ENGINEERING METHODS FOR PREDICTING PRODUCTIVITY
AND LONGEVITY OF HOT-DRY-ROCK GEOTHERMAL
RESERVOIR IN THE PRESENCE OF THERMAL CRACKS

Principal Investigator:

Y. C. Hsu

Department of Mechanical Engineering
The University of New Mexico

Technical Report

by

Y. C. Hsu

Y-M. Lu

F. D. Ju

K. C. Dhingra

Department of Mechanical Engineering
The University of New Mexico

January, 1978

This project was conducted under the auspices of the NEW MEXICO ENERGY INSTITUTE at New Mexico State University. The research project was supported by the New Mexico Energy and Minerals Department as Project No. ERB-75-107.

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ACKNOWLEDGEMENTS

We would like to thank Dr. R. L. San Martin, Director, New Mexico Energy Institute for his encouragement and support.

The principal investigator would like to thank Mr. Y-M. Lu for contributing to chapters 3,5,6 and 7, and for his assistance with chapters 2 and 4, Professor F. D. Ju for contributing to chapter 4 and for helping with chapter 6, Dr. K. C. Dhingra for contributing to chapter 4, Mr. F. Santosa for his assistance with chapter 3. We would also like to thank Mr. H. Murphy and Mr. D. W. Brown at LASL for furnishing the water temperature and pressure of the reservoir, and physical properties of the rock, respectively.

Conclusions, opinions and other evaluative portions of this document solely reflect the views of the authors. Their inclusion herein does not indicate either their acceptance or rejection by the New Mexico Energy Institute at New Mexico State University, project subcontractors or any other cooperating or funding agencies.

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Technical Completion Report

by

Y. C. Hsu, Principal Investigator

Y-M. Lu

F. D. Ju

K. C. Dhingra

Department of Mechanical Engineering
The University of New Mexico

January, 1978

This project took place under the auspices
of the NEW MEXICO ENERGY INSTITUTE at
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ABSTRACT

Additional heat extraction from geothermal energy reservoirs depends on the feasibility to extend the main, hydraulic fracture through secondary thermal cracks of the adjacent hot rock. When the main, hydraulic fracture is cooled sufficiently, these secondary thermal cracks are produced normal to the main fracture surface. As such, both the heat transfer surface area and heat energy available to the fluid circulating through the main, hydraulic fracture system increase.

We have developed methods for predicting the productivity and longevity of a geothermal reservoir. A question is whether a significant long-term enhancement of the heat extraction process is achieved due to these secondary thermal cracks.

In short, the objectives of this investigation are: (1) to study how the main, hydraulic fracture can be extended through these secondary thermal cracks of the rock, and (2) to develop methods for predicting the productivity and longevity of a geothermal reservoir.

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NOMENCLATURE

a	the radius of the main, hydraulic fracture (crack)
$A_{H.C.}$	total heat transfer area of the main, hydraulic fracture
$A_{T.C.}$	total heat transfer area of secondary thermal cracks
$A_1, \dots A_6$	the depth of an element centroid along the z-axis
c	the specific heat of water
c_i	value of c of the crack group denoted by i
c_r	the specific heat of rock
C_o	the compressive strength of rock
d_i	the loading depth of secondary thermal cracks subject to a step loading
D	the depth of secondary thermal cracks
D_i	the ultimate depth of the i -th secondary thermal crack
$D_m, (D_m)_i$	the mean depth of secondary thermal cracks and that of the crack group denoted by i
D_u	the loading depth of secondary thermal cracks subject to a uniform loading
$D_u^1, \dots D_u^6$	the depth of an element centroid along the z-axis
E	Young's modulus of rock
f	normalized stress-intensity factor
F_i	the value of f evaluated at $z = A_i$
g	gravitational acceleration
G	the crack extension force
G_c	a value of G necessary for initiation of fracture (the critical G)

G_o	a value of G necessary for sustained crack propagation
$\bar{G}$	shearing modulus of rock
h	the depth between the earth's surface and the center of the main, hydraulic fracture
k	the permeability of rock
K	buoyancy parameter
K_c	a value of K_I necessary for initiation of fracture (critical K_I or the fracture toughness)
K_I	the stress-intensity factor of opening mode
K_o	a value of K_I necessary for sustained crack propagation
L, L_i	lengths of secondary thermal cracks and the crack group denoted by i
M_i	parameter ($M_i = [(T_i - T_{i-1}) / (t_i - t_{i-1})]$)
P	the pressure of water
P_f	the pressure drop of water from the earth's surface to the water inlet of the main, hydraulic crack due to friction
P_i	the maximum principal stress at element centroid along the x -axis
P_{ij}^1	the water pressure of node ij at the surface of the main, hydraulic fracture
P_{in}	the water pressure at inlet of the main, hydraulic crack
q	parameter ($q = \sqrt{S/\alpha_r}$)
$\dot{Q}_{H.C.}$	the rate of total heat extracted from the main, hydraulic crack
$\dot{Q}_{T.C.}$	the rate of total heat extracted from secondary thermal cracks
s, s_i	the spacing of secondary thermal cracks, and that of the i -th crack
$s_m, (s_m)_i$	the mean spacing of secondary thermal cracks, and that of the crack group denoted by i

s	the Laplace transformation parameter
s_R	stress relief
s_1	overburden earth pressure (the maximum compressive principal stress)
s_2	the maximum horizontal earth principal stress
s_3	the minimum horizontal earth principal stress
s_{ij}	matrix components of normalized tangential stress relief
t	time
t'	modified time parameter
t_i	time at which water temperature and pressure are calculated
$T, \bar{T}$	the temperature of rock, and the Laplace form of T
T_i	water temperature at $t = t_i$
$(T_{avg})_i$	the average temperature of the crack group denoted by i
T_o	the tensile strength of rock
T_{in}, T_{out}	the water inlet and outlet temperature of the main, hydraulic fracture
$T_R, (T_R)_i$	the initial rock temperature, and that of the crack group denoted by i
TS_R	total normalized stress relief
$T_s, \bar{T}_s$	the surface temperature of the main, hydraulic crack, and Laplace form of T_s
u_x	the displacement along the x-axis
U	water temperature
U_i	the value of U_o at $t = t_i$

U_o	the water inlet temperature of secondary thermal cracks
v	the velocity of water through secondary thermal cracks
$\bar{v}$	the temperature of rock, a function of Z and t'
V	the volume flow rate of water through secondary thermal cracks
w	a gap width of the main, hydraulic fracture
W	the weight of water per unit area
x	the horizontal coordinate in the fracture plane
x_i	the value of x of the crack group denoted by i
y	the vertical coordinate in the fracture plane
z	a coordinate orthogonal to the fracture plane
α	thermal expansion coefficient of rock
α_r	thermal diffusivity of rock
β	orientation of the crack at which fracture initiation will commence ($\beta = \pi/2 - (1/2)\tan^{-1}(1/\mu_f)$)
β_i	parameter ($\beta_i = [(U_{i+1} - U_i)/(t_{i+1} - t_i)]$)
γ	surface energy of rock
δ, δ_i	the opening of secondary thermal cracks and that of the i -th crack
$(\delta_m)_i$	mean opening of the crack group denoted by i
ΔP_{ij}	the water pressure difference between nodes ij and the water inlet of the main, hydraulic fracture
ϵ_{xx}	a strain component along the x -axis
η	the angle of the extension of shear fracture measuring from the minimum principal axis
θ	parameter ($\theta = T - T_R$)

θ_i	parameter ($\theta_i = T_i - T_R$)
θ_s	parameter ($\theta_s = T_s - T_R$)
λ_r	thermal conductivity of rock
μ	viscosity of water
μ_f	surface coefficient of friction of rock
ν	Poisson's ratio
ρ	density of water
ρ_r	density of rock
σ_i	the maximum principal stress at element centroid along the x-axis
σ_n	normal stress
$\sigma_{p.c.}$	pre-crack stress
$\sigma_{xx}, \sigma_{xx}^i$	tangential stresses along the x-axis and that of the i-th crack
σ_1, σ_3	the maximum and minimum principal stresses
ϕ	parameter ($\phi = \bar{v} - \theta$)

CHAPTER 1

INTRODUCTION

Recent interest in geothermal energy has been sparked by concern over oil and gas shortages and pollution problems. Dry-hot-rock is by far our largest supply of directly usable clean energy. The entire interior of the earth is at a temperature within or above the melting ranges of the mineral mixtures which compose it. As cited in [1],¹ the western third of the United States possesses the greatest potential for the utilization of such energy. New Mexico is particularly well-favored and benefited for exploitation of this energy due to the following factors: (a) as technology improves, the load centers which are relatively small become economically available almost anywhere in New Mexico; (b) the availability of cooling water for steam condensers for power plants may become critical in the near future; (c) in view of the recent U. S. Supreme Court decision on air-quality standards, people in New Mexico will not tolerate environmental problems and the rapid depletion of a finite state's energy resource such as oil, gas and coal.

A moderate-temperature man-made (hot-dry-rock) geothermal energy reservoir [1 through 4] is being developed beneath the Jemez Plateau of North-Central New Mexico by the Los Alamos Scientific Laboratory (LASL). It has been suggested in Figure 1 that a

¹ Numbers in brackets designate references at the end of the report.

Figure 1. Typical dry-hot-rock geothermal system produced by drilling and hydraulic fracturing.

substantial amount of heat energy could be extracted from regions of essentially impermeable hot rock near the earth's surface by circulating a fluid (for instance, water) from the surface through a large penny-shaped vertical main, hydraulic fracture and back to the surface. In this application, a large hydraulic fracture must be formed and then held open either by using a proppant (with a high resulting driving pressure) or by maintaining the average internal fluid pressure slightly greater than the average normal stress across the fracture. A relatively large fracture diameter is necessary because of the economics associated with this type of moderate-temperature heat source coupled with the typical low conductivity of crystalline rocks.

The propagation and closure stability of such a penny-shaped vertical main hydraulic fracture has been studied [5] subject to realistic stress gradients. For example, as the vertical gradient of the minimum principal horizontal stress exceeds the unit weight of the fluid by two limiting values (779 and 83 dynes/cm³ for the LASL moderate-temperature man-made geothermal reservoir), a stable vertical fracture in granite, which is closed in the lower half and held open in the upper half by the fluid pressure, may not be more than 120 m to 540 m in diameter. This is much less than the proposed size for a commercial moderate-temperature man-made geothermal reservoir. Therefore, in order to achieve a significant heat extraction from the reservoir over a long period of time, it may be essential that the effective heat transfer surface available from the main, hydraulic fracture be enhanced by the creation

of secondary thermal cracks formed as the walls of the main, hydraulic fracture are cooled. Initially, the cooler rock next to the surface of the main, hydraulic fracture will be restrained from contracting by the adjacent hot rock. These restraints will yield tensile stresses. These stresses exceed the tensile strength of the rock after sufficient cooling has occurred. Secondary thermal cracks will then form along the surfaces of the main, hydraulic fracture that will propagate far beyond the cooled region. Both the heat-transfer surface and the heat energy available to a fluid circulating through the reservoir should therefore increase continuously as energy is withdrawn from the reservoir.

We have also investigated whether or not fracture surfaces adjacent to the well bore of a low-temperature geothermal reservoir will thermally crack. Under investigation are two limiting cases where the initial rock temperature is assumed to be 80°C and 180°C , respectively, at a depth of 2,034 m (6,672 ft). The ambient rock surface temperature is assumed to be 10°C at all times. Calculations show that secondary thermal cracks will occur only in the case where the initial rock temperature is 180°C .

In Chapter 1, introduction of this investigation is described. In Chapter 2, a closed-form solution of the rock temperature distribution next to the surface of the main, hydraulic fracture is obtained by use of the Duhamel superposition theorem and water temperatures furnished by LASL. The rock temperature distribution is then calculated by a computer program developed. In Chapter 3, six thermal stress quantities in the rock are calculated by use

of the rock temperatures obtained in Chapter 2 and the SAP IV codes on the IBM 360/370 computer. Then three principal stresses and their principal directions are found from these thermal stress quantities and earth stresses by use of a computer program, "SYMEIG." In Chapter 4, these secondary thermal cracks are predicted based on principal stresses and their principal direction, and fracture mechanics. Furthermore, the additional heat transfer area (surface) is also found. In Chapter 5, the problem of additional heat available to the circulating water due to the additional heat-transfer surface generated by thermal stress cracking is solved. In Chapter 6, the fracture zones in the rock masses next to the geothermal reservoir are studied based on the appropriate fracture criterion, in conjunction with the generated thermal stresses and earth stresses. In Chapter 7, solutions developed in Chapters 2 through 6 are used to estimate the increased heat extraction from the moderate-temperature man-made geothermal reservoir and the increased heat transfer area of the low-temperature geothermal reservoir, resulting from these secondary thermal cracks. In Chapter 8, conclusions are discussed and summarized.

CHAPTER 2

ROCK TEMPERATURE DISTRIBUTION

Heat is extracted from a hot-dry-rock geothermal energy reservoir (Figure 2) by circulating a fluid (for instance, water) through a large penny-shaped vertical hydraulic fracture crack, and back to the surface. During this process, a cooling front is advancing and penetrating from the main, hydraulic fractured surface into the rock. The rock extends infinitely from the plane of the penny-shaped crack. The rock temperature problem in Figure 3 is approximated by the well-known temperature problem of a semi-infinite solid. For constant rock properties, the rock energy equation can be reduced to a one-dimensional heat conduction equation based on the Fourier number in the order of 10^{-3} even for times as great as 30 years [6].

$$\frac{\partial T}{\partial t} = \alpha_r \frac{\partial^2 T}{\partial Z^2} \quad (1)$$

subject to the initial condition

$$T(Z, 0) = T_R \quad (2)$$

and the boundary condition

$$T(0, t) = T_s(t) \quad (3)$$

where α_r is the diffusivity of the rock, T_R is the initial rock temperature which varies with the y-axis in Figure 2, and T_s is the main, hydraulic crack surface temperature which varies with the time, t .

Figure 2. Hydraulically fractured, main geothermal reservoir in the absence of secondary thermal cracks.

Figure 3. The rock temperature, θ , problem approximated by the well-known temperature problem of a semi-infinite solid.

Figure 2 shows the hydraulically fractured, main geothermal reservoir in the absence of any secondary thermal crack. Since the working fluid is single-phase water, it is assumed to be incompressible. Since this fluid is, further, entirely confined to the fracture between assumed impermeable rock surfaces, heat is transferred to this fluid only by thermal conduction through the solid rock surfaces. Considering the small changes in density due to the heating of the fluid, buoyancy effects are accounted for in the body force term in the analyses. The governing equations are written in dimensionless form for a variable fracture gap width ω , and variable fluid properties such as (K, k, μ) in terms of x and y in the plane of the main, hydraulic fracture. These equations, which are derived based on conservation of mass, linear momentum and energy, can then be simplified by neglecting some terms. These simplified equations represent a set of five transient coupled partial differential equations in two spatial dimensions (x and y). Since these equations are highly nonlinear, numerical solutions are obtained, iteratively, by using the finite difference method [6]. Of particular interest here is the water temperature distribution of the present LASL experimental hydraulic fracture at Fenton Hill of a nominal one-thousand-meter diameter. Certain sets of reservoir temperature at every point in the fracture plane and at every time step are furnished by Hugh Murphy, G-3, LASL. These water temperatures $T_s(t)$ are assumed to be the same as the surface temperature of the main crack. Using the above-mentioned $T_s(t)$, Equation (1) can then be solved subject to Equations (2) and (3).

