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QUARK AND GLUON DECAY FUNCTIONS

IN QCD AND RECOMBINATION MODEL

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MASTER

Abstract

Inclusive longitudinal-momentum distributions of pions in jets initiated by quarks and gluons are determined in perturbative QCD and recombination model. The quark and antiquark joint distributions in jets are first calculated in the leading-order approximation at high Q^2 . Gluons in the jets are completely converted to quark pairs. From the overall distribution $q\bar{q}$ pairs with definite quantum numbers then recombine to form pions. The recombination function for the process is well determined in the valon model. No adjustable parameters are involved in these calculations, and no data at low Q^2 are used as phenomenological input. The result for the quark decay functions can be compared with data on e^+e^- annihilation, and the agreement is very good in both shape and normalization. Predictions for the gluon decay functions are presented, but they cannot yet be checked by experiments. The x - and Q^2 -dependences of both type of decay functions have been parametrized in simple form suitable for use in theoretical and experimental applications.

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I. INTRODUCTION

The study of structures of hadron jets associated with quarks or gluons produced at high Q^2 has been regarded as means of testing quantum chromodynamics (QCD). They are hard processes for which QCD has definitive predictions. For example, the jet calculus of Konishi, Ukawa and Veneziano¹ provides a calculational scheme for the determination of quark distributions in a jet initiated by a quark. Theoretical developments in that direction may well be called quark physics, as distinct from hadron physics, in the sense that the initial and final "particles" of the processes considered are quarks and gluons. Since experiments detect hadrons rather than quarks, the problem of hadronization of quarks must necessarily be considered in conjunction with QCD if a complete theoretical description of jet structure is to be given. It is our aim in this paper to provide such a description.

The hadronization problem considered in the framework of valons² has been discussed previously.³⁻⁵ We shall give only a very brief summary of that problem in Section III. The result of that consideration is the determination of the recombination function $R(x_1, x_2, x)$. It specifies the probability for the formation of a pion at x from the recombination of a quark at x_1 and an antiquark at x_2 . The main body of this paper is a description of the QCD calculation of $F(x_1, x_2, Q^2)$, which is the probability of having a quark at x_1 and an antiquark at x_2 in a jet initiated by a quark or a gluon at Q^2 . The decay function $D(x, Q^2)$ can then be obtained in the recombination model⁶ as a convolution of the two probabilities

$$xD(x, Q^2) = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} F(x_1, x_2, Q^2) R(x_1, x_2, x). \quad (1.1)$$

In an earlier paper³ we have given the result of our calculation of the pion spectrum in a quark jet for $Q = 5$ GeV. No adjustable parameters were

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used. Excellent agreement with data was obtained in both normalization and shape. In this paper we shall not only give the details of the calculation but also provide a Q^2 -dependent parametrization of the x distribution of $x D(x, Q^2)$. We shall also present our predictions for gluon jets, which is possible in our approach because we do not need phenomenological inputs at lower Q^2 . Calculation for the gluon jet, being rather similar to that of the quark jet, is described in an Appendix. The results will be presented in terms of u quark or gluon decaying into π^+ , π^- , and π^0 separately. In that way they can be applied to jets produced in neutrino interactions as well as e^+e^- annihilation. As expected, $D_g^{\pi^+}$ has less large- x components than $D_u^{\pi^+}$ but more than $D_u^{\pi^-}$.

In Sec. II we describe the method for obtaining the moments of two-parton distribution in a jet. The hadronization problem is discussed in Sec. III and gluon conversion in Sec. IV. In Sec. V our method for inversion of moments is described and the Q^2 -dependent parametrizations of the decay functions are presented.

II. QUARK-ANTIQUARK DISTRIBUTION FUNCTION

In this section we calculate the $q\bar{q}$ distribution function $F(x_1, x_2, Q^2)$ in a quark jet. First, we define the probability function in noninvariant phase space element $dx_1 dx_2$

$$G(x_1, x_2, Q^2) = (x_1 x_2)^{-1} F(x_1, x_2, Q^2) \quad (2.1)$$

so that with the quark type indices exhibited explicitly, (1.1) takes the form

$$x D_i^{\pi}(x, Q^2) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{k,l} G_{q_i \rightarrow q_k \bar{q}_l}(x_1, x_2, Q^2) R_{k\bar{l}}^{\pi}(x_1, x_2, x), \quad (2.2)$$

in an obvious notation $G_{q_i \rightarrow q_k \bar{q}_l}$ stands for the probability of finding quark q_k (at x_1) and antiquark $\bar{q}_l$ (at x_2) in a jet initiated by quark q_i .

In perturbative QCD to the leading-order approximation in $(\ln Q^2)^{-1}$, $G_{q_i \rightarrow q_k \bar{q}_l}$ has a general expression that can be represented by a diagram shown in Fig. 1, in which there is one bifurcation point.¹ The shaded blobs represent the probabilities of finding one parton (quark, antiquark, or gluon) inside another parton. More precisely, they are the evolution functions^{7,8} for Q^2 evolving from higher to lower values, more appropriately called degradation. Let the first such degradation function (reading from the left of Fig. 1) be $G_{i \rightarrow j}(y, Q^2, k^2)$ where y and k^2 are the momentum fraction and 4-momentum squared, respectively, of parton j just before bifurcation, while Q^2 is a large scale that characterizes the hard process associated with the initial quark i . The bifurcation $j \rightarrow j' + j''$ is described by the combination of factors⁷ $(2\pi k^2)^{-1} \alpha(k^2) P_{j \rightarrow j'}(z/y)$ where $\alpha(k^2)$ is the running coupling constant and z is the momentum fraction of the parton j' . $P_{j \rightarrow j'}(z/y)$ is the decay function associated with the bifurcation vertex; it satisfies the reflection property

$$P_{j \rightarrow j'}(z/y) = P_{j \rightarrow j''}(1 - z/y). \quad (2.3)$$

Subsequent degradations in Fig. 1 are represented by $G_{j' \rightarrow k}(x_1/z, k^2, Q_0^2)$ and $G_{j'' \rightarrow l}(x_2/(y-z), k^2, Q_0^2)$ in obvious notation. Assembling them in terms of the invariant functions [e.g., $yG(y)$, etc.] integrated over invariant

phase space, we have

$$x_1 x_2 G_{i \rightarrow kl}(x_1, x_2, Q^2, Q_0^2) = \sum_{j, j''} \int_{Q_0^2}^{K^2} \frac{dk^2}{k^2} \frac{\alpha(k^2)}{2\pi} \int_0^1 \frac{dy}{y} y G_{i \rightarrow j}(y, Q^2, k^2) \int_{x_1}^{y-x_2} \frac{dz}{y} P_{j \rightarrow i'}(\frac{z}{y}) \frac{x_1}{z} G_{j' \rightarrow k}(\frac{x_1}{z}, k^2, Q_0^2) \frac{x_2}{y-z} G_{j'' \rightarrow l}(\frac{x_2}{y-z}, k^2, Q_0^2). \quad (2.4)$$

We have labeled the destination of Q^2 degradation by Q_0^2 above. At this point it is unnecessary for us to specify its value, except to mention that it will be taken to be rather low. The justification for its choice and for the leading order approximation to be used will be given after the hadronization problem is discussed in Sec. III.

