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Evaluation of Environmental Control Technologies for Magnetic Fields

August 1978

U.S. Department of Energy
Assistant Secretary for Environment
Division of Environmental Control Technology

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Note: A bibliography containing over 1500 references of biological effects of magnetic fields will be published as a separate document.

EVALUATION OF ENVIRONMENTAL CONTROL TECHNOLOGIES FOR MAGNETIC FIELDS

by

W. Hassenzahl, M. W. Mahaffy and W. Weihofen

ABSTRACT

The peripheral magnetic fields of several energy-related technologies are calculated, and shielding options are studied for three field intensities as possible exposure levels: 200 G, 10 G, and 0.3 G.

Seven fusion reactor designs are studied. For a 200-G field level, shielding is not required. For the 10- and 0.3-G levels, land is the most economical shielding method, with shield coils an acceptable alternative at 0.3 G.

Nonnuclear technologies studied are superconducting magnetic energy storage, magnetohydrodynamic (MHD) electric generators, magnetically levitated vehicles, superconducting ac generators, and underground transmission lines.

Superconducting ac generators and underground transmission lines require no shielding.

The superconducting magnetic energy storage coil requires no shielding for 200 G. Both a shield coil and land are needed to meet 10 G or 0.3 G.

The MHD generator needs no shielding to 200 G and 10 G. Land is the most economical means of meeting the 0.3 G level.

Most of the magnetically levitated vehicles require no shielding to 200 G. The field on-board can be reduced to 200-25 G, depending upon the vehicle design, with shield coils. The use of iron, or another permeable material, is necessary to reduce the field to 10 G or 0.3 G. However, iron introduces too much added weight to allow efficient operation.

EXECUTIVE SUMMARY

Several advanced technologies, which are being developed as alternatives to existing technologies, will involve the use of high magnetic fields to increase efficiency, to reduce size, or to make possible new power generation and conversion techniques. These technologies include nuclear fusion reactors using magnetic confinement, magnetohydrodynamics for power generation, superconducting magnets for energy storage, superconducting motors and generators, magnetically levitated transportation systems,* and transmission lines. Fields up to 10 T (100 000 G) can be generated in the magnets for some of these applications.

The major purpose of this study is to calculate the external magnetic fields of these devices and to determine the best method(s) of shielding the environment near them. Specific designs, proposed by various research groups, are considered in this evaluation; complications arose from the dearth of definitive information on the environmental and biological effects of magnetic fields and lack of firm standards for occupational, general, and environmental exposure.

For the purpose of this study, three field intensities are chosen as possible acceptable exposure levels.

- 3×10^{-5} T (0.3 G), the average value of the earth's magnetic field
- 10^{-3} T (10 G), a field slightly below the 12-G level that is reported to affect certain types of cardiac pacemakers
- 2×10^{-2} T (200 G), a field tentatively deemed as acceptable for occupational exposure at some research institutions in the US and USSR.

Three studies are done on each device, that is, one study for each of the three fields mentioned above. For some technologies, shielding to a field of 10^{-4} T (1 G) is also evaluated.

*Magnetic levitation is included because it is expected to improve the energy efficiency of transportation systems.

The three methods of magnetic field control considered are iron, additional land, and shield coils. For most of the technologies, all three shielding methods would be technically effective for the 0.3 and 10-G levels, but some cost less than others. The most economical shielding methods for each device are given in the following table. Cost estimates are based on

MOST ECONOMICAL SHIELDING METHODS FOR SEVERAL ENERGY-RELATED
TECHNOLOGIES AND THREE POTENTIALLY ACCEPTABLE FIELD LEVELS

Energy System	Field Level (G)	Required Land (acres)	Required Iron (10^6 kg)	Active Shield Cost (% of primary magnets' cost)	Most Economical
Tokamak: TEPR	10	NR	NR	NR	NR
	0.3	60	7	15%(30-m radius) ^b	Land/Sc
Tokamak: UWMak I	200	NR	NR	NR	NR
	10	25	80	27% (40-m radius) ^b	Land
	0.3	250			Land/Sc
Tokamak: UWMak II	200	NR	NR	NR	NR
	10	18	40	29%(40-m radius) ^b	Land
	0.3	125			Land/Sc
Tokamak: UWMak III	10	NR	NR	NR	NR
	0.3	50	10	5.9%(40-m radius) ^b	Land/Sc
Yin-Yang Reactor	10	NR	NR	NR	NR
	0.3	122	15.6	4.8%(45-m radius) ^b	Land/Sc
Field-Reversed Mirror Reactor	10	NR	NR	NR	NR
	0.3	64	9.8		Land/Sc
Tandem Mirror Reactor	10	NR	NR	NR	NR
	0.3	46	41.3	3.3%(25-m radius) ^b	Land/Sc
Energy Storage: SMES Coil	200	NR	NR	NR	NR
	10	2000	11 667	4.3%(850-m radius) ^b	Land & Sc
	0.3	14 050			Land & Sc

MOST ECONOMICAL SHIELDING METHODS FOR SEVERAL ENERGY-RELATED
TECHNOLOGIES AND THREE POTENTIALLY ACCEPTABLE FIELD LEVELS

Energy System	Field Level (G)	Required Land (acres)	Required Iron (10^6 kg)	Active Shield Cost (% of primary magnets' cost)	Most Economical
Electric Power Generation:	10	NR	NR	NR	NR
MHD	0.3	112	1.8	Percentage unknown 1.1×10^6 1977 dollars	Land
Magnetically Levitated Vehicles: Magneplane	10 200 0.3	Imp	.1	included in present design specifications	Sc Iron/Imp ^a Iron/Imp ^a
TMLV	200 10 0.3	Imp	.01	42%	NR Iron/Imp ^a
Maglev	200 10 0.3	Imp	.01	43%	NR Iron/Imp ^a
Superconducting ac Generators	0.3	NR	NR	NR	NR
Transmission Lines	0.3	NR	NR	NR	NR

NR - No shielding required, i.e., the field drops to the specified level at the site boundary determined by conditions other than magnetic fields, to which the general public has access.

Imp - Not possible to shield to this level.

Sc - Shield coil.

a - The use of iron in levitated trains for shielding will probably add too much weight to the vehicle to allow efficient operation.

b - See text for discussion of costs of shield coils with different radii.

present costs (1977 dollars) of land, fabricated iron, superconductor, and dewars ("thermos bottles" for superconducting coils). Although we use accepted values, the cost of land, in particular, may vary by a factor of 10 or more from one site to another. Where land and another shielding option are close in cost, the least expensive is listed first and the more expensive second.

The optimum shielding method may be different if the fields near a device are adjusted to reduce occupational exposure. The table will no longer be correct, and active shield coils will be more economical for most of the technologies. It will be possible to provide local shielding, using iron or relatively small superconducting shield coils, to protect workers during periodic or emergency maintenance. These shielding devices could be either fixed or mobile if suitable support structure is included in the design. Calculating the shielding required to reduce occupational exposure levels is beyond the scope of this study because it will depend on design details that are not currently available.

Fusion Power Reactors

Several full-scale fusion power reactors, both experimental and proposed, are considered. Land is generally the most economical means of shielding these devices to limit environmental exposure to magnetic fields. This is mainly because radiation exposure will probably be a more serious concern than magnetic field exposure; thus, remote sites will probably be selected.

Tokamaks

Several Tokamaks, including some experimental reactors, are studied. The pulsed "poloidal fields" always extend farther beyond the reactor than other fields and are the most difficult to shield. Land is the most economical means of shielding these devices to limit environmental magnetic field exposure. Because the reactors may require maintenance, shield coils, for general field reduction, and/or iron, for field control, may be used to reduce occupational exposure.

Field-Reversed and Tandem Mirror Reactors

These reactors will have large external magnetic dipole fields. Additional land will be the most inexpensive shielding method, but shield coils

may be used if 0.3 G is the acceptable field level. Also, shield coils will be very useful if access is required for maintenance.

Hybrid Mirror Reactors with Yin-Yang Magnet Systems

This type of reactor will have large external magnetic dipole and quadrupole fields. Land will provide the least expensive shielding but shield coils may be competitive. A shield coil should have both a dipole and quadrupole field component, and a "wavy" coil will be the best design. The reactor's effectiveness depends in part on the mirror ratio, which is increased by the introduction of a shield coil.

Superconducting Magnetic Energy Storage

This technology, which consists of a superconducting coil buried underground and power, refrigeration, and control components above ground, will not require a shield or additional land if 200 G is an acceptable environmental exposure level. If either 10 or 0.3 G is the maximum acceptable environmental exposure level for magnetic fields, then both a shield coil and land will be necessary.

Transmission Lines

The surface magnetic fields will be almost zero for present and proposed ac and dc transmission lines buried 2 m underground. The dc lines will have a zero external field under all normal operating conditions. Some ac lines will produce ac magnetic fields at the surface of about 1 to 2 G. These fields will be less than 0.3 G outside the right of way.

Magnetohydrodynamic Electric Power Generation

These generators will produce relatively weak magnetic fields beyond the normal site boundaries. However, access for maintenance may require that iron or shield coils be placed near the device to reduce fields to occupationally acceptable levels.

Superconducting Generators and Motors

These devices will be shielded by a stationary iron shield, which makes the device more efficient, enhances the internal magnetic field, and reduces the external magnetic field. Fields near the motors and generators will be a

few gauss at most and should be neither environmentally nor occupationally hazardous.

Magnetically Levitated Vehicles

Magnetically levitated vehicles may be used for high-speed transportation in high-population areas. Active shielding is the only reasonable method of shielding passengers because of the small space available and the weight limits imposed by the technology. Active shielding to 10 G or less is not possible for the designs studied.

I. INTRODUCTION

A. Purpose

Several advanced technologies are now being developed as alternatives to existing energy system components. Among these technologies are controlled thermonuclear fusion reactors (CTR) using magnetic confinement, magnetohydrodynamics (MHD) for power generation, superconducting magnetic energy storage (SMES) systems, superconducting rotating machinery (to replace conventional motors and generators), and transmission lines. Most of these technologies involve the use of large magnets, which, in superconducting devices, can generate both dc and ac fields up to 10 T (100 000 G).

The effects, if any, of high magnetic fields upon biological systems are unclear. However, because magnetic fields may have adverse health and environmental effects, a previous study recommended an early determination of the fields these devices (the storage unit in particular) will generate and an evaluation of the shielding options possible with present technology.

