DE88 017137

Distribution Category:
Magnetic Fusion Energy
(UC-420)

ANL/FPP/TM-215

REVISION 1

Argonne National Laboratory
9700 S. Cass Ave.
Argonne, IL 60439 USA

INTERACTION OF IMPURITY IONS WITH A WEAKLY
NON-MAXWELLIAN SIMPLE HYDROGENIC PLASMA*

by

William K. Terry
Fusion Power Program

August 1988

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

* Work supported by the U.S. Department of Energy/Office of Fusion Energy under contract W-31-109-Eng-38.

MASTER

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT.....	1
I. INTRODUCTION.....	2
II. DESCRIPTION OF THE PROBLEM; NOTATION.....	3
III. THE DISTRIBUTION FUNCTIONS.....	5
A. Electron Distribution Function.....	5
1. Contribution from Electric Current.....	6
2. Contribution from Electron Temperature Gradient.....	8
3. Contribution from Viscous Strain Tensor.....	9
B. Ion Distribution Function.....	12
1. Contribution from Electric Current.....	13
2. Contribution from Ion Temperature Gradient.....	13
3. Contribution from Viscous Strain Tensor.....	15
IV. PROCEDURE FOR DERIVING DYNAMICAL FRICTION AND VELOCITY-SPACE DIFFUSION TENSOR COMPONENTS.....	18
V. RESULTS.....	28
A. Dynamical Friction from Non-Viscous Effects.....	29
B. Contribution of Viscosity to Dynamical Friction.....	30
C. Non-Viscous Part of $\langle \Delta v_{a,p}^2 \rangle$	32
D. Viscous Part of $\langle \Delta v_{a,p}^2 \rangle$	35
E. Non-Viscous Part of $\langle \Delta v_{a,tr}^2 \rangle$	36
F. Viscous Part of $\langle \Delta v_{a,tr}^2 \rangle$	38
VI. SUMMARY.....	40
REFERENCES.....	41

LIST OF FIGURES

	<u>Page</u>
Figure 1. Coordinate System in Velocity Space.....	42
Figure 2. Integration Regions in Velocity Space.....	43
Figure 3. Coordinate System in Derivation of Viscous Effects.....	44

**INTERACTION OF IMPURITY IONS WITH A WEAKLY
NON-MAXWELLIAN SIMPLE HYDROGENIC PLASMA**

by

William K. Terry

Abstract

The average acceleration of an ensemble of "test particles" in a plasma is called the "dynamical friction"; the average rate at which their velocity vectors spread out in velocity space is expressed in a velocity-space diffusion-rate tensor. These quantities are derived for impurity ions interacting with a weakly non-Maxwellian simple hydrogenic plasma. The distribution functions for the plasma ions and electrons are written explicitly.

1. INTRODUCTION

The interaction of a "test" particle with a Maxwellian plasma is a standard topic in introductory plasma physics.^{1,2,3} The study of this interaction gives for an average test particle the acceleration resulting from collisions, customarily written as:

$$\langle \Delta \vec{v} \rangle = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \int_{\vec{v}} P(\vec{v}; \Delta \vec{v}) \Delta \vec{v} d^3 \Delta v, \quad (1)$$

and an average velocity-space diffusion-rate tensor,

$$\langle \Delta \vec{v} \Delta \vec{v} \rangle = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \int_{\vec{v}} P(\vec{v}; \Delta \vec{v}) \Delta \vec{v} \Delta \vec{v} d^3 \Delta v, \quad (2)$$

where $P(\vec{v}; \Delta \vec{v})$ is the probability that a test particle undergoes a change in velocity from $\vec{v}$ to $\vec{v} + \Delta \vec{v}$ in time Δt because of multiple small-angle collisions. These results are used to write the Fokker-Planck equation for the collision integral in the Boltzmann equation, and to obtain certain time scales that can be used to characterize a plasma. Estimates of transport coefficients can also be made from these results, even though the actual plasma in which the transport phenomena occur is non-Maxwellian.

There are circumstances, however, in which the non-Maxwellian character of a plasma is important even in correctly evaluating Equations (1) and (2). One such circumstance involves the transport of impurity ions in the non-Maxwellian divertor plasma in a tokamak. This paper presents the theoretical development of the contributions to Equations (1) and (2), for an impurity particle in a simple hydrogenic plasma, from electric current, temperature gradients, and viscous strain rates. The results presented here will be applied in calculations of impurity particle transport in tokamak divertor plasmas; those calculations will be reported separately.

II. DESCRIPTION OF THE PROBLEM; NOTATION

We consider a simple plasma of electrons and a single ion species, denoted respectively by the subscripts "e" and "i". The subscript "b" is used to refer to either electrons or ions. A small population of impurity ions, denoted by the subscript "a", is present in the plasma. These impurities do not significantly affect the electron density or the distribution functions of the electrons and plasma ions.

We wish to compute the "dynamical friction" $\langle \Delta v_{a,p} \rangle$ and the velocity-space diffusion-rate tensor components $\langle \Delta v_{a,p}^2 \rangle$ and $\langle \Delta v_{a,tr}^2 \rangle$, where the subscripts "p" and "tr" refer to the instantaneous direction of motion of the impurity particle. That is, $\langle \Delta v_{a,p} \rangle$ is the instantaneous acceleration of the average test (impurity) particle parallel to its path; $\langle \Delta v_{a,p}^2 \rangle$ and $\langle \Delta v_{a,tr}^2 \rangle$ are the average velocity-space diffusion rate tensor components parallel to and in the plane transverse to the instantaneous direction of motion. Physically, these tensor components are related to the rates at which the velocity vectors of an ensemble of test particles with equal initial velocity $\vec{v}_a$ spread out in the directions respectively along and across the direction of $\vec{v}_a$.

An analysis¹⁻³ of small-angle Coulomb collision dynamics leads from Equations (1) and (2) to the following expressions (in SI units):

$$\langle \Delta v_{a,p} \rangle_b = - r_{ab} \frac{m_a}{M_{ab}} \int_{\vec{v}_b} f(\vec{v}_b) \frac{v_{a,p} - v_{b,p}}{|\vec{v}_a - \vec{v}_b|^3} d^3 v_b, \quad (3)$$

$$\langle \Delta v_{a,p}^2 \rangle_b = r_{ab} \frac{\partial^2 g_{ab}}{\partial v_{a,p}^2}, \quad (4)$$

and

$$\langle \Delta v_{a,tr}^2 \rangle_b = r_{ab} \frac{2}{v_a} \frac{\partial g_{ab}}{\partial v_a}, \quad (5)$$

where

$$g_{ab} = \int_{\vec{v}_b} f(\vec{v}_b) |\vec{v}_a - \vec{v}_b| d^3 v_b, \quad (6)$$

$$M_{ab} = \frac{m_a m_b}{m_a + m_b}, \quad (7)$$

and

$$r_{ab} = \frac{q_a^2 q_b^2 \ln \Lambda_{ab}}{4\pi \epsilon_0^2 m_a^2}, \quad (8)$$

in which m_a, m_b = masses of particles a and b, q_a, q_b = electric charge of particles a and b, and $\ln \Lambda_{ab}$ = Coulomb logarithm appropriate for an "a" particle in a field of "b" particles.

Strictly speaking, Equation (5) holds only in an isotropic plasma; also, in a non-isotropic plasma the vector $\langle \Delta \vec{v}_a \rangle$ has nonzero terms besides $\langle \Delta v_{a,p} \rangle$, and the velocity-space diffusion-rate tensor has nonzero off-diagonal terms. It is assumed here that none of these effects is important. The goal of the work described here is to find the modified expressions for $\langle \Delta v_{a,p} \rangle$, $\langle \Delta v_{a,p}^2 \rangle$ and $\langle \Delta v_{a,tr}^2 \rangle$ that result from using a non-Maxwellian distribution function in Equations (3) - (5). Further consequences of anisotropy are beyond the scope of this paper.

The subscripts "p" and "tr" are used here instead of the traditional parallel and perpendicular symbols to avoid ambiguity when the traditional parallel and perpendicular symbols are used to refer to directions relative to a magnetic field.

III. THE DISTRIBUTION FUNCTIONS

The distribution functions for electrons and ions in a simple plasma with electric current, temperature gradients, and viscous stresses were found by Braginskii.^{4,5} He used these distribution functions to obtain expressions for macroscopic results such as transport fluxes, but did not actually present the distribution functions explicitly. However, the distribution functions may be deduced from Ref. 4 by filling in some missing steps. The results are presented below.

A. Electron Distribution Function

The distribution function is assumed to be a weakly perturbed Maxwellian distribution:

$$f_e(\vec{v}_e) = f_e^{(0)}(\vec{v}_e) (1 + \phi_e), \quad (9)$$

where $f_e^{(0)}$ is the Maxwellian distribution,

$$f_e^{(0)}(\vec{v}_e) = \frac{n_e}{(2\pi T_e/m_e)^{3/2}} \exp(-m_e v_e^2/2T_e), \quad (10)$$

and where ϕ_e is the perturbation. We write the temperature T_e in units of energy. The electron particle velocity $\vec{v}_e$ is measured in the reference frame of the ion fluid.

The perturbation is found to have the form

$$\phi_e = \vec{\phi}_{e_u} \cdot \vec{v}_e + \vec{\phi}_{e_T} \cdot \vec{v}_e + \phi_{e_w} \quad (11)$$

in which the subscripts u, T, and w refer respectively to the contributions of an electric current, an electron temperature gradient, and viscous stresses in the electron fluid. (The electric current is related to $\vec{U}$, the electron fluid velocity relative to the ion fluid.)