The upper limit of k^2 integration is

$$K^2 = \min(Q^2, Q_1^2) \quad (2.5)$$

where Q_1^2 is some cut-off which will be taken to be 30 GeV^2 for definiteness. The precise value of this cut-off is not crucial but the reason for the cut-off is the following. It is known⁹ that k^2 is a measure of the maximum range of transverse momenta (k_T) of the bifurcation products. Since recombination is not likely to take place between a pair of quark and antiquark that are too far apart in k_T space, k^2 should not be allowed to become arbitrarily large if the resultant G function is to be used in the recombination formula (2.2). We have chosen the cut-off Q_1^2 to be 30 GeV^2 because the average p_T

of pions (relative to jet axis) produced in e^+e^- annihilation at c.m. energy E_{cm} between 5 and 6 GeV is about 0.5 GeV^{10} , a value which we regard as being the maximum range of $\langle k_T \rangle$ for recombining quarks. The final answer will not depend critically on the choice of Q_1^2 .

Because the one-parton distribution functions are known in QCD in terms of their moments, we take the double moment of the two-parton distribution function. Define

$$G_{ij}^n(Q^2, k^2) = \int_0^1 dy y^{n-1} G_{i \rightarrow j}(y, Q^2, k^2), \quad (2.6a)$$

$$G_{i \rightarrow kl}^{mn}(Q^2, Q_0^2) = \int_0^1 dx_1 \int_0^1 dx_2 x_1^{m-1} x_2^{n-1} G_{i \rightarrow kl}(x_1, x_2, Q^2, Q_0^2) \quad (2.6b)$$

$$P_{jj'}^{mn} = \int_0^1 dz z^{m-1} (1-z)^{n-1} P_{j \rightarrow j'}(z). \quad (2.6c)$$

We then obtain from (2.4)

$$G_{i \rightarrow kl}^{mn}(Q^2, Q_0^2) = \sum_{j, j''} \int_{Q_0^2}^{K^2} \frac{dk^2}{k^2} \frac{\alpha(k^2)}{2\pi} G_{i \rightarrow j}^{m+n-1}(Q^2, k^2) P_{jj'}^{mn} G_{j' \rightarrow k}^m(k^2, Q_0^2) G_{j'' \rightarrow l}^n(k^2, Q_0^2) \quad (2.7)$$

Except for the range of integration, (2.7) is in agreement with the basic equation for jet calculus by Konishi, Ukawa and Veneziano.¹

To reckon with the sum in (2.7) over all parton types, we use the subscripts g to denote gluon, Q for quark or antiquark, f for Q-Q transition of the same type (favored), and u for Q-Q transitions between unlike types. Let f (not as a subscript) be the number of flavors of quarks. Consider first the case $k=i \neq l$, such as for π^+ production from a u quark ($u \rightarrow u \bar{d}$). Then

the summation in (2.7), defined by

$$S_{i \rightarrow k \ell}^{mn} = \sum_{j, j'} G_{ij}^p P_{jj'}^{mn} G_{j'k}^m G_{j\ell}^n$$

where $p = m+n-1$, becomes

$$\begin{aligned} S_{i \rightarrow i \ell}^{mn} &= G_f^p (P_{qq}^{mn} G_f^m G_{q\ell}^n + P_{qg}^{mn} G_{q\ell}^m G_u^n) \\ &+ G_u^p \{ P_{qq}^{mn} (2f-1) G_u^m G_{q\ell}^n + P_{qg}^{mn} G_{q\ell}^m [G_f^n + (2f-2) G_u^n] \} \\ &+ G_{qg}^p P_{qq}^{mn} [G_f^m G_u^n + G_u^m G_f^n + (2f-2) G_u^m G_u^n] \\ &+ G_{qg}^p P_{gg}^{mn} G_{q\ell}^m G_{q\ell}^n. \end{aligned} \quad (2.8)$$

The first line corresponds to j being a quark of type i , the second line to j being a quark of a different type, while the last two lines correspond to it being a gluon. To simplify (2.8) we take advantage of the fact that (2.7) is to be used in (2.2) in which $E(x_1, x_2, z)$ is symmetric under the interchange of x_1 and x_2 . Hence, we only need the symmetrized part of (2.8), i.e. under the interchange of m and n . On account of (2.3) we have

$$P_{qg}^{mn} = P_{qq}^{nm} \quad (2.9)$$

Simplification further occurs if the moments are expressed in terms of the singlet and non-singlet components

$$\begin{aligned} G_S^n &= G_f^n + (2f-1) G_u^n \\ G_N^n &= G_f^n - G_u^n \end{aligned} \quad (2.10)$$

We thus obtain

$$\begin{aligned} \bar{S}_{i \rightarrow k \ell}^{mn} &= \frac{1}{2} [S_{i \rightarrow k \ell}^{mn} + S_{i \rightarrow \ell k}^{nm}] \\ &= \frac{1}{2f} \{ P_{qq}^{mn} [G_S^p G_S^m + A_{k\ell} G_N^p G_N^m] G_{q\ell}^n + m \leftrightarrow n \} \\ &+ \frac{1}{2f} P_{qg}^{mn} G_{q\ell}^p [G_S^m G_S^n + B_{k\ell} G_N^m G_N^n] \\ &+ G_{qg}^p P_{gg}^{mn} G_{q\ell}^m G_{q\ell}^n. \end{aligned} \quad (2.11)$$

where for $k=i$, $\ell \neq i$, as we have considered,

$$A_{i\ell} = f-1, \quad B_{i\ell} = -1. \quad (2.12)$$

Similar expressions follow for other processes. Specifically, for $k \neq i$, $\ell \neq k$ (e.g. $u \rightarrow d\bar{u}$, $s\bar{d}$, etc.)

$$A_{k\ell} = -1, \quad E_{k\ell} = -1 \quad (2.13)$$

If $k = \ell = i$ (e.g. $u \rightarrow u\bar{u}$), then

$$A_{ii} = f-1, \quad B_{ii} = 2f-1 \quad (2.14)$$

and if $k = \ell \neq i$ (e.g. $u \rightarrow d\bar{d}$), then

$$A_{kk} = -1, \quad B_{kk} = 2f-1. \quad (2.15)$$

We shall use the leading-order results in QCD for the P and G moments.

The justification will be given in Sec. III. They are collected in Appendix A.

On account of (2.11) the integrand in (2.7) is a sum of terms of the type

$$G_c^p(Q^2, k^2) G_a^m(k^2, Q_0^2) G_b^n(k^2, Q_0^2) \quad (2.16)$$

where a , b , and c can be various combinations of N , S , Qg and gQ . Each of the G moments is a linear combination of terms of the form $\alpha(k^2)$ raised to

some anomalous dimension. The generic form of the integral is therefore

$$\Lambda_{abc}(Q^2) = \frac{1}{2\pi} \int_{Q_0^2}^{K^2} \frac{dk^2}{k^2} \alpha(k^2) \left[\frac{\alpha(Q^2)}{\alpha(k^2)} \right]^{d_c^p} \left[\frac{\alpha(k^2)}{\alpha(Q_0^2)} \right]^{d_a^m + d_b^n} \quad (2.17)$$

where a, b, and c are now combinations of N, +, and -. (See Appendix A.)