The purpose of this study, that is developed and expanded from prior recommendations,¹ is to evaluate several factors for each of the above technologies.

- (1) Magnetic fields produced by the unshielded device.
- (2) Types of shielding required to reduce environmental exposure levels to 0.3, 10, and 200 G.
- (3) Costs of shielding each device to these levels.
- (4) Effects, if any, of the shielding on the field in the device.
- (5) Occupational exposure levels associated with routine operation of the device.

B. Scope

1. We calculate the magnetic fields for seven nuclear fusion reactors that use magnetic confinement, an MHD power generator, a superconducting magnetic energy storage unit for diurnal load leveling, a superconducting generator, three levitated train designs, and both ac and dc transmission lines. The values of the fields produced are discussed in Secs. II and III.

2. We determine the types of shielding required to reduce the fields of each of these devices to 0.3, 10, and 200 G, by using two computer programs for calculating magnetic fields. Because most future energy-related magnet technologies will require coils larger than those produced for previous devices, these programs required modifications. Results of the calculations for various shielding configurations, some of which were based on a trial-and-error optimization, are presented in Secs. II and III. The two modified computer programs used for these calculations are described in Appendixes A and B.

Active shielding (bucking coils) and passive shielding (iron and distance) are considered for each technology and for each of the three chosen exposure levels.

3. We determined the cost of shielding each device to 0.3, 10, and 200 G for several different shielding options. Again, these constraints are arbitrary because they are based on limited and uncertain data.

- 3×10^{-5} T (0.3 G) is the average value of the earth's magnetic field.
- 10^{-3} T (10 G) is a field slightly below the 12-G level that is reported to affect certain types of cardiac pacemakers.
- The Stanford Linear Accelerator Center (SLAC) has set 2×10^{-2} T (200 G) as an interim standard for extended exposure of the whole body for dc magnetic fields. It is the most conservative of all similar standards.

The shield, or shield combination, requiring the least initial capital investment is considered optimum for each field level. Operation and maintenance costs of the various shields were considered briefly, and appear to be small compared to the overall operating costs of the primary devices. The operating costs are difficult to estimate. However, because the shield magnets generally will be in a low field region, a considerable distance from the

the device they are shielding, they will be more easily accessible and probably will require less maintenance per unit initial cost than the primary coil(s).

Iron, generally a very expensive shield, is not an economically attractive shielding option unless it provides field enhancement or accomplishes some other specific purpose, such as shielding instrumentation that must be close to the device.

4. The introduction of either a shield coil or iron will affect the field in the active region of a device. Although the effects vary, if the central field value must be held constant, the primary current can usually be increased to compensate for the field reduction caused by a shield coil. Where feasible, the higher cost of the primary coil caused by the increased current has been included in our cost estimates.

5. It is beyond the scope of this work to evaluate the occupational exposure levels associated with the various technologies or to try to adjust shielding to accommodate workers in special areas near the various devices. It is premature to consider occupational exposure in detail because 10 to 20 years of research may be required before technical and economic feasibility are determined and preliminary engineering designs are completed. Occupational exposures can only be estimated with confidence after a detailed design is completed and operating procedures established. However, we calculate the fields around the various devices and discuss the technically allowable occupational access to most of the devices during operation.

Because the external fields are of secondary importance to the engineers designing a system, they are not included in most publications on the various technologies. In general, the occupational area will be restricted because of radiation, elevated temperature, or other industrial hazards near the devices rather than because of the magnetic fields.

We include graphs of the external fields near both shielded and unshielded devices to allow designers to estimate the occupational magnetic field exposures.

C. Procedure

The necessity of shielding is determined by initially calculating the magnetic fields to which the general public and employees of the facility are

likely to be exposed during operation of the unshielded devices. No shielding is deemed necessary if this level is below 0.3 G for those employed at the facility.

Several types of shielding are examined for the technologies that had field levels greater than 0.3 G outside of the device site. To start, we determine the shield coil required to reduce fields outside the immediate site to acceptable levels, which is generally the shield that, at large distances, cancels the dipole field; a notable exception is the "quadrupole shield" of the Yin-Yang mirror reactor. Most of the shields are designed to cancel the dipole field by setting the dipole moments of the shield and the device equal. This gives

$$I_s = \frac{M_D}{\pi a^2},$$

where I_s is the current through the shield coil in amperes; a is the radius of the shield loop in meters; and M_D , the effective dipole moment of the device, is found from the approximation that at large radial distances

$$B(r) = \frac{CM_D}{r^3},$$

where $B(r)$ is the magnetic field at radial distance r from the center of the device. In these cases, the dipole field of the device, which decreases as $1/r^3$, is cancelled by the shield coil, leaving only the quadrupole field, which decreases as $1/r^5$ and is usually insignificant, and higher order fields.

If the magnetic field must be limited only outside a specific region, such as the site boundary, then the most straightforward, but not necessarily the cheapest, method is to buy enough land so that the field falls below the acceptable level within the boundary. The amount of land necessary to meet the different field constraints is computed. We assume that sufficient land is available at the desired location.

Finally, iron shielding is considered for each of the technologies. Iron shielding, although expensive, can reduce the field outside of the shield to zero.

For the technologies that require shielding, cost estimates are made of the different types of shielding. We compare these costs with the costs of the magnets of the unshielded device to determine a relative cost for the shields. The parameters used in estimating the shield and primary coil costs are open to debate and should not be used as absolute values. However, the relative costs will be useful in determining the cost of shielding when the actual costs of the primary coils are determined because they are independent of inflation. The costs of different kinds of shielding are compared to determine their relative cost effectiveness.

D. Magnetic Forces

The magnetic force per unit length acting on a current-bearing conductor is proportional to the cross-product of the magnetic field strength $\vec{B}$ and the current $\vec{I}$

$$\vec{F} = \vec{I} \times \vec{B}.$$

In the SI system of units, a force of one newton (N) is produced by a field of one tesla (T) acting on a 1-m length of conductor carrying one ampere (A) of current. The conversion factor to obtain the force in pounds (lbf), is $1 \text{ N} = 0.225 \text{ lbf}$.

The force is at right angles to both the magnetic field and the current and only the component of magnetic field perpendicular to the current produces a force. The "right-hand rule" governs the force direction. For example, if the magnetic field is vertically upward on an element of electric current flowing westward, the resulting force on the element is northward. Therefore, a circular coil carrying a current experiences a radially outward force because of its own magnetic field, as shown in Fig. I.1.

The following example illustrates the scale of the forces on the shield coil of a large magnet system. The total outward radial force on a 20-m-radius loop carrying 1 MA in an average magnetic field of 0.1 T, is $1.26 \times 10^7 \text{ N}$ ($2.8 \times 10^6 \text{ lbf}$). Most of the shield coils described in this report experience significantly greater forces.

The average magnetic field used in the force calculation is a vector average of the fields perpendicular to the current flow. The self-field of a

Fig. I.1

Diagram illustrating the magnetic field and the magnetic forces on a coil shielding a primary solenoidal magnet.

conductor reverses direction from one side of the conductor to the other. For a perfectly straight conductor, the vector average of the magnetic field, and hence the net self-induced force, is zero. For a superconductor that has a uniform current density the magnetic field varies linearly through the conductor. Therefore, the average magnetic field is $(B_i + B_o)/2$, where B_i is the magnetic field strength on the inside of the curved segment of conductor, and B_o is the magnetic field strength on the outside of the conductor and has the opposite sign so that $B_i + B_o$ is the difference in the magnitudes of B_i and B_o perpendicular to I , the total current in the coil, the product of the number of turns and the current in each turn. The self-induced outward force, per unit length of conductor, is then

$$F_r = I(B_i + B_o)/2,$$

and the total outward force F is

$$F = 2\pi R F_r = \pi R I (B_i + B_o),$$

where R is the radius of the solenoid. This is an upper bound, because edge effects reduce the vector-average magnetic field and the resulting force.

For the thin circular shield coils being considered, the total radial force can be approximated to about 10% by

$$F = 6\pi^2 C I^2,$$

where C is the constant defined in Appendix C, 0.01 dyn/A^2 . Approximately two-thirds of this force is self-induced, and one-third is due to the magnetic fields being shielded.

The hoop stress σ in the conductor or support bands is obtained from F_r by the relationship

$$\sigma = F_r R/A,$$

where A is the cross-sectional area of the support structure. To calculate the amount of structural material needed to support the magnetic forces, we

must know the working stress σ_w , the maximum elastically supportable tensile force per unit area of the material. The area is thus $F_r R / \sigma_w$, and the minimum volume of the support is $V = 2 \pi R \cdot A = 2 \pi R^2 F_r / \sigma_w = RF / \sigma_w$. We assume throughout this study that the structural support will be stainless steel with an allowable working stress of 207 000 000 Pascals (207 MPa or 30 000 psi), and a density of 7800 kg/m³.

For comparison, we also calculate the amount of structural support required in each primary magnet system. A shield coil has almost no effect on the structural support required for the primary magnet(s) if the current in the primary is adjusted to maintain the magnetic field strength in the interior. The reason is that the fractional decrease in the average magnetic field strength on the conductor of the primary is offset by the same fractional increase in current necessary to maintain the interior field.

E. Cost Analyses

Of the three methods considered for controlling magnetic fields, site area and ferromagnetic material, which are passive controls, have simple, one-parameter cost bases, the price of land and iron, respectively. The active shield coil has three major cost components: the superconductor, the dewar, and the structural support for the magnetic forces.

The most straightforward means of containing a large magnetic field is to have a site area large enough to enclose the region where the magnetic field strength is greater than some desired standard. The cost of this form of field containment is thus the product of the difference between the land necessary to enclose the magnetic field strength and the land required for the facility, in acres, and the price of the land in the vicinity of the site, which will vary greatly from one area to another. We use \$5000 per acre, which should give a rough cost comparison.

