of wave propagation is small and it is the short cycle time which determines the wave growth within the age of the Galaxy.

To examine the constancy of the product $\Sigma r^{1/2} h$, we rely on the detailed models of the nuclear disk for the geometric and kinematic properties of the disk. Burton and Liszt assume a constant surface density in their models and although there is excellent morphological agreement between the predicted and observed moment maps, the agreement between the contour gradients of their model and the observations is not particularly good. We assume that the observed contour gradients provide information on the actual run of density of the disk. The observed antenna temperature (T_A) along the line $b = -l \tan 22^\circ$ should provide a good indication of the run of surface density because: i) This locus of points is the axis of kinematic symmetry (corresponding very nearly to the projected major axis of the circular disk model) and observations along it provide the greatest contrast between the gas in the nuclear disk and the Galaxy at large. ii) The gas is surely optically thin and thus the antenna temperature of the gas should be directly proportional to the volume density. iii) The Burton and Liszt models show good agreement

with the observations under the assumption of a disk of constant thickness. If this assumption is approximately correct, the observed HI antenna temperature should give the run of surface density.

To determine the surface density, we used the improved HI observations (Liszt and Burton 1980) along the line $b = -\ell \tan 22^\circ$. This line intersects the midplane of the disk-like and bar-like models along a unique locus of points, and we determined the apparent LSR velocity along the locus for both models at 0.5° intervals. From the moment map of the HI observations, we found the HI antenna temperature on both sides of $b=0$ and averaged the results. Because the data are satisfactorily modeled by a planar distribution with a constant inclination angle, i , the product $\Sigma r^{1/2}_h \propto T_A r^{3/2} \sin i$ for the disk-like model. For the bar-like model, $r^{1/2}_h \propto r^{3/2} \sin i \cos \theta_0$ where θ_0 is the angle between the major axis of the elliptical streamlines and the projection of the line $b=-\ell \tan 22^\circ$ on the disk.

In Figure 8b we plot T_A as a function of $(r^{1/2}_h)^{-1}$ for both models. The points do not differ markedly from a straight line in either case, as expected from the theory. The dashed lines are the least squares fit to the data for each model. The data plotted in Figure 8b include the HI emission for $\ell \geq 2^\circ$. Observable CO emission extends to nearly $\ell=2^\circ$, and at lower longitudes the H_2 mass is likely to dominate the surface density of the nuclear disk. It is not possible to determine the H_2 mass from CO observations in

the nuclear disk with any confidence, and we have therefore excluded the region of $\ell < 2^\circ$ from our analysis.

It seems reasonable to interpret Figure 8b as implying that the run of density in the nuclear disk agrees with the predictions of the bending wave theory independent of the kinematics of the model used to describe the data. The largest uncertainty probably comes from the assumption of a constant thickness of the gas layer. The disk-like model could have an increasing scale height due to decreased self-gravity of the disk while the bar-like model may have the opposite tendency if it is really a triaxial ellipsoid rather than a thin disk. These possibilities can be tested only with better observations and more refined model making. Nevertheless, for the models we consider, which satisfactorily describe the present observations, the bending-wave theory provides a plausible dynamical basis for the tilt of the nuclear disk.

As can be seen from the references cited, the work described here is the result of a collaborative effort with G. Bertin, L. Blitz, G. Lake and R. P. Sinha. We also appreciate discussions with A. Bosma, W. B. Burton, A. P. Henderson, F. Kerr, P. C. van der Kruit, C. C. Lin, H. S. Liszt, J. P. Ostriker, V. C. Rubin, R. Sancisi, M. Schwarzschild, F. H. Shu, G. de Vaucouleurs, G. Westerhout.

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FIGURE CAPTIONS

Fig. 1. Model of kinematical bending waves.

Fig. 2. Model of bending waves with self-gravity.

Fig. 3. a (Left frame): Illustrates restoring gravitational force. b (Right frame): Dispersion relation in one special case.

Fig. 4. a (Left frame): The disk-halo interaction. b (Right frame): Driven responses amplify as they propagate.

Fig. 5. Higher order bending modes could also exist.

Fig. 6. a (Left frame): Warp in disk with slowly decaying surface density. b (Right frame) Rapidly decaying disk density.

Fig. 7. a (Left frame) The surface density $\Sigma(r)$ against radius in our galaxy as derived from equation (1) using Henderson's (1978) data for the warp. The straight line is an exponential with a scale length of 3.4 kpc, normalized to the Oort limit at the solar radius (10 kpc). b (Right frame): The surface density against radius for galaxy NGC 2841 as derived from equation (1) using the data of Bosma (1978). The solid line is the observed neutral hydrogen surface density. The deviations at distances less than 20 kpc may be due to the contribution of the optical disk.

Fig. 8. a (Left frame): The inner tilted gas disk (< 2 kpc) as depicted by Liszt and Burton. b (Right frame): Plot of HI antenna temperature (a quantity proportional to gas density) as a function of the quantity $(r/h)^{-1}$ for the circular model (filled circles) and bar-like model (open circles) described in the text. The dashed lines are least squares fits to the data.

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Fig. 1

Analogy of
coupled oscillators

Fig. 2

Fig. 3

Fig. 4

Higher bending modes could also exist

Fig. 5

Fig. 6

Fig. 7

Fig. 8