In view of (A.4) we have

$$\frac{dk^2}{k^2} \alpha(k^2) = \frac{1}{b} d\xi \quad (2.18)$$

where

$$\xi(k^2) = \ln[\alpha(Q_0^2)/\alpha(k^2)] \quad (2.19)$$

Thus (2.17) can be trivially integrated, yielding

$$\Lambda_{abc}(Q^2) = [2\pi b(d_c^p - d_a^m - d_b^n)]^{-1} \exp(-d_c^p s) \left\{ \exp[(d_c^p - d_a^m - d_b^n) s_k] - 1 \right\} \quad (2.20)$$

where $s = \zeta(Q^2)$ and $s_k = \zeta(k^2)$.

We have expressed the evolution parameter $\zeta(k^2)$ in terms of $\alpha(k^2)$ because the integrand in (2.17) involves only factors of $\alpha(k^2)$. However, for the final answer we prefer to express it in terms of s written as

$$s = \ln \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right] \quad (2.21)$$

where (A.4) and (A.5) have been used. The reason is that the parameters Q_0 and Λ will be given values later which render $\alpha(Q_0^2)$ meaningless, although the overall evolution parameter s and the G function in terms of it will

retain physical meaning to be described in Sec. III. In view of (2.5) we

obtain from (2.20) for $Q^2 \leq Q_1^2$

$$\Lambda_{abc}(Q^2) = [2\pi b(d_c^p - d_a^m - d_b^n)]^{-1} \left\{ \exp[-(d_a^m + d_b^n) s] - \exp(-d_c^p s) \right\} \quad (2.22)$$

and for $Q^2 > Q_1^2$

$$\Lambda_{abc}(Q^2) = [2\pi b(d_c^p - d_a^m - d_b^n)]^{-1} \exp(-d_c^p s) \left\{ \exp(d_c^p - d_a^m - d_b^n) s_1 - 1 \right\} \quad (2.23)$$

where

$$s_1 = \ln \left[\frac{\ln(Q_1^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right] \quad (2.24)$$

Substituting (A.6) - (A.9) in (2.11), using (2.17) and collecting all

the pieces together for the integral (2.6), we obtain

$$\begin{aligned} G_{i \rightarrow k\ell}^{mn}(s) = & \left\{ (16f^2 \Delta^n)^{-1} P_{gq}^{mn} d_{gq}^n [(1+p^p)(1+p^m)(\Lambda_{++} - \Lambda_{+-}) \right. \\ & + (1+p^p)(1-p^m)(\Lambda_{+-} - \Lambda_{--}) + (1-p^p)(1+p^m)(\Lambda_{+-} - \Lambda_{--}) \\ & + (1-p^p)(1-p^m)(\Lambda_{--} - \Lambda_{--}) + 4A_{k\ell}(\Lambda_{N+N} - \Lambda_{N-N}) \\ & \left. + (m \leftrightarrow n) \right\} \\ & + (8f^2 \Delta^p)^{-1} P_{gq}^{mn} d_{gq}^p [(1+p^m)(1+p^n)(\Lambda_{++} - \Lambda_{+-}) \\ & + (1+p^m)(1-p^n)(\Lambda_{+-} - \Lambda_{--}) + (1-p^m)(1+p^n)(\Lambda_{+-} - \Lambda_{--}) \\ & + (1-p^m)(1-p^n)(\Lambda_{--} - \Lambda_{--}) + 4B_{k\ell}(\Lambda_{N+N} - \Lambda_{N-N}) \\ & + (4f^2 \Delta^p \Delta^m \Delta^n)^{-1} P_{gg}^{mn} d_{gg}^p d_{gq}^m d_{gq}^n [\Lambda_{++} - \Lambda_{+-} \\ & - \Lambda_{+-} + \Lambda_{+-} - \Lambda_{+-} + \Lambda_{+-} + \Lambda_{+-} - \Lambda_{--}], \end{aligned} \quad (2.25)$$

where the Λ 's are given in (2.22) and (2.23), and $A_{x\ell}$ and $B_{k\ell}$ in (2.12) to (2.15). These equations determine the double moment of $q\bar{q}$ distribution of any type in a quark jet of any flavor.

Identical procedure applies to the determination of these G moments for the cases where the initiating parton i is a gluon. We relegate its description to Appendix B.

III. HADRONIZATION OF QUARKS

In the previous section we have determined the $q\bar{q}$ distribution in a jet. We need now the recombination function to convert the quarks to hadrons according to (1.1). That is the hadronization problem which has been discussed in Refs. 3-6. We give here a brief summary.

The problem of hadronization of quarks is obviously related to the wave function of the hadron in terms of the quarks. A simple expression of that wave function cannot be in terms of the quarks as probed in deep inelastic scattering or lepton-pair production since the corresponding Fock space contains an infinite number of quarks, antiquarks and gluons. The valon picture² has been suggested as a finite-dimensional description of a hadron: a nucleon consisting of three valons, and a pion two. A valon is a valence quark cluster, which is a valence quark dressed by sea quarks and gluons due to virtual QCD processes. Valons are defined at low Q^2 at which spatial resolution is not fine enough to discern their internal structures. Being clusters of partons, their mutual interaction in a scattering problem is very different from quark-quark interaction in perturbative QCD. In bound-state problems, however, they are more like

constituent quarks. The structure function of the valons at high Q^2 can be calculated in QCD. This theoretical input combined with the experimental data on the hadron structure functions makes possible a determination of the valon momentum distribution in the hadron. This distribution is the absolute square of the hadron wave function in the valon representation, which obviously is much simpler than a representation in terms of quarks and gluons. For a pion this distribution turns out to be⁴

$$F^\pi(y_1, y_2) = y_1 y_2 \delta(y_1 + y_2 - 1) \quad (3.1)$$

where y_i is the momentum fraction of the i -th valon.