For ferromagnetic, passive shielding, iron is by far the most efficient material, owing to its high magnetic saturation flux intensity, $B_{sat} \approx 2.15$ T, and low cost. We use a cost of \$3/kg fabricated. If scrap iron could be used, the cost would be considerably less. The required radial thickness t of the iron shield is determined to first order by the condition that all of the magnetic field lines of the unshielded device that are outside the shield radius R_s be contained by the iron. By assuming the magnetic field of a

solenoid, which has axial symmetry, has a dipole behavior, i.e., it decreases as $1/r^3$, and it has the value B_1 at a distance R_s , we then have

$$\begin{aligned} 2 \pi R_s t B_{\text{sat}} &= \int_{R_s}^{\infty} B_1 \left(\frac{R_s}{r} \right)^3 2 \pi r dr, \\ &= 2 \pi R_s^2 B_1 \end{aligned}$$

which gives

$$t = \frac{B_1}{B_{\text{sat}}} R_s.$$

The total volume V of iron needed for the shield is then $2 \pi R_s t \ell$, where ℓ is the axial length of the cylindrical shield; *a priori*, one would expect ℓ to be approximately equal to the length of the central magnet system producing the field. Thus, if the field is produced by a magnetic dipole M , the volume of iron shield required at a distance R_s decreases as $1/R_s$:

$$V = 2 \pi R_s \cdot t \cdot \ell = \frac{2 \pi R_s^2 B_1 \ell}{B_{\text{sat}}} = 2 \pi C \frac{\ell}{R_s} \frac{M}{B_{\text{sat}}},$$

where the constant C is a unit conversion factor from the equation for the field, B_1 , in this case, $0.1 \text{ T} \cdot \text{m/MA}$, as described in Appendix C. It may be possible to reduce this volume by segmenting the shield in one or more of its dimensions; however, some additional iron probably will be required near the ends of the primary coils to ensure shielding effectiveness. Detailed three-dimensional calculations will be required to determine the exact shielding requirements, and will not be warranted until the devices have gone from the conceptual to the engineering stage.

The mass of iron required for the shield is obtained by multiplying the shield volume V by the density of iron, $7.88 \times 10^3 \text{ kg/m}^3$. The resulting cost, based on a fabricated material price of $\$3/\text{kg}$, could be reduced significantly by using unfabricated iron, such as iron shot, or scrap in the walls of the facility or as part of the support structure. In general, fabricated iron

does not appear to be an economical shield, but unfabricated iron could be cost effective.

An active shield coil requires the use of superconductors because of the high currents, up to several million ampere-turns (MAT), required. A superconducting shield coil has three component costs.

(1) Conductor (Q) - The quantity of conductor Q needed for a circular shield of radius R_s is $2\pi R_s I$, where I, the total current in the loop, is related to the magnetic dipole moment $M = \pi R_s^2 I$. Thus, if a circular shield coil is designed to cancel the dipole moment M_p produced by a primary magnet system, the quantity of conductor will vary inversely with R_s ; $Q = 2M_p/R_s$. We estimate fabricated superconductor is \$2.60/kAm, therefore the cost of the superconductor in the shield coil is $C_{sc} = \$5.2 \times 10^3 \cdot M_p/R_s$ per MAm.

(2) Dewar (A) - The dewar surface area A for the shield coil is given by $A = 4\pi R_s (\Delta R + \ell + 4\Delta)$, where ΔR and ℓ are the radial thickness and length, respectively, of the shield coil, and Δ is the liquid helium channel and insulation thickness, which we take to be 0.076 m. The dewar cost is based on a unit cost of \$300/m² of surface area. If the cross section of the coil is a square, then $\Delta R = \ell \sqrt{I/J}$. The current density J is assumed to be 50 MA/m² (5000 A/cm²) and the total current I is related to the magnetic moment as above: $I = M_p/\pi R_s^2$. The cost C_D of the dewar is then $C_D \approx \$1100 (\sqrt{M_p/\pi} + R_s)$ where M_p is in MAm² and R_s is in meters.

(3) Support - The cost of the stainless steel support structure for the shield coil is based on \$12/kg for fabricated stainless steel. From Sec. I.D. the mass of a steel hoop of radius R required to support a total radial force F is $\rho R F / \sigma_w$; where ρ is the density, 7.8 Mg/m³, and σ_w is the working stress, which we take as 2×10^8 Pa (30 000 psi). For most of the devices in this study, F is obtained by computing the total magnetic field inside and outside the shield coil loop, B_i and B_o respectively, and using the approximation $F \approx \pi R (B_i + B_o) I$. The total force is approximately $F = 6\pi^2 C I^2 = 6 \times 10^5 M_p^2/R_s^4$. If the radius of the steel support is approximately the same as the radius of the shield coil, the structural support cost, C_s , is about $\$275 M_p^2/R_s^3$.

The cost of an active shield coil should, in principle, include costs incurred by having to increase the current in the primary magnet system or by

altering other parameters to counteract the internal effects of the shield coil. We include this factor whenever feasible; otherwise, we confine ourselves to statements of where and why designs must be changed to allow for the presence of a shield coil. In general, the shield coil will produce an opposing field of magnitude a^3/R_s^3 times the unshielded field in the interior of the device, where a is the radius, or a characteristic dimension, of the primary magnet system. The quantity of conductor in the primary coil will have to be increased by this amount. The amount of structural support will not require alteration, because the effect of the shield coil offsets the increased primary current, as explained in the previous section. For the devices being considered, a^3/R_s^3 is in the range of 10^{-1} to 10^{-3} .

For total shield coil currents less than about 6 MA, the dominant cost is that of the superconductor in the shield coil.

II. ENERGY-RELATED TECHNOLOGIES REQUIRING HIGH MAGNETIC FIELDS:

THERMONUCLEAR FUSION

In the past, magnetic fields with which mankind has come in contact have been restricted to relatively low strengths, less than 1 G, or relatively small regions, less than 1 m³, of field strengths of a few hundred gauss associated with present technologies. With the advent of superconductivity, however, the practicable range of electric currents and resulting magnetic fields has increased enormously. It is now possible to produce stable magnetic fields of 2 T (20,000 G) or more throughout a volume of many cubic meters. Magnetic fields of this size have a side effect with which we have had little or no experience, namely fields significantly larger than the earth's magnetic field, that extend for hundreds of meters and cover many acres. Biological and environmental consequences of these high and/or extensive fields are still undetermined, despite some research and extensive literature on the subject.

The most widely proposed application of the emerging "supermagnet" technology is in the area of energy production, primarily through the control of thermonuclear fusion (refer to Sec. A. below). The magnets will contain a plasma of charged reacting particles in a large volume at temperatures of several hundred million degrees, necessary for successful electric power production. Various devices have been proposed to confine the plasma and

cause it to react. Magnetic fields are comparable for all these devices because of the nature of the process. Below we discuss the most promising design types: the Tokamaks (toroidal plasma confinement), the hybrid mirror reactor (roughly spherical plasma confinement), the field-reversed mirror reactor (plasma confinement in cylindrical cells), and the tandem mirror reactor (extended cylindrical plasma confinement).

A. General Description of Controlled Thermonuclear Fusion

The objective of a thermonuclear fusion reactor is to provide an environment in which energy can be produced in a controlled manner from the "burning" of nuclear fuels. In this process, two (or more) light nuclei are fused to form a heavier, more stable nucleus, with energy released. The most promising reaction seems to be the burning of a fuel composed of equal amounts of Deuterium (D) and Tritium (T), by the following process

This energy, 17.6 MeV/atom or 3.4×10^{11} J/g of fuel, appears as the kinetic energies of the two product particles in inverse proportion to their masses: 20% is in the helium nucleus and 80% in the neutron. In addition, the neutron may be used to breed additional tritium for fuel by the following reaction in a lithium-rich "blanket" surrounding the D-T plasma:

The primary problem is containment of the reacting D-T plasma. To overcome the mutual repulsion of the positive deuterons and tritons, the particles must have kinetic energies corresponding to temperatures in excess of about 10^9 K. The resulting pressure, which is proportional to the absolute temperatures and the number density of particles, is quite large.

*By comparison one barrel of oil releases only 5.5×10^9 J when burned, and one ton of coal only 1.4 to 3.0×10^{10} J when burned.

This pressure must be balanced, or at least partly balanced, depending on the type of reactor design, by the electromagnetic pressure, which is proportional to the square of the magnetic field strength. The number density (nuclear concentration) obtainable for both deuterons and tritons is thus proportional to the square of the magnetic field strength in the active plasma. The rate of energy production (power output) of the reactor is proportional to the fourth power of the obtainable magnetic field strength because the "burn" rate is proportional to the product of the deuteron and triton concentrations. For cost effective power production, very high magnetic fields, 4 T (40 000 G) or higher, must be available over many cubic meters of plasma volume. Such fields can be produced efficiently only through the recently developed technology of superconducting magnet coils.

B. General Description of Tokamak Fusion Reactors

Tokamak fusion reactors are designed to confine, heat, and burn a D-T plasma within the bore of a toroidal configuration of magnets. At present, the larger Tokamak reactors, commercial and experimental, are still primarily in the conceptual design stage and their parameters are not fixed. The magnetic fields of several proposed designs are considered in this study. The analyses and the shielding techniques presented here should be applicable to present as well as to future designs. There is some redundancy in this study of Tokamaks but we felt it best to have a broad representative set of reactors analyzed and drop the obsolete designs when this document is updated.

The Tokamak comprises several sets of magnets. The main set of coils consists of D-shaped magnets arranged in a circle to form a torus. Figure II.1a shows a cross section of such a torus, with axes marked for reference; the x-axis of the figure is perpendicular to the page. Magnets having this configuration produce the primary plasma-containment field.

Two other sets of magnet coils are present in the Tokamaks. One set of coils sets up a relatively weak, constant "poloidal field" that is parallel to the z-axis, which is vertical in all proposed Tokamaks. The field produced by these coils determines the position of the plasma within the bore. The other poloidal set of coils creates a strong pulsed field encircling the plasma. These coils are the primary of a transformer and the plasma is a single turn

secondary. When pulsed, they induce a voltage in the plasma, driving a plasma current that heats the plasma to temperatures approaching those at which fusion will occur. Both sets of poloidal coils are axisymmetric and are centered on the z-axis. Figure II.1b is a cross-sectional view of the UWMAK III Tokamak reactor,² showing the spatial relationship of the coils in the reactor.

The values of the fields, and the configuration of the magnets, vary considerably for the different Tokamak designs. Poloidal field coils can be placed inside or outside the central conductor of the toroidal field coils. These magnets are superconducting and operate at about 4 K. The magnetic fields and possible shields are considered below for several Tokamak designs. The magnetic fields are calculated using the computer program P8MAFCO, described in Appendix B. The results are summarized in Table II.1.

C. Tokamak Experimental Power Reactor (TEPR)

The Tokamak Experimental Power Reactor (TEPR) was proposed by ANL³ to study the feasibility of the Tokamak fusion reactor. The toroid of the reactor consists of 16 D-shaped magnets with a peak field of 8 to 10 T, a major radius of 6.25 m and a minor horizontal diameter d of 7.78 m.⁴ The design for the poloidal field coils calls for 14 pairs of coils arranged around the outside of the toroid, as given in Table II.2, which is taken from Ref. 5.