The results for the contributions $\vec{\phi}_{eu}$, $\vec{\phi}_{eT}$, and ϕ_{ew} are presented in the following subsections.

1. Contribution From Electric Current

The perturbation resulting from the electric current is written as:

$$\vec{\phi}_{eu} = A_{eu} \vec{U}_{\parallel} + \hat{A}_{eu} \vec{U}_{\perp} + \hat{\hat{A}}_{eu} (\vec{\omega}_e \times \vec{U}), \quad (12)$$

where the perpendicular and parallel symbols refer to the direction of a magnetic field $\vec{B}$, and where $\vec{\omega}_e$ is a vector along $\vec{B}$ with magnitude equal to the electron gyrofrequency.

The coefficients in Equation (12) are given by

$$\hat{A}_{eu} = -\frac{m_e}{T_e} \sum_{k=1}^{\infty} \hat{a}_{eu_k} L_k^{(3/2)}(x_e), \quad (13)$$

$$\hat{\hat{A}}_{eu} = -\frac{m_e}{T_e} \sum_{k=1}^{\infty} \hat{\hat{a}}_{eu_k} L_k^{(3/2)}(x_e), \quad (14)$$

and

$$A_{eu} = \lim_{\omega_e \rightarrow 0} \hat{A}_{eu}. \quad (15)$$

In Equations (13) - (15), $x_e = m_e v_e^2 / 2T_e$, and $L_k^{(3/2)}(x_e)$ is the k^{th} Sonine polynomial (or Laguerre polynomial) of order 3/2.

Following Braginskii, we present the expansion coefficients in Equations (13) - (15) only for $k = 1, 2$. Higher expansion orders are usually not necessary. These coefficients are:

$$\hat{a}_{eu_1} = (2.04 x_e^2 + 1.0724) / \Delta_e, \quad (16a)$$

$$a_{eu_1}'' = \tau_e (0.6 x_e^2 + 1.221) / \Delta_e, \quad (16b)$$

$$a_{eu_1} = \lim_{\omega_e \rightarrow 0} a_{eu_1}' = 0.2844 \quad (16c)$$

$$a_{eu_2}' = (1.7895 x_e^2 + 0.1218) / \Delta_e, \quad (16d)$$

$$a_{eu_2}'' = \tau_e (0.4288 x_e^2 - 0.5612) / \Delta_e, \quad (16e)$$

and

$$a_{eu_2} = \lim_{\omega_e \rightarrow 0} a_{eu_2}' = 0.03231, \quad (16f)$$

where

$$x_e = \omega_e \tau_e,$$

$$\Delta_e = x_e^4 + 14.79 x_e^2 + 3.7703,$$

and τ_e is the 90-degree deflection time of an electron in collisions with ions, or

$$\tau_e = \frac{12\pi^2}{(2\pi)^{1/2}} \frac{m_e^{1/2} \epsilon_o^2 \tau_e^{3/2}}{\ln \Lambda_{ei} q_e^2 q_i^2 n_i}. \quad (17)$$

2. Contribution From Electron Temperature Gradient

The contribution from $\vec{\nabla} T_e$ is:

$$\vec{\Phi}_{eT} = A_{eT} \vec{\nabla} \ln T_e + \hat{A}_{eT} \vec{\nabla} \ln T_e + \hat{\hat{A}}_{eT} (\vec{\omega}_e \times \vec{\nabla} \ln T_e), \quad (18)$$

with

$$\hat{A}_{eT} = \tau_e \sum_{k=1}^{\infty} \hat{a}_{eT_k} L_k^{(3/2)}(x_e), \quad (19)$$

$$\hat{\hat{A}}_{eT} = \tau_e \sum_{k=1}^{\infty} \hat{\hat{a}}_{eT_k} L_k^{(3/2)}(x_e), \quad (20)$$

and

$$A_{eT} = \lim_{\omega_e \rightarrow 0} \hat{A}_{eT}. \quad (21)$$

The expansion coefficients for $k = 1, 2$ are:

$$\hat{a}_{eT_1} = (1.8656 x_e^2 + 4.768)/\Delta_e, \quad (22a)$$

$$\hat{\hat{a}}_{eT_1} = \tau_e (x_e^2 + 8.668)/\Delta_e, \quad (22b)$$

$$a_{eT_1} = \lim_{\omega_e \rightarrow 0} \hat{a}_{eT_1} = 1.2646, \quad (22c)$$

$$\hat{a}_{eT_2} = (1.2281 x_e^2 - 2.3845)/\Delta_e, \quad (22d)$$

$$\ddot{a}_{eT_2} = - 6.9344 \tau_e / \Delta_e, \quad (22e)$$

and

$$a_{eT_2} = \lim_{\omega_e \rightarrow 0} \dot{a}_{eT_2} = - 0.6325. \quad (22f)$$

3. Contribution From Viscous Strain Tensor

The perturbation in $f_e(\vec{v}_e)$ resulting from viscous effects is broken into three parts:

$$\phi_{ew} = \phi_{e(0)} + \phi_{e(1)} + \phi_{e(2)}, \quad (23)$$

where

$$\phi_{e(0)} = - B_{eo} (X_e) W_{eo} s_{e_{jk}}, \quad (24a)$$

$$\phi_{e(1)} = - (B_{e1} (X_e) \dot{W}_{e1} + \ddot{B}_{e1} (X_e) \ddot{W}_{e1}) s_{e_{jk}}, \quad (24b)$$

and

$$\phi_{e(2)} = - (B_{e2} (X_e) \dot{W}_{e2} + \ddot{B}_{e2} (X_e) \ddot{W}_{e2}) s_{e_{jk}}. \quad (24c)$$

In Equation (24), the Einsteinian summation notation is used to denote summation over the indices of a Cartesian coordinate system. If the magnetic field (if present) is aligned with the z-axis, the tensors $\tilde{W}_{eo}$, $\tilde{W}_{e1}$, $\tilde{W}_{e2}$, $\tilde{W}_{e1}''$ and $\tilde{W}_{e2}''$ are

$$\tilde{W}_{eo} = \begin{pmatrix} 1/2(W_{e_{xx}} + W_{e_{yy}}) & 0 & 0 \\ 0 & 1/2(W_{e_{xx}} + W_{e_{yy}}) & 0 \\ 0 & 0 & W_{e_{zz}} \end{pmatrix}, \quad (25a)$$

$$\tilde{W}_{e1} = \begin{pmatrix} 1/2(W_{e_{xx}} - W_{e_{yy}}) & W_{e_{xy}} & 0 \\ W_{e_{yx}} & -1/2(W_{e_{xx}} - W_{e_{yy}}) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (25b)$$

$$\tilde{W}_{e2} = \begin{pmatrix} 0 & 0 & W_{e_{xz}} \\ 0 & 0 & W_{e_{yz}} \\ W_{e_{zx}} & W_{e_{zy}} & 0 \end{pmatrix}, \quad (25c)$$

$$\tilde{W}_{e1}'' = W_e \begin{pmatrix} -2W_{e_{xy}} & W_{e_{xx}} - W_{e_{yy}} & 0 \\ W_{e_{xx}} - W_{e_{yy}} & 2W_{e_{xy}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (25d)$$

and

$$\ddot{W}_{e2} = \omega_e \begin{pmatrix} 0 & 0 & -W_{e\ yz} \\ 0 & 0 & W_{e\ xz} \\ -W_{e\ zy} & W_{e\ zx} & 0 \end{pmatrix}, \quad (25e)$$

in which $\ddot{W}_e$ is the rate-of-strain tensor:

$$W_{e\ jk} = \frac{\partial v_{ej}}{\partial x_k} + \frac{\partial v_{ek}}{\partial x_j} - \frac{2}{3} \delta_{jk} \vec{v}_e \cdot \vec{v}_e, \quad (25f)$$

with $\vec{v}_e$ the electron fluid velocity and $(x_1, x_2, x_3) \equiv (x, y, z)$.

The coefficients in Equation (24) are given by:

$$B_{eo} (x_e) = 0.733 \tau_e L_o^{(5/2)} (x_e), \quad (26a)$$

$$\dot{B}_{e1} (x_e) = \frac{0.5125 x_e^2 + 0.5313}{x_e^4 + 3.45 x_e^2 + 0.725} \tau_e L_o^{(5/2)} (x_e), \quad (26b)$$

$$\ddot{B}_{e1} (x_e) = \frac{x_e (0.25 x_e^2 + 0.4944)}{x_e^4 + 3.45 x_e^2 + 0.725} \frac{\tau_e}{\omega_e} L_o^{(5/2)} (x_e), \quad (26c)$$

$$\dot{B}_{e2} (x_e) = \frac{2.05 x_e^2 + 8.50}{x_e^4 + 13.8 x_e^2 + 11.6} \tau_e L_o^{(5/2)} (x_e), \quad (26d)$$

and

$$B_{e2}^{(2)}(x_e) = \frac{x_e^2 (x_e^2 + 7.91)}{x_e^4 + 13.8 x_e^2 + 11.6} - \frac{\tau_e}{\omega_e} L_0^{(5/2)}(x_e), \quad (26e)$$

where $L_0^{(5/2)}(x_e)$ is the zeroth Sonine (Laguerre) polynomial of order 5/2.