The recombination function is directly related to the valon distribution since the amplitude for recombination is the complex conjugate of the valon wave function in the hadron.⁴ Hence, the recombination function for a pion is

$$R^\pi(x_1, x_2, x) = \frac{x_1 x_2}{x^2} \delta\left(\frac{x_1}{x} + \frac{x_2}{x} - 1\right). \quad (3.2)$$

It is necessary to stress that valons are parton clusters defined at some low Q^2 so that the confinement problem is restricted only to the question of the wave function of the valons in a hadron, leaving the question about the internal structure of a valon to be independent of the hadron. Eqs. (3.1) and (3.2) refer only to the properties of valons in a pion but not about the structure of valons themselves. The latter, when probed at high Q^2 , is given by perturbative QCD. In extrapolating from high Q^2 to low Q^2 the leading order approximation would fail to be valid at some point. If we had a calculational scheme to account for all higher-order and higher-twist effects, we would use it to guide us to the low Q^2 region where eventually at some value, Q_v^2 say, all moments of the valon structure function become one. That is, at Q_v^2 the momentum distribution is $\delta(\xi-1)$ where ξ is the fraction of momentum of the valon. It means that

the internal structure of a valon can no longer be resolved at Q_v^2 . The momentum of a valon is therefore the momentum of the quark at Q_v^2 . Of course, at present a calculational scheme, that gives such as reliable accuracy at low Q^2 is non-existent. Thus, we define a Q_0^2 as starting point of the evolution from $\delta(\xi-1)$ such that by using leading-order calculation only we obtain at high Q^2 a valon structure that makes phenomenological sense.²

It is important to recognize that Q_0^2 is defined in conjunction with leading-order expression for the valon structure function at high Q^2 . Therefore, when we come to the jet problem at hand we should use leading-order calculation for the degradation to Q_0^2 and then use (3.2) to describe recombination because (3.2) follows from (3.1) which is defined at Q_0^2 . The hadronization problem is the reverse of the hadron structure problem. In Sec. II we have determined the $q\bar{q}$ distribution at Q_0^2 after degradation from Q^2 . At Q_0^2 we identify those q and $\bar{q}$ as valons because at that low value of Q_0^2 one cannot distinguish a quark from a valon. Further dressing of those quarks would not change the momenta of the whole clusters when identified as valons. It is the momentum distribution of those objects that is of calculational concern in (1.1).

The result for $D(x, Q^2)$ is expressible entirely in terms of s , given in (2.21). Q_0 and Λ enter the picture only in relating s to Q^2 . In Refs. 2 and 3 we used

$$Q_0 = 0.82 \text{ GeV}, \quad \Lambda = 0.74 \text{ GeV} \quad (3.3)$$

as inferred from the analysis of the neutrino experiment by ABCLOS¹¹ on the nucleon structure. Subsequently, an analysis of the more accurate data using muons¹² as well as neutrinos¹³ reveals that the more appropriate values should be¹⁴

$$Q_0 = 0.80 \text{ GeV}, \quad \Lambda = 0.65 \text{ GeV} \quad (3.4)$$

The discrepancies from (3.3) reflect the discrepancies between data from the different experiments. Our numerical calculations have been done in

terms of s but the results will be shown in terms of Q^2 using (3.4).

IV. GLUON CONVERSION

In Sec. II we have determined the distribution for $q_i \rightarrow q_k \bar{q}_l$, and similarly in Appendix B for $g \rightarrow q_k \bar{q}_l$. In the following discussion we refer to quark jets for definiteness, although the same consideration applies to gluon jets as well. In Sec. III the recombination of q_k and $\bar{q}_l$ to form a pion is discussed. It appears that the stage is set for substitution of the results so far obtained into (2.2) for the evaluation of the decay function $D(x, Q^2)$. However, if we do, we would be wrong by not accounting for the hadronization of the gluons in the quark jet. Before hadronization the quark jet consists of quarks, antiquarks and gluons, the latter taking a sizable fraction of the jet momentum. Those gluons must hadronize too; it is this problem that concerns us in this section.

If glueball existed, we would consider hadronization of gluons into glueballs, which presumably decay subsequently into pions. The pion spectrum would therefore be enhanced by such gluon conversions. However, we know of no glueballs, let alone their structure function or recombination function. Yet the gluons get converted into hadrons somehow lest the momentum balance would not be satisfied. Pion formation discussed in Sec. III is in terms of valons, or quark and antiquark at Q_0 which effectively dress themselves up to become clusters. Thus our model for gluon hadronization is to convert each gluon into a $q\bar{q}$ pair at Q_0 and add to the $q\bar{q}$ distribution obtained above. To ensure that hadronization is complete we require that the conversion process $g \rightarrow q\bar{q}$ be described by a probability function $\bar{P}_{g \rightarrow q\bar{q}}(z)$ that satisfies the momentum sum rule

$$\sum_{j=1}^3 \int_0^1 dz z \bar{P}_{g \rightarrow \bar{q}_j \bar{q}_j}(z) = \frac{1}{2} \quad (4.1)$$

where z is the momentum fraction of one of the quark products. For simplicity we take the z dependence of $\bar{P}(z)$ to be that given by lowest order perturbation result, i.e.

$$\bar{P}_{g \rightarrow \bar{q} \bar{q}}(z) = \frac{3}{2f} [z^2 + (1-z)^2] \quad (4.2)$$

The process for including the gluon conversion in the quark jet is then the following. First the calculation in Sec. II must be repeated but with (a) $k = q$, $\ell = g$, (b) $k = \bar{q}$, $\ell = g$, and (c) $k = g_1$, $\ell = g_2$. In each of these three cases the gluons are converted by a convolution integral such as

$$G_{i \rightarrow \bar{q} \bar{q}}^{(a)}(x_1, x_2, Q) = \int_{x_2}^{1-x_1} \frac{dy}{y} G_{i \rightarrow \bar{q} g}(x_1, y, Q) \bar{P}_{g \rightarrow \bar{q} \bar{q}}\left(\frac{x_2}{y}\right) \quad (4.3)$$

for case (a). For case (c) both gluons must be converted. The double moments of $G_{i \rightarrow k \ell}$ for the three cases are given in Appendix C. Adding these three contributions to the one determined in (2.25), denoted by $G^{(c)}$ for the present purpose, yields the total $q\bar{q}$ distribution

$$G_{i \rightarrow q \bar{q}} = G^{(a)} + G^{(c)} + G^{(b)} + G^{(c)} \quad (4.4)$$

It is this distribution that should be used in (2.2) for the determination of $D_{q_i}^{\pi}(x, s)$ for a quark jet.

Exactly the same procedure is to be applied to the gluon jet.

VI. THE DECAY FUNCTIONS

Let the moments of the decay function of a parton p into a pion be defined by

$$D_p^{\pi}(n, s) = \int_0^1 dx x^{n-1} D_p^{\pi}(x, s) \quad (5.1)$$

The application of (3.2) into (2.2) then yields with the help of (5.1) and (2.6b)

$$D_p^{\pi}(n, s) = \sum_{m=0}^{n-3} \binom{n-3}{m} G_{p \rightarrow k \ell}^{m+2, n-m-1}(s), \quad (5.2)$$

where $k = u$, $\ell = \bar{d}$ for π^+ and $k = d$, $\ell = \bar{u}$ for π^- ; p can be either a quark of any flavor or a gluon. Color and spin components of the initial parton have been averaged over. Flavor independence of QCD implies a number of identities among the decay functions; for example

$$D_u^{\pi^+} = D_{\bar{d}}^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} \quad (5.3)$$

$$D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_u^{\pi^-} = D_{\bar{d}}^{\pi^-} = D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} \quad (5.4)$$

$$D_u^{\pi^0} = D_{\bar{d}}^{\pi^0} = D_d^{\pi^0} = D_{\bar{u}}^{\pi^0} = D_{\bar{s}}^{\pi^0} = D_s^{\pi^0} \quad (5.5)$$

$$D_g^{\pi^+} = D_g^{\pi^-} \quad (5.6)$$

We present here the results of our calculation of five decay functions: $D_u^{\pi^+}$, $D_u^{\pi^-}$, $D_u^{\pi^0}$, $D_g^{\pi^+}$, and $D_g^{\pi^0}$. Their moments are shown in Figs. 2-6 for four values of s . In the calculation we have taken $f = 3$.