1. Magnetic Fields. An ideal torus produces no external fields. The toroid of the TEPR, comprising 16 discrete coils, is not ideal. The toroidal field outside the windings, at distances greater than about 10 m from the axis, drops off very rapidly. Figure II.2 shows the total magnetic field, B_T , in gauss, generated by the toroidal field coils of the TEPR, as a function of distance from the axis in the midplane of the torus. It shows that although the toroidal field outside the torus is nonzero, it is well below the earth's magnetic field 10 m from the device. The poloidal fields of the device, however, will fall off at large distances as a dipole, i.e., as $1/r^3$, where r is distance from the reactor center. Environmental shielding need be considered only for the poloidal field.

TABLE II.1

COMPARISON OF ACTIVE AND PASSIVE SHIELDING COSTS FOR TOKAMAKS

Reactor	25-m Shield Coil Cost (\$10 ⁶)	30-m Shield Coil Cost (\$10 ⁶)	35-m Shield Coil Cost (\$10 ⁶)	40-m Shield Coil Cost (\$10 ⁶)	Cost of Iron Shield ^a (\$10 ⁶)	Cost of Land ^b (\$10 ⁶)
TEPR	1.8	1.4	-	-	21	.3
UWMAK I	-	23.6	19.9	15.0	240	1.25
UWMAK II	-	8.3	6.2	4.9	160	.6
UWMAK III	-	2.2	1.6	1.2	31	.25

^aComputed from a fabricated cost of \$3/kg

^bComputed from a cost of \$5000/acre

The poloidal field of the TEPR was calculated using the program P8MAFCO (see Appendix B) and the parameters for the coil pairs given in Table II.2. Figure II.3 shows the total magnetic field contours of the TEPR without shielding. The 200-G contour is about 30 m from the center of the device, and probably will not be accessible to the public, although it may encompass part of the occupational area. Occupational and public access to, or exclusion from, the area around this experimental reactor has not been suggested by the designers. The 10-G contour is less than 85 m from the reactor center, and the 0.3-G contour is 265 m from the center.

The poloidal field of a Tokamak reactor is a pulsed field. Environmental effects of pulsed magnetic fields is less understood than constant fields. It is not known what standards will be set on time-varying magnetic fields.

2. Shielding. Site boundaries for the TEPR are unlikely to be closer than 80 m from the reactor center. Therefore, for the TEPR, environmental shielding is necessary only if the acceptable exposure level is below 10 G. For the 0.3-G case, we have calculated the results of several shielding options. Because the poloidal field coils are pulsed, the shield coils must also be pulsed.

Fig. II.1a

Cross section of a Tokamak reactor showing the arrangement of D-shaped magnets. The z-axis is vertical. The major radius of the D-shaped magnets is R , and the minor diameter is d .

Fig. II.2

The total toroidal magnetic field, B_T , in gauss, of the TEPR. The midplane field is plotted as a function of distance from the reactor center.

Fig. II.1b

Cross-sectional view of the UWMAK III fusion reactor, showing the spatial relationship of the toroidal and poloidal coils, from Ref. 2.

Fig. II.3

Contours for the maximum total magnetic field during a burn cycle of the unshielded TEPR.

TABLE II.2

DESCRIPTION OF TEPR POLOIDAL COILS AND SHIELD COIL CHOICES

Coil Pair	Mean Radius (m)	Axial Length (m)	Radial Thickness (m)	Mean Axial Position (m)	Current Density (A/cm ²)	Current (MA _t)
1	12.075	0.334	0.15	± 3.417	2946	2.95
2	11.625	0.267	0.15	± 4.284	2946	2.36
3	8.925	0.50	0.15	± 7.25	2946	4.42
4	7.175	0.50	0.15	± 8.05	2946	4.42
5	5.675	0.50	0.15	± 8.15	2946	4.42
6	2.325	0.28	0.15	± 7.30	-2946	-2.48
7	2.175	1.092	0.15	± 7.026	-2946	-9.642
8	2.025	0.731	0.15	± 6.716	-2946	-6.46
9 ^a	1.695	13.0	0.15	0.0	2640	51.48
10	1.775	0.40	0.15	± 6.70	2640	3.17
11	1.825	0.40	0.15	± 7.10	2640	3.17
12	1.875	0.30	0.15	± 7.45	2640	2.38
13	3.875	0.80	0.15	± 8.20	2640	6.34
14	11.625	0.07	0.15	± 4.065	2640	0.55
SCH1 ^a	25.0	0.226	0.25	0.0	-5000	-2.82
SHC2 ^a	30.0	0.157	0.25	0.0	-5000	-1.96

^aSingle coil

The simplest choice of active shielding for the TEPR and the other Tokamak designs is a large solenoidal shield coil coaxial with the poloidal field coils and in the midplane of the reactor. Because spacing of peripheral equipment is unknown, we chose two coils of different radii (25-m and 30-m radii). The design currents in these coils were chosen to cancel the dipole field of the TEPR at large distances. Table II.2 gives the parameters of these shield coils; the currents given correspond to the maximum TEPR poloidal field.

Figure II.4 gives the contours of the total magnetic field at its maximum value during a burn cycle for the TEPR shielded with the 25-m-radius coil. The shielded field falls to 10 G by 60 m and to 0.3 G by 110 m.

Figure II.5 gives the contours of the radius total magnetic field at its maximum value during a burn cycle with the 30-m-radius shield coil. This shield is slightly less effective than the other because of its larger size. However, the field falls to 10 G by 65 m and to 0.3 G by 120 m.

Because the TEPR poloidal field is pulsed during a burn cycle,⁵ the shield coil must also be pulsed. The cycle of the shield coil must be the same as that for the TEPR poloidal coils.

Even with the above shields it is possible that additional land, i.e. more than that required if magnetic fields were not a consideration, might be required if the acceptable level of public exposure is less than 1 G. If 0.3 G is the acceptable level then, with a shield, 10 to 12 acres will be required for the installation.

Choices of passive shielding are (1) buying enough land so that the magnetic field is below an acceptable level outside the site fence and (2) shielding with iron or some other magnetically permeable metal.

Six acres is required to enclose the 10-G contour. Sixty acres is required to enclose the field to 0.3 G.

A 38-cm thick iron shield (see Sec. I.E) is required at a distance of 25 m to shield the TEPR. A cylinder of iron of this thickness and the height of the TEPR (~15 m) requires about 900 m^3 or $7 \times 10^6 \text{ kg}$ of iron.

3. Forces. Forces on the TEPR shield coil will result primarily in a hoop stress in the coil and support structure. The force calculation described in Sec. I.D gives a good estimate of the force on the shield coils during operation.

Fig. II.4

Contours for the maximum total magnetic field during a burn cycle of the TEPR with the 25-m-radius shield coil.

Fig. II.5

Contours for the maximum total magnetic field during a burn cycle of the TEPR with the 30-m-radius shield coil.

The forces on the poloidal coils of the device are both radial and axial, the values depending on the location of the coils and the phase of the burn cycle. Values for the radial force on each coil were computed from the pressure values given by Wang et al.⁵ The radial forces on the poloidal coils of the device are given in Table II.3. The maximum radial force on the 25-m-radius shield coil is 4.73×10^7 N (10.6×10^6 lbf). The radial force on the 30-m-radius shield coil is 2.36×10^7 N (5.3×10^6 lbf) at maximum field.

TABLE II.3
RADIAL FORCE ACTING ON TEPR POLOIDAL
COILS AND SHIELD COILS

Coil Pair	Approximate Total Radial Force	
	(10^6 N)	(10^6 lbf)
1	2.67	0.60
2	50.0	11.24
3	32.2	7.24
4	77.71	17.47
5	130.82	29.41
6	-84.10	-18.90
7	-15.8	-3.55
8	232.6	52.29
^a 9	773.05	173.79
10	-74.60	-16.77
11	-43.64	-9.81
12	-34.8	-7.82
13	213.02	47.89
14	14.9	3.34

^aSHC1 47.3 10.63

^aSHC2 23.62 5.31

^asingle coil

4. Costs. It is difficult to compute active shielding costs for Tokamak reactors. Poloidal shielding of these devices has a serious effect on the poloidal field in the plasma region. A good illustration is the TEPR. The poloidal coils partially shield one another in the plasma region of the reactor. Outside of the reactor, except for coil pairs 6, 7, and 8, the poloidal fields are additive. As a result, the fields within the bore are smaller than the poloidal fields immediately outside the reactor. Because the shield coils were designed to shield the large fields outside the reactor, they create comparatively large fields within the plasma region, relative to the field of the device. The 25-m-radius shield coil on the TEPR reduces the field in the plasma region by 20%, a totally unacceptable figure. The only reasonable solution is to redesign the poloidal field coils so that the shield coil is an integral part of that design, in effect, another poloidal field coil. We compute the cost of the shield coils relative to that of the poloidal coils of the unshielded TEPR to estimate the shielding costs. Table II.4 gives the parameters used to determine the cost of active shielding. Table II.5 gives the costs of the unshielded poloidal coils and the shield coils and the cost of the shield coils as a percentage of the unshielded poloidal coils.

We also computed the cost of passive shielding for the TEPR. A fabricated iron shield the height of the TEPR is about $\$2.1 \times 10^7$, about 10 times more expensive than active shielding. Sixty acres of land would be required to enclose the area exposed to a field greater than 0.3 G. In most areas, it will be cheaper to acquire this land than to shield the device. Other forms of shielding will be cost effective only if the land costs more than the break-even cost of about \$23 000/acre. Active and passive shielding costs are given in Table II.1.

D. University of Wisconsin Tokamak Reactor (UWMAK I)

The University of Wisconsin has three designs for commercial-sized fusion power reactors. We have calculated the exterior fields that would exist near the UWMAK I, UWMAK II, and UWMAK III reactors with the computer program, P8MAFCO (see Appendix B).