Introduce the variables $\theta(Z, t)$ and $\theta_s(Z, t)$ such that $\theta(Z, t) \equiv T(Z, t) - T_R$ and $\theta_s(Z, t) \equiv T_s(Z, t) - T_R$. Equations (1) to (3) become then

$$\frac{\partial \theta}{\partial t} = \alpha_r \frac{\partial^2 \theta}{\partial Z^2} \quad (4)$$

subject to

$$\theta(Z, 0) = 0 \quad (5)$$

$$\theta(0, t) = \theta_s(t). \quad (6)$$

The general solution of Equation (4) is, based on [7]

$$\theta(Z, t) = \frac{Z}{2\sqrt{\pi\alpha_r}} \int_0^t \frac{\theta_s(t') e^{-Z^2/(4\alpha_r(t-t'))}}{(t-t')^{3/2}} dt'. \quad (7)$$

If we break up the variation of $\theta_s(t)$ into the finite numbers of time intervals (Figure 3), say n , then the method of superposition can be used to obtain the general solution. Here the semi-infinite slab of heat conducting rock initially at zero uniform temperature is suddenly exposed to its surface temperature which equals to the water temperature, $\theta_s(t)$. Here $\theta_s(t)$ varies linearly from its initial value zero to the final value θ_1 over a time interval $0 \leq t \leq t_1$. In Figure 3, a surface temperature is approximated in a linear fashion over n 's time interval.

$$\left. \begin{array}{l} \theta_s = M_1 t \\ \theta_s = M_2 t \\ \vdots \\ \theta_s = M_n t \end{array} \right\} \begin{array}{l} 0 < t \leq t_1 \\ t_1 < t \leq t_2 \\ \vdots \\ t_{n-1} < t \leq t_n \end{array} \quad (8)$$

where

$$\left. \begin{aligned} M_1 &= \theta_1/t_1 = (T_1 - T_R)/t_1 \\ M_2 &= (\theta_2 - \theta_1)/(t_2 - t_1) = (T_2 - T_1)/(t_2 - t_1) \\ &\vdots \\ M_n &= (\theta_n - \theta_{n-1})/(t_n - t_{n-1}) = (T_n - T_{n-1})/(t_n - t_{n-1}). \end{aligned} \right\} \quad (9)$$

During the first time interval $0 < t \leq t_1$, the solution of the rock temperature θ is,

$$\theta(Z, t) = 4M_1 t i^2 \operatorname{erfc} \left(\frac{Z}{\sqrt{4\alpha_r t}} \right) \quad (10)$$

where $i^2 \operatorname{erfc}(\quad)$ is the second integral of the complementary error function. Continuing on to the second time interval, $t_1 < t \leq t_2$, an application of the Duhamel superposition theorem gives the rock temperature

$$\theta(Z, t) = 4M_1 t i^2 \operatorname{erfc} \left(\frac{Z}{\sqrt{4\alpha_r t}} \right) + 4(M_2 - M_1)(t - t_1) i^2 \operatorname{erfc} \left(\frac{Z}{\sqrt{4\alpha_r (t - t_1)}} \right). \quad (11)$$

In the second time interval, $t_1 < t \leq t_2$, let $t' = t - t_1$. Then the rock temperature $\bar{v}(Z, t')$ is governed by

$$\frac{\partial \bar{v}}{\partial t'} = \alpha_r \frac{\partial^2 \bar{v}}{\partial Z^2}, \quad 0 < t' \leq t_2 - t_1 \quad (12)$$

subject to

$$\bar{v}(Z, 0) = \theta(Z, t_1) \quad (13)$$

$$\bar{v}(0, t') = M_2 t' + M_1 t_1. \quad (14)$$

Introduce $\phi(Z, t') \equiv \bar{v}(Z, t') - \theta(Z, t)$ and rewrite Equations (12)

to (14) as

$$\frac{\partial \phi}{\partial t'} = \alpha_r \frac{\partial^2 \phi}{\partial Z^2}, \quad 0 < t' \leq t_2 - t_1 \quad (15)$$

subject to

$$\phi(Z, 0) = 0 \quad (16)$$

$$\phi(0, t') = (M_2 - M_1)t'. \quad (17)$$

The solution of $\phi(Z, t')$ in Equation (15) is

$$\phi(Z, t') = 4(M_2 - M_1)t' i^2 \operatorname{erfc}\left(\frac{Z}{\sqrt{4\alpha_r t'}}\right). \quad (18)$$

Then, the corresponding $\bar{v}(Z, t - t_1)$ is

$$\begin{aligned} \bar{v}(Z, t - t_1) &= \bar{v}(Z, t') = \theta(Z, t) + \phi(Z, t') \\ &= 4M_1 t i^2 \operatorname{erfc} \frac{Z}{\sqrt{4\alpha_r t}} \\ &\quad + 4(M_2 - M_1)(t - t_1) i^2 \operatorname{erfc}\left(\frac{Z}{\sqrt{4\alpha_r (t - t_1)}}\right), \\ t_1 &< t \leq t_2. \end{aligned} \quad (19)$$

By using the principle of superposition, the rock temperature solution for the n -th time interval, $t_{n-1} < t \leq t_n$, is

$$\begin{aligned} \theta(Z, t) &= 4M_1 t i^2 \operatorname{erfc}\left(\frac{Z}{\sqrt{4\alpha_r t}}\right) + \\ &\quad 4 \sum_{j=1}^{n-1} (M_{j+1} - M_j)(t - t_j) i^2 \operatorname{erfc}\left(\frac{Z}{\sqrt{4\alpha_r (t - t_j)}}\right). \end{aligned} \quad (20)$$

Substituting $\theta \equiv T - T_R$ and Equation (9) into (20), with $n = 5$, gives the closed-form solution as

$$T(Z, t) = T_R + 4(T_1 - T_R) \frac{t}{t_1} i^2 \operatorname{erfc}\left(\frac{Z}{\sqrt{4\alpha_r t}}\right) + 4 \sum_{j=1}^4 \left[\frac{T_{j+1} - T_j}{t_{j+1} - t_j} - \frac{T_j - T_{j-1}}{t_j - t_{j-1}} \right] (t - t_j) i^2 \operatorname{erfc}\left(\frac{Z}{\sqrt{4\alpha_r (t - t_j)}}\right). \quad (21)$$

Here $T(Z, t)$ is the transient temperature anywhere in the rock and can be easily calculated from the time history of the water temperature. A computer program called "TMPR" as attached in Appendix A, was developed for calculating $T(Z, t)$.

Because of symmetry of the surface temperature distribution T_s with respect to the y-axis, of particular study is only one half of the penny-shaped crack as shown in Figure 4. Columns A to E extend infinitely from the plane (paper) of the penny-shaped crack into the rock (outward from the paper). The transient temperature anywhere in the rock is plotted in Figures 5 to 9 when the crack surface is kept at T_s . Time periods of interest range from one day to three months. Here T_s is assumed to be equal to the water temperature which is calculated at n time steps (see Appendix B) by using LASL's fracture heat and mass codes called "TEACH" and "HOT FRAC" [6]. Further, to investigate the cooling history of the rock, variation of $T(Z, t)$ vs. t at locations A to E are plotted in Figures 10 to 13 corresponding to the depth of 0, 1.0, 1.5 and 2.0 m respectively. The cooling of the rock is most rapid during the initial one month period.

Figure 4. Location of points of interest in the penny-shaped crack.

Figure 5. The transient rock temperature along column A.

Figure 6. The transient rock temperature along column B

Figure 7. The transient rock temperature along column C.

Figure 8. The transient rock temperature along column D.

Figure 9. The transient rock temperature along column E.

Figure 10. Variation of $T(Z, t)$ with time at the surface ($Z = 0$ m).

Figure 11. Variation of $T(Z, t)$ with time at $Z = 1.0$ m.

Figure 12. Variation of $T(Z,t)$ with time at $Z = 1.5$ m.

Figure 13. Variation of $T(Z, t)$ with time at $Z = 2.0$ m

The above-obtained $T(Z, t)$ will be used to calculate the thermal stress of the rock by using SAP IV codes on the IBM 360/370 here at UNM. A thick shell element is used in conjunction with the boundary element.

CHAPTER 3

THERMAL STRESSES

The problem of computing thermal stresses in the rock, resulting from arbitrary temperature distributions on the rock surface, can only be solved numerically. Of all the numerical methods available, the finite element method is most suitable for the type of geometry on hand.

In brief, the basis of the finite element method is the representation of the continuum describing the structure by an assemblage of subdivisions called finite elements. These elements are considered interconnected at joints which are called nodes.

Different material properties are assigned to a particular finite element and some simple functions are chosen to approximate the distribution of the actual displacements over each finite element. The equilibrium conditions for the entire body are satisfied in such a way that the continuity of displacements is preserved at the interconnecting nodes. Then, the unknown displacements are solved for the given boundary conditions. With manipulations, it is possible to obtain the desired solution in terms of the strains or stresses rather than the displacements.

SAP IV is a finite element computer code with capability to analyze large three dimensional systems. Since it has provision to deal with large varieties of elements, depending upon the problem, a most suitable type of the element could be selected. Again, since both the brick and the thick shell elements offered

considerable promise in dealing with our current problem, these elements are thoroughly investigated with different mesh sizes. Different mesh sizes refer to different numbers of nodal points used. For example, cases of 282, 420, 462 and 612 nodal points have been tried. Based upon the consideration of aspect ratio of the element, the convergence of the solution of stresses, and the capability of the IBM 360/370 at the University of New Mexico, it was decided that the case of 462 nodal points and the thick shell element are well suited for our particular problem.

Having chosen the thick shell element in conjunction with 462 nodal points, thermal stresses are then computed. Because the rock temperature distribution is a function of time, six thermal stress quantities at each element of the rock are calculated for time periods ranging from ten days to three months. Table 1 lists all the physical constants pertaining to the rock used.

In addition to the thermal stresses produced by the cooling of the rock, the overburden earth pressure S_1 and the minimum horizontal principal earth stress of the rock and the fluid pressure are also considered for calculating the principal stresses and their corresponding directions. Theoretical values of S_1 , S_2 and S_3 are obtained based on [8]. Table 2 summarized the values of S_1 , S_2 and S_3 and the fluid (water) pressure, furnished by Hugh Murphy at LASL.

At $h = 2.95$ km, values of S_1 , S_2 , S_3 and P_{ij} are, approximately,

Young's Modulus	E	3.8×10^{10}	Pa	(5.5×10^6)	psi)
Shearing Modulus	$\bar{G}$	1.46×10^{10}	Pa	(2.11×10^6)	psi)
Poisson's Ratio	ν	0.3			
Density of Rock	ρ_r	2.4×10^3	Kg/m ³	(3.84×10^4)	lb/ft)
Density of Fluid	ρ	0.96×10^3	kg/m ³	(1.54×10^4)	lb/ft)
Thermal expansion coefficient	α	0.8×10^{-5}	/°C	(0.44×10^{-5})	/°F)
Diffisivity of the rock	α_r	1.0×10^{-6}	m ² /sec	(1.08×10^{-5})	ft ² /sec)
Tensile Strength of the rock	T_o	8.0×10^6	Pa	(1.16×10^3)	psi)
Critical Stress Intensity Factor	$\frac{K_c}{\sqrt{\pi}}$	1.56×10^6	N/m ^{3/2}	(3.79×10^4)	lb _f /ft)
Fracture Surface Energy	γ	100	J/m ²	(8.82×10^{-3})	Btu/ft)

Table 1. Physical constants of the rock.

	<u>Theoretical Value, Pa</u>	<u>Experimental Value, Pa [9]</u>
Overburden pressure (the maximum compressive principal stress), S_1	$\rho_r gh$	
The maximum horizontal principal stress, S_2	$\frac{\nu}{1-\nu} \rho_r gh = 0.43 \rho_r gh$	$1.38 \text{ MPa} + S_3$
The minimum horizontal principal stress, S_3	$\frac{\nu}{1-\nu} \rho_r gh = 0.43 \rho_r gh$	$0.5 \rho gh$
Fluid pressure at inlet, P_{in}	$\rho gh - P_f$	
Fluid pressure at nodes ij , P_{ij}	$\Delta P_{ij} + P_{in}$	
Fluid pressure difference between nodes ij and the inlet, ΔP_{ij}	See Appendix C	
Fluid frictional pressure drop from the earth's surface to the inlet, P_f	$1.4 \times 10^6 \text{ Pa}$	

Table 2. Values of earth stresses, S_1 , S_2 , S_3 and the fluid pressure, P_{ij} .

$$S_1 = 0.69 \times 10^8 \text{ Pa.}, S_2 = 0.3 \times 10^8 \text{ Pa.}, S_3 = 0.3 \times 10^8 \text{ Pa.},$$

$$P_{in} = 0.27 \times 10^8 \text{ Pa.}, (\Delta P_{ij})_{\text{maximum}} \approx 0.38 \times 10^5 \text{ Pa.},$$

$$P_{ij} = \Delta P_{ij} + P_{in}.$$

Here, the values of $P_{ij} \approx 0.27 \times 10^8 \text{ Pa.}$ are obtained in the pressure tables in Appendix C. As it turns out the fluid pressure P_{ij} in the main, hydraulic fracture is very nearly equal to the minimum horizontal principal stress, S_3 , in the Z-direction and thus approximately nullifies it.

In order to study secondary thermal cracks, it is necessary to calculate the principal stresses and their respective directions. This is done by making use of the computer code called "SYMEIG" (Appendix D).

CHAPTER 4

SECONDARY THERMAL CRACKS

4.1 Introduction

In order to locate the position and the direction of the secondary thermal cracks, we need both the magnitudes and the directions of the principal stresses. The rock is assumed to be homogeneous and isotropic. A secondary thermal crack will form at a location where the maximum principal stresses exceed the tensile strength T_0 of the rock. The normal direction to the crack plane is in the direction of the maximum principal stress. If T_0 is exceeded by principal stresses in two directions, then we should expect to have two perpendicular cracks at the same location.

A thorough assessment of the beneficial aspects of these secondary thermal cracks resulting from the cooling of the rock requires need of computation for the depth, the opening, and the length of these cracks as well as spacings between them. These results enable us to compute the additional heat transfer surface area generated, and hence, additional heat extracting from the rock can be calculated.

This chapter is divided into seven sections dealing with secondary thermal cracks. In the second section, a procedure for the computation of ultimate depths (D_i , $i = 1, 2, \dots, n$) of these cracks in a tensile and/or compressive stress field is described. In the third and fourth sections, the solution for the problem of the spacing (s_{i+1}) between the adjacent cracks and the opening of these

cracks is sought. In the fifth section, the length of these cracks is predicted. In the sixth section, the mean depth, the mean spacing and the mean opening of cracks are predicted. In the last section, the availability of the additional surface area resulting from these secondary thermal cracks is estimated.

4.2 Depth of Secondary Thermal Cracks

In order to compute to what depth a secondary thermal crack will propagate in a tensile and/or compressive stress field, the stress-intensity factors need to be found at the tip of these cracks. Consider such a crack as a long tensile one with the depth D_1 (Figure 14) in an ideally elastic semi-infinite medium. The rate of the release of the strain energy associated with the crack extension G , Young's modulus E , and Poisson's ratio ν are related to the stress-intensity factor of model I, K_I in [10],

$$K_I = \{EG/[\pi(1 - \nu^2)]\}^{1/2}. \quad (22)$$

Here K_I is a measure of G which represents the energy available to derive the crack extension process. If K_I is increased to a critical value K_C , a material dependent constant, the crack will propagate without any further increase in the applied stress. Here K_C represents the molecular cohesive strength of the material, and is therefore a measure of resistance to the initiation of a crack. The value of K_I depends upon the depth-distribution of stress and the changes of the depth of the crack due to the crack propagation. Define K_O as a measure of resistance to propagation of a crack in the range such that

Figure 14. Secondary thermal cracks resulting from the cooling of the rock.

$$0 \leq K_0 \leq K_c. \quad (23)$$

The corresponding value of G is G_0 which should then be between zero and G_c . When K_I becomes equal to a certain value K_0 , there would not be any further increase in the crack depth and the crack is then arrested.

By using the principle of linear superposition, the stress-intensity factor of $\sigma_{xx}(0, y, Z)$ at a given value of y , can be obtained for any arbitrary stress distribution over the depth D of a crack. Let $f(Z/D)$ be the normalized stress-intensity factor for a two dimensional crack of depth D in an elastic semi-infinite medium with a uniform loading over a specified depth D_u . Then, by making use of the function $f(Z/D)$, the normalized stress-intensity factor was obtained for an arbitrary loading $\sigma_{xx}(0, y, Z)$ in the Z -direction at a given value of y [11]

$$\begin{aligned} K_I/(\pi D)^{1/2} &= \int_0^D \sigma_{xx}(0, y, Z) df(Z/D) \\ &= \int_0^D \sigma_{xx}(0, y, Z) (df/dZ) dZ. \end{aligned} \quad (24)$$

Integration by parts would yield, with the small value of $f(0)$ neglected,

$$K_I/(\pi D)^{1/2} = \sigma_{xx}(0, y, D) f(1) - \int_0^D (d\sigma_{xx}/dZ) f dZ. \quad (25)$$

For a uniform stress distribution with $(d\sigma_{xx}/dz) = 0$, Equation (25) reduces to

$$K/(\pi D)^{1/2} = \sigma_{xx}(0, y, D)f(1). \quad (26)$$

An Adequate closed-form representation for $f(Z/D)$ is given in [11] as

$$f(Z/D) = 0.80(Z/D) + 0.04(Z/D)^2 + 3.62 \times 10^{-6} e^{11.18(Z/D)}. \quad (27)$$

For $Z = D$, $f(1) = 1.1$ and Equation (26) reduces to

$$K_I / \left[\sigma_{xx}(0, y, D) (\pi D)^{1/2} \right] = 1.1 \quad (28)$$

which is valid for the case of a uniform loading over the entire depth D of a crack. This indeed is the correct answer in [12] for the normalized stress-intensity factor for a uniform loading over the entire depth D of the crack.

Secondary thermal cracks would occur whenever the maximum principal stress, generated by the cooling of the rock at a given location, exceeds its T_o . We must, however, point it out that in our computation the value of T_o used is of competent homogeneous rock.

In our attempt to compute the crack depth of a specified period, we first obtain the depth-distribution of the principal stress at a given location. Since the SAP IV computer code uses the finite element method, the knowledge of the principal stresses along the Z -axis in the rock is limited to only a few points based

on the capability of the IBM 360/370. It is therefore reasonable to approximate $\sigma_{xx}(0, y, Z)$ by several straight lines of different slopes, as far as the computation of K_I is concerned. A typical depth-distribution of stress is plotted in Figure 15 with five straight lines of different slopes. Here $\sigma_{xx}^1, \sigma_{xx}^2, \dots, \sigma_{xx}^6$, are the maximum principal stresses along the x-axis (within approximately 10°) at depths of $Z = 0, 0.25, 0.75, 1.5, 3.0$, and 5.0 m respectively. These depths are denoted by $D_u^1, D_u^2, \dots, D_u^6$. When any of the principal directions deviate more than 10° from the x-axis, an interpolation technique is used to obtain the corresponding approximate principal stress.

Once the depth-distribution of the stress is specified, the computation of K_I can be readily carried out by using Equation (25). Define $(j + 1)$ as the number of the points at which stresses are given. Define $\sigma_{xx}^1 = P_1, \sigma_{xx}^2 = P_2, \dots, \sigma_{xx}^{j+1} = P_{j+1}$; $D_u^1 = A_1, D_u^2 = A_2, \dots, D_u^{j+1} = A_{j+1}$. Define F_{j+1} as the value of $f(Z/D_u)$ evaluated at $Z = A_{j+1}$. Hence, Equation (25) becomes, for a given stress distribution

$$K_I/(\pi D)^{1/2} = P_{j+1}F_{j+1} - P_1F_1 - \sum_{N=1}^j \left(\frac{P_{N+1} - P_N}{A_{N+1} - A_N} \right) (G_{N+1} - G_N) \quad (29)$$

where

$$G_{N+1} - G_N = \int_{A_N}^{A_{N+1}} f(Z/D) dZ. \quad (30)$$

Figure 15. A typical depth-distribution of stress $\sigma_{xx}(0, y, Z)$.

Now using the above formalism, the depth of these cracks can then be calculated. Starting with $j = 1$, the corresponding value of K_I is calculated by using Equation (29). If this obtained value of K_I exceeds K_O , it implies that the rock depth is greater than A_2 . Increase the integer j by one, calculate the corresponding new value for K_I by using Equation (29) and then test against K_O . The processes of increasing j continues until the resulting K_I calculated from Equation (29) is equal to K_O . Obviously, if the condition is satisfied between two adjacent j 's, the interpolation technique is used to compute the corresponding value of A_{N+1} and P_{N+1} . In practice, we do not know the value of K_O exactly, but, we do know in Equation (23) that its value lies between zero and K_C . Thus, the value of K_I obtained by using Equation (29) is tested against these two limiting values, namely, $K_O = 0$ and $K_O = K_C$. We therefore have two corresponding depths and the actual depth will be somewhere in between them. Since the procedure mentioned is complicated, a computer program was written for the above computation (see Appendix E).