To invert the moments a commonly adopted approach is Yndurain's method,¹⁵ which is what is used in Ref. 3. The method, however, does not give accurate result at small values of x . We have found that good fits of all the moments can be achieved with errors well within 1% by using two beta functions for quark decay functions and three beta functions for gluons. That is, we assume the following form for the x distribution of a decay function from a parton p to a hadron h :

$$x D_p^h(x, s) = \sum_i a_i(s) (1-x)^{b_i(s)}, \quad (5.7)$$

where i runs over two terms for $p = u$ and three terms for $p = g$. The results of our fits of the moments shown in Figs. 2 to 6 can be summarized by the parameters $a_i(s)$ and $b_i(s)$ for every decay function. They are shown as dots in Figs. 7-9. These parameters can be well approximated by straight lines in s . We parametrize them by

$$a_i(s) = a_i(0) + a'_i s \quad (5.8a)$$

$$b_i(s) = b_i(0) + b'_i s \quad (5.8b)$$

and give in Table 1 the parameters $a_i(0)$, a'_i , etc. for each of the five decay functions.

We calculate the x distribution using (5.7) and (5.8). The results are exhibited in Figs. 10-14 for four values of Q^2 which is related to

s by (2.21) and (3.4). For a clear comparison between the shapes of the quark and gluon decay functions we show $D_u^{\pi^+}$, $D_g^{\pi^+}$, and $D_d^{\pi^+} = D_u^{\pi^-}$ in Fig. 15 for $Q^2 = 100 \text{ GeV}^2$, and $D_u^{\pi^0}$ with $D_g^{\pi^0}$ in Fig. 16. As expected, $D_g^{\pi^+}$ is more rapidly damped than $D_u^{\pi^+}$ at high x but less so than $D_u^{\pi^-}$ since the leading quark in a u -quark jet can hadronize into π^+ . In either quark or gluon jet there is a sharp peak at very small x . Effects due to finite hadron masses and high quark masses which have not been taken into account would round out the peak. Nevertheless, it is a consequence of abundant soft gluon emissions. Such peaks should be looked for experimentally in very high Q^2 jets.

For contacts with experiments e^+e^- annihilation processes offer the most direct test of jet structures as predicted here. Associated with two-jet events are quark jets only, so if we define

$$\begin{aligned} N_g^{\pi^+} &= (2\sigma_T)^{-1} d\sigma^{\pi^+}/dx \\ &= \frac{5}{12} (D_u^{\pi^+} + D_d^{\pi^+}) + \frac{1}{6} D_s^{\pi^+} \end{aligned} \quad (5.9)$$

we have

$$N_g^{\pi^+\pi^-} \equiv N_g^{\pi^+} + N_g^{\pi^-} = \frac{5}{6} D_u^{\pi^+} + \frac{7}{6} D_u^{\pi^-} \quad (5.10)$$

for the fragmentation of an "average" quark into charged pions in e^+e^- annihilation. To compare with data it is necessary to add the contribution from resonance formation and decay. The probability for resonance formation is unknown in the recombination model, so we adopt the phenomenological description²⁶ that puts the formation of ρ meson at a rate of 0.45 that of pion. With this input we can calculate the pion spectra arising from

resonances using the method described in Ref. 16; we find that such contributions are small especially in the large x region. In Fig. 17 we present the results of our calculation for $N_{e^+e^-}^{\pi^+\pi^-}$ at $Q = 5$ GeV exhibiting separately the four contributions in (4.4) and their sum (dashed line) before the resonance contribution is added. The total inclusive distribution of charge pions including resonance decay shown in solid line agrees well with data in both normalization and shape for the bulk of the x range where data exist.¹⁷ A slight disagreement exists between the calculated result in Fig. 17 and that shown in Ref. 3. That is because we have used Yndurain's method¹⁵ for moment inversion in Ref. 3 which is less accurate than our present method in the low x region. Experiments at PETRA for higher values of Q^2 unfortunately do not yet have particle identification. It should be stressed that the theoretical calculations essentially has no adjustable parameters. All parameters that appear are fixed by other considerations. The agreement between theory and experiment where data exist therefore lends support to both the QCD description of quark proliferation and the recombination description of quark hadronization.

The results that we have given in this section for the five decay functions are amenable to experimental tests in either e^+e^- annihilation processes or deep inelastic scattering where jet production is analyzed. Experimentally, it is difficult to make unambiguous identification of gluon jets, even in e^+e^- annihilation processes where jet events are cleanest. To determine the Q^2 value associated with a gluon jet is even more difficult. An exception to this situation is, of course, the decay of Υ into 3 gluons, but $Q^2 \approx 10 \text{ GeV}^2$ for each gluon jet is unfortunately too low to serve as a definitive test of our predictions. Nevertheless, an experimental determination of the pion inclusive distribution of those jets would be of great

interest.

VII. CONCLUSION

We have determined the quark and gluon decay functions without using any quark decay function at lower Q^2 as phenomenological input, nor making any guesses at the gluon decay function. This has been possible because we have used a model for hadronization which makes its phenomenological contact through deep inelastic scattering. The model has been found to be reliable in low- p_T hadronic reactions^{4,6} as well as in quark jets.³ Indeed, it is because we do not need phenomenological inputs to serve as boundary conditions for the evolution equation⁷ that a calculation of the gluon decay function has been possible.

In the absence of data to check our results, except for the case of quark jet at $Q \sim 5$ GeV, we can only mention that they are qualitatively in accordance to general expectations concerning large x behavior and Q^2 dependences. Quantitatively the quark jets behave as $(1-x)^{1-2}$ at large x while the gluon jets behave as $(1-x)^{2-3.5}$, as can be seen in Fig. 7. At small x the decay functions rise sharply although $x D(x)$ is finite at $x = 0$ in all cases. It implies that soft gluon emission and subsequent conversion to quark pairs are very effective. Such small- x peaks should be looked for in processes where they are not smeared out by finite mass and finite-energy effects.

We have provided simple parametrizations of the decay functions. The Q^2 -dependent parameters are listed in Table 1. These compact results should be useful in both theoretical and experimental applications.