TABLE II.4
PARAMETERS FOR TEPR SHIELDING COSTS

Coil Pair (unshielded values)	Total Conductor Length (Mam)	Dewar Surface Area (m ²)	Volume of Stainless Steel (m ³)
1	223.96	214.2	0.16
2	172.36	210.9	2.84
3	247.80	214.2	1.42
4	199.20	172.2	2.72
5	157.56	136.2	3.64
6	36.16	42.9	0.91
7	131.76	84.6	0.16
8	82.20	60.3	2.36
^a 9	548.28	286.6	6.54
10	35.32	38.1	6.13
11	36.12	39.2	0.37
12	28.00	35.6	0.30
13	154.28	122.2	4.07
14	40.48	153.3	0.83

^aSHC1 442.97 245.3 5.77

^aSHC2 369.45 268.3 3.46

^aSingle coil

TABLE II.5
RELATIVE COSTS OF TEPR ACTIVE SHIELDING

	<u>Coil System Cost^a (\$10⁶)</u>	<u>% Relative to Poloidal Field Coil</u>
<u>Poloidal Field Coils Unshielded</u>		
Conductor costs	5.57	
Dewar costs	0.54	
Structural support costs	<u>3.04</u>	
Total	9.15	100
<u>25-m-radius shield:</u>		
Conductor costs	1.18	21
Dewar costs	0.07	13
Structural support costs	<u>0.54</u>	<u>18</u>
Total	1.79	20
<u>30-m-radius shield:</u>		
Conductor costs	0.98	18
Dewar costs	0.08	15
Structural support costs	<u>0.33</u>	<u>11</u>
Total	1.39	15

^aCosts computed in 1977 dollars by method described in Sec. I.E.

1. Magnetic Fields. UWMAK I has 12 D-shaped toroidal magnets and 9 pairs of poloidal field coils. The major radius R (see Fig. II.1) of the toroid is about 13.5 m, with a minor horizontal diameter of about 18 m.⁶ Table II.6 gives the magnet parameters used in calculating the poloidal magnetic field. The toroidal field falls off rapidly, as in the case of the TEPR, and will not require shielding for environmental purposes. In fact the field falls to 0.3 G within the confines of the proposed 50-m-radius building.⁷

Poloidal field contours from the unshielded UWMAK I reactor are given in Fig. II.6. The building that houses the reactor has been drawn in for reference. The 200-G contour is about 10 m outside the boundary of the building, and the 10-G contour is about 115 m outside the building or about 165 m from the center of the device. The field falls to 0.3 G at about 540 m from the reactor center.

2. Shielding. Because spacing of peripheral equipment is unknown, several shield-coil radii were chosen; 30 m, 35 m, and 40 m, so that designers can select a coil that does not interfere with reactor operation and maintenance. The shield coils lie in the x-y plane with their axis coincident with the z-axis of the reactor. Parameters for these shield coils are given in Table II.6.

Figure II.7 gives poloidal field contours at the axial midplane of the UWMAK I reactor with a 30-m-radius shield coil. The 200-G contour is less than 5 m from the outside boundary of the reactor building. The 10-G contour is within 90 m of the center of the device, and the total field falls to 0.3 G by 180 m from the reactor center.

Figure II.8 gives the contours of the total magnetic field of the device and the 35-m-radius shield coil. Here the contours are spread out farther than in the 30-m-radius shield case. The 200-G contour is about 57 m, and the 10-G contour is 96 m from the reactor center. The field falls to 0.3 G within 190 m.

Figure II.9 gives the contours of the total magnetic field of the UWMAK I and the 40-m-radius shield coil. Again the contours are similar to the other two shielded cases, but spread out slightly (about 5 m farther from the reactor center than the 35-m-radius shield case).

The active shielding described above would require redesigning the UWMAK I poloidal coil system. The poloidal coils are arranged around the outside of the toroid such that they partially shield each other in the plasma region. Thus a shield coil large enough to shield the very high fields outside of the device will have an appreciable effect upon the relatively small poloidal field in the plasma. The only reasonable solution to this dilemma for active shielding is to redesign the poloidal field coils of the reactor using the shield coil as an extra poloidal coil. This solution will probably reduce the cost of the poloidal field coils and the shield coil.

Fig. II.6

Contours of the maximum total magnetic field during a burn cycle of the unshielded UWMAK I reactor.

Fig. II.7

Contours of the maximum total magnetic field during a burn cycle of the UWMAK I with the 30-m-radius shield coil.

Fig. II.8

Contours of the maximum total magnetic field during a burn cycle of the UWMAK I reactor with the 35-m-radius shield coil.

Fig. II.9

Contours of the maximum total magnetic field during a burn cycle of the UWMAK I reactor with the 40-m-radius shield coil.

TABLE II.6

DESCRIPTION OF UWMAK I POLOIDAL COILS AND ACTIVE SHIELD CHOICES

Coil Pair	Mean Radius (m)	Axial Length (m)	Radial Thickness (m)	Mean Axial Position (m)	Current Density (A/cm ²)	Current (MA)
1	8.0	1.43	1.43	±13.0	1200	49.0
2	13.0	1.29	1.29	±13.5	739	24.6
3	18.0	1.85	1.85	±12.0	716	-49.0
4	22.0	0.254	0.208	±8.0	-4730	-5.0
5	3.89	1.07	1.07	±3.0	-1530	-35.0
6	4.3	0.995	0.995	±5.97	-1540	-30.4
7	4.95	0.817	0.817	±8.9	-1770	-23.6
8	6.13	0.875	0.875	±12.7	-1440	-22.0
9	8.04	0.49	0.49	±17.3	-3460	-16.3
SHC1 ^a	30.0	0.50	0.648	0.0	5000	16.2
SHC2 ^a	35.0	0.50	0.476	0.0	5000	11.9
SHC3 ^a	40.0	0.50	0.364	0.0	5000	9.1

^aSingle coil

The two alternative forms of passive shielding are to buy and fence off enough land from public access to allow the fields to fall to acceptable levels within the site area, or to install a shield of permeable material between the reactor and the public. For the UWMAK I, about 25 acres are required to allow the field to drop to below 10 G. About 250 acres are required to allow the field to drop to below 0.3 G.

At a distance of 50 m, an 81-cm-thick iron cylinder (see Sec. I.E) is required to shield the UWMak I. An iron shield built as a cylindrical shell the height of the reactor (about 40 m) would require $10\,200\text{ m}^3$ or 8×10^7 kg of iron.

3. Forces. Shield coil forces will vary greatly over a burn cycle. Because the poloidal fields are pulsed fields, the shield coils must also be pulsed. The shield coil forces calculated in this section are the maximum forces the coils will experience during a burn cycle. The maximum forces experienced by the poloidal coils of the UWMak I were taken from Ref. 6.

Maximum coil forces are given in Table II.7. Negative values for coils 2, 4, and 8 mean that the force is compressive. A compressive force is more difficult to support because it may buckle the coil. Thus, more material is required to support this type of load. Buckling was not considered when figuring the structural support material for the three pairs of coils above; therefore, the amount of material computed is low and considered a minimum.

4. Costs. The necessity of redesigning the UWMak I poloidal field coils for active shielding introduces uncertainty into the cost estimates. As for the TEPR above, we compute only the cost of the shield coils relative to the cost of the unshielded poloidal coils to estimate the shielding costs. Table II.8 gives the parameters used to compute the active shield costs. The poloidal coil costs, and the shield coil cost as a percentage of the poloidal coil cost, are given in Table II.9. Shield coil costs range from 42% to 27% of the poloidal coil cost.

The cost of passive shielding for the UWMak I is also found. The cylindrical iron shield described in Sec. 2 costs about $\$2.4 \times 10^8$, far more expensive than the active shields. For the TEPR, additional land will usually be the cheapest shielding alternative. The 250 acres required to enclose the 0.3-G contour must cost more than about $\$60\,000/\text{acre}$ to make the active shield cheaper. Thus, simply increasing the site area seems to be the best shielding method. However, there may be other reasons to reduce the site dimensions, or there could be concern about the field within the site boundary.

Active and passive shielding costs are compared in Table II.1.

TABLE II.7
MAXIMUM RADIAL FORCE EXPERIENCED BY UWMAK I COILS
(UNSHIELDED) AND SHIELD COILS

Coil Pair	Maximum Radial Force ^a (Total)	
	(10 ⁶ N)	(10 ⁶ lbf)
1	544	122
2	-446	-100.2
3	914	205.5
4	-5.6	-1.3
5	304	68.3
6	264	59.3
7	178	40
8	-328	-73.7
9	64	14.4
SHC1 ^b	1345	302.3
SHC2 ^b	808	181.6
SHC3 ^b	502	112.8

^a Maximum force occurs at maximum field in the poloidal coils.

^b Single coil

E. UWMAK II

1. Magnetic Fields. The UWMAK II has 24 D-shaped toroidal magnets and 15 pairs of poloidal field coils.⁶ The major radius of the UWMAK II reactor is about 14 m, with a minor horizontal diameter of about 20 m.

In calculating the external magnetic field from the UWMAK II, the toroidal field is again neglected because of its rapid fall-off with distance compared to the poloidal field. Table II.10 gives the parameters of the poloidal field coils.

TABLE II.8
PARAMETERS FOR COST ESTIMATES OF UWMAK I
ACTIVE SHIELDING

Coil Pair	Total Conductor Length (MAm)	Dewar Outer Surface Area (m ²)	Volume of Stainless Steel (m ³)
1	2463.0	636.4	25.0
2	2009.4	942.6	24.8
3	5541.8	1811.8	88.4
4	691.1	424.0	0.6
5	855.8	239.0	7.3
6	821.2	248.0	6.9
7	733.9	241.2	5.2
8	847.6	316.6	7.9
9	823.7	259.6	2.6
SHC1 ^a	3053.6	547.7	162.5
SHC2 ^a	2617.0	563.3	136.7
SHC3 ^a	2287.1	587.5	97.0

^aSingle coil

Figure II.10 gives the maximum poloidal field contours of the unshielded reactor. If the building that houses the UWMAK II has the same dimensions (50-m radius) as that of the UWMAK I, then the 200-G contour would be within the confines of the building. The 10-G contour is 125 m and the 0.3-G contour about 400 m from the center of the reactor.

When comparing the unshielded field of the UWMAK II with that of the UWMAK I, we find that the designers had some success in redesigning the poloidal field coils so that the exterior of the device is partially shielded. UWMAK III (Sec. II.F) continues this trend.