Finally,

$$s_{e_{jk}} = s_{e_j} s_{e_k} - (s_e^2/3) \delta_{jk}, \text{ where } \vec{s}_e = \vec{v}_e (m_e/2T_e)^{1/2}, \text{ and } s_e = |\vec{s}_e| = x_e^{1/2}.$$

B. Ion Distribution Function

The distribution is the perturbed Maxwellian

$$f_i(\vec{v}_i) = f_i^{(0)}(\vec{v}_i)(1 + \phi_i), \quad (27)$$

$$\text{where } f_i^{(0)}(\vec{v}_i) = \frac{n_i \exp(-m_i v_i^2/2T_i)}{(2\pi T_i/m_i)^{3/2}}, \quad (28)$$

$$\text{and where } \phi_i = \vec{\phi}_{i_u} \cdot \vec{v}_i + \vec{\phi}_{i_T} \cdot \vec{v}_i + \phi_{i_w}. \quad (29)$$

The terms in ϕ_i have meanings analogous to their counterparts in Equation (11). The results for $\vec{\phi}_{i_u}$, $\vec{\phi}_{i_T}$, and ϕ_{i_w} are presented below.

1. Contribution From Electric Current

The ion distribution function is determined only by ion-ion collisions,⁵
so

$$\vec{\Phi}_{i_u} = 0 \quad . \quad (30)$$

2. Contribution From Ion Temperature Gradient

The contribution from $\vec{\nabla} T_i$ is

$$\vec{\Phi}_{i_T} = A_{iT} \vec{\nabla} \ln T_i + A'_{iT} \vec{\nabla} \ln T_i + A''_{iT} (\vec{\omega}_i \times \vec{\nabla} \ln T_i), \quad (31)$$

with

$$A'_{iT} = \tau_i \sum_{k=1}^{\infty} a'_{iT_k} L_k^{(3/2)}(x_i), \quad (32)$$

$$A''_{iT} = \tau_i \sum_{k=1}^{\infty} a''_{iT_k} L_k^{(3/2)}(x_i), \quad (33)$$

and

$$A_{iT} = \lim_{\omega_i \rightarrow 0} A'_{iT}, \quad (34)$$

where

$$x_i = m_i v_i^2 / 2 T_i$$

and

$$\tau_i = 12\pi^{3/2} \frac{m_i^{1/2} \epsilon_0^2 T_i^{3/2}}{\ln \Lambda_{ii} q_i^4 n_i} \quad (35)$$

The expansion coefficients for $k = 1, 2$ are

$$a'_{iT_1} = (0.8 x_i^2 + 1.059) / \Delta_i, \quad (36a)$$

$$a''_{iT_1} = -\tau_i (x_i^2 + 1.86) / \Delta_i, \quad (36b)$$

$$a_{iT_1} = \lim_{\omega_i \rightarrow 0} a'_{iT_1} = 1.56, \quad (36c)$$

$$a'_{iT_2} = (0.343 x_i^2 - 0.282) / \Delta_i, \quad (36d)$$

$$a''_{iT_2} = 0.716 \tau_i / \Delta_i, \quad (36e)$$

and

$$a_{iT_2} = \lim_{\omega_i \rightarrow 0} a'_{iT_2} = -0.4165, \quad (36f)$$

where

$$x_i = \omega_i \tau_i,$$

and

$$\Delta_i = x_i^4 + 2.7 x_i^2 + 0.677.$$

3. Contribution From Viscous Strain Tensor

The perturbation in $f_i(\vec{v}_i)$ resulting from viscous effects is written:

$$\phi_{i_w} = \phi_{i(o)} + \phi_{i(1)} + \phi_{i(2)}, \quad (37)$$

where

$$\phi_{i(o)} = - B_{i_o}(X_i) W_{i_o jk} s_{i_jk}, \quad (38a)$$

$$\phi_{i(1)} = - (B_{i1}(X_i) \hat{W}_{i1 jk} + B_{i1}''(X_i) \hat{\hat{W}}_{i1 jk}) s_{i_jk}, \quad (38b)$$

and

$$\phi_{i(2)} = - (B_{i2}(X_i) \hat{W}_{i2 jk} + B_{i2}''(X_i) \hat{\hat{W}}_{i2 jk}) s_{i_jk}. \quad (38c)$$

In Equation (38), the Einsteinian summation notation is used again. If the magnetic field (if present), is aligned with the z-axis, the tensors $\tilde{W}_{10}$, $\tilde{W}_{11}$, $\tilde{W}_{12}$, $\tilde{W}_{11}^{\prime\prime}$ and $\tilde{W}_{12}^{\prime\prime}$ are

$$\tilde{W}_{10} = \begin{pmatrix} 1/2(W_{1xx} + W_{1yy}) & 0 & 0 \\ 0 & 1/2(W_{1xx} + W_{1yy}) & 0 \\ 0 & 0 & W_{1zz} \end{pmatrix}, \quad (39a)$$

$$\tilde{W}_{11} = \begin{pmatrix} 1/2(W_{1xx} - W_{1yy}) & W_{1xy} & 0 \\ W_{1yx} & -1/2(W_{1xx} - W_{1yy}) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (39b)$$

$$\tilde{W}_{12} = \begin{pmatrix} 0 & 0 & W_{1xz} \\ 0 & 0 & W_{1yz} \\ W_{1zx} & W_{1zy} & 0 \end{pmatrix}, \quad (39c)$$

$$\tilde{W}_{11}^{\prime\prime} = \omega_i \begin{pmatrix} -2W_{1xy} & W_{1xx} - W_{1yy} & 0 \\ W_{1xx} - W_{1yy} & 2W_{1xy} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (39d)$$

and

$$\tilde{W}_{i2} = \omega_i \begin{pmatrix} 0 & 0 & -W_{i\ yz} \\ 0 & 0 & W_{i\ xz} \\ -W_{i\ zy} & W_{i\ zx} & 0 \end{pmatrix}, \quad (39e)$$

in which

$$W_{i\ jk} = \frac{\partial V_{ij}}{\partial x_k} + \frac{\partial V_{ik}}{\partial x_j} - \frac{2}{3} \delta_{jk} \vec{v} \cdot \vec{v}_i, \quad (39f)$$

with $\vec{v}_i$ the ion fluid velocity and $(x_1, x_2, x_3) = (x, y, z)$.

The coefficients in Equation (38) are given by:

$$B_{i0}(X_i) = 0.96 \tau_i L_o^{(5/2)}(X_i), \quad (40a)$$

$$B_{i1}(X_i) = \frac{0.3 x_i^2 + 0.1394}{x_i^4 + 1.0075 x_i^2 + 0.1456} \tau_i L_o^{(5/2)}(X_i), \quad (40b)$$

$$B_{11}''(x_1) = -x_1 \frac{0.25 x_1^2 + 0.1488}{x_1^4 + 1.0075 x_1^2 + 0.1456} \frac{\tau_1}{\omega_1} L_o^{(5/2)}(x_1), \quad (40c)$$

$$B_{12}'(x_1) = \frac{1.2 x_1^2 + 2.23}{x_1^4 + 4.03 x_1^2 + 2.23} \tau_1 L_o^{(5/2)}(x_1), \quad (40d)$$

and

$$B_{12}''(x_1) = -\frac{x_1 (x_1^2 + 2.38)}{x_1^4 + 4.03 x_1^2 + 2.23} \frac{\tau_1}{\omega_1} L_o^{(5/2)}(x_1). \quad (40e)$$

Also,

$$s_{ijk} = s_{ij} s_{ik} - (s_i^2/3) \delta_{jk}, \quad \vec{s}_i = \vec{v}_i (m_i/2T_i)^{1/2}, \text{ and } s_i = |\vec{s}_i| = x_i^{1/2}.$$

IV. PROCEDURE FOR DERIVING DYNAMICAL FRICTION AND VELOCITY-SPACE DIFFUSION TENSOR COMPONENTS

The derivation of $\langle \Delta v_{a,p} \rangle$, $\langle \Delta v_{a,p}^2 \rangle$, and $\langle \Delta v_{a,tr}^2 \rangle$ is accomplished by substituting the distribution functions presented in Section III into Equations (3) and (6), performing the integrations, and summing the results for electrons and plasma ions. That is:

$$\langle \Delta v_{a,p} \rangle = \langle \Delta v_{a,p} \rangle_e + \langle \Delta v_{a,p} \rangle_i \quad (41a)$$

$$\langle \Delta v_{a,p}^2 \rangle = \langle \Delta v_{a,p}^2 \rangle_e + \langle \Delta v_{a,p}^2 \rangle_i \quad (41b)$$

and

$$\langle \Delta v_{a,tr}^2 \rangle = \langle \Delta v_{a,tr}^2 \rangle_e + \langle \Delta v_{a,tr}^2 \rangle_i. \quad (41c)$$