APPENDIX A

We collect here some of the well-known results from leading-order calculations in QCD.^{7,18} Assuming SU(3) color, we have:

$$P_{Q+Q}(x) = \frac{4}{3} \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{Q+g}(x) = \frac{4}{3x} [1 + (1-x)^2]$$

$$P_{g+Q}(x) = \frac{1}{2} [x^2 + (1-x)^2]$$

$$P_{g+g}(x) = 6 \left[\frac{1-x}{x} + \left(\frac{x}{1-x} \right)_+ + x(1-x) \right] - \delta(x-1) \left(\frac{1}{2} + \frac{f}{3} \right)$$

where $[F(x)]_+ = F(x) - \delta(x-1) \int_0^1 dx F(x)$

The corresponding moments are therefore

$$P_{\bar{q}q}^{mn} = P_{q\bar{q}}^{mn} = \frac{1}{3} \left[\frac{(m-1)!(n-2)!}{(m+n-2)!} + \frac{(m+1)!(n-2)!}{(m+n)!} \right] \quad (A.1)$$

$$P_{gg}^{mn} = \left[2(m+n+1)^{-1} \right] \left[(m+1)!(n-1)! + (m-1)!(n+1)! \right] \quad (A.2)$$

$$P_{gg}^{mn} = 6 \left[\frac{(m-2)!n! + m!(n-2)!}{(m+n-1)!} + \frac{m!n!}{(m+n+1)!} \right] \quad (A.3)$$

The running coupling constant is

$$d(k^2) = d(\mu^2) [1 + b d(\mu^2) \ln(k^2/\mu^2)]^{-1} \\ = [b \ln(k^2/\Lambda^2)]^{-1} \quad (A.4)$$

where $B = (33-2f)/12\pi$. In terms of the anomalous dimensions and the evolution parameter ξ , where

$$\xi \equiv \ln [\alpha(\mu_0^2)/\alpha(k^2)], \quad (A.5)$$

the solutions of the renormalization group equations yield

$$G_N^n(k^2, Q_0^2) = \exp(-d_N^n \xi) \quad (A.6)$$

$$G_S^n(k^2, Q_0^2) = \frac{1}{2}(1+p^n) e^{-d_+^n \xi} + \frac{1}{2}(1-p^n) e^{-d_-^n \xi} \quad (A.7)$$

$$G_{Tq}^n(k^2, Q_0^2) = \frac{d_{Tq}^n}{2f\Delta^n} [e^{-d_-^n \xi} - e^{-d_+^n \xi}] \quad (A.8)$$

$$G_{Qq}^n(k^2, Q_0^2) = \frac{d_{Qq}^n}{\Delta^n} [e^{-d_-^n \xi} - e^{-d_+^n \xi}] \quad (A.9)$$

where

$$f^n = (d_N^n - d_{gg}^n) / \Delta^n$$

$$\Delta^n = d_+^n - d_-^n = [(d_N^n - d_{gg}^n)^2 + 4d_{gQ}^n d_{Qg}^n]^{1/2}$$

$$d_N^n = \frac{1}{3\pi b} \left[1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^n \frac{1}{j} \right]$$

$$d_{Qg}^n = \frac{-2}{3\pi b} \cdot \frac{(2+n+n^2)}{n(n^2-1)}$$

$$d_{gQ}^n = \frac{-f}{2\pi b} \cdot \frac{(2+n+n^2)}{n(n+1)(n+2)}$$

$$d_{gg}^n = \frac{-3}{\pi b} \left[\frac{-1}{12} + \frac{1}{n(n-1)} + \frac{1}{(n+1)(n+2)} - \frac{f}{18} - \sum_{j=2}^n \frac{1}{j} \right]$$

$$d_{\pm}^n = \frac{1}{2} [d_N^n + d_{gg}^n \pm \Delta^n]$$

Because g and Q indices for the channel matrix are ordered in accordance to the labelling of (2.6C), the matrix indices in the above equations as well as in (A.1), (A.2), (A.8) and (A.9) are the transpose of the usual notation in the literature.

APPENDIX B

Using the same procedure as in Section II, we consider here the $q\bar{q}$ distributions in a gluon jet. The probability of finding a quark q_k at x_1 and an antiquark $q_{\bar{k}}$ at x_2 in a gluon jet is

$$G_{g \rightarrow k\bar{k}}(x_1, x_2, Q^2, Q_0^2) = \sum_{j, j', j''} \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha(k^2)}{2\pi} \int_{x_1+x_2}^1 \frac{dy}{y} G_{g \rightarrow j}(y, Q^2, k^2) \cdot \int_{x_1}^{y-x_2} dz P_{j \rightarrow j'}(\frac{z}{y})^{\frac{1}{2}} G_{j' \rightarrow k}(\frac{x_1}{z}, k^2, Q_0^2) \frac{1}{y-z} G_{j'' \rightarrow \bar{k}}(\frac{x_2}{y-z}, k^2, Q_0^2). \quad (B.1)$$

After taking double moments and defining $G_{ij}^n(Q^2, k^2)$, $G_{g \rightarrow k\bar{k}}^{mn}(Q^2, Q_0^2)$ and $P_{jj'}^{mn}$, as in the case of quark jet, we obtain

$$G_{g \rightarrow k\bar{k}}^{mn}(Q^2, Q_0^2) = \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha(k^2)}{2\pi} S_{g \rightarrow k\bar{k}}^{mn}(Q^2, k^2) \quad (B.2)$$

where

$$S_{g \rightarrow k\bar{k}}^{mn} \equiv \sum_{j, j', j''} G_{ji}^p P_{jj'}^{mn} G_{j'k}^m G_{j''\bar{k}}^n \quad (B.3)$$

and $p = m + n - 1$. For the case $k \neq \bar{k}$, we have

$$S_{g \rightarrow k\bar{k}}^{mn} = G_{gQ}^p \{ P_{QQ}^{mn} [G_f^m G_{gQ}^n + (2f-1)G_u^m G_{gQ}^n] + P_{Qg}^{mn} [G_{gQ}^m G_f^n + (2f-1)G_{gQ}^m G_u^n] \} + G_{gg}^p \{ P_{gQ}^{mn} [G_f^m G_u^n + G_u^m G_f^n + (2f-2)G_u^m G_u^n] + P_{gg}^{mn} G_{gQ}^m G_{gQ}^n \}. \quad (B.4)$$

We note that (B.4) is already symmetric under the interchange of m and n .

Using (2.9) and (2.10), we get

$$S_{g \rightarrow k\ell}^{mn} = (P_{\alpha\alpha}^{mn} G_{g\alpha}^p G_S^m G_{g\alpha}^n + m \leftrightarrow n) \\ + \frac{1}{2f} P_{g\alpha}^{mn} G_{gg}^p (G_S^m G_S^n + A_{k\ell} G_N^m G_N^n) \\ + P_{gg}^{mn} G_{gg}^p G_{g\alpha}^m G_{g\alpha}^n$$

(B.5)

where

$$A_{k\ell} = -1, \quad (k \neq \ell).$$

(B.6)

For the case $k = \ell$, we follow the same procedure as above and obtain the same expression as (B.5) but

$$A_{k\ell} = 2f - 1, \quad (k = \ell).$$

(B.7)