TABLE II.9
RELATIVE COSTS OF THREE SHIELD COIL CANDIDATES
FOR THE UWMAK I

	Coil System Cost ^a (\$10 ⁶)	% Relative to Poloidal Field Coil Costs
<u>Unshielded Poloidal Field Coils</u>		
Conductor costs	38.3	
Dewar costs	1.5	
Structural support costs	15.8	
Total	<u>55.6</u>	100
<u>30-m-Radius Shield</u>		
Conductor costs	8.2	21
Dewar costs	0.2	13
Structural support costs	15.2	96
Total	<u>23.6</u>	<u>42</u>
<u>35-m-Radius Shield</u>		
Conductor costs	6.9	18
Dewar costs	0.2	13
Structural support costs	12.8	81
Total	<u>19.9</u>	<u>36</u>
<u>40-m-Radius Shield</u>		
Conductor costs	5.7	15
Dewar costs	0.2	13
Structural support costs	9.1	58
Total	<u>15.0</u>	<u>27</u>

^aCosts computed in 1977 dollars by method described in Sec. I.E.

2. Shielding. Shield coils of 30-, 35-, and 40-m radii were chosen, coaxial with the poloidal coils. Table II.10 gives the parameters of the different shield coils.

TABLE II.10

PARAMETERS OF POLOIDAL FIELD COILS OF UWMAK II AND SHIELD COILS

Coil Pair	Mean Radius (m)	Axial Length (m)	Radial Thickness (m)	Mean Axial Position (m)	Current Density (A/cm ²)	Current (MA)
1	8.2	0.32	0.32	±6.5	-3613.3	-7.40
2	8.8	0.34	1.36	±10.0	1053.2	9.74
3	11.5	0.22	0.22	±12.3	3016.5	2.92
4	11.5	0.31	0.31	±9.5	3538.0	6.80
5	14.6	0.22	0.22	±8.0	-3243.8	-3.14
6	19.0	0.44	0.44	±5.0	-3522.7	-13.64
7	3.61	1.00	0.32	±1.0	-1290.6	-8.26
8	3.72	1.00	0.32	±3.06	-1243.7	-7.96
9	3.94	0.80	0.36	±4.99	-1253.5	-7.22
10	4.25	0.78	0.34	±6.96	-1285.8	-6.82
11	4.66	1.32	0.20	±8.92	-1079.6	-5.70
12	5.43	0.60	2.60	±13.50	-215.4	-6.72
13	6.50	0.36	1.00	±15.05	-927.8	-6.68
14	7.25	0.34	0.65	±17.0	-1135.8	-5.02
15	8.22	0.45	0.46	±19.3	-1062.8	-4.40
SHC1 ^a	30.0	0.30	0.43	0.0	5000	7.16
SHC2 ^a	35.0	0.30	0.35	0.0	5000	5.26
SHC3 ^a	40.0	0.30	0.27	0.0	5000	4.03

^aSingle coil

Figure II.11 gives the maximum poloidal field contours of the UWMAK II with the 30-m-radius shield coil during a burn cycle. The 200-G contour is practically unchanged from its unshielded position. The other contours, however, have been brought in considerably. The 10-G contour is 75 m and the 0.3-G contour within 150 m of the reactor center.

Figures II.12 and II.13 show the maximum poloidal field contours of the UWMAK II shielded with the 35- and 40-m-radius shield coils respectively. Although slightly farther from the center with each successively larger shield coil, the contours are similar to those of the 30-m-radius shield. With the largest shield coil, the 0.3-G contour is about 165 m from the reactor center.

As for the TEPR and the UWMAK I, the active shields given above have a large effect upon the field within the plasma region. The 30-m-radius shield decreases the poloidal field in the plasma bore of the reactor by 37%. Boosting the currents of the poloidal coils to compensate for the effect is unacceptable because of the large increase required. The same argument applies to the 35-m-radius shield coil, which decreases the field by 22%. The 40-m-radius shield decreases the field by 15%, which may be a borderline case. The most practical solution, if active shielding is desired, is to redesign the poloidal coils with the shield coil as an integral part of the design.

The alternatives to active shielding are passive shielding with a magnetically permeable metal, such as iron, or buying sufficient land to ensure that the fields have dropped to acceptable levels outside the site boundaries.

A cylindrical 40-cm-thick iron shell is necessary to absorb the field at a distance of 50 m. An iron shell of this thickness and the height of the reactor would require about 4950 m^3 or $4 \times 10^7 \text{ kg}$ of iron.

If only land is used as a shield, about 18 acres is required to ensure that the field drops below 10 G, and 125 acres is required to enclose the field above a level of 0.3 G.

3. Forces. The maximum radial force on each of the poloidal field coils of the UWMAK II was taken from Ref. 6. We computed the maximum radial force acting on the shield coils using the method described in Sec. I.D. Table II.11 gives the maximum radial forces acting upon the poloidal field coils and shield coils during a burn cycle.

Fig. II.10

Contours of the maximum total magnetic field during a burn cycle of the unshielded UWMAK II reactor.

Fig. II.11

Contours of the maximum midplane magnetic field during the burn cycle of a UWMAK II reactor with the 30-m-radius shield coil.

Fig. II.12

Contours of the maximum midplane magnetic field during the burn cycle of a UWMAK II reactor with the 35-m-radius shield coil.

Fig. II.13

Contours of the maximum midplane magnetic field during the burn cycle of a UWMAK II reactor with the 40-m-radius shield coil.

TABLE II.11
MAXIMUM TOTAL RADIAL FORCES ON UWMAK II POLOIDAL
FIELD COILS AND SHIELD COILS

Coil Pair	Maximum Radial Force	
	(10^6 N)	(10^6 lbf)
1	-13.5	-3.0
2	49.0	11.0
3	3.12	0.7
4	-14.9	-3.3
5	20.2	4.5
6	85.6	19.2
7	47.8	10.7
8	45.8	10.3
9	40.4	9.1
10	33.6	7.6
11	16.7	3.8
12	34.8	7.8
13	19.0	4.3
14	13.1	2.9
15	7.1	1.6
SHC1 ^a	332	74.6
SHC2 ^a	181	40.7
SHC3 ^a	107	24.1

^aSingle coil

4. Costs. Costs of the different shield coils are computed relative to the poloidal field coil cost. Table II.12 gives the parameters used in

TABLE II-12
PARAMETERS USED IN COST ESTIMATES OF UWMAK II
ACTIVE SHIELDING

Coil Pair	Total Conductor Length (MAm)	Dewar Outer Surface Area (m ²)	Volume Stainless Steel (m ³)
1	381.26	194.7	0.5
2	538.53	443.4	2.4
3	210.99	215.3	0.2
4	491.35	267.3	0.8
5	288.02	273.3	1.4
6	1628.35	565.8	8.1
7	187.37	147.4	0.9
8	186.05	151.9	0.9
9	178.76	145.0	0.9
10	182.15	152.2	0.7
11	166.88	213.7	0.4
12	229.27	478.3	1.4
13	272.82	272.0	0.7
14	228.71	235.9	0.5
15	227.26	251.0	0.3
Total	5397.77	4007.2	20.1
SHC1 ^a	1349.63	780.2	48.9
SHC2 ^a	1156.73	839.9	31.0
SHC3 ^a	1012.85	879.4	20.9

^aSingle coil

estimating the active shielding costs of the UWMAK II. Costs of the coils and shields, and the relative cost of the shield coils vs the poloidal field coil system are given in Table II.13. The shield coil costs are between 29% and 48% of the poloidal coil cost.

Passive shielding is an attractive alternative. The 50-m-radius iron shell described above costs about $\$1.6 \times 10^8$, far more expensive than active shielding. However, the 125 acres required to ensure that the field outside of the site boundary is below 0.3 G must be more than \$40 000/acre to make active shielding cost-effective. If the land is available, it is usually much less expensive than this.

Table II.1 compares the costs of active and passive shielding.

F. UWMAK III

1. Magnetic Fields. The UWMAK III reactor has 18 D-shaped toroidal magnets and 34 pairs of poloidal field coils. The major radius of the toroid is about 8 m, with a minor horizontal diameter of about 14 m. Table II.14 gives the parameters of the coils used to calculate the total poloidal magnetic field.⁶ The toroidal field was again neglected because of its rapid decrease with distance relative to the poloidal field.

Figure II.14 gives the maximum magnetic field contours of the unshielded poloidal coils. The poloidal field of the UWMAK III is already better shielded at large distances from the reactor because of the poloidal field coil design than the other Tokamaks in this study. The 200-G contour is less than 35 m from the center of the reactor. The 10-G contour is approximately 80 m and the 0.3-G contour is 250 m from the reactor center.

2. Shielding. Shield coils of 30-, 35-, and 40-m radii were again chosen to examine the cost of active shielding and to determine its effect on the reactor. Table II.14 gives the parameters of the shield coils.

The 30-m-radius shield coil shields the reactor most effectively. Figure II.15 gives the maximum poloidal field contours of the shielded device. The 10-G contour is within 55 m and the 0.3-G contour within 105 m of the reactor center. The 30-m-radius shield coil decreases the poloidal field in the plasma region by only 5.8%. This can be compensated by raising the total number of ampere-turns in the poloidal field coils by that factor and by some minor repositioning of the conductors. Of the Tokamaks studied, this is

FIG. II.14

Contours of the maximum total magnetic field during a burn cycle of the unshielded UWMAK III reactor.

FIG. II.15

Contours of the maximum midplane magnetic field during the burn cycle of a UWMAK III reactor with a 30-m-radius shield coil.

FIG. II.16

Contours of the maximum midplane magnetic field during the burn cycle of a UWMAK III reactor with a 35-m-radius shield coil.

FIG. II.17

Contours of the maximum midplane magnetic field during the burn cycle of a UWMAK III reactor with a 40-m-radius shield coil.

TABLE II.13

RELATIVE COSTS OF ACTIVE SHIELDING OF THE UWMAK II

	Coil System ^a Cost (\$10 ⁶)	% Relative to Poloidal Field Coil Costs
<u>Unshielded Poloidal Field Coils</u>		
Conductor costs	14.0	
Dewar costs	1.2	
Structural support costs	1.9	
Total	17.1	100
<u>30-m-Radius Shield</u>		
Conductor costs	3.5	25
Dewar costs	0.2	17
Structural support costs	4.6	242
Total	8.3	48.5
<u>35-m-Radius Shield</u>		
Conductor costs	3.0	21
Dewar costs	0.3	25
Structural support costs	2.9	153
Total	6.2	36
<u>40-m-Radius Shield</u>		
Conductor costs	2.6	19
Dewar costs	0.3	25
Structural support costs	2.0	105
Total	4.9	29

^aCost computed in 1977 dollars by method described in Sec. I.E.

the only design for which this method of maintaining the field in the plasma region will not be prohibitively expensive.