Each of the terms in Equations (41) is broken into contributions from the various parts of the distribution functions of Section III. For $b = e, i$, these are written:

$$\begin{aligned} \langle \Delta v_{a,p} \rangle_b &= \langle \Delta v_{a,p} \rangle_{b,o} + \langle \Delta v_{a,p} \rangle_{b,u_{||}} + \langle \Delta v_{a,p} \rangle_{b,u_{\perp}} \\ &\quad + \langle \Delta v_{a,p} \rangle_{b,u_{\wedge}} + \langle \Delta v_{a,p} \rangle_{b,T_{||}} + \langle \Delta v_{a,p} \rangle_{b,T_{\perp}} \\ &\quad + \langle \Delta v_{a,p} \rangle_{b,T_{\wedge}} + \langle v_{a,p} \rangle_{b,w} \end{aligned} \quad (42a)$$

$$\begin{aligned} \langle \Delta v_{a,p}^2 \rangle_b &= \langle \Delta v_{a,p}^2 \rangle_{b,o} + \langle \Delta v_{a,p}^2 \rangle_{b,u_{||}} + \langle \Delta v_{a,p}^2 \rangle_{b,u_{\perp}} \\ &\quad + \langle \Delta v_{a,p}^2 \rangle_{b,u_{\wedge}} + \langle \Delta v_{a,p}^2 \rangle_{b,T_{||}} + \langle \Delta v_{a,p}^2 \rangle_{b,T_{\perp}} \\ &\quad + \langle \Delta v_{a,p}^2 \rangle_{b,T_{\wedge}} + \langle \Delta v_{a,p}^2 \rangle_{b,w} \end{aligned} \quad (42b)$$

and

$$\begin{aligned}
\langle \Delta v_{a, tr}^2 \rangle_b &= \langle \Delta v_{a, tr}^2 \rangle_{b, o} + \langle \Delta v_{a, tr}^2 \rangle_{b, u_{\parallel}} + \langle \Delta v_{a, tr}^2 \rangle_{b, u_{\perp}} \\
&+ \langle \Delta v_{a, tr}^2 \rangle_{b, u_{\wedge}} + \langle \Delta v_{a, tr}^2 \rangle_{b, T_{\parallel}} + \langle \Delta v_{a, tr}^2 \rangle_{b, T_{\perp}} \\
&+ \langle \Delta v_{a, tr}^2 \rangle_{b, T_{\wedge}} + \langle \Delta v_{a, tr}^2 \rangle_{b, W} \quad .
\end{aligned} \tag{42c}$$

In equation (42a),

$$\langle \Delta v_{a, p} \rangle_{b, o} = - \frac{r_{ab} m_a}{M_{ab}} \int_{\vec{v}_b} f_b^{(o)}(\vec{v}_b) \frac{(v_{a, p} - v_{b, p})}{|\vec{v}_a - \vec{v}_b|^3} d^3 v_b \tag{43a}$$

$$\langle \Delta v_{a, p} \rangle_{b, u_{\parallel}} = - \frac{r_{ab} m_a}{M_{ab}} \int_{\vec{v}_b} f_b^{(o)} A_{bu} \vec{U}_{\parallel} \cdot \vec{v}_b \frac{(v_{a, p} - v_{b, p})}{|\vec{v}_a - \vec{v}_b|^3} d^3 v_b \tag{43b}$$

$$\langle \Delta v_{a, p} \rangle_{b, u_{\perp}} = - \frac{r_{ab} m_a}{M_{ab}} \int_{\vec{v}_b} f_b^{(o)} A_{bu} \vec{U}_{\perp} \cdot \vec{v}_b \frac{(v_{a, p} - v_{b, p})}{|\vec{v}_a - \vec{v}_b|^3} d^3 v_b \tag{43c}$$

$$\langle \Delta v_{a, p} \rangle_{b, u_{\wedge}} = - \frac{r_{ab} m_a}{M_{ab}} \int_{\vec{v}_b} f_b^{(o)} A_{bu} (\vec{\omega}_b \times \vec{U}) \cdot \vec{v}_b \frac{(v_{a, p} - v_{b, p})}{|\vec{v}_a - \vec{v}_b|^3} d^3 v_b \tag{43d}$$

$$\langle \Delta v_{a,p} \rangle_{b,T_{||}} = - \frac{r_{ab} m_a}{M_{ab}} \int_{\vec{v}_b} f_b^{(o)} A_{bT_{||}} \vec{v}_b \ln T_b \cdot \vec{v}_b \frac{(v_{a,p} - v_{b,p})}{|\vec{v}_a - \vec{v}_b|^3} d^3 v_b \quad (43e)$$

$$\langle \Delta v_{a,p} \rangle_{b,T_{\perp}} = - \frac{r_{ab} m_a}{M_{ab}} \int_{\vec{v}_b} f_b^{(o)} A_{bT_{\perp}} \vec{v}_b \ln T_b \cdot \vec{v}_b \frac{(v_{a,p} - v_{b,p})}{|\vec{v}_a - \vec{v}_b|^3} d^3 v_b \quad (43f)$$

$$\langle \Delta v_{a,p} \rangle_{b,T_{\wedge}} = - \frac{r_{ab} m_a}{M_{ab}} \int_{\vec{v}_b} f_b^{(o)} A_{bT_{\wedge}} (\vec{\omega}_b \times \vec{v}_b \ln T_b) \cdot \vec{v}_b \frac{(v_{a,p} - v_{b,p})}{|\vec{v}_a - \vec{v}_b|^3} d^3 v_b \quad (43g)$$

and

$$\begin{aligned} \langle \Delta v_{a,p} \rangle_{b,w} = & - \frac{r_{ab} m_a}{M_{ab}} \int_{\vec{v}_b} f_b^{(o)} [-B_{bo}(\chi_b) W_{bojk} \\ & - B_{b1}(\chi_b) W_{b1jk} - B_{b1}^{\wedge}(\chi_b) W_{b1jk}^{\wedge} - B_{b2}(\chi_b) W_{b2jk} \\ & - B_{b2}^{\wedge}(\chi_b) W_{b2jk}^{\wedge}] s_{jk} \frac{(v_{a,p} - v_{b,p})}{|\vec{v}_a - \vec{v}_b|^3} d^3 v_b \end{aligned} \quad (43h)$$

In Equations (42b) and (42c), each term comes from differentiating the result of substituting the appropriate part of $f_b(\vec{v}_b)$ into Equation (6), in a manner

analogous to the construction of Equations (43). These expressions are straightforward to obtain, and are omitted for brevity.

The evaluation of Equations (43) involves sequential integrations by parts. The approach to these integrations is adequately presented by using Equation (43c) as an example; note only $b = e$ is nontrivial in this case.

From Equation (13) for A'_{eu} , and noting that

$$L_1^{(3/2)}(x_e) = \frac{5}{2} - x_e \quad (44a)$$

and

$$L_2^{(3/2)}(x_e) = \frac{35}{8} - \frac{7}{2}x_e + \frac{1}{2}x_e^2, \quad (44b)$$

we can write Equation (43c) for electrons as:

$$\begin{aligned} \langle \Delta v_{a,p} \rangle_{e,u_1} = & - \frac{r_{ae} m_a}{M_{ae}} \int_{\vec{v}_e} f_e^{(0)} \left\{ - \frac{m_e}{T_e} \left[a'_{eu1} \left(\frac{5}{2} - \frac{m_e v_e^2}{2T_e} \right) \right. \right. \\ & \left. \left. + a'_{eu2} \left(\frac{35}{8} - \frac{7m_e v_e^2}{4T_e} + \frac{m_e^2 v_e^4}{8T_e^2} \right) \right] \frac{\vec{U}_1 \cdot \vec{v}_e (v_{a,p} - v_{e,p})}{|\vec{v}_a - \vec{v}_e|^3} \right\} d^3 v_e. \quad (45) \end{aligned}$$

Next, we construct a velocity-space coordinate system, illustrated in Fig. 1. The polar axis is chosen to lie along $\vec{v}_a$. Then ψ is the angle between $\vec{v}_a$ and $\vec{U}_1$, and θ is the angle between $\vec{v}_a$ and $\vec{v}_e$. The azimuthal angle ϕ is measured between a reference axis and the projection of $\vec{v}_e$ in the plane perpendicular to $\vec{v}_a$; the reference axis is taken to be perpendicular to $\vec{v}_a$ in the plane of $\vec{v}_a$ and $\vec{U}_1$. A right-handed set of unit basis vectors, $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$, is defined so that $\hat{e}_3$ is along $\vec{v}_a$ and $\hat{e}_2$ is along the azimuthal reference axis.

Then

$$\vec{U}_1 = U_1 \sin \psi \hat{e}_2 + U_1 \cos \psi \hat{e}_3 \quad (46a)$$

and

$$\vec{v}_e = v_e \sin \theta \sin \phi \hat{e}_1 + v_e \sin \theta \cos \phi \hat{e}_2 + v_e \cos \theta \hat{e}_3. \quad (46b)$$

Equations (46) are substituted into Equation (45), the ranges of integration are displayed for the velocity-space coordinates (v_e, θ, ϕ) , and Equation (10) is used for $f_e^{(0)}$:

$$\begin{aligned} \langle \Delta v_{a,p} \rangle_{e,u_1} &= \frac{\Gamma_{ae} m_a m_e}{M_{ae} T_e} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{v_e=0}^{\infty} \frac{n_e \exp \left(-\frac{m_e v_e^2}{2 T_e} \right)}{(2\pi T_e / m_e)^{3/2}} \left[a'_{eu_1} \left(\frac{5}{2} - \frac{m_e v_e^2}{2 T_e} \right) \right. \\ &\quad \left. + a'_{eu_2} \left(\frac{35}{8} - \frac{7 m_e v_e^2}{4 T_e} + \frac{m_e^2 v_e^4}{8 T_e^2} \right) \right] (U_1 v_e \cos \psi \cos \theta \\ &\quad + U_1 v_e \sin \psi \sin \theta \cos \phi) \frac{(v_a - v_e \cos \theta) v_e^2 \sin \theta d\phi d\theta dv_e}{(v_a^2 + v_e^2 - 2 v_a v_e \cos \theta)^{3/2}}. \quad (47) \end{aligned}$$