Substituting (A.6)-(A.9) into (B.5) and performing the integration in (B.2) we have

$$G_{g \rightarrow k\ell}^{mn} = \left\{ (8f^2 \Delta^p \Delta^n)^{-1} P_{\alpha\alpha}^{mn} d_{g\alpha}^p d_{g\alpha}^n [(1+p^m)(\Lambda_{++} - \Lambda_{+-} \right. \\ + \Lambda_{+-} - \Lambda_{--}) + (1-p^m)(\Lambda_{++} - \Lambda_{+-} + \Lambda_{--} - \Lambda_{--})] \\ + (m \leftrightarrow n) \left. \right\} \\ + (16f)^{-1} P_{g\alpha}^{mn} \left\{ (1-p^p) [(1+p^m)(1+p^n) \Lambda_{++} \right. \\ + (1+p^m)(1-p^n) \Lambda_{+-} + (1-p^m)(1+p^n) \Lambda_{+-} + \left. \right.$$

$$(1-p^m)(1-p^n) \Lambda_{--} + 4 A_{k\ell} \Lambda_{NN} + (1+p^p) [(1+p^m)(1+p^n) \\ \cdot \Lambda_{+-} + (1+p^m)(1-p^n) \Lambda_{+-} + (1-p^m)(1+p^n) \Lambda_{+-} \\ + (1-p^m)(1-p^n) \Lambda_{--} + 4 A_{k\ell} \Lambda_{NN} - 1] \left. \right\} \\ + (8f^2 \Delta^m \Delta^n)^{-1} P_{gg}^{mn} d_{g\alpha}^m d_{g\alpha}^n \left\{ (1-p^p) (\Lambda_{++} - \Lambda_{+-} \right. \\ - \Lambda_{+-} + \Lambda_{--}) + (1+p^p) (\Lambda_{++} - \Lambda_{+-} - \Lambda_{+-} + \Lambda_{--}) \left. \right\}$$

(B.8)

where the various symbols are as in Section II except for $A_{k\ell}$ which are given in (B.6) and (B.7).

APPENDIX C

We list here our results for different cases of gluon conversion after repeating the procedure in Section II.

(I) For the quark jet:

(a) $k = q$, $\ell = g$ and (b) $k = \bar{q}$, $\ell = g$

$$\begin{aligned}
 G_{i \rightarrow k g}^{mn} = & \frac{1}{2} \left\{ (16f)^{-1} P_{qq}^{mn} P_{gq}^n [(1+p^p)(1+p^m)(1-p^n)\Lambda_{++} \right. \\
 & + (1-p^p)(1+p^m)(1-p^n)\Lambda_{+-} + (1+p^p)(1+p^m)(1+p^n)\Lambda_{-+} \\
 & + (1-p^p)(1+p^m)(1+p^n)\Lambda_{--} + (1+p^p)(1-p^m)(1-p^n)\Lambda_{++} \\
 & + (1-p^p)(1-p^m)(1-p^n)\Lambda_{+-} + (1+p^p)(1-p^m)(1+p^n)\Lambda_{-+} \\
 & + (1-p^p)(1-p^m)(1+p^n)\Lambda_{--} + 4A_{ik}(1-p^n)\Lambda_{N+N} \\
 & \left. + 4A_{ik}(1+p^n)\Lambda_{N-N} \right] \\
 & + (4f\Delta^p\Delta^m)^{-1} P_{qg}^{mn} P_{gq}^n d_{qg}^p d_{gq}^m [(1+p^p) \cdot \\
 & (\Lambda_{++} + \Lambda_{--} - \Lambda_{+-} - \Lambda_{-+})] + (1-p^p)(\Lambda_{++} + \Lambda_{--} \\
 & - \Lambda_{+-} - \Lambda_{-+})] \\
 & + (2\Delta^p\Delta^m)^{-1} P_{gq}^{mn} P_{gq}^n d_{qg}^p d_{gq}^m [(1+p^m) \cdot \\
 & (\Lambda_{++} - \Lambda_{+-} - \Lambda_{-+} + \Lambda_{--})] + (1-p^m)(\Lambda_{++} - \Lambda_{+-} -
 \end{aligned}$$

$$\begin{aligned}
 & \Lambda_{--} + \Lambda_{--})] \\
 & + (4f\Delta^p\Delta^m)^{-1} P_{gg}^{mn} P_{gq}^n d_{qg}^p d_{gq}^m [(1-p^n)(\Lambda_{++} - \Lambda_{+-} \\
 & - \Lambda_{-+} + \Lambda_{--}) + (1+p^n)(\Lambda_{+-} - \Lambda_{-+} - \Lambda_{--} + \Lambda_{--})] \\
 & + (m \leftrightarrow n) \}
 \end{aligned}$$

(C.1)

where

$$A_{ik} = 2f - 1, \quad (i = k)$$

and

$$A_{ik} = -1, \quad (i \neq k)$$

(C.2)

(c) $k = z_1$ and $\ell = g_2$

$$\begin{aligned}
 G_{i \rightarrow z_1 g_2}^{mn} = & \left\{ (4\Delta^m)^{-1} P_{qq}^{mn} P_{gq}^m P_{gq}^n d_{qg}^m [(1+p^p)(1-p^n) \cdot \right. \\
 & (\Lambda_{++} - \Lambda_{+-}) + (1+p^p)(1+p^n)(\Lambda_{+-} - \Lambda_{--}) + (1-p^p)(1-p^n) \cdot \\
 & (\Lambda_{+-} - \Lambda_{-+}) + (1-p^p)(1+p^n)(\Lambda_{-+} - \Lambda_{--})] + (m \leftrightarrow n) \} \\
 & + 2f(\Delta^p\Delta^m\Delta^n)^{-1} P_{gq}^{mn} P_{gq}^m P_{gq}^n d_{qg}^p d_{qg}^m d_{qg}^n \\
 & \cdot (\Lambda_{++} + \Lambda_{--} - \Lambda_{+-} - \Lambda_{-+} - \Lambda_{-+} - \Lambda_{--} + \Lambda_{+-} + \Lambda_{-+}) +
 \end{aligned}$$

$$(4\Delta^P)^{-1} P_{gg}^{mn} P_{gq}^m P_{gq}^n d_{qg}^P [(1-P^m)(1-P^n)(\Lambda_{++}-\Lambda_{+-}) \\ + (1-P^m)(1-P^n)(\Lambda_{+-}-\Lambda_{--}) + (1+P^m)(1-P^n)(\Lambda_{+-}-\Lambda_{--}) \\ + (1+P^m)(1+P^n)(\Lambda_{--}-\Lambda_{--})]$$

(C.3)

(II) For the gluon jet:

(a) $k = q$, $l = g$ and (b) $k = \bar{q}$, $l = g$

$$G_{g \rightarrow kg}^{mn} = \frac{1}{2} \left\{ (8f\Delta^P)^{-1} P_{qq}^{mn} P_{gq}^m P_{gq}^n d_{qg}^P [(1+P^m)(1-P^n) \cdot (\Lambda_{++}-\Lambda_{+-}) + (1+P^m)(1+P^n)(\Lambda_{+-}-\Lambda_{--}) + (1-P^m)(1-P^n) \cdot (\Lambda_{++}-\Lambda_{+-}) + (1-P^m)(1+P^n)(\Lambda_{+-}-\Lambda_{--})] \right. \\ \left. + (4f^2\Delta^P\Delta^m\Delta^n)^{-1} P_{ag}^{mn} P_{gq}^m P_{gq}^n d_{gq}^P d_{gq}^m d_{gq}^n \cdot (\Lambda_{++}-\Lambda_{+-}-\Lambda_{--}-\Lambda_{+-}-\Lambda_{+-}+\Lambda_{+-}-\Lambda_{+-}+\Lambda_{+-}) \right. \\ \left. + (4\Delta^n)^{-1} P_{gq}^{mn} P_{gq}^m P_{gq}^n d_{qg}^n [(1+P^P)(1+P^m) \cdot (\Lambda_{+-}-\Lambda_{--}) + (1+P^P)(1-P^m)(\Lambda_{+-}-\Lambda_{--}) + (1-P^P)(1+P^m) \cdot (\Lambda_{+-}-\Lambda_{+-}) + (1-P^P)(1-P^m)(\Lambda_{+-}-\Lambda_{+-})] + \right.$$

$$(8f\Delta^m)^{-1} P_{gg}^{mn} P_{gq}^m P_{gq}^n d_{gq}^m [(1+P^P)(1+P^n)(\Lambda_{+-}-\Lambda_{--}) \\ + (1+P^P)(1-P^n)(\Lambda_{+-}-\Lambda_{+-}) + (1-P^P)(1+P^n)(\Lambda_{+-}-\Lambda_{+-}) \\ + (1-P^P)(1-P^n)(\Lambda_{++}-\Lambda_{++})] \\ + (m \leftrightarrow n) \}$$

(C.4)

(c) $k = g_1$, $l = g_2$

$$G_{g \rightarrow g_1 g_2}^{mn} = \left\{ (2\Delta^P\Delta^m)^{-1} P_{qq}^{mn} P_{gq}^m P_{gq}^n d_{qg}^P d_{qg}^m [(1-P^n) \cdot (\Lambda_{++}-\Lambda_{+-}-\Lambda_{++}+\Lambda_{+-}) + (1+P^n)(\Lambda_{+-}-\Lambda_{+-}-\Lambda_{--}+\Lambda_{--})] + (m \leftrightarrow n) \right\} \\ + f(\Delta^m\Delta^n)^{-1} P_{gq}^{mn} P_{gq}^m P_{gq}^n d_{qg}^m d_{qg}^n [(1-P^P) \cdot (\Lambda_{++}+\Lambda_{--}-\Lambda_{+-}-\Lambda_{+-}) + (1+P^P)(\Lambda_{+-}+\Lambda_{--}-\Lambda_{+-}-\Lambda_{+-})] \\ + \frac{1}{8} P_{gg}^{mn} P_{gq}^m P_{gq}^n [(1-P^P)(1-P^m)(1-P^n)\Lambda_{++} \\ + (1+P^P)(1-P^m)(1-P^n)\Lambda_{+-} + (1-P^P)(1-P^m)(1+P^n)\Lambda_{+-} \\ + (1+P^P)(1+P^m)(1-P^n)\Lambda_{+-} + (1-P^P)(1+P^m)(1+P^n)\Lambda_{--} \\ + (1+P^P)(1-P^m)(1+P^n)\Lambda_{--} + (1-P^P)(1+P^m)(1-P^n)\Lambda_{++} \\ + (1+P^P)(1+P^m)(1+P^n)\Lambda_{--}].$$

(C.5)

Table 1. Parameters for Decay Functions

	$a_1(0)$	a_1	$b_1(0)$	b_1	$a_2(0)$	a_2	$b_2(0)$	b_2	$a_3(0)$	a_3	$b_3(0)$	b_3
$D_{u\pi}^{\pi-}$	0.390	-0.051	-0.39	0.65	10.1	-1.68	34.1	-2.93	--	--	--	--
$D_{u\pi}^{\pi-}$	0.203	-0.033	2.12	0.84	4.91	0.355	15.6	4.65	--	--	--	--
$D_{u\pi}^{\pi^0}$	0.198	-0.024	-0.39	0.66	13.5	-2.85	38.6	-5.16	--	--	--	--
$D_{g\pi}^{\pi^+}$	0.269	-0.066	1.14	0.67	2.90	-0.678	4.64	1.32	193.	-38.6	32.3	10.8
$D_{g\pi}^{\pi^0}$	0.161	-0.031	-1.91	1.45	3.31	-0.786	3.26	1.75	222.	-46.0	41.4	8.74

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FIGURE CAPTIONS

- Fig. 1 Schematic diagram for the inclusive distribution of a quark and an antiquark in a quark or gluon jet. Gluons and quarks leaking from the blobs are not indicated.
- Fig. 2 Moments of the decay function for a u quark into π^+ mesons versus the moment order n at different values of the evolution parameter s .
- Fig. 3 Moments of the decay function for a u quark into π^- mesons versus the moment order n at different values of the evolution parameter s .
- Fig. 4 Moments of the decay function for a u quark into π^0 mesons versus the moment order n at different values of the evolution parameter s .
- Fig. 5 Moments of the decay function for a gluon into π^+ mesons versus the moment order n at different values of the evolution parameter s .
- Fig. 6 Moments of the decay function for a gluon into π^0 mesons versus the moment order n at different values of the evolution parameter s .
- Fig. 7 Parameters $a_1(s)$ and $b_1(s)$ in Eq. (5.7) versus the evolution parameter s for decay functions $D_u^{\pi^+}$, $D_u^{\pi^-}$, $D_u^{\pi^0}$, $D_g^{\pi^+}$, and $D_g^{\pi^0}$. The dots represent values calculated directly from moment inversion. The solid lines are the results from straight line fits of the dots.

Fig. 8

Parameters $a_2(s)$ and $b_2(s)$ in Eq. (5.7) versus the evolution parameter s for decay functions $D_u^{\pi^+}$, $D_u^{\pi^-}$, $D_u^{\pi^0}$, $D_g^{\pi^+}$, and $D_g^{\pi^0}$. The dots represent values calculated directly from moment inversion. The solid lines are the results from straight line fits of the dots.

Fig. 9

Parameters $a_3(s)$ and $b_3(s)$ in Eq. (5.7) versus the evolution parameter s for decay function $D_g^{\pi^+}$ and $D_g^{\pi^0}$. The dots represent values calculated directly from moment inversion. The solid lines are the results from straight line fits of the dots.

Fig. 10

Invariant decay function for $u \rightarrow \pi^+$ at different values of Q^2 .

Fig. 11

Invariant decay function for $u \rightarrow \pi^-$ at different values of Q^2 .

Fig. 12

Invariant decay function for $u \rightarrow \pi^0$ at different values of Q^2 .

Fig. 13

Invariant decay function for $g \rightarrow \pi^+$ at different values of Q^2 .

Fig. 14

Invariant decay function for $g \rightarrow \pi^0$ at different values of Q^2 .

Fig. 15

Comparison of invariant decay functions for u-quark, d-quark, and gluon decaying into π^+ .

Fig. 16

Comparison of invariant decay functions for u-quark and gluon decaying into π^0 .

Fig. 17

Charged pion inclusive distribution in e^+e^- annihilation with (solid line) and without (dashed line) resonance contribution. The four dot-dashed lines give the different components of gluon conversion discussed in Eq. (4.4). Data are from Ref. 17.

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