As an illustration, the depth of a secondary thermal crack is calculated by using Equation (29) in conjunction with a depth distribution of stress in Figure 15.

$$\begin{aligned}
 P_1 &= 4.4 \times 10^7 \text{ Pa}, & Z = A_1 &= 0 \\
 P_2 &= 3.92 \times 10^7 \text{ Pa}, & Z = A_2 &= 0.25 \text{ m} \\
 P_3 &= 2.32 \times 10^7 \text{ Pa}, & Z = A_3 &= 0.75 \text{ m} \\
 P_4 &= 4.13 \times 10^6 \text{ Pa}, & Z = A_4 &= 1.5 \text{ m} \\
 P_5 &= -1.69 \times 10^7 \text{ Pa}, & Z = A_5 &= 3.0 \text{ m}
 \end{aligned}$$

$$P_6 = -2.80 \times 10^7 \text{ Pa}, \quad Z = A_6 = 5.0 \text{ m}.$$

Hence, Equation (29) becomes with $K = K_0$ and $j = 5$

$$\begin{aligned} K_0/(\pi D)^{1/2} = & P_6 F_6 - P_1 F_1 + \left[(P_2 - P_1)/(A_2 - A_1) \right] (G_2 - G_1) + \\ & \left[(P_3 - P_2)/(A_3 - A_2) \right] (G_3 - G_2) + \left[(P_4 - P_3)/(A_4 - A_3) \right] (G_4 - G_3) + \\ & \left[(P_5 - P_4)/(A_5 - A_4) \right] (G_5 - G_4) + \left[(P_6 - P_5)/(A_6 - A_5) \right] (G_6 - G_5), \end{aligned} \quad (31)$$

where $F_1, F_2, \dots, F_6$ corresponds to the value of $f(Z/D)$ in Equation (27) at $Z = 0, 0.25, 0.75, 1.5, 3.0$, and 5.0 m respectively. Here $G_1, G_2, \dots, G_6$ are defined in Equation (30). The computer tries different values of Z in order to satisfy Equation (31). Obviously, this calls for the computation of different F_N 's and G_N 's at each of these trials for Z . The values of depths as well as the maximum principal stresses, beside the one specified by the above depth-distribution of the stress, are obtained by the interpolation technique. These two values of depths, obtained by using two limiting values of $K_0 = 0$ and $K_0 = K_c$, are found to yield an upper limit (3.40 m) and a lower limit (3.30 m). The approximate value of the depth is suggested to be $(3.40 + 3.30)/2 = 3.35$ m.

4.3 Spacing Between Adjacent Thermal Cracks

Formation of a secondary thermal crack (for instance, a crack at $x = 0$) reduces the tangential stress in its vicinity. This reduction in stress, so called stress relief, S_R , depends on the distance from the first thermal crack at $x = 0$. Define $\sigma_{p.c.}(x,$

y, 0) as the pre-crack stress at a certain distance x from the first thermal crack. As $\left[\sigma_{p.c.} - |S_R| \right]_{z=0} < T_o$, no thermal crack is formed here. However, if

$$\left[\sigma_{p.c.} - |S_R| \right]_{z=0} \geq T_o \quad (32)$$

then the second thermal crack would occur and S_1 can then be computed. The same procedure is repeated such that the spacings between adjacent cracks are obtained. A computer program is developed for calculating various spacings (see Appendix E).

In the process, we compute S_R at a particular distance x due to a step (uniform) loading σ_1 applied on a crack of depth of D_1 , and then, add up their respective contributions (σ_2 with D_2 , etc.) at such particular distance x. After repeating the above process at several distances from the first thermal crack, a plot of S_R vs. x is made. If, in the vicinity of the first crack, S_R is sufficiently high to reduce the pre-crack stress state to less than T_o of the rock, then no crack is possibly formed there. However, at a certain distance, S_R becomes equal to the pre-crack stress minus T_o . This is the location of the second thermal crack, and thus, the spacing between the first and the second crack can then be computed from Equation (32). The procedure is repeated and the spacings between other adjacent cracks are obtained in a similar manner. For actual computation of various spacings, a computer program is written.

The problem of predicting the spacing between such adjacent

cracks is, therefore, directly related to the computation of S_R as a function of the location. It can be shown that for a given depth D of the crack, S_R is strongly dependent upon the depth-distribution of stress. For a given stress distribution, the stress relief domain (zone) increases appreciably with the depth of the crack.

The general problem of obtaining the solution for S_R as produced by a secondary thermal crack subjected to arbitrary loading conditions, may be obtained by approximating the given depth-distribution of stress into a superposition of step functions of different magnitudes. The general problem then reduces to the superposition of results from problems of S_R , as produced by a given crack with the depth D subjected to various step (uniform) loadings extended to a specific depth. The numerical solution for a single step loading problem computed by an iteration method is available in [12]. In [12], the stress relief on the surface $Z = 0$ in the domain in the vicinity of a tensile crack of the depth D and subject to a step loading, was computed in a semi-infinite elastic medium. These results are depicted in Table 3.

To take advantage of these results, in conjunction with the current problem, a given depth-distribution of S_R is subdivided into several step loading functions with loading depths of d_i ($i = 1, 2, \dots, n$) and the corresponding S_R 's magnitudes of $\sigma_i(Z)$ ($i = 1, 2, \dots, n$). Clearly, $D = d_n$. The stress relief for each individual step loading function can be computed with the aid of data in Table 3. By using the principle of superposition, the totality of such individual solutions yields S_R for the current problem.

x/D	Z/D						
	0.05	0.1	0.2	0.3	0.5	0.75	1.0
0.05	(-0.99)						
0.125	-0.68	(-0.98)					
0.25	-0.36	-0.68	-0.98				
0.5	-0.18	-0.36	-0.68	-0.89			
0.75	-0.12	-0.24	-0.50	-0.67	-0.87		
1.0	-0.087	-0.18	-0.37	-0.51	-0.71	-0.88	-0.94
1.5	-0.051	-0.11	-0.21	-0.31	-0.45	-0.60	-0.65
2.0	-0.033	-0.065	-0.13	-0.20	-0.31	-0.40	-0.45
2.5	-0.024	-0.050	-0.10	-0.15	-0.22	-0.29	-0.32
3.0	-0.017	-0.034	-0.075	-0.11	-0.16	-0.22	-0.24
4.0	-0.010	-0.020	-0.040	-0.059	-0.088	-0.12	-0.15
5.0	-0.006	-0.012	-0.025	-0.036	-0.061	-0.082	-0.097
7.0	-0.003	-0.006	-0.014	-0.021	-0.029	-0.042	-0.049
10.0	-0.001	-0.003	-0.005	-0.008	-0.014	-0.020	-0.023

Table 3. Normalized tangential stress relief $S_R (\pi/2)/\sigma_i(Z)$ on the free surface ($Z = 0$) near a crack of depth D in a semi-infinite elastic medium.

The purpose of the following calculations is to provide an illustration of the procedure involved in the computation of the typical spacing between two adjacent thermal cracks. Assume that the depth of the first crack is known and these pre-crack stress values $\sigma_{p.c.}(x, y, 0)$ in its vicinity are specified. Consider those values of $P_1, P_2, \dots, P_6$; and $A_1, A_2, \dots, A_6$ in Figure 15. They are listed in Section 4-2. This distribution function in solid lines can be equivalently represented by several step functions in dotted lines of magnitudes $\sigma_1, \sigma_2, \sigma_3$, and σ_4 such that

$$\begin{aligned}\sigma_1(Z) &= P_2 - P_3 = 1.61 \times 10^7 \text{ Pa, } 0 < Z \leq 0.5 \text{ m} \\ \sigma_2(Z) &= P_3 - P_4 = 1.907 \times 10^7 \text{ Pa, } 0 < Z \leq 1.0 \text{ m} \\ \sigma_3(Z) &= P_4 - P_5 = 2.103 \times 10^7 \text{ Pa, } 0 < Z \leq 2.0 \text{ m} \\ \sigma_4(Z) &= P_5 = -1.69 \times 10^7 \text{ Pa, } 0 < Z \leq 3.25 \text{ m}\end{aligned}$$

The above step functions are represented in Figure 16. Their resultant is identical to the original depth-distribution of stress at the average points. Thus, the problem of a single crack of depth $D = 3.25 \text{ m}$ and with an arbitrary stress loading, as specified by P_1, P_2, P_3, P_4, P_5 , and P_6 , is reduced to four problems of equal crack depth of 3.25 m . Each, however, is subjected to a uniform stress loading of magnitudes $\sigma_1, \sigma_2, \sigma_3$, and σ_4 distributed over depths $d_i = 0.50, 1.0, 2.0$, and 2.5 m respectively. Therefore, the quantity d_i/D for these first, second, third, and fourth hypothetical cracks will be obtained by dividing the respective d_i 's by $D = 3.25 \text{ m}$. This results in values of (d_i/D) as $0.154, 0.308, 0.616$, and 1.0 in Table 4.

Figure 16. The depth-distribution of stress equivalently represented by several step functions.

x/D	d ₁ /D			
	0.154	0.308	0.616	1.00
0	-1.00	-1.00	-1.00	-1.00
0.25	-0.84	-1.00	-1.00	-1.00
0.50	-0.52	-0.89	-1.00	-1.00
1.00	-0.28	-0.51	-0.79	-0.94

Table 4. A matrix of normalized S_R .

Since S_R at a given location x depends upon the ratios d_1/D and x/D only, by using the above four d_1/D values and Table 3, a matrix, S_{ij} , of the normalized S_R is then constructed in Table 4. Here i and j represent the number of rows and columns. Thus, the number of -0.89 which is in the third row and the second column is the normalized S_R provided by a crack of depth $D = 3.25$ m at a dimensionless distance of $x/D = 0.5$ with $x = 1.625$ m. This crack is subjected to a uniform stress of a unit magnitude extending to a depth of $d_2 = 1.0$ m. For this case, $d_2/D = 0.308$ with $D = 3.25$ m. The total normalized stress relief $TS_R(x/D)$ becomes

$$TS_R(x/D) = \sum_{j=1}^n \sigma_j S_{ij} \quad (33)$$

where n is the total number of step loads. For our example problem at $y = 225$ m, $n = 4$. To illustrate the use of Equation (33),

first let $x/D = 0$,

$$TS_R(0) = \sigma_1 S_{11} + \sigma_2 S_{12} + \sigma_3 S_{13} + \sigma_4 S_{14}$$

Substituting appropriate values in Table 4 into the above equation gives

$$\begin{aligned} TS_R(0) &= \left[1.61(-1.00) + 1.907(-1.00) + \right. \\ &\quad \left. 2.103(-1.00) - 1.69(-1.00) \right] \times 10^7 \\ &= -3.92 \times 10^7 \text{ Pa.} \end{aligned}$$

Similarly, for $x/D = 0.25$,

$$\begin{aligned} TS_R(0.25) &= \left[1.61(-0.84) + 1.907(-1.00) + \right. \\ &\quad \left. 2.103(-1.00) - 1.69(-1.00) \right] \times 10^7 \\ &= -3.67 \times 10^7 \text{ Pa.} \end{aligned}$$

Again, for $x/D = 0.5$,

$$TS_R(0.5) = -2.94 \times 10^7 \text{ Pa.}$$

Finally, for $x/D = 1.0$,

$$TS_R(1.0) = -1.94 \times 10^7 \text{ Pa.}$$

Now in order to obtain the location x of the second crack, we also need to know $\sigma_{p.c.}(x, y, 0)$ in the vicinity of the first crack.

Assume that stresses are not different from one another within a region of a few meters. Thus, we can take 3.92×10^7 Pa also as $\sigma_{p.c.}(x, y, 0)$ in the vicinity of the first crack. This means that the second crack will form at a location x such that where $[3.92 \times 10^7 + TS_R(x/D)]$ becomes equal to $T_0 = 0.80 \times 10^7$ Pa. We now plot a graph of $TS_R(x/D)$ vs. (x/D) in Figure 17. From this figure, the value of x/D , at which TS_R is equivalent to -3.12×10^7 Pa, is 0.45. Since $D = 3.25$ m, the second crack will form at a distance of $x = 1.46$ m from the first crack.

After $S_1 = 1.46$ m is determined, the same procedure is repeated to obtain D_2 . A computer program was already developed to calculate $D_1, D_2, \dots$.

4.4 Opening, δ , of a Thermal Crack at $Z = 0$

From the knowledge of the spacing between two adjacent cracks and S_R at these two locations, the opening, δ , of the crack can then be calculated. This is because δ is nothing but the reduction in the displacement, u_x , of the rock along the x -axis at $Z = 0$, as a result of its cooling. The stress-strain relation is

$$\frac{|\text{Stress relief}|}{|\text{Strain relief}|} = \frac{|TS_R|}{|\epsilon_{xx}|} = E. \quad (34)$$

By using $\epsilon_{xx} = \frac{\partial u_x}{\partial x}$, the resulting contraction of the rock between x_1 and x_2 , due to S_R produced by a thermal crack at x_1 , is, based on Equation (34),

$$u_x = \frac{1}{E} \int_{x_1}^{x_2} TS_R dx. \quad (35)$$

Figure 17. Total normalized stress relief $TS_R(x/D)$ vs. (x/D) .

The same amount of u_x is also produced, due to TS_R resulting from a thermal crack at x_2 . Since the pre-crack stress values at x_1 and x_2 are about the same, TS_R assumes the same when cracks form at x_1 and x_2 . As such, δ is

$$\delta = 2u_x = \frac{2}{E} \int_{x_1}^{x_2} TS_R dx. \quad (36)$$

The negative value of δ indicates that the rock contracts as a result of cooling. The integral in Equation (36) is just the shaded area, A , of TS_R versus distance curve in Figure 17.

At the bottom tip of the crack, its opening approaches to zero. Hence, the crack is of the wedge shape.

Once x_2 is obtained, the same procedure is repeated to complete the opening of other thermal cracks.

For the purpose of illustration, the above procedure is carried out for a typical thermal crack with the crack depth $D = 3.25$ m, the spacing $s_1 = x_2 - x_1 = 1.46 - 0 = 1.46$ m, and $E = 3.8 \times 10^{10}$ Pa. Plot the computed TS_R as a function of distance (x/D) , as shown in Figure 17. Compute the area A , between the first crack ($x/D = 0$) and the second crack ($x/D = 0.45$), to be equal to $|-5.278 \times 10^7 \text{ Pa}|$. Thus, $|u_x|$ in Equation (36) becomes

$$|u_x| = \frac{|-5.278 \times 10^7|}{3.8 \times 10^{10}} \text{ m} = |-1.388| \text{ mm} = 1.388 \text{ mm}.$$

The negative sign of u_x means the rock contracts as a result of cooling. The opening of such a typical thermal crack at $x = x_1 = 0$

and $Z = 0$ is about 2.8 mm.

4.5 Length, L , of the Crack along the y -axis

To compute the length, L , of the crack at a given x , we need to find two locations (points) along the y -axis where the maximum principal stress falls just below T_0 of the rock. The distance between these two end points is the length of the crack at that particular x . Since stresses are calculated only at the centroid of each element, the above-computed length is the one over the corresponding element. When any of the end points of the crack does not fall exactly at the centroid of a particular element, interpolation and/or extrapolation technique is used to obtain the stresses.

4.6 Mean Depth D_m , Mean Spacing s_m , and Mean Opening δ_m

By using the interpolated and/or extrapolated values of the maximum principal stresses at the centroid of the elements along the x -axis these thermal cracks can be, for convenience, viewed in several groups such as $0 \leq x \leq x_I$ for group I, $x_I \leq x \leq x_{II}$ for group II, etc. The numbers of groups, N , are chosen, based on the precision approximating thermal cracks distribution.

The mean depth of a given group is obtained first by taking the mean value of depths along the x -axis at a given y , and then by calculating the mean value of the above-mentioned mean values at various values of y . This definition is also valid for the mean spacing and the mean opening.

4.7 Additional Heat Transfer Surface Area, $A_{T.C.}$, Induced By

Thermal Cracks

Define $A_{T.C.}$ as

$$A_{T.C.} = 4 \left\{ 2 \left[D_m L_m \right]_I + \sum_{i=I}^N \left[(x_i - x_{i-1}) (2 D_m L_m / s_m)_i \right] \right\}. \quad (37)$$

Define the total heat transfer area, $A_{H.C.}$, of the main, hydraulic crack as

$$A_{H.C.} = 2 (\pi a^2) \quad (38)$$

where a is the radius of a penny-shaped main, hydraulic crack.

CHAPTER 5

EXTRACTING ADDITIONAL HEAT FROM
SECONDARY THERMAL CRACKS

Of study is the problem for extracting the additional heat $\dot{Q}_{T.C.}$ afforded by secondary thermal cracks. These cracks are parallel to the yZ -plane and extend into the rock from the surface at $Z = 0$. For simplicity, Figure 18 shows only one secondary thermal crack. As a first approximation of the system in Figure 18, consider a vertical, rectangular thermal crack (Figure 19) which is in perfect thermal contact with the water traveling at the constant velocity v , and with the inlet water temperature $U_0(t)$. This approximate system in Figure 19 is governed by the rock energy equation and the energy balance equation at the interface, which is generated by thermal cracks between the flowing water and the hot rock. These equations are subject to the proper boundary conditions and the initial condition. By using the Laplace transformation, the unsteady state solution of the water temperature is found. Subsequently, the closed-form solution for $\dot{Q}_{T.C.}$ is obtained.

Consider a semi-infinite medium (solid or rock), $x > 0$, initially at temperature T_R in perfect thermal contact at $x = 0$ with the water flowing at the velocity v in the y -axis. The water temperature $U_0(t)$ at inlet is a linear function of time for different time steps. Let W be the weight of the water in contact with a unit area of the solid, c the specific heat of the water, U the water temperature depending on y and t , and T_S the surface temperature of the wall at y and t (see Figure 19).

Figure 18. Secondary thermal cracks resulting from the cooling of the rock.