Figure II.16 and II.17 give the maximum poloidal field contours with the 35- and 40-m-radius shield coils respectively. The contours are approximately the same, although slightly farther out with increasing shield coil radius.

TABLE II.14

PARAMETERS OF THE POLOIDAL FIELD COILS
OF THE UWMAK III REACTOR AND SHIELD COILS

Coil Pair	Mean Radius (m)	Axial Length (m)	Radial Thickness (m)	Mean Axial Position (m)	Maximum Current I (MAT)
1 ^a	3.9	0.104	0.100	± 0.0	-0.40
2	3.9	0.104	0.100	± 1.0	0.80
3 ^b	4.0	0.410	0.100	± 4.5	4.00
4 ^b	4.6	0.124	0.124	± 5.5	-1.20
5	3.9	0.403	0.100	± 9.0	-2.40
6 ^b	5.9	0.438	0.438	± 9.5	4.40
7 ^b	6.8	0.936	0.936	± 9.0	31.00
8 ^b	6.8	0.194	0.194	± 13.3	-2.60
9 ^b	8.3	0.440	0.440	± 11.3	8.60
10 ^b	8.6	0.446	0.446	± 9.9	-6.00
11 ^b	8.8	0.314	0.314	± 10.0	-3.06
12 ^b	8.8	0.358	0.358	± 9.3	-3.30
13 ^b	8.8	0.296	0.296	± 9.0	-2.20
14 ^b	8.9	0.380	0.380	± 7.7	-3.20
15 ^b	9.3	0.266	0.266	± 7.5	-3.24
16 ^b	10.8	0.266	0.266	± 7.3	-3.10
17 ^b	11.8	0.238	0.238	± 6.8	-3.20
18 ^b	12.8	0.242	0.242	± 5.3	-3.30
19 ^b	13.0	0.246	0.246	± 4.3	-3.30
20 ^b	13.5	0.246	0.246	± 2.3	-3.04
21 ^{ab}	13.5	0.262	0.262	0.0	-1.52
22	2.502	0.60	0.25	± 1.0	10.30
23	2.508	0.60	0.25	± 3.0	10.26
24	2.522	0.60	0.25	± 5.0	10.20
25	2.546	0.55	0.25	± 7.0	9.96
26	2.591	0.53	0.25	± 9.0	9.68
27	2.675	0.35	0.35	± 10.997	9.14
28	2.854	0.33	0.33	± 12.989	8.2
29	3.227	0.33	0.33	± 14.950	6.52
30	3.857	0.26	0.26	± 16.844	4.5
31	4.725	0.18	0.18	± 18.643	2.9
32	5.489	0.10	0.10	± 19.932	1.06
33 ^b	9.0	0.18	0.18	± 10.5	0.38
34 ^b	19.0	0.14	0.14	± 5.0	0.40
SHC1 ^a	30.0	0.22	0.15	0.0	1.66
SHC2 ^a	35.0	0.16	0.15	0.0	1.22
SHC3 ^a	40.0	0.12	0.15	0.0	0.93

^aSingle coils^bCoils with circular cross section

For the 40-m-radius shield, the 0.3-G contour is 120 m from the reactor center, about 15 m farther than in the 30-m-radius shield case. The 35-m-radius shield coil decreases the poloidal field in the plasma region by 3.5%, and the 40-m-radius shield coil decreases it by 2.3%. Both shields will be easier to compensate for than the 30-m-radius shield.

Passive shielding was also attractive for the UWMAK III. A 10.5-cm-thick cylindrical iron shell is required to shield the UWMAK III at a distance of 50 m. A cylindrical 50-m-radius iron shell as high as the reactor will require about 1320 m^3 or $1 \times 10^7 \text{ kg}$ of iron.

Five acres are required to ensure that the magnetic field is below 10 G outside of the site area. Fifty acres are required to ensure that the field will be below 0.3 G outside of the site area.

3. Forces. The maximum radial force on each of the UWMAK III poloidal field coils was taken from Ref. 6. For the shield coils, we computed the maximum radial force by the method described in Section I.D. Table II.15 gives the maximum radial force acting upon the poloidal coils and the shield coils during a burn cycle.

4. Costs. Because of the partial shielding of the area outside of the UWMAK III afforded by the poloidal field coil design, the amount of shielding and the cost are quite low relative to the other Tokamak designs discussed above. Table II.16 gives the parameters used in the cost estimate of active shielding. The cost of the poloidal field coils, the shield coils, and the relative cost of the shield coils vs the unshielded poloidal field coils are given in Table II.17. The cost increase of the poloidal field coils is the amount required to compensate for the shielding of the magnetic field in the plasma region. We assume that the currents are increased in the poloidal field coils by the percentage change in the uncompensated magnetic field. The added cost is computed from increased conductor length and dewar costs.

Active shielding costs from 1.2 to 2.2 million 1977 dollars, or about 5.9 to 10.8% of the cost of the unshielded poloidal field coils.

The cost of iron shielding is much greater. The cylindrical shield described above costs about $\$3.1 \times 10^7$.

Enclosing the site boundary with enough land to ensure that the field outside the site boundary is below a certain value costs less than the above alternatives at most sites. To make active shielding cost effective, the

TABLE II.15
MAXIMUM TOTAL RADIAL FORCES ON UWMAK III
POLOIDAL FIELD COILS AND SHIELD COILS

Coil Pair	Maximum Radial Force	
	(10^6 N)	(10^6 lbf)
1 ^a	-1.08	-0.24
2	-2.16	-0.49
3	-3.76	-0.85
4	-2.02	-0.45
5	-13.18	-2.96
6	72.8	16.37
7	189.4	42.58
8	4.76	1.07
9	26.0	5.85
10	74.0	16.64
11	35.6	8.0
12	48.6	10.93
13	34.0	7.64
14	58.0	13.0
15	19.1	4.3
16	20.16	4.53
17	8.24	1.85
18	12.22	2.75
19	12.9	2.9
20	15.3	3.44
21 ^a	10.0	2.25
22	43.8	9.85
23	43.4	9.76
24	42.8	9.62
25	41.4	9.31
26	39.6	8.9
27	36.2	8.14
28	30.4	6.83
29	20.8	4.68
30	10.8	2.43
31	4.6	1.03
32	0.64	0.14
33	1.84	0.41
34	-0.52	0.12
SHC1 ^a	19.57	4.40
SHC2 ^a	10.45	2.35
SHC3 ^a	6.09	1.37

^aSingle Coil

TABLE II.16

PARAMETERS FOR COST ESTIMATES OF
UWMAK III ACTIVE SHIELDING

Coil Pair	Total Conductor Length (MAm)	Dewar Outer Surface Area (m ²)	Volume of Stainless Steel (m ³)
1 ^a	9.8	24.9	0.02
2	19.6	49.9	0.04
3	100.5	81.9	0.07
4	34.7	49.6	0.04
5	58.8	79.2	0.22
6	81.6	110.1	1.94
7	1324.5	212.1	6.41
8	111.1	85.2	0.16
9	448.5	155.4	1.07
10	324.2	162.3	3.11
11	169.2	136.9	1.55
12	182.5	146.6	2.11
13	121.6	132.9	1.48
14	178.9	153.2	2.52
15	189.3	133.4	0.88
16	210.4	154.9	1.07
17	237.3	161.0	0.48
18	265.4	175.9	0.77
19	269.5	180.0	0.83
20	257.9	186.9	1.02
21 ^a	128.9	96.2	0.67
22	161.8	72.6	0.58
23	161.2	72.8	0.58
24	161.6	73.2	0.57
25	159.3	70.7	0.55
26	157.6	70.6	0.53
27	153.6	67.6	0.49
28	147.0	69.2	0.44
29	132.2	78.2	0.33
30	109.1	80.0	0.21
31	86.1	78.9	0.11
32	36.6	69.6	0.02
33	21.5	109.7	0.08
34	47.8	212.4	0.05
Total	6259.6	3793.8	30.99
SHC1 ^a	312.9	254.8	2.86
SHC2 ^a	268.3	271.5	1.78
SHC3 ^a	233.7	291.1	1.19

^aSingle coil

TABLE II.17

COMPARATIVE COSTS OF UWMAK III ACTIVE SHIELDING

	^a Coil System Cost (\$10 ⁶)	% Relative to Poloidal Field Coil Costs
<u>Unshielded Poloidal Field Coils</u>		
Conductor costs	16.3	
Dewar costs	1.1	
Structural support costs	2.9	
Total	20.3	100
<u>30-m-radius Shield Coil</u>		
Cost increase of poloidal coils	1.0	5
Conductor cost	0.8	4.9
Dewar costs	0.1	9.1
Structural support costs	0.3	10.3
Total	2.2	10.8
<u>35-m-radius Shield Coil</u>		
Cost increase of poloidal coils	0.6	3
Conductor costs	0.7	4.3
Dewar costs	0.1	9.1
Structural support costs	0.2	6.9
Total	1.6	7.9
<u>40-m-radius Shield Coil</u>		
Cost increase of poloidal coils	0.4	2
Conductor costs	0.6	3.7
Dewar costs	0.1	9.1
Structural support costs	0.1	3.4
Total	1.2	5.9

^aCost computed in 1977 dollars by method described in Section I.E.

price of land must be above \$24 000/acre, assuming 0.3 G to be an acceptable field level.

Table II.1 summarizes the costs of shielding the various Tokamaks.

G. Yin-Yang Magnet Systems; Hybrid Mirror Reactor

1. Magnetic Fields. The Yin-Yang magnet system, shown in Fig. II.18, consists of two flattened, cupped loops of conductor, resembling the seam of a baseball. The two loops, oriented at right angles to one another, enclose a roughly spherical cavity, a "magnetic bottle." The Yin-Yang configuration provides a relatively efficient system for magnetic plasma confinement, with a mirror ratio (ratio of magnetic field strength at the boundary of the "magnetic bottle" to field strength in the interior) of 10 or more possible, depending on the coil geometry.

We consider the design for the primary Yin-Yang system of the proposed mirror hybrid reactor discussed in LLL preprint UCRL-79548.⁸ This magnet system has an outside diameter of about 22.5 m and an interior diameter of about 15 m. The conductors have a cross section of 1.5 x 3.6 m, and each half of the pair carries a total current of about 5.5×10^7 ampere-turns. The LLL design includes a secondary, smaller Yin-Yang system within the primary magnets, which we ignore because it has little effect on the external field and is not typical of other Yin-Yang designs. Further, the effects of the secondary coils could be produced by altering the primary coil design parameters.

