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PHASE SPACE ANALYSIS OF CONVECTION IN A ^3He - SUPERFLUID ^4He SOLUTION[†]

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ABSTRACT

Observations have been made on thermal convection below 1K in a dilute solution of ^3He in superfluid ^4He contained in a cylindrical cell of aspect ratio $\Gamma \approx 1.20$. Complicated oscillatory phenomena were observed with a high degree of reproducibility using two temperature sensors. Phase-space analysis suggests a description in terms of strange-attractor dynamics.

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We present some preliminary results for the time-dependent behavior of a convecting dilute solution of ^3He in superfluid ^4He . A new oscillatory state has been observed, with complex behavior which we interpret in terms of strange attractor concepts. In previous work¹ using similar apparatus, we reported the steady-state behavior of this system.²

Our convection cell was filled with a solution of 1.3 mole % ^3He in superfluid ^4He and operated at 0.7 K. This solution is believed to be effectively a one-component fluid with a calculated Prandtl number of about 0.05 and a negative thermal expansion coefficient. The cell was constructed with top and bottom plates of copper and an annular spacer fitting between the plates made of a graphite-resin material of high thermal resistivity.³ The inner wall of this removable spacer, together with the top and bottom plates, defined the cell geometry. This spacer fitted snugly into a thin-walled stainless steel tube which served to confine the fluid. The cell height was 0.80 cm, and for the data presented here the fluid cavity was cylindrical with aspect ratio (radius/height) $r = 1.20$. A small copper piece was inserted into a hole in the center of the top plate and thermally isolated from the rest of the top plate by a 0.05 mm thick Mylar sheet. This piece, which we call the "probe," had a diameter of 0.30 cm and an area comprising 2.5% of the total exposed area of the top plate. Two differential thermocouples were used to infer the fluid motion; one was attached between the top and bottom plates (top-bottom TC) and the other between the probe and the top plate (probe TC). Each thermocouple was connected to a SQUID ammeter.

The bottom plate was maintained at a fixed temperature of 0.700 K by means of a ^3He refrigerator and a resistance heater. The power applied to the top

plate heater ($\dot{Q}_{TOP}$) was kept constant while the top-bottom and probe TC outputs were recorded. We use the time-averaged temperature difference across the cell (ΔT) as a measure of the strength of the drive. We define ΔT_c as the value of ΔT at which convection starts. Above ΔT_c two separate steady states exist,⁴ characterized by different relations between ΔT and $\dot{Q}_{TOP}$. Both states have time-dependent regimes; it was more difficult to prepare the time-dependent state with higher conductance ($\dot{Q}_{TOP}/\Delta T$). As we increased $\dot{Q}_{TOP}$ slowly, only the state with higher conductance exhibited a reproducible sequence of oscillations having low dimensional characteristics. The sequence included regimes of large and small amplitude oscillations with very similar frequencies, followed by a regime in which both the large and small amplitudes appeared in an alternating manner.

We concentrate here on the value $\Delta T/\Delta T_c = 4.15$, at which aperiodic character is evident but the structure of the attractor is still relatively simple. Figure 1 shows the time series for both thermocouple sensors. The oscillation amplitudes of ca. 1 mK p-p may be compared to $\Delta T = 28.0$ mK. The comparable size of the two oscillations suggests a coherent structure extending throughout the cell rather than a localized effect. In Fig. 2 we show the power spectral density of the probe TC output. Here and in all succeeding analyses we filtered the TC outputs with a two-pole filter cutting off at 0.1 Hz. For all computer generated figures (Fig. 2, 4, 5, and 6) the sampling period was 0.7 sec. Other data taken at smaller $\Delta T/\Delta T_c$ show a signal with a comparable amplitude but a noise floor below 20 dB. Thus, the spectrum in Fig. 2 suggests that there is appreciable broadband power not caused by noise in the system parameters (e.g., the bottom plate temperature) or the TC

sensor noise. However, with the present frequency resolution, which is typified by the width of the sharp peaks in Fig. 2, it is unfortunately not possible unambiguously to distinguish multiple peaks from broadband noise.

We employed two methods of projecting the system trajectory onto a two-dimensional phase space: in Fig. 3 we use top-bottom and probe TC outputs, and in Fig. 4 we use the probe TC output alone by means of delay coordinates.⁶ These figures suggest the trajectory lies on a sheet-like attractor and that a three-dimensional phase space suffices to closely approximate the observed dynamics. Using three delay coordinates generated from the probe TC output and taking a Poincaré section (Fig. 5), we confirm this. Next, we parameterize the Poincaré section by associating each point in the section with its value T_n on the horizontal axis. The subscript n labels the points in the section in order of increasing time. Figure 6(a) is the return map generated by plotting T_{n+1} against T_n . Figure 6(b) is a return map for a larger drive of $\Delta I/\Delta I_C = 4.32$. Although T_{n+1} does not appear in these figures to be uniquely determined by T_n , there is clearly a correlation. The two point clusters in Fig. 6(a) become spread out along a curve in Fig. 6(b). This merging of bands with increasing drive has been seen in numerical simulations of certain attractors.⁷

Although it is not certain at this point whether the oscillations observed are a quality of low Prandtl-number, one-component fluids or a result of superfluid or multi-component fluid characteristics, the interesting dynamics and good signal-to-noise ratio found in this system make further study promising. We are continuing our efforts toward a full characterization of the evolution of the attractor with increasing $\Delta I/\Delta I_C$.

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FIGURE CAPTIONS

Fig. 1. Time series at $\Delta T/\Delta T_c = 4.15$: (a) the probe thermocouple and (b) the top-bottom thermocouple. The origins are chosen arbitrarily.

Fig. 2. Power spectral density of the probe thermocouple output at $\Delta T/\Delta T_c = 4.15$. A 2048-point time series was used with a Blackman window.⁵

Fig. 3. Two dimensional phase space plot at $\Delta T/\Delta T_c = 4.15$. The probe and top-bottom thermocouple outputs are plotted on the vertical and horizontal axes respectively.

Fig. 4. Delay map at $\Delta T/\Delta T_c = 4.15$. The probe thermocouple output $T(t)$ is plotted against $T(t + \tau)$. $\tau = 4.9$ sec is seven times the sampling period. 500 data points from the time series are used.

Fig. 5. Poincaré section of the three dimensional delay map with axes $T(t)$, $T(t + \tau)$ and $T(t + 2\tau)$ at $\Delta T/\Delta T_c = 4.15$. Points are shown whenever the trajectory crosses the plane $T(t + 2\tau) = 0$ with increasing $T(t + 2\tau)$. A 2100 point time series is used to obtain 149 points on the Poincaré section.

Fig. 6. Return maps (a) at $\Delta T/\Delta T_c = 4.15$ and (b) at $\Delta T/\Delta T_c = 4.32$. The value of $T(t)$ on the Poincaré section for point n is plotted against its succeeding value. 2100 point time series are used.

TEMPERATURE (mK)

PROBE THERMOCOUPLE

TOP-BOTTOM THERMOCOUPLE

$t | c, b,$

c

$(L+1) L$

(a) T_{n+1}