Figure 19. Simplified system of Figure 18.

In the region $x > 0$, the rock energy equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha_r} \frac{\partial T}{\partial t} \quad (x > 0, y > 0, t > 0) \quad (39)$$

where $\alpha_r = \lambda_r / (\rho_r c_r)$. Assume that the heat conduction of the rock in the y -axis is negligible. As such, energy flows only perpendicular to the direction of the fluid flow and Equation (39) then reduces to

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha_r} \frac{\partial T}{\partial t} \quad (x > 0, y > 0, t > 0)$$

subject to conditions

$$\left. \begin{aligned} T(x, y, 0) &= T_R & (x > 0, y > 0, t = 0) \\ T(0, y, t) &= T_S = U & (x = 0, y > 0, t > 0) \\ T(\infty, y, t) &= T_R & (x \rightarrow \infty, y > 0, t > 0) \end{aligned} \right\} \quad (41)$$

At the interface, an energy balance Equation [13] is, with λ_r as the thermal conductivity of the rock,

$$wc \left(\frac{\partial U}{\partial t} + v \frac{\partial U}{\partial y} \right) = \lambda_r \frac{\partial T}{\partial x} \quad (x = 0, y > 0, t > 0) \quad (42)$$

subject to conditions

$$\begin{aligned} T(0, 0, t) &= U(0, t) = U_i + \beta_i (t - t_i) \\ & \quad (t_i \leq t < t_{i+1}). \end{aligned} \quad (43)$$

Here, the water temperature $U(y, t)$ has values U_i at $y = 0$ for $t = t_i$. Define β_i as $\beta_i = (U_{i+1} - U_i)/(t_{i+1} - t_i)$ with $t_0 = 0$.

In terms of Laplace transforms $\bar{T}(x, y, S)$ and $\bar{U}(y, S)$, the above equations become, with S as the transform parameter,

$$\frac{\partial^2 \bar{T}}{\partial x^2} - \frac{S}{\alpha_r} \bar{T} = -\frac{T_R}{\alpha_r} \quad (x > 0, y > 0) \quad (44)$$

$$\left. \begin{aligned} \bar{T}(0, y, S) &= \bar{U}(0, S) \\ \bar{T}(\infty, y, S) &= T_R/S \end{aligned} \right\} \quad (y > 0) \quad (45)$$

$$v \frac{d\bar{U}}{dy} + S\bar{U} = \frac{\lambda_r}{Wc} \left. \frac{\partial \bar{T}}{\partial x} \right|_{x=0} \quad (x=0, y > 0) \quad (46)$$

$$\bar{T}(0, 0, S) = \bar{U}(0, S) = \frac{U_i - \beta_i t_i}{S} + \frac{\beta_i}{S^2} \quad (47)$$

The solution of Equation (44) subject to (45) is

$$\bar{T}(x, y, S) = \left(\bar{U} - \frac{T_R}{S} \right) e^{-qx} + \frac{T_R}{S} \quad (48)$$

where $q = \sqrt{S/\alpha_r}$. Substituting Equation (48) into (46) gives

$$v \frac{d\bar{U}}{dy} + \left(S + \frac{\lambda_r q}{Wc} \right) \bar{U} = \frac{\lambda_r T_R}{Wc} \frac{q}{S} \quad (49)$$

The solution $\bar{U}(y, S)$ of Equation (49) subject to (47) is

$$\bar{U}(y, S) = \bar{U}(0, S) e^{-\left(\frac{S}{v} + \frac{\lambda_r q}{Wcv}\right)y} + \frac{\left(\frac{\lambda_r T_R q}{WcS}\right)}{\left(S + \frac{\lambda_r q}{Wc}\right)} \left[1 - e^{-\left(\frac{S}{v} + \frac{\lambda_r q}{Wcv}\right)y} \right]$$

$$\begin{aligned}
&= (U_i - \beta_i t_i) e^{-\frac{Y}{v} S} \frac{\left(e^{-\frac{\lambda_r Y \sqrt{S}}{Wc v \alpha_r}} \right)}{S} + \frac{\left(\frac{\lambda_r T_R}{Wc \sqrt{\alpha_r}} \right)}{S \left(\sqrt{S} + \frac{K_r}{Wc v} \right)} \\
&- \frac{\left(\frac{\lambda_r T_R}{Wc \sqrt{\alpha_r}} \right)}{S \left(\sqrt{S} + \frac{\lambda_r}{Wc \sqrt{\alpha_r}} \right)} e^{-\frac{Y}{v} S - \frac{\lambda_r Y \sqrt{S}}{Wc v \alpha_r}} + \frac{\beta_i}{S^2} e^{-\left(\frac{S}{v} + \frac{\lambda_r q}{Wc v} \right) Y} \quad (50)
\end{aligned}$$

Taking the inverse transform of Equation (50) and using the table of integral transforms [14], the solution $U(y, t)$ is obtained as

$$\begin{aligned}
U(y, t) &= (U_i - \beta_i t_i - T_R) \operatorname{erfc} \left[\frac{\lambda_r Y}{2 Wc v \sqrt{\alpha_r} \left(t - \frac{Y}{v} \right)} \right] \\
&+ T_R \left\{ 1 - e^{\left(\frac{\lambda_r}{Wc \sqrt{\alpha_r}} \right)^2 \left[\operatorname{erfc} \left(\frac{\lambda_r}{Wc} \sqrt{\frac{t}{\alpha_r}} \right) - \operatorname{erfc} \left(\frac{\lambda_r Y}{2 Wc v \sqrt{\alpha_r} \left(t - \frac{Y}{v} \right)} \right) \right.} \right. \\
&\left. \left. + \frac{\lambda_r}{Wc} \sqrt{\frac{t - \frac{Y}{v}}{\alpha_r}} \right] \right\} + \beta_i \left\{ \left[\left(t - \frac{Y}{v} \right) + \frac{\left(\frac{\lambda_r Y}{Wc v} \right)^2}{2 \alpha_r} \right] \operatorname{erfc} \left[\frac{\lambda_r Y}{2 \sqrt{\alpha_r} \left(t - \frac{Y}{v} \right)} \right] \right. \\
&\left. - \frac{\left(\frac{\lambda_r Y}{Wc v} \right)^2}{4 \alpha_r \left(t - \frac{Y}{v} \right)^{\frac{1}{2}}} - \frac{\lambda_r Y}{Wc v} \left(\frac{t - \frac{Y}{v}}{\pi \alpha_r} \right) e \right\} \quad (51)
\end{aligned}$$

For $t \gg (y/v)$ and $t \gg (\lambda_r y / Wc v)^2 / 4\alpha_r$, Equation (51) is approximated as

$$U(y, t) \cong U_i + \beta_i(t - t_i) + \left[T_R - (U_i - \beta_i t_i) \right] \operatorname{erf} \left(\frac{\lambda_r y}{2Wc v \sqrt{\alpha_r t}} \right). \quad (52)$$

At the outlet point ($y = L$) of a vertical thermal crack, the water temperature and the rate of energy extraction, $\dot{Q}_{T.C.}$, are, as shown in Figure 19,

$$U(L, t) = U_i + \beta_i(t - t_i) + \left[T_R - (U_i - \beta_i t_i) \right] \operatorname{erf} \left(\frac{\lambda_r L}{2Wc v \sqrt{\alpha_r t}} \right) \quad (53)$$

$$\begin{aligned} \dot{Q}_{T.C.} &= Wc v D \left[U(L, t) - U(0, t) \right] \\ &= Wc v D \left[T_R - (U_i - \beta_i t_i) \right] \operatorname{erf} \left(\frac{\lambda_r L}{2Wc v \sqrt{\alpha_r t}} \right) \\ &= \rho c v \left[T_R - (U_i - \beta_i t_i) \right] \operatorname{erf} \left(\frac{\lambda_r L D}{\rho c v \sqrt{\alpha_r t}} \right) \\ &\quad (t_i \leq t \leq t_{i+1}). \end{aligned} \quad (54)$$

For $i = 0$, $t_0 = 0$, set $\beta_0 = 0$ at $t_0 = 0$ and simplify Equation (54) as

$$\dot{Q}_{T.C.} = \rho c v (T_R - U_0) \operatorname{erf} \left(\frac{\lambda_r L D}{\rho c v \sqrt{\alpha_r t}} \right). \quad (55)$$

CHAPTER 6

FRACTURE ZONES IN ROCK MASSES
NEXT TO GEOTHERMAL RESERVOIRS

The transient temperature gradients and the thermo-elastic stress distributions have been solved numerically for a rock mass subject to the convective flux over the main, hydraulic fracture. The induced stress fields, which motivate crack growth, are related to the brittle fracture of rock according to the Griffith [15] and McClintock-Walsh theories [16]. By use of interpolation techniques for the principal stresses, it is allowed to predict fracture zones and, hence, the extent of internal weakening thermally induced within the rock mass.

Define the compressive stresses as positive. As the normal stress σ_n , acting across the face of a pre-existing secondary thermal crack, causes the crack to close, i.e., when

$$\sigma_n = \frac{1}{2} \left[(\sigma_1 + \sigma_3) + (\sigma_1 - \sigma_3) \cos 2\beta \right] > 0 \quad (56)$$

or

$$\sigma_1 \left[\left(\mu_f^2 + 1 \right)^{\frac{1}{2}} - \mu_f \right] + \sigma_3 \left[\left(\mu_f^2 + 1 \right)^{\frac{1}{2}} + \mu_f \right] > 0, \quad (57)$$

then a compressive fracture [16] will occur when

$$\sigma_1 - \sigma_3 \left[\frac{\left(\mu_f^2 + 1 \right)^{\frac{1}{2}} + \mu_f}{\left(\mu_f^2 + 1 \right)^{\frac{1}{2}} - \mu_f} \right] = C_o \quad (58)$$

where σ_1 is the maximum principal stress, σ_3 is the minimum principal stress, C_0 is the uniaxial compressive strength of the material (positive number), μ_f is the surface coefficient of friction and β is the orientation of the crack at which fracture initiation will commence. As shown in Figure 20, the value of β is calculated from the expression $\beta = \frac{\pi}{2} - \frac{1}{2} \tan^{-1}(1/\mu_f)$ and is measured from the axis of the maximum principal stress. Under a compressive stress state, crack propagation follows a curved path out of the plane of a pre-existing crack toward a direction which is parallel to the axis of the maximum principal compressive stress.

When σ_n causes the crack to open, i.e., when

$$\sigma_n = \sigma_1 \left[\left(\mu_f^2 + 1 \right)^{1/2} - \mu_f \right] + \sigma_3 \left[\left(\mu_f^2 + 1 \right)^{1/2} + \mu_f \right] < 0 \quad (59)$$

then the brittle fracture governed by the Griffith criterion [15] applies. As such, as the stress state is

$$\sigma_1 + 3 \sigma_3 < 0, \quad (60)$$

fracture initiation occurs when

$$\sigma_3 = -T_0. \quad (61)$$

For such a tensile fracture, the crack begins to extend in its own plane in a direction which is perpendicular to that of the minimum principal stress.

As the stress state is

$$\sigma_1 + \sigma_3 > 0, \quad (62)$$

Figure 20. Normal stress σ_n acting across the face of a pre-existing secondary thermal crack.

fracture initiation [15] occurs when

$$(\sigma_1 - \sigma_3)^2 = 8T_o(\sigma_1 + \sigma_3). \quad (63)$$

Under this condition of shear fracture, the grain boundaries of cracks begin to extend in a plane, whose direction measured from the axis of the minimum principal stress is found from the relation of $\cos 2\eta = 0.5(\sigma_3 - \sigma_1)(\sigma_3 + \sigma_1)^{-1}$.

The physical properties of granite used are listed, based on [17],

$$T_o = 8 \times 10^6 \text{ pa.}, C_o = 8.24 \times 10^7 \text{ pa.}, \mu_f = 0.64. \quad (64)$$

A computer program is developed to interpolate the principal stress fields in the Z-direction of the depth of these secondary thermal cracks, and to solve Equations (56) through (63).

Figures 21 through 23 represent fracture zones (at $x = 62.5 \text{ m}$, 225 m and 412.5 m) of rock of a main, hydraulic fracture with $a = 500 \text{ m}$, for time periods from ten days to three months. In these figures, the tensile zone exists on the left region of the curve and the compressive zone exists on the right region of the curve. The tensile zone extends gradually from the region around the inlet of the water as time increases. All of these curves in the tensile zone in the three figures, are equistress curves. They are perpendicular to the axis of the maximum principal stress and represent the tensile-induced fracture surface. However, in the compressive zone, fracture initiation does not occur because of the large uniaxial compressive strength of granite. For a smaller main fracture

Figure 21. Fracture zones at $x = 62.5$ m.

Figure 22. Fracture zones at $x = 225$ m.

Figure 23. Fracture zone at $x = 412.5$ m.

with $a = 100$ m, results indicate that only a small region of the tensile zone is induced and the rest of the region is dominated by the compressive zone. Due to the increasing compressive stress, the compressive failure is initiated in the small area around the inlet of the water and around the periphery of the main, hydraulic fracture. However, those phenomena proceed very slowly even up to two years.

CHAPTER 7

APPLICATIONS

7.1 Description of Geothermal Reservoirs

The moderate-temperature man-made and low-temperature geothermal reservoirs are denoted by model I and model II, and depicted in Figure 24. The corresponding reservoirs characteristics are listed in Table 5. The physical constants of rock (granite) used were listed in Table 1.

	<u>Model I</u>	<u>Model II</u>
a (m)	500	140
h (m)	2,950	2,034
S_1 (Pa)	0.69×10^8	0.69×10^8
S_2 (Pa)	0.30×10^8	0.30×10^8
S_3 (Pa)	$S_3 \cong P_{ij}$	$S_3 \cong P_{ij}$
Flow rate (Kgm/sec)	160	130
Location of inlet	(0, 425, 0)	(0, 133, 0)
Location of outlet	(0, 875, 0)	(0, 217, 0)
T_{in} (°C)	67	40
T_{out} (°C)	To vary with time	To vary with time
Time periods	10 days ~ 3 mon.	1 mon. ~ 3 yrs.

Table 5. Geothermal reservoirs characteristics.

Figure 24. The moderate-temperature man-made (Model I) and the low-temperature geothermal (Model II) reservoirs.

7.2 Results of Secondary Thermal Cracks of Model I at 24 Days

By use of the interpolation and/or extrapolation values of the principal stresses along the x-axis at the centroid of the element, these secondary thermal cracks are divided into several groups, for convenience of numerical calculation, such as $0 < x < 60$ m for group I, $60 \text{ m} < x < 100$ m for group II, etc. The number of the group to be chosen depends upon the precision for approximation. Table 6 and Figure 25 show these mean geometric quantities of the secondary thermal cracks at 24 days.

No. of Groups, N	x_i (m)	$(D_m)_i$ (m)	L_i (m)	$(s_m)_i$ (m)	$(\delta_m)_i$ (mm)
I	60	2.95	475(29~504)*	1.52	1.50
II	100	2.80	475(29~504)	1.57	1.58
III	225	2.35	400(90~490)	1.65	1.65
IV	280	1.65	350(140~490)	1.75	1.73
V	325	1.00	350(140~490)	1.80	1.75
VI	380	0.40	200(275~475)	1.80	1.76

*Numbers in parentheses refer to two end points of the secondary thermal crack.

Table 6. D_m , L , s_m , δ_m of Model I.

Figure 25. D_m , s_m , δ_m , L of Model I.

7.3 Results of Additional Heat Transfer Surface Area, $A_{T.C.}$, of Model I at 24 Days

By using Equation (37) and Table 6, $A_{T.C.}$ generated is

$$\begin{aligned}
 A_{T.C.} &= 4 \left\{ 2(D_m L)_I + \sum_{i=I}^{VI} (x_i - x_{i-1}) \left[\frac{2D_m L}{s_m} \right]_i \right\} \\
 &= 8 \left(2.95 \times 475 + \frac{60 \times 2.95 \times 475}{1.52} + \frac{40 \times 2.8 \times 475}{1.57} \right. \\
 &\quad + \frac{125 \times 2.35 \times 400}{1.65} + \frac{55 \times 1.65 \times 350}{1.75} + \frac{45 \times 1.0 \times 350}{1.70} \\
 &\quad \left. + \frac{55 \times 0.4 \times 200}{1.60} \right) m^2 \\
 &= 1.536 \times 10^6 m^2.
 \end{aligned}$$

By using Equation (38), $A_{H.C.}$ is

$$A_{H.C.} = 2\pi a^2 = 2\pi \times (500)^2 = 1.571 \times 10^6 m^2.$$

Therefore the ratio of $A_{T.C.}$ to $A_{H.C.}$ is

$$\frac{1.536 \times 10^6}{1.571 \times 10^6} = 97.77\%.$$

This means that after these secondary thermal cracks initiate, the total heat transfer area ($A_{T.C.} + A_{H.C.}$) is about twice of the original area ($A_{H.C.}$).

7.4 Results of the Extracted Heat from Secondary Thermal Cracks of Model I at 24 Days

For the current problem, Equation (55) is valid for $t \gg 2$ hours and $t \gg 20$ days as calculated from Equation (51). By using Equation (55), $(\dot{Q}_{T.C.})_i$ for each group is calculated. The required data are specified in Table 7. Here, U_o , T_R , and dP/dy for each group are obtained by use of interpolation values in Appendices B and C. The value of T_{avg} is assumed to be approximately equal to $(U_o + T_R)/2$ and is used as the reference temperature for calculating all those fluid constants, such as ρ , c and μ .