The integration over ϕ eliminates the second term in U_1 , and gives a factor of 2π in the first term. Then Equation (47) can be written:

$$\langle \Delta v_{a,p} \rangle_{e,u_1} = \Omega_{ae} \int_{\theta=0}^{\pi} \int_{v_e=0}^{\infty} \frac{F(v_e) (v_a - v_e \cos \theta) v_e \cos \theta \sin \theta d\theta dv_e}{(v_a^2 + v_e^2 - 2 v_a v_e \cos \theta)^{3/2}}, \quad (48)$$

$$\text{where } \Omega_{ae} = \frac{2\pi \Gamma_{ae} m_a m_e U_1 \cos \psi}{M_{ae} T_e} \frac{n_e}{(2\pi T_e / m_e)^{3/2}} \quad (49a)$$

and

$$F(v_e) = v_e^2 \exp\left(-\frac{m_e v_e^2}{2T_e}\right) \left[a'_{eu_1} \left(\frac{5}{2} - \frac{m_e v_e^2}{2T_e} \right) + a'_{eu_2} \left(\frac{35}{8} - \frac{7m_e v_e^2}{4T_e} + \frac{m_e^2 v_e^4}{8T_e^2} \right) \right]. \quad (49b)$$

The integration over θ and v_e must be divided into three regions in velocity space, according to whether the electrons in the given region tend to speed up or slow down the impurity particle. These regions are illustrated in Fig. 2. Electrons in Region 2 stream past the impurity particle along its direction of motion and tend to speed it up. Electrons in Regions 1 and 3 either move more slowly than the impurity particle and get in its way, or they actually counter-stream against it. In either case, these electrons slow the impurity down. These considerations are used to select the proper sign of square-root terms in the results of integrations over θ .

The regions can be described as follows:

Region 1: $v_e \leq v_a$

Region 2: $v_a < v_e$, $0 \leq \theta \leq \cos^{-1}(v_a/v_e)$

Region 3: $v_a < v_e$, $\cos^{-1}(v_a/v_e) < \theta \leq \pi$.

The result of integrating Equation (48) over θ is:

$$\begin{aligned}
\langle \Delta v_{a,p} \rangle_{e,u_1} &= \Omega_{ae} \int_{v_e=0}^{v_a} F(v_e) \left\{ \frac{\cos \theta \left(\frac{v_e}{v_a} \cos \theta - 1 \right)}{(v_a^2 + v_e^2 - 2v_a v_e \cos \theta)^{1/2}} \right. \\
&\quad + \left(\frac{2 \cos \theta}{v_a^2} - \frac{1}{v_a v_e} \right) (v_a^2 + v_e^2 - 2v_a v_e \cos \theta)^{1/2} \\
&\quad + \left. \frac{2(v_a^2 + v_e^2 - 2v_a v_e \cos \theta)^{3/2}}{3v_a^3 v_e} \right\} \pi \, dv_e \\
&\quad + \Omega_{ae} \int_{v_e=v_a}^{\infty} F(v_e) \left\{ \frac{\cos \theta \left(\frac{v_e}{v_a} \cos \theta - 1 \right)}{(v_a^2 + v_e^2 - 2v_a v_e \cos \theta)^{1/2}} \right. \\
&\quad + \left(\frac{2 \cos \theta}{v_a^2} - \frac{1}{v_a v_e} \right) (v_a^2 + v_e^2 - 2v_a v_e \cos \theta)^{1/2} \\
&\quad + \left. \frac{2(v_a^2 + v_e^2 - 2v_a v_e \cos \theta)^{3/2}}{3v_a^3 v_e} \right\} \cos^{-1}(v_a/v_e) \, dv_e \\
&\quad + \Omega_{ae} \int_{v_e=v_a}^{\infty} F(v_e) \left\{ \frac{\cos \theta \left(\frac{v_e}{v_a} \cos \theta - 1 \right)}{(v_a^2 + v_e^2 - 2v_a v_e \cos \theta)^{1/2}} \right.
\end{aligned}$$

$$\begin{aligned}
& + \left(\frac{2 \cos \theta}{v_a^2} - \frac{1}{v_a v_e} \right) (v_a^2 + v_e^2 - 2v_a v_e \cos \theta)^{1/2} \\
& + \frac{2(v_a^2 + v_e^2 - 2v_a v_e \cos \theta)^{3/2}}{3v_a^3 v_e} \Bigg]_{\theta = \cos^{-1}(v_a/v_e)}^{\pi} dv_e \quad (50)
\end{aligned}$$

Since the integral over θ is a continuous function at $\theta = \cos^{-1}(v_a/v_e)$, the last two integrals in Equation (50) cancel at their common integration limit, and the remaining terms can be written:

$$\begin{aligned}
\langle \Delta v_{a,p} \rangle_{e,u_1} &= n_{ae} \int_{v_e=0}^{v_a} (\pm)_1 F(v_e) \left\{ \frac{2}{v_a} - \frac{(2v_e + v_a)(v_a + v_e) + (2v_e - v_a)(v_a - v_e)}{v_a^2 v_e} \right. \\
&+ \left. \frac{2[(v_a + v_e)^3 - (v_a - v_e)^3]}{3v_a^3 v_e} \right\} dv_e \\
&+ n_{ae} \int_{v_e=v_a}^{\infty} (\pm)_2 F(v_e) \left[\frac{1}{v_a} - \frac{(2v_e - v_a)(v_a - v_e)}{v_a^2 v_e} - \frac{2(v_a - v_e)^3}{3v_a^3 v_e} \right] dv_e \\
&+ n_{ae} \int_{v_e=v_a}^{\infty} (\pm)_3 F(v_e) \left[\frac{1}{v_a} - \frac{(2v_e + v_a)(v_a + v_e)}{v_a^2 v_e} + \frac{2(v_a + v_e)^3}{3v_a^3 v_e} \right] dv_e, \quad (51)
\end{aligned}$$

in which $(\pm)_1$, $(\pm)_2$, and $(\pm)_3$ are the signs of such radicals as $(v_a^2 + v_e^2 \pm 2 v_a v_e)^{1/2} = \pm (v_a \pm v_e)$, in Regions 1, 2, and 3, respectively. The most straightforward way to see how to choose the proper values for these signs is to note that these radicals always arise from the factor $|\vec{v}_a - \vec{v}_b|^3$ in the denominators of Equations (43), which is always a positive quantity. In Region 1, $v_a > v_e$, so the factors $(v_a + v_e)$ and $(v_a - v_e)$ are both positive, and $(\pm)_1 = +1$. In Region 2, only the form $(\pm)_2 (v_a - v_e)$ appears, and since $v_a < v_e$ in this region, $(\pm)_2 = -1$. In Region 3, only the form $(\pm)_3 (v_a + v_e)$ appears, so that $(\pm)_3 = +1$. The same conclusions may be reached in evaluating the integral of Equation (43a) by carrying $(\pm)_1$, $(\pm)_2$, and $(\pm)_3$ until the very last step, and confirming that these choices are necessary to give the well-known correct result for the unperturbed Maxwellian distribution function.

After the proper signs are substituted in Equation (51), and the equation is reduced further, the expression for $F(v_e)$ from Equation (49b) is reinserted, with the result:

$$\begin{aligned} \langle \Delta v_{a,p} \rangle_{e,u_1} &= n_{ae} \int_{v_e=0}^{v_a} v_e \exp(-\frac{1}{2} \frac{v_e^2}{v_e^2}) \left[a_{eu_1}' \left(\frac{5}{2} - \frac{1}{2} \frac{v_e^2}{v_e^2} \right) \right. \\ &+ a_{eu_2}' \left(\frac{35}{8} - \frac{7}{2} \frac{v_e^2}{v_e^2} + \frac{1}{2} \frac{v_e^4}{v_e^4} \right) \left. \left(\frac{4v_e^3 - 6v_a^3}{3v_a^3} \right) \right] dv_e \\ &+ n_{ae} \int_{v_e=v_a}^{\infty} \frac{4v_e}{3} \exp(-\frac{1}{2} \frac{v_e^2}{v_e^2}) \left[a_{eu_1}' \left(\frac{5}{2} - \frac{1}{2} \frac{v_e^2}{v_e^2} \right) \right. \end{aligned}$$

$$+ a_{eu} \left(\frac{35}{8} - \frac{7 \lambda_e^2 v_e^2}{2} + \frac{\lambda_e^4 v_e^4}{2} \right) dv_e, \quad (52)$$

where $\lambda_e = (m_e/2 T_e)^{1/2}$.

The products in Equation (52) are expanded, and the integration is carried out, term-by-term, through sequential integrations by parts. Then Equation (49a) is used to replace the factor Ω_{ae} . The final result is:

$$\begin{aligned} \langle \Delta v_{a,p} \rangle_{e,u_1} &= \frac{4n_e \Gamma_{ae} m_a U_1 \cos(\vec{U}_1, \vec{v}_a)}{T_e (2\pi T_e/m_e)^{1/2}} \{ a_{eu_1} \left[\exp(-\lambda_e^2 v_a^2) \left(\frac{5}{4} - \frac{1}{2} \lambda_e^2 v_a^2 \right) - \frac{3}{4} \right] \right. \\ &\quad \left. + a_{eu_2} \left[\exp(-\lambda_e^2 v_a^2) \left(\frac{1}{4} \lambda_e^4 v_a^4 - \frac{3}{2} \lambda_e^2 v_a^2 + \frac{25}{16} \right) - \frac{15}{16} \right] \right\}. \end{aligned} \quad (53)$$

All the remaining terms in Equation (42a) are found in an analogous manner. Because of the complexity of the viscous term, it is necessary to choose a local coordinate system aligned in a specific manner when this term is considered. The coordinate system is described in the summary of results.