As an illustration, group I is taken as a typical example. First, we calculate the average velocity of group I, v_I based upon [18]

$$v_I = - \left[\frac{(\delta_m/2)^2}{3\mu} \frac{dP}{dy} \right]_I = - \left[\frac{(0.0015)^2}{3 \times 1.571 \times 10^{-4}} \times (-37.895) \right]$$

$$= 0.181 \text{ m/sec.}$$

The volume flow rate, V_I , is

$$V_I = (vD \delta_m)_I = (0.181 \times 2.95 \times 0.0015)$$

$$= 8.01 \times 10^{-4} \text{ m}^3/\text{sec.}$$

By using Equation (55), we obtain $(\dot{Q}_{T.C.})_I$, with t expressed in year,

No. of Groups	$(U_o)_i$	$(T_R)_i$	$(T_{avg})_i$	$(\rho)_i$	$(c)_i$	$(\mu)_i$	$(dP/dy)_i$	$(\dot{Q}_{T.C.})_i$
	°C	°C	°C	$10^3 \frac{\text{Kgm}}{\text{m}^3}$	$10^3 \frac{\text{joule}}{\text{Kg-}^\circ\text{K}}$	$10^4 \frac{\text{Kgm}}{\text{m-sec}}$	Pa/m	M.W.
I	157.6	189.2	173.4	0.94	4.42	1.57	-37.9	3.254
II	188.6	198.5	193.6	0.93	4.47	1.43	-29.6	0.575
III	150.71	188.7	169.7	0.94	4.41	1.60	-16.6	3.224
IV	155.8	196.2	176.0	0.91	4.42	1.57	-1.2	0.071
V	167.6	180.2	173.9	0.94	4.42	1.57	-12.0	0.126
VI	176	199.8	188.0	0.93	4.47	1.43	-23.1	0.139

Table 7. Data of Model I for calculating $\dot{Q}_{T.C.}$.

$$\begin{aligned}
(\dot{Q}_{T.C.})_I &= \frac{x_I}{s_I} \left[\rho c V (T_R - U_O) \operatorname{erf} \left(\frac{\lambda_r^{LD} m}{\rho c V \sqrt{\alpha_r t}} \right) \right]_I \\
&= 4.15 \operatorname{erf} \left(\frac{0.2244}{\sqrt{t}} \right) \\
&= 3.254 \text{ M.W.}
\end{aligned}$$

By using the above-mentioned procedure, $(\dot{Q}_{T.C.})_i$, of other groups can also be calculated and results are listed in the last column of Table 7.

The total $\dot{Q}_{T.C.}$ at 24 days is then the summation of $(\dot{Q}_{T.C.})_i$

$$\begin{aligned}
\dot{Q}_{T.C.} &= \sum_{i=1}^{VI} (\dot{Q}_{T.C.})_i \\
&= (3.254 + 0.575 + 3.224 + 0.071 + 0.126 + 0.139) \\
&= 7.389 \text{ M.W.}
\end{aligned}$$

The values of $\dot{Q}_{T.C.}$ at one month, two months, and three months are also calculated. The corresponding values are 9.92 M.W., 12.43 M.W., and 6.69 M.W., respectively. These data are plotted in Figure 26 to compare to the heat, $\dot{Q}_{H.C.}$, extracted from the main, hydraulic fracture (crack). Here, $\dot{Q}_{H.C.}$ is represented by the form of $\dot{Q}_{H.C.} = \rho c V (T_{out} - T_{in})$ in [6]. Considering the time period from one month to three months, the ratio of $\left(\int_{1 \text{ mon.}}^{3 \text{ mon.}} \dot{Q}_{T.C.} dt / \int_{1 \text{ mon.}}^{3 \text{ mon.}} \dot{Q}_{H.C.} dt \right)$ is 25.67%. This means that the beneficial effect

Figure 26. Power histories for a 500-m-radius fracture with or without secondary thermal cracks.

in the heat energy extracted from the main fracture, in the presence of the secondary thermal cracks, is 25.67%. Figure 26 represents power histories for a 500-m-radius fracture with or without secondary thermal cracks.

7.5 Some Results of Model II

We have investigated whether or not fracture surfaces adjacent to the well bore of a low-temperature geothermal reservoir will thermally crack. Under investigation are two limiting cases where the initial rock temperature is assumed to be 80°C and 180°C, respectively. The ambient rock surface is assumed to be 10°C at all times. The depth of the thermal well used is 2,034 m (6,672 ft) for our current investigation. Calculations show that the secondary thermal cracks occur only in the case where the initial rock temperature is 180°C. The mean values of the depth, the spacing and the opening were calculated for one day, ten days, two months, and three months, respectively. However, due to the lack of the field data of the water temperature and pressure which vary with time, only the additional heat transfer area is determined due to these secondary thermal cracks and is then compared to the heat transfer area of the main, hydraulic crack. Results show that the beneficial effect in the heat transfer area generated by the main, hydraulic crack, in the presence of those secondary thermal cracks, is 242%.

CHAPTER 8

CONCLUSIONS

Based on the conservation of mass, the principle of momentum and the first law of thermodynamics, solutions and procedures were developed for estimating the additional heat energy, which was extracted from stable secondary thermal cracks in a geothermal energy reservoir. The reservoir assumes the form of a large penny-shaped vertical main, hydraulic fracture (crack). Initially, the cooler rock next to the surface of the main, hydraulic fracture is restrained from contracting by the adjacent rock. These restraints yield tensile stresses, which then exceed the tensile strength of the rock after sufficient cooling has occurred. Stable secondary thermal cracks then form along the surfaces of the main, hydraulic fracture that will propagate far beyond the cooled region. Both the additional heat transfer surface and the additional heat energy available to a fluid circulating through the crack of rock should therefore increase continuously as energy is withdrawn from the geothermal energy reservoir.

By using the Duhamel superposition theorem and the calculated water temperatures, a closed-form solution was obtained for the rock temperature distribution and a computer program was developed to calculate it. Six thermal stress quantities were calculated by using the SAP IV codes, in conjunction with the rock temperature distribution. The three principal stresses and their principal directions were calculated by using the computer program, "SYEMIG", in conjunction with thermal stresses and earth stresses.

The depths, spacings, openings of secondary thermal cracks, and the additional heat transfer area and the additional heat energy resulting from these secondary thermal cracks were calculated based on the principal stresses and their principal directions, fracture mechanics and rock properties. The fracture zones in the rock masses next to the geothermal reservoir were investigated based on the fracture criterion, in conjunction with the generated thermal stresses and earth stresses.

The solutions obtained were used to investigate the increased heat extraction from both moderate-temperature man-made and low-temperature geothermal reservoirs resulting from these stable secondary thermal cracks.

Model I tested is a moderate-temperature man-made (hot-dry-rock) geothermal energy reservoir which is being developed beneath the Jemez Plateau of North-Central New Mexico by the LASL. It has been suggested that a substantial amount of heat energy could be extracted from regions of essentially impermeable hot rock, with secondary thermal cracks, near the earth's surface by circulating a fluid (for instance, water) from the surface through a large penny-shaped vertical main, hydraulic fracture and back to the surface. Of study is a hydraulic fracture at Fenton Hill of a nominal one-thousand-meter diameter. Model II treated is a low-temperature geothermal reservoir which occurs in those places with low rock temperature gradient at approximately 1500 to 3000 m depth. Thermal stresses, fracture zones and additional heat transfer area, which are generated due to thermal cooling, were investigated for

both models. The additional heat energy extracted from those secondary thermal cracks was further calculated for Model I.

Calculation results indicate that the 100% and 242% beneficial effects for Model I and Model II, in the additional heat transfer area of the main, hydraulic crack can be obtained in the presence of these secondary thermal cracks. The corresponding beneficial effects in the additional heat energy for Model I is 26%. However, due to the lack of the field data of the water temperature and pressure varying with time, only the additional heat transfer area was determined for Model II. A tensile fracture initiates in the tensile zone of Model I during the first three month period. The compressive fracture initiates in the compressive zone of Model II as times increase until two years.

Currently, an improvement in the heat extraction process is mainly due to the additional heat transfer surface afforded by these stable secondary thermal cracks. However, the reservoir thermal performance may be further increased by reservoir management techniques which enhance the growth and interaction of these secondary thermal cracks. In other words, in a dynamic geothermal reservoir extending by the growth and interaction of these secondary thermal cracks [19 through 21], the predicted heat extraction process would undoubtedly be improved, but should be examined analytically and experimentally.

The additional heat energy calculated is obtained by considering the heat flow perpendicular to the secondary thermal crack surface. However, the practical problem must be dealt with by simultaneously considering the interactions of the flowing fluid

and the energy extraction process in both the main, hydraulic fracture and the resulting secondary thermal cracks. The dynamic interaction between these growing secondary thermal cracks should also be investigated to determine how the productivity and longevity of geothermal reservoir is improved.

APPENDIX A

A COMPUTER PROGRAM, "TMPR," FOR
AN ELASTIC SOLID TEMPERATURE SOLUTION

This computer program is written in FORTRAN IV language and
can be run on an IBM 360/370 computer.

```

      MAIN *TMPR*
      REAL*8 X(30),Y(30),E(30),W(30),H(30),C(30),D(30)
      REAL*8 TR,ALFA,T(50),U(50),TIME(50),SLOPE(50),Z,DZ,TEMP,TE,R,
      *TEMPER(50),EX,EY
      REAL*8 ZR(10)
      READ(5,14) NR,NB
14  FORMAT(2I5)
      READ(5,12) (ZF(I),I=1,NF)
12  FORMAT (6F10.2)
      READ(5,2)(U(I),I=1,4)
      READ(5,6)(TIME(I),I=1,12)
6   FORMAT(6D10.4)
      IFIN=0
98  READ(5,22) EX,EY
      READ (5,97) (T(I),I=1,11)
22  FCFMAT(2F10.2)
97  FCFMAT (3X,11F7.1)
2   FCFMAT(4D12.4)
      R=7./25.
      EX=EX*R
      EY=EY*R
      TR=T(1)
      N=10
      ALFA=1.00          D-6
      D09L=1,N
      SLOPE(L)=(T(L+1)-T(L))/(TIME(L+1)-TIME(L))
9   CONTINUE
      DO 3 J=4,4
      Z=0.0
      DO 6 K=1,6
      Z=ZR(K)
      DO4 IB=2,N
      IF(U(J).GT.TIME(IB).AND.U(J).LT.TIME(IB+1)) GOTO 5
4   CONTINUE
      IB=1
5   S=Z/(DSQRT(4.*ALFA*U(J)))
      TEMPER(K)=TR+4.*SLOPE(1)*U(J)*F(S)
      IF (IB.EQ.1) GOTO 19
      TE=C.0
      IA=IB-1
      DO7 IC=1,IA
      S=Z/DSQRT(4.*ALFA*(U(J)-TIME(IC+1)))
      TE=TE+4.*((SLOPE(IC+1)-SLOPE(IC))*(U(J)-TIME(IC+1))*F(S)
7   CONTINUE
      TEMPER(K)=TEMPER(K)+TE
      IF(K.EQ.6) GC TC 20
19  WRITE(7,10) NR,EX,EY,Z,TEMPER(K)
10  FORMAT(15,19X,'1',2(4X,'1'),3(F10.2),5X,F10.2)
      GC TC 15
20  WRITE(7,11) NR,EX,EY,Z,TEMPER(K)
11  FCFMAT(15,6(4X,'1'),3F10.2,5X,F10.2)
15  NB=NB+1
8   CONTINUE
3   CONTINUE
      IFIN=IFIN+1
      IF(IFIN.EQ.37) GC TO 99
      GOTO 98
99  STOP
      END

```

```

REAL FUNCTION F*8 (S)
REAL*8      X(30),Y(30),E(30),W(30),B(30),C(30),D(30)
REAL*8 G
REAL*8 U,S
U=S
E(1)=0.0
DC1001=1.29
100 E(1+1)=E(1)+.10
W(1)=1.00000
W(2)=.793573
W(3)=.622654
W(4)=.482842
W(5)=.369906
W(6)=.279859
W(7)=.209021
W(8)=.154061
W(9)=.112021
W(10)=.080329
W(11)=.056790
W(12)=.039571
W(13)=.027169
W(14)=.018375
W(15)=.012239
W(16)=.0080263
W(17)=.0051814
W(18)=.0032919
W(19)=.0020580
W(20)=.0012660
W(21)=.0007656
W(22)=.0004555
W(23)=.0002665
W(24)=.0001532
W(25)=.00008664
W(26)=.00004814
W(27)=.00002630
W(28)=.00001411
W(29)=.000007438
W(30)=.000003853
IF(U.GT.2.5) GOTO 102
IF(U.LE.E(2)) GOTO 103
IF(U.GE.2.90) GOTO 102
DC104I=3.30
IF(U.LE.E(1)) GOTO 106
104 CONTINUE
106 X(1)=E(I-2)
X(2)=E(I-1)
X(3)=E(I)
X(4)=E(I+1)
Y(1)=W(I-2)
Y(2)=W(I-1)
Y(3)=W(I)
Y(4)=W(I+1)
N=4
CALL      SPLINE(N,X,Y,B,C,D)
G=SEVAL(N,U,X,Y,B,C,D)
F=G/4.C
GOTO 107
103 X(1)=E(1)
X(2)=E(2)

```



```

      X(3)=E(3)
      X(4)=E(4)
      Y(1)=W(1)
      Y(2)=W(2)
      Y(3)=W(3)
      Y(4)=W(4)
      N=4
      CALL      SPLINE(N,X,Y,B,C,D)
      G=SEVAL(N,U,X,Y,B,C,D)
      F=G/4.*C
      GCTC 107
108  X(4)=E(30)
      X(3)=E(29)
      X(2)=E(28)
      X(1)=E(27)
      Y(4)=W(30)
      Y(3)=W(29)
      Y(2)=W(28)
      Y(1)=W(27)
      CALL      SPLINE(N,X,Y,B,C,D)
      G=SEVAL(N,U,X,Y,B,C,D)
      F=G/4.*C
      GCTC 107
102  F=0.*0
107  RETURN
      END

```

```

SUBROUTINE SPLINE (N, X, Y, B, C, D)
  INTEGER N
  REAL*8 X(N), Y(N), B(N), C(N), D(N)
  INTEGER NM1, IB, I
  REAL*8 T
  NM1 = N-1
  IF ( N.LT. 3 ) GO TO 50
  D(1) = X(2) - X(1)
  C(2) = (Y(2) - Y(1))/D(1)
  DO 10 I = 2, NM1
    D(I) = X(I+1) - X(I)
    B(I) = 2.*(D(I-1) + D(I))
    C(I+1) = (Y(I+1) - Y(I))/D(I)
    C(I) = C(I+1) - C(I)
10  CONTINUE
  B(1) = -D(1)
  B(N) = -D(N-1)
  C(1) = 0.
  C(N) = 0.
  IF ( N.EQ. 3 ) GO TO 15
  C(1) = C(3)/(X(4)-X(2)) - C(2)/(X(3)-X(1))
  C(N) = C(N-1)/(X(N)-X(N-2)) - C(N-2)/(X(N-1)-X(N-3))
  C(1) = C(1)*D(1)**2/(X(4)-X(1))
  C(N) = -C(N)*D(N-1)**2/(X(N)-X(N-3))
15  DO 20 I = 2, N
    T = D(I-1)/B(I-1)

```

```

      B(I) = B(I) - T*D(I-1)
      C(I) = C(I) - T*C(I-1)
20  CONTINUE
      C(N) = C(N)/B(N)
      DO 30 IB = 1, NM1
        I = N-IB
        C(I) = (C(I) - D(I)*C(I+1))/B(I)
30  CONTINUE
      B(N) = (Y(N) - Y(NM1))/D(NM1) + D(NM1)*(C(NM1) + 2.*C(N))
      DO 40 I = 1, NM1
        B(I) = (Y(I+1) - Y(I))/D(I) - D(I)*(C(I+1) + 2.*C(I))
        D(I) = (C(I+1) - C(I))/D(I)
        C(I) = 3.*C(I)
40  CONTINUE
      C(N) = 3.*C(N)
      D(N) = D(N-1)
      RETURN
50  IF ( N .LT. 2 ) RETURN
      B(1) = (Y(2)-Y(1))/(X(2)-X(1))
      C(1) = 0.

      D(1) = 0.
      RETURN
      END

```

```

      REAL FUNCTION SEVAL*B (N, U, X, Y, B, C, D)
      INTEGER N
      REAL*8 U, X(N), Y(N), B(N), C(N), D(N)
      INTEGER I, J, K
      REAL*8 DX
      DATA 1/1/
      IF ( I.GE.N ) I = 1
      IF ( U .LT. X(I) ) GO TO 10
      IF ( U .LE. X(I+1) ) GO TO 30
10  I = 1
      J = N + 1
20  K = (I+J)/2
      IF ( U .LT. X(K) ) J = K
      IF ( U .GE. X(K) ) I = K
      IF ( J .GT. I+1 ) GO TO 20
30  DX = U - X(I)
      SEVAL = Y(I) + DX*(B(I) + DX*(C(I) + DX*D(I)))
      RETURN
      END

```


APPENDIX B

DATA OF WATER TEMPERATURE

The water temperature is calculated at five time steps by using LASL's fracture heat and mass transfer codes called "TEACH" and "HOT FRAC."

TIME = .030 YEARS TEMPERATUR												
1	2	3	4	5	6	7	8	9	10	11	12	Y =
J												
22	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	1.02E+03
21	2.19E+02	2.19E+02	2.19E+02	2.19E+02	2.19E+02	2.18E+02	2.18E+02	2.18E+02	2.18E+02	2.18E+02	2.18E+02	9.75E+02
20	2.25E+02	2.25E+02	2.23E+02	2.24E+02	2.24E+02	2.22E+02	2.22E+02	2.21E+02	2.21E+02	2.21E+02	2.21E+02	9.25E+02
19	2.29E+02	2.27E+02	2.29E+02	2.29E+02	2.28E+02	2.27E+02	2.25E+02	2.25E+02	2.24E+02	2.24E+02	2.24E+02	8.75E+02
18	2.31E+02	2.31E+02	2.31E+02	2.31E+02	2.31E+02	2.31E+02	2.30E+02	2.28E+02	2.28E+02	2.27E+02	2.27E+02	8.25E+02
17	2.31E+02	2.31E+02	2.31E+02	2.31E+02	2.32E+02	2.32E+02	2.33E+02	2.33E+02	2.31E+02	2.31E+02	2.30E+02	7.75E+02
16	2.31E+02	2.31E+02	2.31E+02	2.30E+02	2.31E+02	2.32E+02	2.34E+02	2.36E+02	2.35E+02	2.34E+02	2.33E+02	7.25E+02
15	2.31E+02	2.29E+02	2.29E+02	2.29E+02	2.30E+02	2.31E+02	2.34E+02	2.37E+02	2.39E+02	2.37E+02	2.36E+02	6.75E+02
14	2.29E+02	2.27E+02	2.25E+02	2.25E+02	2.26E+02	2.28E+02	2.32E+02	2.36E+02	2.41E+02	2.41E+02	2.39E+02	6.25E+02
13	2.27E+02	2.27E+02	2.19E+02	2.19E+02	2.19E+02	2.22E+02	2.24E+02	2.34E+02	2.41E+02	2.44E+02	2.42E+02	5.75E+02
12	2.14E+02	2.09E+02	2.04E+02	2.07E+02	2.10E+02	2.15E+02	2.22E+02	2.31E+02	2.40E+02	2.47E+02	2.45E+02	5.25E+02
11	5.70E+01	1.41E+02	1.93E+02	1.93E+02	1.99E+02	2.07E+02	2.17E+02	2.27E+02	2.39E+02	2.48E+02	2.48E+02	4.75E+02
10	7.27E+01	1.36E+02	1.58E+02	1.74E+02	1.88E+02	1.99E+02	2.12E+02	2.24E+02	2.37E+02	2.47E+02	2.51E+02	4.25E+02
9	7.72E+01	1.25E+02	1.47E+02	1.63E+02	1.78E+02	1.93E+02	2.08E+02	2.22E+02	2.36E+02	2.48E+02	2.54E+02	3.75E+02
8	8.67E+01	1.24E+02	1.44E+02	1.59E+02	1.74E+02	1.91E+02	2.07E+02	2.22E+02	2.37E+02	2.54E+02	2.57E+02	3.25E+02
7	8.57E+01	1.25E+02	1.43E+02	1.59E+02	1.75E+02	1.93E+02	2.10E+02	2.25E+02	2.40E+02	2.60E+02	2.60E+02	2.75E+02
6	1.07E+02	1.31E+02	1.47E+02	1.64E+02	1.83E+02	2.01E+02	2.17E+02	2.33E+02	2.59E+02	2.63E+02	2.63E+02	2.25E+02
5	1.21E+02	1.44E+02	1.57E+02	1.77E+02	1.97E+02	2.13E+02	2.29E+02	2.51E+02	2.66E+02	2.66E+02	2.66E+02	1.75E+02
4	1.41E+02	1.59E+02	1.81E+02	2.00E+02	2.18E+02	2.31E+02	2.53E+02	2.69E+02	2.69E+02	2.69E+02	2.69E+02	1.25E+02
3	1.72E+02	1.93E+02	2.16E+02	2.29E+02	2.45E+02	2.60E+02	2.72E+02	2.72E+02	2.72E+02	2.72E+02	2.72E+02	7.50E+01
2	2.34E+02	2.56E+02	2.68E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.50E+01

TIME = .068 YEARS TEMPERATUR												
1	2	3	4	5	6	7	8	9	10	11	12	Y =
J												
22	2.13E+02	2.13E+02	2.14E+02	2.13E+02	2.13E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	1.02E+03
21	2.18E+02	2.19E+02	2.19E+02	2.19E+02	2.19E+02	2.18E+02	2.18E+02	2.18E+02	2.18E+02	2.18E+02	2.18E+02	9.75E+02
20	2.19E+02	2.27E+02	2.25E+02	2.21E+02	2.22E+02	2.22E+02	2.22E+02	2.21E+02	2.21E+02	2.21E+02	2.21E+02	9.25E+02
19	2.19E+02	2.18E+02	2.19E+02	2.21E+02	2.22E+02	2.24E+02	2.25E+02	2.24E+02	2.24E+02	2.24E+02	2.24E+02	8.75E+02
18	2.18E+02	2.17E+02	2.17E+02	2.19E+02	2.21E+02	2.23E+02	2.25E+02	2.27E+02	2.27E+02	2.27E+02	2.27E+02	8.25E+02
17	2.16E+02	2.15E+02	2.15E+02	2.17E+02	2.19E+02	2.21E+02	2.23E+02	2.26E+02	2.30E+02	2.36E+02	2.36E+02	7.75E+02
16	2.14E+02	2.12E+02	2.12E+02	2.12E+02	2.14E+02	2.16E+02	2.19E+02	2.24E+02	2.30E+02	2.33E+02	2.32E+02	7.25E+02
15	2.14E+02	2.09E+02	2.07E+02	2.06E+02	2.07E+02	2.08E+02	2.12E+02	2.19E+02	2.27E+02	2.37E+02	2.34E+02	6.75E+02
14	2.05E+02	2.02E+02	1.98E+02	1.96E+02	1.96E+02	1.98E+02	2.03E+02	2.11E+02	2.23E+02	2.39E+02	2.41E+02	6.25E+02
13	1.94E+02	1.89E+02	1.84E+02	1.82E+02	1.82E+02	1.85E+02	1.91E+02	2.01E+02	2.17E+02	2.39E+02	2.40E+02	5.75E+02
12	1.71E+02	1.62E+02	1.64E+02	1.64E+02	1.67E+02	1.72E+02	1.80E+02	1.92E+02	2.10E+02	2.37E+02	2.53E+02	5.25E+02
11	5.70E+01	1.45E+02	1.42E+02	1.47E+02	1.52E+02	1.59E+02	1.69E+02	1.83E+02	2.03E+02	2.33E+02	2.57E+02	4.75E+02
10	6.85E+01	9.00E+01	1.25E+02	1.31E+02	1.39E+02	1.48E+02	1.60E+02	1.77E+02	1.97E+02	2.26E+02	2.59E+02	4.25E+02
9	7.03E+01	8.64E+01	9.83E+01	1.15E+02	1.20E+02	1.40E+02	1.55E+02	1.72E+02	1.93E+02	2.24E+02	2.61E+02	3.75E+02
8	7.25E+01	8.63E+01	9.67E+01	1.09E+02	1.25E+02	1.35E+02	1.51E+02	1.70E+02	1.92E+02	2.40E+02	2.76E+02	3.25E+02
7	7.53E+01	8.67E+01	9.64E+01	1.07E+02	1.20E+02	1.35E+02	1.52E+02	1.72E+02	1.96E+02	2.60E+02	2.81E+02	2.75E+02
6	7.70E+01	8.97E+01	9.81E+01	1.03E+02	1.20E+02	1.40E+02	1.58E+02	1.80E+02	2.35E+02	2.63E+02	2.68E+02	2.25E+02
5	8.42E+01	9.34E+01	1.04E+02	1.10E+02	1.30E+02	1.51E+02	1.71E+02	2.02E+02	2.66E+02	2.66E+02	2.67E+02	1.75E+02
4	9.24E+01	1.02E+02	1.19E+02	1.35E+02	1.53E+02	1.70E+02	2.02E+02	2.65E+02	2.69E+02	2.69E+02	2.69E+02	1.25E+02
3	1.08E+02	1.28E+02	1.49E+02	1.64E+02	1.84E+02	2.41E+02	2.72E+02	2.72E+02	2.72E+02	2.72E+02	2.72E+02	7.50E+01
2	1.58E+02	1.94E+02	2.30E+02	2.71E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.50E+01

25.000 75.000 125.000 175.000 225.000 275.000 325.000 375.000 425.000 475.000 525.000

TIME= .115 YEARS TEMPERATUR												
1	2	3	4	5	6	7	8	9	10	11	12	Y =
J												
22	2.10E+02	2.14E+02	2.17E+02	2.16E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.16E+02	2.16E+02	1.02E+03
21	2.10E+02	2.14E+02	2.17E+02	2.16E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.16E+02	2.16E+02	9.75E+02
20	2.10E+02	2.14E+02	2.17E+02	2.16E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.16E+02	2.16E+02	9.25E+02
19	2.10E+02	2.14E+02	2.17E+02	2.16E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.16E+02	2.16E+02	8.75E+02
18	2.10E+02	2.14E+02	2.17E+02	2.16E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.15E+02	2.16E+02	2.16E+02	8.25E+02
17	1.97E+02	1.94E+02	1.97E+02	1.99E+02	2.01E+02	2.05E+02	2.09E+02	2.15E+02	2.29E+02	2.30E+02	2.31E+02	7.75E+02
16	1.92E+02	1.92E+02	1.91E+02	1.92E+02	1.93E+02	1.95E+02	1.99E+02	2.05E+02	2.17E+02	2.32E+02	2.37E+02	7.25E+02
15	1.87E+02	1.86E+02	1.84E+02	1.83E+02	1.83E+02	1.83E+02	1.86E+02	1.92E+02	2.04E+02	2.36E+02	2.44E+02	6.75E+02
14	1.82E+02	1.77E+02	1.74E+02	1.71E+02	1.67E+02	1.70E+02	1.73E+02	1.80E+02	1.94E+02	2.29E+02	2.55E+02	6.25E+02
13	1.66E+02	1.65E+02	1.56E+02	1.54E+02	1.53E+02	1.55E+02	1.58E+02	1.66E+02	1.82E+02	2.19E+02	2.60E+02	5.75E+02
12	1.43E+02	1.34E+02	1.35E+02	1.36E+02	1.36E+02	1.41E+02	1.46E+02	1.54E+02	1.71E+02	2.09E+02	2.64E+02	5.25E+02
11	6.73E+01	1.10E+02	1.10E+02	1.22E+02	1.25E+02	1.29E+02	1.35E+02	1.45E+02	1.61E+02	1.99E+02	2.62E+02	4.75E+02
10	6.77E+01	9.37E+01	1.07E+02	1.11E+02	1.15E+02	1.20E+02	1.27E+02	1.38E+02	1.54E+02	1.88E+02	2.54E+02	4.25E+02
9	6.85E+01	8.29E+01	9.45E+01	1.02E+02	1.06E+02	1.13E+02	1.22E+02	1.34E+02	1.49E+02	1.81E+02	2.54E+02	3.75E+02
8	6.96E+01	7.98E+01	8.55E+01	9.27E+01	1.00E+02	1.08E+02	1.18E+02	1.31E+02	1.47E+02	2.17E+02	3.05E+02	3.25E+02
7	7.09E+01	7.99E+01	8.46E+01	9.02E+01	9.74E+01	1.06E+02	1.17E+02	1.31E+02	1.48E+02	2.60E+02	3.25E+02	2.75E+02
6	7.27E+01	7.92E+01	8.44E+01	9.04E+01	9.84E+01	1.08E+02	1.19E+02	1.35E+02	1.80E+02	2.63E+02	2.92E+02	2.25E+02
5	7.55E+01	8.06E+01	8.58E+01	9.35E+01	1.03E+02	1.13E+02	1.25E+02	1.46E+02	2.66E+02	2.64E+02	2.67E+02	1.75E+02
4	7.85E+01	8.30E+01	8.77E+01	1.02E+02	1.13E+02	1.24E+02	1.39E+02	2.16E+02	2.69E+02	2.69E+02	2.69E+02	1.25E+02
3	8.47E+01	9.41E+01	1.00E+02	1.10E+02	1.20E+02	1.70E+02	2.72E+02	2.72E+02	2.72E+02	2.72E+02	2.72E+02	7.50E+01
2	1.05E+02	1.24E+02	1.51E+02	2.25E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.50E+01

TIME= .173 YEARS TEMPERATUR												
1	2	3	4	5	6	7	8	9	10	11	12	Y =
J												
22	2.23E+02	2.23E+02	2.20E+02	2.20E+02	2.19E+02	2.16E+02	2.15E+02	2.16E+02	2.16E+02	2.14E+02	2.15E+02	1.02E+03
21	2.17E+02	2.18E+02	2.18E+02	2.18E+02	2.18E+02	2.19E+02	2.19E+02	2.19E+02	2.19E+02	2.19E+02	2.19E+02	9.75E+02
20	2.04E+02	2.08E+02	2.12E+02	2.10E+02	2.12E+02	2.20E+02	2.21E+02	2.21E+02	2.21E+02	2.22E+02	2.21E+02	9.25E+02
19	1.89E+02	1.87E+02	1.90E+02	1.95E+02	2.02E+02	2.08E+02	2.18E+02	2.24E+02	2.24E+02	2.24E+02	2.24E+02	8.75E+02
18	1.93E+02	1.82E+02	1.83E+02	1.87E+02	1.92E+02	1.97E+02	2.04E+02	2.17E+02	2.27E+02	2.27E+02	2.28E+02	8.25E+02
17	1.77E+02	1.77E+02	1.77E+02	1.80E+02	1.82E+02	1.86E+02	1.90E+02	1.97E+02	2.24E+02	2.30E+02	2.32E+02	7.75E+02
16	1.72E+02	1.71E+02	1.71E+02	1.72E+02	1.73E+02	1.74E+02	1.76E+02	1.81E+02	1.92E+02	2.33E+02	2.50E+02	7.25E+02
15	1.65E+02	1.65E+02	1.64E+02	1.62E+02	1.62E+02	1.61E+02	1.62E+02	1.65E+02	1.73E+02	2.31E+02	2.40E+02	6.75E+02
14	1.58E+02	1.56E+02	1.52E+02	1.50E+02	1.48E+02	1.47E+02	1.47E+02	1.51E+02	1.61E+02	1.90E+02	2.45E+02	6.25E+02
13	1.42E+02	1.30E+02	1.35E+02	1.32E+02	1.32E+02	1.32E+02	1.33E+02	1.37E+02	1.47E+02	1.75E+02	2.54E+02	5.75E+02
12	1.23E+02	1.17E+02	1.16E+02	1.17E+02	1.18E+02	1.20E+02	1.22E+02	1.27E+02	1.36E+02	1.61E+02	2.47E+02	5.25E+02
11	6.79E+01	1.01E+02	1.04E+02	1.06E+02	1.08E+02	1.10E+02	1.14E+02	1.19E+02	1.28E+02	1.52E+02	2.37E+02	4.75E+02
10	6.75E+01	9.83E+01	9.58E+01	9.84E+01	1.01E+02	1.03E+02	1.07E+02	1.13E+02	1.21E+02	1.44E+02	2.27E+02	4.25E+02
9	6.80E+01	7.79E+01	8.91E+01	9.17E+01	9.46E+01	9.81E+01	1.02E+02	1.09E+02	1.18E+02	1.37E+02	2.12E+02	3.75E+02
8	6.86E+01	7.57E+01	7.94E+01	8.53E+01	8.98E+01	9.44E+01	9.96E+01	1.07E+02	1.16E+02	1.50E+02	2.84E+02	3.25E+02
7	6.94E+01	7.51E+01	7.89E+01	8.23E+01	8.70E+01	9.21E+01	9.79E+01	1.06E+02	1.16E+02	2.60E+02	3.73E+02	2.75E+02
6	7.04E+01	7.50E+01	7.85E+01	8.21E+01	8.67E+01	9.22E+01	9.82E+01	1.08E+02	1.32E+02	2.63E+02	3.47E+02	2.25E+02
5	7.17E+01	7.56E+01	7.88E+01	8.32E+01	8.86E+01	9.44E+01	1.00E+02	1.12E+02	2.66E+02	2.64E+02	2.67E+02	1.75E+02
4	7.36E+01	7.70E+01	8.14E+01	8.72E+01	9.33E+01	9.93E+01	1.05E+02	1.45E+02	2.69E+02	2.69E+02	2.69E+02	1.25E+02
3	7.58E+01	8.54E+01	8.85E+01	9.56E+01	1.00E+02	1.23E+02	2.72E+02	2.72E+02	2.72E+02	2.72E+02	2.72E+02	7.50E+01
2	1.51E+02	9.38E+01	1.03E+02	1.40E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.75E+02	2.50E+01
X=	25.000	75.000	125.000	175.000	225.000	275.000	325.000	375.000	425.000	475.000	525.000	

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----- TIME= .247 YEARS TEMPERATUR -----
I = 2 3 4 5 6 7 8 9 10 11 12 Y =
J
22 2.31E+02 2.22E+02 2.25E+02 2.27E+02 2.25E+02 2.17E+02 2.16E+02 2.16E+02 2.16E+02 2.16E+02 2.16E+02 1.02E+03
21 2.19E+02 2.18E+02 2.18E+02 2.18E+02 2.18E+02 2.19E+02 2.19E+02 2.19E+02 2.19E+02 2.19E+02 2.19E+02 9.75E+02
20 1.98E+02 2.04E+02 2.06E+02 2.02E+02 2.03E+02 2.18E+02 2.21E+02 2.21E+02 2.21E+02 2.22E+02 2.22E+02 9.25E+02
19 1.73E+02 1.71E+02 1.74E+02 1.80E+02 1.87E+02 1.94E+02 2.08E+02 2.24E+02 2.24E+02 2.24E+02 2.24E+02 8.75E+02
18 1.66E+02 1.65E+02 1.66E+02 1.70E+02 1.75E+02 1.80E+02 1.87E+02 2.02E+02 2.27E+02 2.27E+02 2.28E+02 8.25E+02
17 1.59E+02 1.59E+02 1.60E+02 1.61E+02 1.64E+02 1.67E+02 1.71E+02 1.77E+02 2.07E+02 2.30E+02 2.37E+02 7.75E+02
16 1.53E+02 1.53E+02 1.53E+02 1.53E+02 1.54E+02 1.55E+02 1.56E+02 1.59E+02 1.64E+02 2.33E+02 2.75E+02 7.25E+02
15 1.46E+02 1.46E+02 1.45E+02 1.44E+02 1.44E+02 1.43E+02 1.42E+02 1.43E+02 1.46E+02 2.00E+02 2.90E+02 6.75E+02
14 1.39E+02 1.37E+02 1.34E+02 1.32E+02 1.30E+02 1.28E+02 1.27E+02 1.28E+02 1.32E+02 1.49E+02 2.35E+02 6.25E+02
13 1.24E+02 1.22E+02 1.18E+02 1.16E+02 1.15E+02 1.15E+02 1.15E+02 1.17E+02 1.20E+02 1.31E+02 2.04E+02 5.75E+02
12 1.08E+02 1.04E+02 1.03E+02 1.04E+02 1.04E+02 1.05E+02 1.06E+02 1.08E+02 1.13E+02 1.23E+02 1.87E+02 5.25E+02
11 6.70E+01 9.25E+01 9.41E+01 9.56E+01 9.68E+01 9.81E+01 9.97E+01 1.02E+02 1.07E+02 1.17E+02 1.76E+02 4.75E+02
10 6.73E+01 8.56E+01 8.84E+01 8.99E+01 9.13E+01 9.29E+01 9.49E+01 9.77E+01 1.02E+02 1.13E+02 1.66E+02 4.25E+02
9 6.77E+01 7.47E+01 8.42E+01 8.52E+01 8.70E+01 8.90E+01 9.14E+01 9.51E+01 9.96E+01 1.09E+02 1.55E+02 3.75E+02
8 6.81E+01 7.32E+01 7.59E+01 8.07E+01 8.35E+01 8.62E+01 8.91E+01 9.32E+01 9.83E+01 1.20E+02 2.02E+02 3.25E+02
7 6.86E+01 7.27E+01 7.55E+01 7.81E+01 8.11E+01 8.42E+01 8.76E+01 9.23E+01 9.76E+01 2.60E+02 4.05E+02 2.75E+02
6 6.93E+01 7.27E+01 7.52E+01 7.76E+01 8.05E+01 8.37E+01 8.73E+01 9.26E+01 1.07E+02 2.63E+02 3.95E+02 2.25E+02
5 7.01E+01 7.29E+01 7.52E+01 7.80E+01 8.12E+01 8.46E+01 8.80E+01 9.41E+01 2.66E+02 2.66E+02 2.67E+02 1.75E+02
4 7.13E+01 7.37E+01 7.65E+01 8.00E+01 8.35E+01 8.70E+01 8.99E+01 1.10E+02 2.69E+02 2.69E+02 2.70E+02 1.25E+02
3 7.32E+01 7.55E+01 8.01E+01 8.43E+01 8.67E+01 9.97E+01 2.72E+02 2.72E+02 2.72E+02 2.72E+02 2.72E+02 7.50E+01
2 7.77E+01 8.16E+01 8.60E+01 1.03E+02 2.75E+02 2.75E+02 2.75E+02 2.75E+02 2.75E+02 2.75E+02 2.75E+02 2.50E+01
X= 25.000 75.000 125.000 175.000 225.000 275.000 325.000 375.000 425.000 475.000 525.000

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APPENDIX C

DATA OF WATER PRESSURE

The water pressure is calculated at five time steps by using LASL's fracture heat and mass transfer codes called "TEACH" and "HOT FRAC."