For the terms in Equations (42b) and (42c), the integration of Equation (6) also proceeds in a fashion analogous to the procedure given above. Then the application of Equations (4) and (5) is straightforward.

V. RESULTS

The results presented below are expressed for the reference frame of the ion fluid.

A. Dynamical Friction from Non-viscous Effects

Some condensed notation is convenient:

$$Q_{bu} = \frac{4n_b \Gamma_{ab} m_a}{T_b (2\pi T_b / m_b)^{1/2}} \quad (54a)$$

$$Q_{bT} = \frac{4n_b \Gamma_{ab} m_a \tau_b}{M_{ab} (2\pi T_b / m_b)^{1/2}} \quad (54b)$$

$$\xi_{b1} = \exp(-\ell_{ba}^2 v_a^2) \left(\frac{5}{4} - \frac{\ell_{ba}^2 v_a^2}{2} \right) - \frac{3}{4} \quad (54c)$$

$$\xi_{b2} = \exp(-\ell_{ba}^2 v_a^2) \left(\frac{\ell_{ba}^4 v_a^4}{4} - \frac{3\ell_{ba}^2 v_a^2}{2} + \frac{25}{16} \right) - \frac{15}{16} . \quad (54d)$$

The non-viscous dynamical friction components follow:

$$\langle \Delta v_{a,p} \rangle_{b,o} = - \Gamma_{ab} n_b \ell_b^2 \left(1 + \frac{m_a}{m_b} \right) \frac{\text{erf}(\ell_{ba} v_a) - (2/\pi)^{1/2} \ell_{ba} v_a}{\ell_{ba}^2 v_a^2} \exp(-\ell_{ba}^2 v_a^2) \quad (55a)$$

$$\langle \Delta v_{a,p} \rangle_{e,u_{\parallel}} = Q_{eu} U_{\parallel} \cos(\vec{U}_{\parallel}, \vec{v}_a) (a_{eu1} \xi_{e1} + a_{eu2} \xi_{e2}) \quad (55b)$$

$$\langle \Delta v_{a,p} \rangle_{i,u_1} = 0 \quad (55c)$$

$$\langle \Delta v_{a,p} \rangle_{e,u_1} = Q_{eu} U_1 \cos(\vec{U}_1, \vec{v}_a) (a_{eu_1}' \xi_{e1} + a_{eu_2}' \xi_{e2}) \quad (55d)$$

$$\langle \Delta v_{a,p} \rangle_{i,u_1} = 0 \quad (55e)$$

$$\langle \Delta v_{a,p} \rangle_{e,u_1} = Q_{eu} \omega_e U_1 \cos(\vec{\omega}_e \times \vec{U}_1, \vec{v}_a) (a_{eu_1}'' \xi_{e1} + a_{eu_2}'' \xi_{e2}) \quad (55f)$$

$$\langle \Delta v_{a,p} \rangle_{i,u_1} = 0 \quad (55g)$$

$$\langle \Delta v_{a,p} \rangle_{b,T_1} = -Q_{bT} |\vec{v}_1| \ln T_b \cos(\vec{v}_1, T_b, \vec{v}_a) (a_{bT_1} \xi_{b1} + a_{bT_2} \xi_{b2}) \quad (55h)$$

$$\langle \Delta v_{a,p} \rangle_{b,T_1} = -Q_{bT} |\vec{v}_1| \ln T_b \cos(\vec{v}_1, T_b, \vec{v}_a) (a_{bT_1}' \xi_{b1} + a_{bT_2}' \xi_{b2}) \quad (55i)$$

$$\langle \Delta v_{a,p} \rangle_{b,T_1} = -Q_{bT} \omega_b |\vec{v}_1| \ln T_b \cos(\vec{\omega}_b \times \vec{v}_1, \vec{v}_a) (a_{bT_1}'' \xi_{b1} + a_{bT_2}'' \xi_{b2}). \quad (55j)$$

B. Contribution of Viscosity to Dynamical Friction

The viscous effects are much more complicated than the effects of electric currents and temperature gradients. In order to obtain the viscous effects in a practical manner, one must choose a coordinate system. The results below are derived in the coordinate system shown in Fig. 3. In this coordinate system, the magnetic field $\vec{B}$ must point along the z-axis, in the positive direction. The angle ψ is the angle from $\vec{B}$ to $\vec{v}_a$. The angle τ is the angle to the x-axis from the projection of $\vec{v}_a$ onto the x-y plane, in the direction determined by the right-hand rule. That is, if a right-hand

screw aligned with the z-axis is turned in the direction of positive τ , it will advance in the positive z-direction. The x-axis may be chosen arbitrarily in the plane perpendicular to $\hat{B}$.

The viscosity contribution to the dynamical friction can then be written as:

$$\begin{aligned} \langle \Delta v_{a,p} \rangle_{b,w} &= \langle \Delta v_{a,p} \rangle_{b,w,xx} + \langle \Delta v_{a,p} \rangle_{b,w,yy} + \langle \Delta v_{a,p} \rangle_{b,w,zz} \\ &+ 2 \langle \Delta v_{a,p} \rangle_{b,w,xy} + 2 \langle \Delta v_{a,p} \rangle_{b,w,xz} + 2 \langle \Delta v_{a,p} \rangle_{b,w,yz} . \end{aligned} \quad (56)$$

It is once again convenient to introduce some simplifying notation:

$$Q_{bw} = \frac{\pi n_b \Gamma_{ab} m_a \tau_b}{M_{ab} (2\pi T_b / m_b)^{3/2}} \quad (57a)$$

$$\epsilon_{b3} = \frac{2}{l_b^4 v_a^3} \exp(-l_b^2 v_a^2) \left(\frac{3}{4} + \frac{l_b^2 v_a^2}{2} + \frac{l_b^4 v_a^4}{3} \right) - \frac{3\pi^{1/2} \text{erf}(l_b v_a)}{4 l_b^5 v_a^4} \quad (57b)$$

$$\epsilon_{b4} = \frac{\exp(-l_b^2 v_a^2)}{l_b^4 v_a^3} \left(\frac{9}{2} + 3 l_b^2 v_a^2 + 2 l_b^4 v_a^4 \right) - \frac{9\pi^{1/2} \text{erf}(l_b v_a)}{4 l_b^5 v_a^4} \quad (57c)$$

$$\tilde{W}_{b,jk} = \beta_{b0} W_{b0,jk} + \beta_{b1} \hat{W}_{b1,jk} + \beta_{b1} \hat{\hat{W}}_{b1,jk} + \beta_{b2} \hat{W}_{b2,jk} + \beta_{b2} \hat{\hat{W}}_{b2,jk} \quad (57d)$$

$$\beta_{bn}^{(m)} = B_{bn}^{(m)} (X_b) / \tau_{bo} L_o^{(5/2)} (X_1). \quad (57e)$$

In Equation (57e), $n = 0, 1, 2$, and the superscript (m) refers to primes. See Equations (25), (26), (39), and (40) to determine the components of Equation (57d).

The terms in Equation (56) are written below.

$$\langle \Delta v_{a,p} \rangle_{b,w,xx} = Q_{bw} \tilde{W}_{b,xx} \xi_{b3} (\sin^2 \zeta + \cos^2 \psi \cos^2 \zeta - 2 \sin^2 \psi \cos^2 \zeta) \quad (58a)$$

$$\langle \Delta v_{a,p} \rangle_{b,w,yy} = Q_{bw} \tilde{W}_{b,yy} \xi_{b3} (\cos^2 \zeta + \cos^2 \psi \sin^2 \zeta - 2 \sin^2 \psi \sin^2 \zeta) \quad (58b)$$

$$\langle \Delta v_{a,p} \rangle_{b,w,zz} = Q_{bw} \tilde{W}_{b,zz} \xi_{b3} (\sin^2 \psi - 2 \cos^2 \psi) \quad (58c)$$

$$\langle \Delta v_{a,p} \rangle_{b,w,xy} = Q_{bw} \tilde{W}_{b,xy} \xi_{b4} \sin^2 \psi \sin \zeta \cos \zeta \quad (58d)$$

$$\langle \Delta v_{a,p} \rangle_{b,w,xz} = -Q_{bw} \tilde{W}_{b,xz} \xi_{b4} \sin \psi \cos \psi \cos \zeta \quad (58e)$$

$$\langle \Delta v_{a,p} \rangle_{b,w,yz} = Q_{bw} \tilde{W}_{b,yz} \xi_{b4} \sin \psi \cos \psi \sin \zeta \quad (58f)$$

C. Non-viscous Part of $\langle \Delta v_{a,p}^2 \rangle$

Simplifying notation for the parallel part of the velocity-space diffusion tensor is given below:

$$Q_{bu}' = Q_{bT}' = \frac{4n_b \Gamma_{ab}}{3\pi^{1/2}} \quad (59a)$$

$$\begin{aligned} \xi_{b5} = \exp(-\xi_{ba}^2 \xi_{va}^2) & \left(\frac{9}{4 \xi_{va}^3} + \frac{3}{2 \xi_{ba} \xi_{va}} + \frac{3 \xi_{ba} \xi_{va}}{2} + \frac{25 \xi_{ba}^3 \xi_{va}^3}{2} \right. \\ & \left. - 12 \xi_{ba}^5 \xi_{va}^5 + 2 \xi_{ba}^7 \xi_{va}^7 \right) - \frac{9 \pi^{1/2} \operatorname{erf}(\xi_{ba} \xi_{va})}{8 \xi_{ba}^4 \xi_{va}^4} \end{aligned} \quad (59b)$$

$$\begin{aligned} \xi_{b6} = \exp(-\xi_{ba}^2 \xi_{va}^2) & \left(\frac{567}{32 \xi_{ba}^3 \xi_{va}^3} + \frac{189}{16 \xi_{ba} \xi_{va}} + 5.8484 \xi_{ba} \xi_{va} + 22.477 \xi_{ba}^3 \xi_{va}^3 \right. \\ & \left. - 32.95 \xi_{ba}^5 \xi_{va}^5 + 10.9 \xi_{ba}^7 \xi_{va}^7 + 4.2 \xi_{ba}^9 \xi_{va}^9 - \xi_{ba}^{11} \xi_{va}^{11} \right) \\ & - 567 \pi^{1/2} \operatorname{erf}(\xi_{ba} \xi_{va}) / (64 \xi_{ba}^4 \xi_{va}^4) \end{aligned} \quad (59c)$$