TIME = .060 YEARS PRESSURE												
1	2	3	4	5	6	7	8	9	10	11	12	Y =
22	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.02E+03
21	-4.91E+03	-9.47E+03	-1.15E+04	-9.66E+03	-8.29E+03	-7.16E+03	-6.13E+03	-5.55E+03	-5.20E+03	-5.02E+03	0.	9.75E+02
20	-4.80E+03	-9.55E+03	-1.18E+04	-9.54E+03	-8.23E+03	-6.90E+03	-5.53E+03	-5.15E+03	-4.85E+03	-4.68E+03	0.	9.55E+02
19	-4.38E+03	-9.31E+03	-1.24E+04	-9.09E+03	-7.34E+03	-6.13E+03	-5.20E+03	-4.49E+03	-4.14E+03	-3.94E+03	0.	9.75E+02
18	-7.48E+03	-7.72E+03	-8.15E+03	-7.19E+03	-6.12E+03	-5.11E+03	-4.21E+03	-3.45E+03	-3.45E+03	-3.03E+03	0.	9.25E+02
17	-6.41E+03	-6.44E+03	-6.33E+03	-5.85E+03	-5.09E+03	-4.23E+03	-3.36E+03	-2.52E+03	-1.98E+03	-1.86E+03	0.	7.75E+02
16	-5.55E+03	-5.50E+03	-5.31E+03	-4.87E+03	-4.23E+03	-3.45E+03	-2.58E+03	-1.69E+03	-8.10E+02	-8.10E+02	0.	7.25E+02
15	-4.95E+03	-4.75E+03	-4.51E+03	-4.04E+03	-3.44E+03	-2.66E+03	-1.79E+03	-8.71E+02	1.38E+00	2.41E+02	0.	6.75E+02
14	-4.35E+03	-4.12E+03	-3.83E+03	-3.29E+03	-2.59E+03	-1.79E+03	-9.02E+02	9.55E+00	8.79E+02	1.65E+03	0.	6.25E+02
13	-4.14E+03	-3.78E+03	-3.13E+03	-2.37E+03	-1.56E+03	-7.23E+02	1.33E+02	9.69E+02	1.74E+03	2.44E+03	0.	5.75E+02
12	-4.13E+03	-3.37E+03	-2.08E+03	-1.14E+03	-3.19E+02	4.68E+02	1.20E+03	1.84E+03	2.37E+03	2.74E+03	0.	5.25E+02
11	0.	-1.59E+03	-1.34E+03	-6.07E+02	3.22E+01	5.93E+02	1.05E+03	1.35E+03	1.48E+03	1.42E+03	0.	4.75E+02
10	1.10E+03	2.34E+02	-5.58E+02	-6.18E+02	4.87E+02	-3.77E+02	-3.99E+02	-5.93E+02	-9.85E+02	-1.64E+03	0.	4.25E+02
9	2.91E+03	1.91E+03	8.92E+02	-4.43E+01	-6.45E+02	-1.20E+03	-1.07E+03	-2.69E+03	-3.70E+03	-5.14E+03	0.	3.75E+02
8	4.43E+03	3.37E+03	2.47E+03	7.43E+02	-5.29E+02	-1.82E+03	-3.23E+03	-4.78E+03	-6.40E+03	-7.20E+03	0.	3.25E+02
7	5.37E+03	4.84E+03	3.23E+03	1.50E+03	-3.57E+02	-2.30E+03	-4.56E+03	-6.91E+03	-9.62E+03	-9.62E+03	0.	2.75E+02
6	7.54E+03	6.25E+03	4.46E+03	2.21E+03	-2.89E+02	-3.04E+03	-6.02E+03	-9.15E+03	-1.18E+04	-1.20E+04	0.	2.25E+02
5	9.13E+03	7.71E+03	5.54E+03	2.75E+03	-4.74E+02	-6.02E+03	-7.74E+03	-1.17E+04	-1.17E+04	-1.44E+04	0.	1.75E+02
4	1.09E+04	9.23E+03	6.48E+03	2.93E+03	-9.72E+02	-6.21E+03	-9.90E+03	-1.48E+04	-1.72E+04	-1.98E+04	0.	1.25E+02
3	1.25E+04	1.04E+04	6.99E+03	2.57E+03	-1.72E+03	-7.80E+03	-1.65E+04	-1.98E+04	-2.22E+04	-2.35E+04	0.	7.50E+01
2	1.35E+04	1.10E+04	6.89E+03	9.10E+02	-4.95E+03	-1.19E+04	-1.86E+04	-2.26E+04	-2.54E+04	-2.66E+04	0.	2.50E+01

TIME = .068 YEARS PRESSURE												
1	2	3	4	5	6	7	8	9	10	11	12	Y =
22	-4.91E+03	-9.47E+03	-1.15E+04	-9.66E+03	-8.29E+03	-7.16E+03	-6.13E+03	-5.55E+03	-5.20E+03	-5.02E+03	-5.02E+03	1.02E+03
21	-7.27E+03	-7.87E+03	-8.71E+03	-7.57E+03	-5.65E+03	-3.81E+03	-2.09E+03	-1.06E+03	-4.12E+02	-7.85E+01	-5.02E+03	9.75E+02
20	-7.04E+03	-7.72E+03	-9.07E+03	-7.49E+03	-5.73E+03	-3.74E+03	-1.46E+03	-7.26E+02	-1.42E+02	2.08E+02	-4.54E+03	9.25E+02
19	-6.35E+03	-7.41E+03	-1.29E+04	-6.98E+03	-4.79E+03	-3.15E+03	-1.73E+03	-4.68E+02	1.39E+02	6.19E+02	-3.96E+03	8.75E+02
18	-5.06E+03	-5.41E+03	-5.93E+03	-4.79E+03	-3.52E+03	-2.26E+03	-1.02E+03	2.14E+02	2.14E+02	1.17E+03	-3.02E+03	8.25E+02
17	-3.81E+03	-3.87E+03	-3.91E+03	-3.26E+03	-2.43E+03	-1.43E+03	-3.14E+02	9.11E+02	1.95E+03	2.27E+03	-1.86E+03	7.75E+02
16	-2.78E+03	-2.74E+03	-2.56E+03	-2.14E+03	-1.47E+03	-6.05E+02	4.53E+02	1.68E+03	3.14E+03	3.14E+03	-8.10E+02	7.25E+02
15	-2.05E+03	-1.94E+03	-1.48E+03	-1.22E+03	-5.64E+02	3.02E+02	1.35E+03	2.58E+03	3.93E+03	4.50E+03	2.41E+02	6.75E+02
14	-1.73E+03	-1.44E+03	-9.79E+02	-3.37E+02	4.60E+02	1.40E+03	2.40E+03	3.69E+03	5.01E+03	6.58E+03	1.65E+03	6.25E+02
13	-1.78E+03	-1.13E+03	-2.24E+02	7.11E+02	1.76E+03	2.73E+03	3.80E+03	4.91E+03	6.04E+03	7.30E+03	2.44E+03	5.75E+02
12	-2.23E+03	-3.94E+02	8.28E+02	1.98E+03	3.06E+03	4.08E+03	5.04E+03	5.94E+03	6.72E+03	7.46E+03	2.78E+03	5.25E+02
11	0.	6.80E+02	1.74E+03	2.83E+03	3.77E+03	4.62E+03	5.34E+03	5.89E+03	6.19E+03	6.13E+03	1.42E+03	4.75E+02
10	2.15E+03	2.44E+03	2.48E+03	3.24E+03	3.82E+03	4.31E+03	4.61E+03	4.65E+03	4.37E+03	3.43E+03	-1.68E+03	4.25E+02
9	5.45E+03	4.95E+03	4.22E+03	4.04E+03	4.03E+03	3.96E+03	3.69E+03	3.12E+03	2.17E+03	2.81E+02	-5.10E+03	3.75E+02
8	7.22E+03	6.65E+03	5.91E+03	5.16E+03	4.49E+03	3.74E+03	2.79E+03	1.52E+03	-1.36E+02	-1.10E+03	-7.20E+03	3.25E+02
7	8.76E+03	8.17E+03	7.36E+03	6.26E+03	5.97E+03	5.66E+03	5.04E+03	-1.23E+02	-2.97E+03	-2.97E+03	-9.62E+03	2.75E+02
6	1.02E+04	9.56E+03	8.56E+03	7.21E+03	5.52E+03	4.43E+03	3.71E+02	-1.72E+03	-5.05E+03	-5.51E+03	-1.20E+04	2.25E+02
5	1.17E+04	1.11E+04	9.75E+03	7.96E+03	5.83E+03	4.79E+03	-4.82E+02	-4.44E+03	-4.44E+03	-8.73E+03	-1.44E+04	1.75E+02
4	1.34E+04	1.25E+04	1.08E+04	8.31E+03	5.10E+03	4.47E+03	-2.99E+03	-9.17E+03	-1.34E+04	-1.60E+04	-1.08E+04	1.25E+02
3	1.50E+04	1.39E+04	1.12E+04	7.70E+03	4.03E+03	-2.91E+03	-1.33E+04	-1.09E+04	-2.31E+04	-2.52E+04	-2.35E+04	7.50E+01
2	1.70E+04	1.44E+04	9.65E+03	8.77E+03	-2.77E+03	-1.22E+04	-2.14E+04	-2.78E+04	-3.20E+04	-3.41E+04	-2.80E+04	2.50E+01
1	20.000	75.000	125.000	175.000	225.000	275.000	325.000	375.000	425.000	475.000	525.000	

PIB = .115 YEARS PRESSURE												
I =	2	3	4	5	6	7	8	9	10	11	12	Y =
J												
22	-7.27E+03	-7.86E+03	-9.71E+03	-7.57E+03	-5.65E+03	-3.81E+03	-2.09E+03	-1.06E+03	-4.12E+02	-7.85E+01	-7.85E+01	1.02E+03
21	-6.27E+03	-6.65E+03	-7.35E+03	-6.09E+03	-3.93E+03	-1.54E+03	6.80E+02	2.05E+03	2.93E+03	3.30E+03	-7.85E+01	9.75E+02
20	-5.95E+03	-6.67E+03	-7.74E+03	-6.04E+03	-4.09E+03	-1.71E+03	1.18E+03	2.21E+03	3.02E+03	3.50E+03	2.08E+02	9.25E+02
19	-5.12E+03	-6.13E+03	-9.38E+03	-5.54E+03	-3.28E+03	-1.49E+03	1.77E+02	1.82E+03	2.66E+03	3.33E+03	6.19E+02	8.75E+02
18	-3.71E+03	-4.57E+03	-4.48E+03	-3.44E+03	-2.24E+03	-1.01E+03	2.66E+02	1.69E+03	1.69E+03	3.06E+03	1.17E+03	8.25E+02
17	-2.41E+03	-2.47E+03	-2.45E+03	-2.01E+03	-1.34E+03	-4.98E+02	4.98E+02	1.69E+03	2.96E+03	3.45E+03	2.27E+03	7.75E+02
16	-1.45E+03	-1.39E+03	-1.29E+03	-1.01E+03	-5.48E+02	9.80E+01	9.39E+02	2.01E+03	3.48E+03	3.48E+03	3.14E+03	7.25E+02
15	-7.97E+02	-7.38E+02	-5.75E+02	-2.71E+02	1.90E+02	8.16E+02	1.62E+03	2.63E+03	3.87E+03	4.45E+03	4.50E+03	6.75E+02
14	-4.59E+02	-4.64E+02	-1.07E+02	3.89E+02	1.03E+03	1.73E+03	2.57E+03	3.56E+03	4.73E+03	6.29E+03	6.58E+03	6.25E+02
13	-4.19E+02	-3.75E+02	1.69E+02	1.18E+03	1.90E+03	2.80E+03	3.66E+03	4.57E+03	5.57E+03	6.79E+03	7.30E+03	5.75E+02
12	-1.42E+03	-4.40E+02	1.11E+03	2.09E+03	2.87E+03	3.79E+03	4.57E+03	5.34E+03	6.09E+03	6.88E+03	7.40E+03	5.25E+02
11	.	9.47E+02	1.83E+03	2.60E+03	3.42E+03	4.10E+03	4.73E+03	5.27E+03	5.71E+03	6.01E+03	6.13E+03	4.75E+02
10	2.37E+03	2.16E+03	2.58E+03	2.94E+03	3.47E+03	4.12E+03	4.76E+03	4.55E+03	4.58E+03	4.17E+03	3.43E+03	4.25E+02
9	4.03E+03	3.64E+03	3.45E+03	3.53E+03	3.70E+03	3.85E+03	3.89E+03	3.77E+03	3.34E+03	1.94E+03	2.81E+02	3.75E+02
8	5.25E+03	4.84E+03	4.48E+03	4.20E+03	4.01E+03	3.79E+03	3.43E+03	2.87E+03	2.01E+03	1.05E+03	-1.10E+03	3.25E+02
7	6.24E+03	5.80E+03	5.43E+03	4.93E+03	4.39E+03	3.76E+03	3.26E+03	1.88E+03	1.36E+02	1.36E+02	-2.97E+03	2.75E+02
6	7.12E+03	6.77E+03	6.23E+03	5.55E+03	4.73E+03	3.60E+03	2.42E+03	8.76E+02	-1.45E+03	-2.00E+03	-5.51E+03	2.25E+02
5	8.31E+03	7.63E+03	7.05E+03	6.09E+03	4.90E+03	3.30E+03	1.59E+03	-6.67E+02	-6.67E+02	-5.24E+03	-8.73E+03	1.75E+02
4	9.02E+03	8.09E+03	7.74E+03	6.44E+03	4.76E+03	2.65E+03	2.57E+02	-4.10E+03	-1.00E+04	-1.32E+04	-1.60E+04	1.25E+02
3	1.01E+04	8.44E+03	7.69E+03	6.15E+03	4.15E+03	1.80E+02	-1.01E+04	-1.59E+04	-2.15E+04	-2.44E+04	-2.53E+04	7.50E+01
2	1.07E+04	8.12E+03	7.20E+03	5.27E+02	-4.60E+03	-1.30E+04	-7.30E+04	-2.99E+04	-3.50E+04	-3.76E+04	-3.41E+04	2.50E+01

PIB = .173 YEARS PRESSURE												
I =	2	3	4	5	6	7	8	9	10	11	12	Y =
J												
22	-6.27E+03	-6.65E+03	-7.35E+03	-6.09E+03	-3.93E+03	-1.54E+03	6.80E+02	2.05E+03	2.93E+03	3.39E+03	3.39E+03	1.02E+03
21	-4.43E+03	-4.57E+03	-4.91E+03	-3.77E+03	-1.79E+03	8.98E+02	3.43E+03	5.09E+03	6.17E+03	6.74E+03	3.39E+03	9.75E+02
20	-4.12E+03	-4.54E+03	-5.23E+03	-3.74E+03	-1.98E+03	4.54E+02	3.63E+03	5.00E+03	6.03E+03	6.64E+03	3.50E+03	9.25E+02
19	-3.34E+03	-4.26E+03	-6.35E+03	-3.39E+03	-1.50E+03	1.08E+02	1.82E+03	3.79E+03	4.84E+03	5.69E+03	3.33E+03	8.75E+02
18	-2.11E+03	-2.32E+03	-2.62E+03	-1.84E+03	-9.14E+02	6.95E+01	1.15E+03	2.55E+03	2.55E+03	4.30E+03	3.06E+03	8.25E+02
17	-1.01E+03	-1.55E+03	-1.55E+03	-7.78E+02	-3.39E+02	2.17E+02	9.06E+02	1.77E+03	3.10E+03	3.73E+03	3.45E+03	7.75E+02
16	-2.31E+02	-2.33E+02	-1.99E+02	-6.42E+01	1.77E+02	5.28E+02	1.01E+03	1.67E+03	2.70E+03	2.70E+03	3.48E+03	7.25E+02
15	1.47E+02	1.69E+02	2.38E+02	3.88E+02	6.29E+02	9.72E+02	1.43E+03	2.04E+03	2.84E+03	3.32E+03	4.45E+03	6.75E+02
14	6.42E+01	1.84E+02	4.17E+02	7.35E+02	1.13E+03	1.60E+03	2.14E+03	2.76E+03	3.53E+03	4.67E+03	6.29E+03	6.25E+02
13	-3.40E+02	5.31E+01	5.95E+02	1.17E+03	1.76E+03	2.34E+03	2.92E+03	3.53E+03	4.19E+03	5.07E+03	6.79E+03	5.75E+02
12	-9.02E+02	1.75E+02	1.61E+03	1.73E+03	2.38E+03	2.96E+03	3.52E+03	4.05E+03	4.57E+03	5.20E+03	6.88E+03	5.25E+02
11	.	7.34E+02	1.47E+03	2.09E+03	2.63E+03	3.12E+03	3.57E+03	3.98E+03	4.31E+03	4.68E+03	6.01E+03	4.75E+02
10	1.50E+03	1.58E+03	1.91E+03	2.28E+03	2.65E+03	3.00E+03	3.29E+03	3.53E+03	3.62E+03	3.53E+03	4.17E+03	4.25E+02
9	2.03E+03	2.54E+03	2.48E+03	2.62E+03	2.81E+03	2.97E+03	3.08E+03	3.09E+03	2.90E+03	1.90E+03	1.94E+03	3.75E+02
8	3.66E+03	3.45E+03	3.14E+03	3.03E+03	2.98E+03	2.93E+03	2.80E+03	2.56E+03	2.12E+03	1.15E+03	1.05E+03	3.25E+02
7	4.20E+03	4.56E+03	3.77E+03	3.48E+03	3.21E+03	2.90E+03	2.50E+03	1.95E+03	9.44E+02	9.44E+02	1.34E+02	2.75E+02
6	4.83E+03	4.65E+03	4.27E+03	3.88E+03	3.42E+03	2.86E+03	2.19E+03	1.42E+03	-1.20E+02	-1.03E+03	-2.00E+03	2.25E+02
5	5.35E+03	5.12E+03	4.74E+03	4.22E+03	3.54E+03	2.70E+03	1.73E+03	5.44E+02	5.44E+02	-4.14E+03	-5.24E+03	1.75E+02
4	5.50E+03	5.64E+03	5.20E+03	4.47E+03	3.53E+03	2.30E+03	1.07E+03	-1.09E+03	-8.89E+03	-1.23E+04	-1.32E+04	1.25E+02
3	6.54E+03	6.19E+03	5.45E+03	4.35E+03	3.20E+03	1.32E+03	-8.98E+03	-1.39E+04	-2.08E+04	-2.41E+04	-2.44E+04	7.50E+01
2	7.07E+03	6.47E+03	5.37E+03	2.39E+03	-6.33E+03	-1.37E+04	-2.38E+04	-3.07E+04	-3.62E+04	-3.92E+04	-3.76E+04	2.50E+01
X =	25.000	75.000	125.000	175.000	225.000	275.000	325.000	375.000	425.000	475.000	525.000	

APPENDIX D

SUBROUTINE, "SYEMIG," FOR COMPUTING
EIGENVALUES AND EIGENVECTORS OF REAL SYMMETRIC MATRIX

This computer program is written in FORTRAN IV language and
can be run on an IBM 360/370 computer.


```

      DO 5 I=K,N
      E(I)=E(I)-KAPPA*D(I)
      DO 4 J=K,I
      A(I,J)=A(I,J)-D(I)*E(J)-E(I)*D(J)
4     CONTINUE
5     CONTINUE

C
6     A(K,K-1)=D(K)
      D(K)=A(K,K)
      A(K,K)=BETA
      E(K)=-ALPHA
7     CONTINUE
8     D(N)=A(N,N)
      E(N)=A(N,N-1)

C
C     ACCUMULATE TRANSFORMATIONS
C     PRODUCES X SO THAT XT*(INPUT A)*X IS TRIDIAGONAL
C
      IF(WANTX.EQ.0) GO TO 20
      X(N,N)=1.
      DO 15 KB=1,NM1
      K=N-KB
      BETA=A(K,K)
      DO 11 I=K,N
      X(I,K)=0.
      X(K,I)=1.
11     CONTINUE
      X(K,K)=1.
      IF(K.EQ.1) GO TO 15
      IF(BETA.EQ.0.) GO TO 15
      DO 14 J=K,N
      GAMMA=0.
      DO 12 I=K,N
      GAMMA=GAMMA+X(I,J)*A(I,K-1)
12     CONTINUE
      GAMMA=GAMMA/BETA
      DO 13 I=K,N
      X(I,J)=X(I,J)-GAMMA*A(I,K-1)
13     CONTINUE
14     CONTINUE
15     CONTINUE

C
C     TRIDIAGONAL QR ALGORITHM
C     IMPLICIT SHIFT FROM LOWER 2-BY-2
C
20    DO 27 MB=2,N
      M=N+2-MB
      MM1=M-1
      L=1
21    E(L)=0.

C
C     FIND L SUCH THAT E(L) IS NEGLIGIBLE
C
      L=M
22    S=DABS(D(L-1))+DABS(D(L))
      T=S+DABS(E(L))
      IF(T.EQ.S) GO TO 23
      L=L-1
      IF(L.GE.2) GO TO 22

```

```

C      IF E(M) IS NEGLIGIBLE, THEN D(M) IS AN EIGENVALUE, SO..
C
C      23 IF(L.EQ.M) GO TO 27
C      FORM IMPLICIT SHIFT
C      T=(D(M-1)-D(M))/(E(M)+E(M))
C      S=DSORT(1.+T*T)
C      IF(T.LT.0.) S=-S
C      E(L)=D(L)-D(M)+E(M)/(T+S)
C      F=E(L+1)
C
C      CHASE NONZERO F DOWN MATRIX
C
C      DO 26 J=L,M+1
C      T=DABS(E(J))+DABS(F)
C      ALPHA=T*DSORT((E(J)/T)**2+(F/T)**2)
C      C=E(J)/ALPHA
C      S=F/ALPHA
C      BETA=S*(D(J+1)-D(J))+2.*C*E(J+1)
C      E(J)=ALPHA
C      E(J+1)=E(J+1)-C*BETA
C      T=S*BETA
C      D(J)=D(J)+T
C      D(J+1)=D(J+1)-T
C      IF(J.EQ.M+1) GO TO 24
C      F=S*E(J+2)
C      E(J+2)=-C*E(J+2)
C
C      24 IF(WANTX.EQ.0) GO TO 26
C      DO 25 I=1,N
C      T=X(I,J)
C      X(I,J)=C*T+S*X(I,J+1)
C      X(I,J+1)=S*T-C*X(I,J+1)
C      25 CONTINUE
C
C      26 CONTINUE
C      GO TO 21
C
C      27 CONTINUE
C      28 CONTINUE
C      REORDER THE EIGENVALUES AND EIGENVECTORS
C      DO 32 I=2,3
C      IF(D(1).LT.D(I)) GO TO 32
C      S=D(1)
C      D(1)=D(I)
C      D(I)=S
C      DO 31 J=1,3
C      S=X(J,1)
C      X(J,1)=X(J,I)
C      X(J,I)=S
C      31 CONTINUE
C      32 CONTINUE
C
C      IF(D(2).GT.D(3)) RETURN
C      S=D(2)
C      D(2)=D(3)
C      D(3)=S

```

```
DO 33 I=1,3  
S=X(1,2)  
X(1,2)=X(1,3)  
X(1,3)=S  
33 CONTINUE  
RETURN  
END
```


APPENDIX E

A COMPUTER PROGRAM FOR COMPUTING
DEPTH, SPACING AND OPENING OF
SECONDARY THERMAL CRACKS

This computer program is written in FORTRAN IV language and
can be run on an IBM 360/370 computer.

```

MAIN 'CRACK'
DIMENSION P(7),A(7),C(7),F(7),J(7),K(7)
DIMENSION STR1(10),STR2(10),STR3(10),STR4(10),STR5(10),STR6(10)
DIMENSION ESTR(10)
DIMENSION S(10,10)
C S(10,10) IS THE STRESS RELIEF MATRIX AND IS DERIVED FROM
C TABLE # 3.
C TO COMPUTE THE VALUES OF DEPTH SPACING AND OPENING OF A GIVEN
C CRACK AT A GIVEN LOCATION, WE NEED TO SUPPLY THE THE STRESS
C VALUES AS A FUNCTION OF DEPTH. THIS IS DONE WITH THE HELP OF
C TWO CARDS, ON FIRST CARD WE INPUT SEVEN PRINCIPAL STRESS VALUES
C REFERRED TO AS P(1),P(2),P(3),P(4),P(5),P(6),P(7).
READ(5,130) ((S(I,J),I=1,10),J=1,10)
IF IN=0
139 READ(5,131) (P(I),I=1,7)
A(1)=0.00
A(2)=0.25
A(3)=0.75
A(4)=1.50
A(5)=3.00
A(6)=6.00
A(7)=10.00
C TSR IS THE TENSILE STRENGTH OF THE ROCK IN PASCALS
TSR=0.3E7
C THIS PORTION OF THE PROGRAM COMPUTES DEPTH
C CK IS IN ENGG. UNITS
CK=1.55E+6
I=1
L=0
10 SUM=0
DO 20 N=1,1
C B IS THE DEPTH OF THE CRACK
B=A(I+1)
R(N)=(P(N)-P(N+1))/(A(N)-A(N+1))
F(N)=(0.30*A(N)/B)+(0.04*(A(N)/B)**2)+(3.62E-6)*EXP(11.13*A(N)/B)
F(N+1)=(0.30*A(N+1)/B)+(0.04*(A(N+1)/B)**2)+(3.62E-6)*EXP(11.13*A(N+1)/B)
G(N)=0.4*((A(N)**2)/B)+(0.01333*((A(N)**3)/(B**2)))+
10.324E-6*B*EXP(11.13*A(N)/B)
G(N+1)=0.4*((A(N+1)**2)/B)+(0.01333*((A(N+1)**3)/(B**2)))+
10.324E-6*B*EXP(11.13*A(N+1)/B)
SUM=SUM-R(N)*(G(N+1)-G(N))
20 CONTINUE
CSUM=(P(I+1)*F(I+1)-P(1)*F(1+SUM))*0.005
IF(CSUM) 32,33,26
26 IF(CSUM.GE.0.90*CK.AND.CSUM.LE.1.10*CK) GO TO 31
GO TO 32
31 DEPTH1=A(I+1)
GO TO 50
32 IF(ABS(CSUM).LE.0.1*CK) GO TO 30
IF(CSUM.LT.0.1) GO TO 40
IF(L.GT.0) GO TO 40
I=I+1
GO TO 10
33 DEPTH2=A(I+1)
40 L=L+1
IF(L.GT.1) GO TO 45
PK=P(I+1)
AK=A(I+1)

```

```

      X=(PK-P(1))/(AK-A(1))
45  V = PK-.01*X*L
      U = AK-.01*L
      P(I+1)=V
      A(I+1)=U
      GO TO 10
50  D=(DEPTH1+DEPTH2)/2
C   D IS THE AVERAGE DEPTH OF THE CRACK AT A GIVEN LOCATION
C   THIS PORTION OF THE PROGRAM COMPUTES THE SPACING BETWEEN
C   TWO ADJACENT CRACKS.
      IF(D.LE.0.5) GO TO 100
      IF(D.GT.0.5.AND.D.LE.1.0) GO TO 200
      IF(D.GT.1.0.AND.D.LE.2.0) GO TO 300
      IF(D.GT.2.0.AND.D.LE.4.0) GO TO 400
      IF(D.GT.4.0.AND.D.LE.6.0) GO TO 500
      IF(D.GT.6.0.AND.D.LE.15.0) GO TO 600
100  I=1
C   ESTR(I) IS THE TOTAL STRESS RELIEF AS PRODUCED BY A CRACK
C   AT A DISTANCE OF 0.25*I METERS.
C   STR1(I),STR2(I),STR3(I),STR4(I) AND STR5(I) ARE STRESS RELIEFS
C   AS PRODUCED BY A CRACK OF GIVEN DEPTH WITH UNIFORM LOADINGS
C   EXTENDING FROM THE SURFACE(Z=0) TO Z=1.50,1.0,2.0,4.0 AND 10
C   METERS RESPECTIVELY, AT A DISTANCE OF 0.25*I.
C   IN CASE, FOR EXAMPLE, D IS LESS THAN 4.0 METERS, THEN, STR4(I)
C   WILL BE STRESS RELIEF FOR A CRACK WITH UNIFORM LOADING
C   EXTENDING FROM Z=0.0 TO Z=D METERS AND SO ON.
101  ESTR(I)=P(2)*S(1,1)
      IF(P(2)-ESTR(I)-TSR) 4,5,2
2   CRSP=.25*D*((I-1)-(P(2)-ESTR(I-1)-TSR)/(ESTR(I-1)-ESTR(I)))
      GO TO 51
      J CRSP=0.25*I*D
      GO TO 51
4   I=I+1
      GO TO 101
200  I=1
201  J=5.0/D
      FJ=0.50*10/D
      STR1(I)=(S(1,J)+(S(1,J+1)-S(1,J))*(FJ-J))*(P(2)-P(3))
      STR2(I)=P(3)*S(1,10)
      ESTR(I)=STR1(I)+STR2(I)
      IF(P(2)-ESTR(I)-TSR) 7,6,5
6   CRSP=0.25*I*D
      GO TO 51
5   CRSP=.25*D*((I-1)-(P(2)-ESTR(I-1)-TSR)/(ESTR(I-1)-ESTR(I)))
      GO TO 51
7   I=I+1
      GO TO 201
300  I=1
301  J=5.0/D
      FJ=0.50*10/D
      STR1(I)=(S(1,J)+(S(1,J+1)-S(1,J))*(FJ-J))*(P(2)-P(3))
      J=10.0/D
      FJ=1.0*10/D
      STR2(I)=(S(1,J)+(S(1,J+1)-S(1,J))*(FJ-J))*(P(3)-P(4))
      STR3(I)=P(4)*S(1,10)
      ESTR(I)=STR1(I)+STR2(I)+STR3(I)
      IF(P(2)-ESTR(I)-TSR) 11,10,9
9   CRSP=.25*D*((I-1)-(P(2)-ESTR(I-1)-TSR)/(ESTR(I-1)-ESTR(I)))
      GO TO 51

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19 CRSP=0.25*I*D
GO TO 51
11 I=I+1
GO TO 301
400 I=1
401 J=5.0/D
FJ=5.0/D
STR1(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(2)-P(3))
J=10.0/D
FJ=10.0/D
STR2(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(3)-P(4))
J=20.0/D
FJ=20.0/D
STR3(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(4)-P(5))
STR4(I)=P(5)*S(I,10)
ESTR(I)=STR1(I)+STR2(I)+STR3(I)+STR4(I)
IF(P(2)-ESTR(I)-TSR) 16,15,14
16 I=I+1
GO TO 401
15 CRSP=0.25*I*D
GO TO 51
14 CRSP=.25*D*((I-1)-(P(2)-ESTR(I-1)-TSR)/(ESTR(I-1)-ESTR(I)))
GO TO 51
500 I=1
501 J=5.0/D
FJ=5.0/D
IF(D.GE.5.0) GO TO 502
STR1(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(2)-P(3))
GO TO 503
502 STR1(I)=(1.-(1.-S(I,1))*FJ)*(P(2)-P(3))
503 J=10.0/D
FJ=10.0/D
STR2(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(3)-P(4))
J=20.0/D
FJ=20.0/D
STR3(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(4)-P(5))
J=40.0/D
FJ=40.0/D
STR4(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(5)-P(6))
STR5(I)=P(6)*S(I,10)
ESTR(I)=STR1(I)+STR2(I)+STR3(I)+STR4(I)+STR5(I)
IF(P(2)-ESTR(I)-TSR) 21,22,21
21 CRSP=.25*D*((I-1)-(P(2)-ESTR(I-1)-TSR)/(ESTR(I-1)-ESTR(I)))
GO TO 51
22 CRSP=0.25*I*D
GO TO 51
23 I=I+1
IF(1.EQ.3) GO TO 52
GO TO 501
600 I=1
601 J=5.0/D
FJ=0.50*10/D
IF(D.GE.5.0) GO TO 602
STR1(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(2)-P(3))
602 STR1(I)=(1.-(1.-S(I,1))*FJ)*(P(2)-P(3))
J=10.0/D
FJ=10.0/D
STR2(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(3)-P(4))
J=20.0/D

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FJ=2.0*10/D
STR3(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(4)-P(5))
J=40.0/D
FJ=4.0*10/D
STR4(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(5)-P(6))
J=60.0/D
FJ=6.0*10/D
STR5(I)=(S(I,J)+(S(I,J+1)-S(I,J))*(FJ-J))*(P(6)-P(7))
STR6(I)=P(7)*S(I,10)
ESTR(I)=STR1(I)+STR2(I)+STR3(I)+STR4(I)+STR5(I)+STR6(I)
IF(P(2)-ESTR(I)-TSR) 81,27,28
28 CRSP=.25*D*((I-1)-(P(2)-ESTR(I-1)-TSR)/(ESTR(I-1)-ESTR(I)))
GO TO 51
27 CRSP=0.25*I*D
GO TO 51
81 I=I+1
GO TO 601
C THIS PORTION OF THE PROGRAM COMPUTES THE OPENING WIDTH OF
C THE CRACK AT A GIVEN LOCATION.
51 EFF=(ESTR(I)-(ESTR(1)-ESTR(I))*(CRSP-0.25*D)/(0.25*(I-1)*D))
STAR=(P(2)+ESTR(I))*0.25*J+(ESTR(I)+EFF)*(CRSP-0.25*D)
C OPWTH IS THE OPENING WIDTH OF THE CRACK.
C YOUNMOD IS THE YOUNG'S MODULUS OF THE ROCK.
YOUNMOD=3.8E10
OPWTH=STAR/YOUNMOD
GO TO 52
52 WRITE(6,134) D,OPWTH,CRSP
130 FORMAT(17F7.2)
131 FORMAT(7E10.2)
134 FORMAT(6X,3F15.4)
135 FORMAT(7F10.4)
136 FORMAT(110)
IFIN=IFIN+1
IF (IFIN.EQ.35) GO TO 140
GO TO 139
140 STOP
END

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