The results are given below:

$$\langle \Delta v_{a,p}^2 \rangle_{b,o} = \Gamma_{ab} n_b \xi_b \left[\frac{\operatorname{erf}(\xi_{ba} \xi_{va}) - (2/\pi^{1/2}) \xi_{ba} \xi_{va} \exp(-\xi_{ba}^2 \xi_{va}^2)}{\xi_{ba}^3 \xi_{va}^3} \right] \quad (60a)$$

$$\langle \Delta v_{a,p}^2 \rangle_{e,u_{\parallel}} = - \frac{m_e Q_{eu} U_{\parallel} \cos(\vec{U}_{\parallel}, \vec{v}_a)}{T_e} (a_{eu_1} \xi_{e5} + a_{eu_2} \xi_{e6}) \quad (60b)$$

$$\langle \Delta v_{a,p}^2 \rangle_{i,u_{\parallel}} = 0 \quad (60c)$$

$$\langle \Delta v_{a,p}^2 \rangle_{e,u_{\perp}} = - \frac{m_e Q_e U_{\perp} \cos(\vec{U}_{\perp}, \vec{v}_a)}{T_e} (a_{eu_1} \xi_{e5} + a_{eu_2} \xi_{e6}) \quad (60d)$$

$$\langle \Delta v_{a,p}^2 \rangle_{i,u_{\perp}} = 0 \quad (60e)$$

$$\langle \Delta v_{a,p}^2 \rangle_{e,u_{\perp}} = - \frac{m_e Q_e \omega_e U_{\perp} \cos(\vec{\omega}_e \times \vec{U}_{\perp}, \vec{v}_a)}{T_e} (a_{eu_1} \xi_{e5} + a_{eu_2} \xi_{e6}) \quad (60f)$$

$$\langle \Delta v_{a,p}^2 \rangle_{i,u_{\perp}} = 0 \quad (60g)$$

$$\langle \Delta v_{a,p}^2 \rangle_{b,T_{\parallel}} = Q_b \tau_b |\vec{v}_{\parallel}| \ln T_b |\cos(\vec{v}_{\parallel} T_b, \vec{v}_a)| (a_{bT_1} \xi_{b5} + a_{bT_2} \xi_{b6}) \quad (60h)$$

$$\langle \Delta v_{a,p}^2 \rangle_{b,T_{\perp}} = Q_b \tau_b |\vec{v}_{\perp}| \ln T_b |\cos(\vec{v}_{\perp} T_b, \vec{v}_a)| (a_{bT_1} \xi_{b5} + a_{bT_2} \xi_{b6}) \quad (60i)$$

$$\langle \Delta v_{a,p}^2 \rangle_{b,T_{\perp}} = Q_b \tau_b \omega_b |\vec{v}_{\perp}| \ln T_b |\cos(\vec{\omega}_b \times \vec{v}_{\perp}, \vec{v}_a)| (a_{bT_1} \xi_{b5} + a_{bT_2} \xi_{b6}). \quad (60j)$$

D. Viscous Part of $\langle \Delta v_{a,p}^2 \rangle$

As in the case of dynamical friction, the results for the viscous effects are expressed in the coordinate system shown in Figure 3. The viscous contribution to $\langle \Delta v_{a,p}^2 \rangle_b$ is written

$$\begin{aligned} \langle \Delta v_{a,p}^2 \rangle_{b,w} = & \langle \Delta v_{a,p}^2 \rangle_{b,w,xx} + \langle \Delta v_{a,p}^2 \rangle_{b,w,yy} + \langle \Delta v_{a,p}^2 \rangle_{b,w,zz} \\ & + 2 \langle \Delta v_{a,p}^2 \rangle_{b,w,xy} + 2 \langle \Delta v_{a,p}^2 \rangle_{b,w,xz} + 2 \langle \Delta v_{a,p}^2 \rangle_{b,w,yz} . \quad (61) \end{aligned}$$

We introduce the simplifying notation

$$Q_{bw} = \frac{n_b \Gamma_{ab} \tau_b}{(2\pi T_b / m_b)^{1/2}} \quad (62a)$$

$$\xi_{b7} = \exp(-\frac{1}{2} \frac{v_a^2}{v_b^2}) \left(\frac{1}{\frac{2}{3} \frac{v_a^2}{v_b^2}} + \frac{2}{5} + \frac{\frac{4}{3} \frac{v_a^4}{v_b^4}}{5} \right) - \frac{\pi^{1/2} \operatorname{erf}(\frac{1}{3} \frac{v_a}{v_b})}{2 \frac{v_a^3}{v_b^3}} \quad (62b)$$

$$\xi_{b8} = \exp(-\frac{1}{2} \frac{v_a^2}{v_b^2}) \left(\frac{3}{\frac{4}{3} \frac{v_a^2}{v_b^2}} + \frac{2}{\frac{2}{3} \frac{v_a^2}{v_b^2}} + \frac{4}{5} + \frac{\frac{4}{3} \frac{v_a^4}{v_b^4}}{15} \right) - \frac{3\pi^{1/2} \operatorname{erf}(\frac{1}{3} \frac{v_a}{v_b})}{2 \frac{v_a^5}{v_b^5}} \quad (62c)$$

$$\xi_{b9} = \exp(-\frac{1}{2} \frac{v_a^2}{v_b^2}) \left(2 + \frac{5}{\frac{2}{3} \frac{v_a^2}{v_b^2}} + \frac{9}{\frac{4}{3} \frac{v_a^2}{v_b^2}} \right) + \left(\frac{1}{2 \frac{v_a^3}{v_b^3}} - \frac{9}{2 \frac{v_a^5}{v_b^5}} \right) \pi^{1/2} \operatorname{erf}(\frac{1}{3} \frac{v_a}{v_b}) . \quad (62d)$$

Equations (57d) and (57e) are also applicable here. The terms in Equation (61) are written below:

$$\begin{aligned} \langle \Delta v_{a,p}^2 \rangle_{b,w,xx} &= Q_{bw}' \tilde{W}_{b,xx} [(\sin^2 \zeta + \cos^2 \psi \cos^2 \zeta - 2/3) \epsilon_{b7} \\ &+ (3 \sin^2 \psi \cos^2 \zeta - 1) \epsilon_{b8}] \end{aligned} \quad (63a)$$

$$\begin{aligned} \langle \Delta v_{a,p}^2 \rangle_{b,w,yy} &= Q_{bw}' \tilde{W}_{b,yy} [(\cos^2 \zeta + \cos^2 \psi \sin^2 \zeta - 2/3) \epsilon_{b7} \\ &+ (3 \sin^2 \psi \sin^2 \zeta - 1) \epsilon_{b8}] \end{aligned} \quad (63b)$$

$$\langle \Delta v_{a,p}^2 \rangle_{b,w,zz} = Q_{bw}' \tilde{W}_{b,zz} [(\sin^2 \psi - 2/3) \epsilon_{b7} + (3 \cos^2 \psi - 1) \epsilon_{b8}] \quad (63c)$$

$$\langle \Delta v_{a,p}^2 \rangle_{b,w,xy} = - Q_{bw}' \tilde{W}_{b,xy} (\sin^2 \psi \sin \zeta \cos \zeta) \epsilon_{b9} \quad (63d)$$

$$\langle \Delta v_{a,p}^2 \rangle_{b,w,xz} = Q_{bw}' \tilde{W}_{b,xz} (\sin \psi \cos \psi \cos \zeta) \epsilon_{b9} \quad (63e)$$

$$\langle \Delta v_{a,p}^2 \rangle_{b,w,yz} = - Q_{bw}' \tilde{W}_{b,yz} (\sin \psi \cos \psi \sin \zeta) \epsilon_{b9} . \quad (63f)$$

E. Non-viscous Part of $\langle \Delta v_{a,tr}^2 \rangle$

We introduce the following notation to simplify the results:

$$Q_{bu}'' = Q_{bT}'' = \frac{8n_b \Gamma_{ab}}{3\pi^{1/2}} \quad (64a)$$

$$\begin{aligned} \xi_{b10} = & \frac{3\pi^{1/2} \operatorname{erf}(\xi_{ba} v)}{8 \xi_{ba}^4 v^4} - \exp(-\xi_{ba}^2 v^2) \left(\frac{3}{4 \xi_{ba}^3 v^3} + \frac{1}{\xi_{ba} v} \right. \\ & \left. + \frac{\xi_{ba} v}{4} - 3 \xi_{ba}^3 v^3 + \xi_{ba}^5 v^5 \right) \end{aligned} \quad (64b)$$

$$\begin{aligned} \xi_{b11} = & \frac{189\pi^{1/2} \operatorname{erf}(\xi_{ba} v)}{64 \xi_{ba}^4 v^4} + \exp(-\xi_{ba}^2 v^2) \left(\frac{-189}{32 \xi_{ba}^3 v^3} - \frac{69}{16 \xi_{ba} v} \right. \\ & - 1.3883 \xi_{ba} v + 4.925 \xi_{ba}^3 v^3 - 3.85 \xi_{ba}^5 v^5 \\ & \left. + \frac{2 \xi_{ba}^7 v^7}{5} + \frac{\xi_{ba}^9 v^9}{2} \right) \end{aligned} \quad (64c)$$

The results follow:

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{b,0} = 2 r_{ab} n_b \xi_b \left[\frac{\operatorname{erf}(\xi_{ba} v)}{\xi_{ba} v} + \frac{\exp(-\xi_{ba}^2 v^2)}{\pi^{1/2} \xi_{ba}^2 v^2} - \frac{\operatorname{erf}(\xi_{ba} v)}{2 \xi_{ba}^3 v^3} \right] \quad (65a)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{e, u_{\parallel}} = - \frac{Q_{eu}^{\text{tr}} m_e u_{\parallel} \cos(\vec{U}_{\parallel}, \vec{v}_a)}{T_e} (a_{eu_1} \xi_{e10} + a_{eu_2} \xi_{e11}) \quad (65b)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{i, u_{\parallel}} = 0 \quad (65c)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{e, u_{\perp}} = - \frac{Q_{eu}'' m_e U_{\perp} \cos(\vec{U}_{\perp}, \vec{v}_a)}{T_e} (a_{eu_1}' \xi_{e10} + a_{eu_2}' \xi_{e11}) \quad (65d)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{i, u_{\perp}} = 0 \quad (65e)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{e, u_{\parallel}} = - \frac{Q_{eu}'' m_e \omega_e U_{\perp} \cos(\vec{\omega}_e \times \vec{U}_{\perp}, \vec{v}_a)}{T_e} (a_{eu_1}'' \xi_{e10} + a_{eu_2}'' \xi_{e11}) \quad (65f)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{i, u_{\parallel}} = 0 \quad (65g)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{b, T_{\parallel}} = Q_{bT}'' \tau_b |\vec{v}_{\parallel}| \ln T_b |\cos(\vec{v}_{\parallel}, \vec{v}_a)| (a_{bT_1}' \xi_{b10} + a_{bT_2}' \xi_{b11}) \quad (65h)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{b, T_{\perp}} = Q_{bT}'' \tau_b |\vec{v}_{\perp}| \ln T_b |\cos(\vec{v}_{\perp}, \vec{v}_a)| (a_{bT_1}' \xi_{b10} + a_{bT_2}' \xi_{b11}) \quad (65i)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{b, T_{\parallel}} = Q_{bT}'' \tau_b \omega_b |\vec{v}_{\perp}| \ln T_b |\cos(\vec{\omega}_b \times \vec{v}_{T_b}, \vec{v}_a)| (a_{bT_1}'' \xi_{b10} + a_{bT_2}'' \xi_{b11}) \quad (65j)$$

F. Viscous Part of $\langle \Delta v_{a, \text{tr}}^2 \rangle$

We use the simplifying notation given below:

$$Q_{bw}'' = \frac{2n_b \Gamma_{ab} \tau_b}{(2\pi T_b / m_b)^{1/2}} \quad (66a)$$

$$\xi_{b12} = \frac{\pi^{1/2} \operatorname{erf}(k_{ba} v_a)}{4 k_{ba}^3 v_a^3} - \exp(-k_{ba}^2 v_a^2) \left(\frac{1}{2 k_{ba}^2 v_a^2} + \frac{3}{5} + \frac{2 k_{ba}^2 v_a^2}{5} \right) \quad (66b)$$

$$\xi_{b13} = \frac{3\pi^{1/2} \operatorname{erf}(k_{ba} v_a)}{8 k_{ba}^5 v_a^5} - \exp(-k_{ba}^2 v_a^2) \left(\frac{3}{4 k_{ba}^4 v_a^4} + \frac{1}{2 k_{ba}^2 v_a^2} + \frac{1}{5} + \frac{2 k_{ba}^2 v_a^2}{15} \right) \quad (66c)$$

$$\begin{aligned} \xi_{b14} = & \left(\frac{9}{8 k_{ba}^5 v_a^5} - \frac{1}{4 k_{ba}^3 v_a^3} \right) \pi^{1/2} \operatorname{erf}(k_{ba} v_a) \\ & - \exp(-k_{ba}^2 v_a^2) \left(\frac{1}{2 k_{ba}^2 v_a^2} + \frac{9}{4 k_{ba}^4 v_a^4} \right) \end{aligned} \quad (66d)$$

Then the final results are as follows:

$$\begin{aligned} \langle \Delta v_{a, \text{tr}}^2 \rangle_{b, w, xx} = & Q_{bw}'' \tilde{w}_{b, xx} \left[(\sin^2 \zeta + \cos^2 \psi \cos^2 \zeta - \frac{2}{3}) \xi_{b12} \right. \\ & \left. + (3 \sin^2 \psi \cos^2 \zeta - 1) \xi_{b13} \right] \end{aligned} \quad (67a)$$

$$\langle \Delta v_{a, \text{tr}}^2 \rangle_{b, w, yy} = Q_{bw}'' \tilde{w}_{b, yy} \left[(\cos^2 \zeta + \cos^2 \psi \sin^2 \zeta - \frac{2}{3}) \xi_{b12} \right]$$

$$+ (3 \sin^2 \psi \sin^2 \zeta - 1) \epsilon_{b13}] \quad (67b)$$

$$\langle \Delta v_{a, tr}^2 \rangle_{b, w, zz} = Q_{bw}'' \tilde{w}_{b, zz} \left[(\sin^2 \psi - \frac{2}{3}) \epsilon_{b12} + (3 \cos^2 \psi - 1) \epsilon_{b13} \right] \quad (67c)$$

$$\langle \Delta v_{a, tr}^2 \rangle_{b, w, xy} = - Q_{bw}'' \tilde{w}_{b, xy} (\sin^2 \psi \sin \zeta \cos \zeta) \epsilon_{b14} \quad (67d)$$

$$\langle \Delta v_{a, tr}^2 \rangle_{b, w, xz} = - Q_{bw}'' \tilde{w}_{b, xz} (\sin \psi \cos \psi \cos \zeta) \epsilon_{b14} \quad (67e)$$

$$\langle \Delta v_{a, tr}^2 \rangle_{b, w, yz} = - Q_{bw}'' \tilde{w}_{b, yz} (\sin \psi \cos \psi \sin \zeta) \epsilon_{b14}. \quad (67f)$$

VI. SUMMARY

The dynamical friction and velocity-space diffusion tensor components have been found for an impurity ion interacting with a weakly non-Maxwellian simple hydrogenic plasma. The distribution functions for the non-Maxwellian plasma ions and electrons are presented explicitly, based on work by Braginskii.^{4,5} The method by which the results of this paper were derived is illustrated for one sample case.

References

1. L. Spitzer, Jr., Physics of Fully Ionized Gases, 2nd ed., Wiley (1962).
2. G. Schmidt, Physics of High Temperature Plasmas, Academic Press (1966).
3. T. J. M. Boyd and J. J. Sanderson, Plasma Dynamics, Barnes and Noble (1969).
4. S. I. Braginskii, Sov. Phys. JETP, 6, 358 (1958).
5. S. I. Braginskii, in Reviews of Plasma Physics, Vol. I, M. A. Leontovich, ed., Consultant, Bureau (1965).

Figure 1. Coordinate System in Velocity Space

REGION 2: $v_{e,p} > v_d$

REGION 3: $v_{e,p} > v_d$

Figure 2. Integration Regions in Velocity Space

Figure 3. Coordinate System for Derivation of Viscous Effects

DISTRIBUTION LIST FOR ANL/FPP/TM-215

Internal

C. Baker	D. Gruen	C. Reed
M. Billone	A. Hassanein	D. Smith
J. Brooks (5)	T. Hua	H. Stevens
Y. Cha	A. Hull	D. Sze
O. Chopra	C. Johnson	W. Terry (10)
R. Clemmer	A. Krauss	L. Turner
D. Ehst	Y. Liu	T. Yule
K. Evans	B. Loomis	FPP Files (25)
P. Finn	S. Majumdar	ANL Contract File
Y. Gohar	R. Mattas	ANL Library
L. Greenwood	B. Picologlou	ANL Patent Dept.
	K. Porges	TIS Files (3)

External:

DOE-OSTI, for distribution per UC-420 (37)
Manager, Chicago Operations Office
P. Lykoudis, Purdue University
M. Petravie, Princeton University
D. Post, Princeton University
C. Singer, University of Illinois, Champaign-Urbana
W. Stacey, Jr., Georgia Institute of Technology
P. Stangeby, University of Toronto
T. Tomabechi, Japan Atomic Energy Research Institute, JAPAN
W. Verbeeck, CEC, BELGIUM
Prof. Vetter, Kernforschungszentrum Karlsruhe und Verwaltung, WEST GERMANY
Bibliothek, Max-Planck-Institute fur Plasmaphysik, WEST GERMANY
Bibliothek, Institute fur Plasmaphysik, KFA Julich, WEST GERMANY
C.E.A. Library, Fontenay-aux-Roses, FRANCE
Librarian, Culham Laboratory, ENGLAND
Library laboratorio Gas Ionizati, ITALY
Thermonuclear Library, Japan Atomic Energy Research Institute, JAPAN