The peak magnetic field strength near the primary coils is about 10 T (100 000 G) and the field at the reactor center is 2.8 T. The mirror ratio is about 2.3, because the field strength at the plasma exit point, the "neck" of the "magnetic bottle", is about 6.3 T.

The tortuous current path of the Yin-Yang system results in a complicated magnetic field. To calculate the fields, each loop of the Yin-Yang was separated into four arcs, and each arc section was divided into an array of $3 \times 7 = 21$ current-bearing elements. The program P8MAFCO, discussed in App. B, computed the resulting magnetic fields. This program assumes a zero-cross-section conductor, but the magnetic fields at large distances should be a close approximation to those produced by a continuous current distribution. In fact, a single current-bearing element can be used for each arc section, with errors of .3% or less in the field calculations at distances 100 m or more from the primary magnet system.

Figure II.19 shows the vertical and horizontal magnetic field components as functions of horizontal distance and Fig. II.20 shows the total field as a

FIG. II.18

Yin-Yang Mirror Reactor Coils.

FIG. II.19

Horizontal and vertical field components in the vertical plane of an unshielded Yin-Yang fusion reactor.

FIG. II.20

Total magnetic field along the axis of an unshielded Yin-Yang fusion reactor.

function of vertical distance. The dominant magnetic dipole is produced by the circular, horizontal component of current in the coils, i.e., there is a net current that is in the same sense for both halves. In the horizontal plane the field is vertical and decreases as $1/R^3$ with distance R from the device. In addition, the vertical components of the current in the coils create magnetic fields in the horizontal plane which vary in direction and decrease faster than $1/R^3$ with distance from the device. The external field contours are plotted in Fig. II.21. Contours in the horizontal plane, the occupation level, are drawn for 200 G, 10 G and 1 G. The magnetic field strength drops to 200 G about 44 m from the reactor center. The 10-G contour is at 110 m, and the 1-G contour is at 240 m from the reactor center. The small arrows in Fig. II.21 indicate the direction of the magnetic field component in the plane, $B_{||}$ (see Fig. II.19). The additional field has the angular dependence of a magnetic quadrupole. Although it is not a pure quadrupole, we shall refer to that part of the magnetic field which is not magnetic dipole as "the quadrupole field."

2. Shielding. The simplest and most economical shielding method is to increase the site area. About 57 acres of land is required to include the area with magnetic fields of higher than 1 G and about 122 acres is required to include the 0.3-G contour.

A passive magnetic shield, consisting of an iron shell around the magnet system, would probably be too expensive to be practicable. There are two possible locations for such a shield:

- 1) inside or outside of the Yin-Yang support structure, whose exact location has not been determined, at a radius of 20 m
- 2) in the wall of the cylindrical 45-m-radius building containing the reactor.

A 1.57-m radial thickness is required for a 20-m-radius shield, and a 0.31-m thickness for a 45-m-radius shield. If the shield extends a vertical distance equal to the height of the Yin-Yang, 22.5 m, the total weight of iron required is 35×10^6 kg for the 20-m-radius shield and 15.6×10^6 kg for the 45-m-radius shield, resulting in material costs of $\$105 \times 10^6$ and $\$47 \times 10^6$, respectively (see to Sec. I.E for cost analyses).

If land is unavailable, then the most promising shielding option appears to be an active shield coil. There are two approaches to designing

FIG. II.21

Contours of the total magnetic field
near an unshielded Yin-Yang reactor.

the shield coil. One is to derive the effective dipole moment of the Yin-Yang from the large-distance behavior of the vertical magnetic field component in the horizontal plane, and construct a simple current loop with an equal and opposite magnetic dipole moment. We shall refer to this as a "dipole shield." The second approach is to try to shield the nondipole, or "quadrupole," field as well as the dipole field, by making the shield in the form of a "wavy" current loop, the quadrants of which dip up and down alternately from the horizontal plane. We shall refer to this type of shield as a "quadrupole shield." As can be seen from Fig. II.19, the quadrupole field is about one sixth as strong as the dipole field even at distances of 200 m, and this fraction does not decrease very rapidly with distance. Thus, a quadrupole shield can be much more effective in reducing the external magnetic field strength than a dipole shield, which cannot be more than 86% (6/7) effective, because it cannot cancel out $B_{||}$. The best locations for an active shield coil are the same as for the passive iron shields discussed above, that is, as part of the Yin-Yang structural support with a 20-m radius, or in the wall of the reactor facility with a 45-m radius.

The effective magnetic dipole moment of the Yin-Yang is $1.354 \times 10^4 \text{ MA}\cdot\text{m}^2$. A 20-m-radius loop with an equal dipole moment has a current of 10.775 MA and a 45-m-radius loop has 2.128 MA. Figures II.22 and II.23 show the fields for the Yin-Yang reactor shielded with the 20- and 45-m-radius dipole shield coils respectively. The two shield coils are almost equally effective, but the quadrupole field strength is still 1 G at distances of about 150 m.

To shield the field more completely, we need a quadrupole shield that can be made most simply by bending the shield coil quadrants alternately up and down from the horizontal plane. Through trial-and-error we determined that 75° bends in the 20-m-radius shield coil and 30° bends in the 45-m-radius shield coil give the same relative amounts of dipole and quadrupole fields as those produced by the reactor. Since the bending reduces the effective dipole moment of the shield coils, the currents must be changed correspondingly. For these "quadrupole" shield coils, the currents should be 14.37 MA in the 20-m-radius coil and 2.0 MA in the 45-m-radius coil. The current in the 45-m-radius coil is reduced to take advantage of the fact that, for distances from between 45 m and about 180 m, the fields from the shield coil are

FIG. II.22

Contours of the total magnetic field near a Yin-Yang fusion reactor with a circular, 20-m-radius shield coil.

FIG. II.23

Contours of the total magnetic field near a Yin-Yang fusion reactor with a circular, 45-m-radius shield coil.

larger and decreasing more rapidly than those from the Yin-Yang, and at larger distances the field is too small to be significant.

The use of "quadrupole" shields constitutes a great improvement over "dipole" shields. Figures II.24 and II.25 show the magnetic field contours for the 20- and 45-m-radius quadrupole shields respectively. The 1-G contour is 75 m from the reactor in the former case and 110 m in the latter, and the fields are decreasing very rapidly with distance from the reactor.

All the shield coils reduce the magnetic field in the reactor. The amount of opposing magnetic field produced by the dipole shield coils decreases as $1/r^3$ with the radius r of the shield coil. The interior fields are reduced by 12% for the circular 20-m-radius shield coil, and by 1% for the 45-m-radius shield coil. The 20-m quadrupole shield coil reduces the interior field by 17%, while the 45-m quadrupole shield coil has a 1% effect. For many types of magnetic confinement devices the adverse effect of a shield coil can be overcome by increasing the current in the primary coils. However, the mirror ratio of the magnetic fields within the Yin-Yang cannot be so restored. The mirror ratio, which is the ratio of the magnetic field strength at the plasma exit point to the field strength at the center, is changed by the presence of a shield coil because the shield produces an almost constant subtractive field in the entire plasma interaction region. This results in an increase in the mirror ratio from 2.31 for the unshielded Yin-Yang to 2.32, 2.33, 2.52, and 2.63 for the 45-m-radius dipole and quadrupole, and 20-m-radius dipole and quadrupole shield coils, respectively. This is not necessarily a problem, because an increased mirror ratio generally means better plasma confinement. However, the Yin-Yang magnet parameters which govern the value of the mirror ratio must be designed integrally with the shield coil to produce the desired mirror ratio.

3. Forces. The amount of structural support needed to withstand the large electromagnetic forces on the shield coils contributes significantly to the total cost of active shielding. We assumed a 5000 A/cm^2 current density and a square conductor cross section to find the average magnetic field in a shield coil. The computer program P8MAFCO, described in App. B, computed the magnetic fields on the inner and outer surface of the conductor, and the average of these two values was used as the average field within the conductor (refer to Sec. I.D). Table II.18 gives the forces for the four shield coils

FIG. II.24

Contours of the total magnetic field near a Yin-Yang fusion reactor with a 20-m-radius "quadrupole" or "wavy" shield coil.

FIG. II.25

Contours of the total magnetic field near a Yin-Yang fusion reactor with a 45-m-radius "quadrupole" or "wavy" shield coil.

discussed in the preceding section and for the Yin-Yang primary magnets. The currents, average internal magnetic fields, and volume and mass of structural steel needed to support the forces are also given in Table II.18. The preliminary Yin-Yang design⁸ uses prestressed concrete rather than steel to support the forces, but for cost comparison purposes we assume that stainless steel is used for the Yin-Yang as well as for the shield coil. The 45-m-radius shield coils need about 10% as much support as the 20-m-radius coils.

4. Costs. The only cost components for passive shielding are the prices of land and iron. Using \$5000/acre of land, we find that the 57 acres necessary to encompass the 1-G contour costs about \$285 000. Since the magnetic field decreases as $1/R^3$ with distance R and land area increases as R^2 , the cost of the site area increases as $R^{2/3}$. The 122 acres required to enclose the 0.3-G contour costs about \$610 000. Using \$3/kg of iron, a 20-m-radius shield costs $\$105 \times 10^6$, and a 45-m-radius shield costs $\$47 \times 10^6$, as explained in the previous section. The amount of iron required, and hence the cost, decreases as $1/r$ with the radius r of the shield.

Active shielding devices, superconducting current loops, have three cost components, discussed in Sec. I.E: quantity of conductor, dewar cost, and mass of structural support. We have calculated present component costs and total costs for the four shield coils discussed above, using the values: \$2.60/kAm of conductor, \$300/m² of dewar surface area, and \$12/kg of stainless steel. Table II.19 gives these costs and the costs of the Yin-Yang magnet system for comparison. Table II.20 gives relative component costs with respect to the primary magnets costs for the four shield coils, to allow for the possibility of changing costs. Despite the effectiveness of the 20-m-radius shield coils, their cost, 20% or 30% of the cost of the Yin-Yang magnet system, makes them less cost effective than the other shield coils. The most cost-effective active shield is a 45-m-radius quadrupole shield coil. Costing less than 5% of the reactor, located within the wall of the facility, it can reduce the land area requirements for the .3-G magnetic field to 14.7 acres, a very reasonable figure for a facility site. The simple expedient of making the site very large, however, would be cheaper unless the additional 107 acres of land needed to enclose the .3-G contour without an active shield cost more than \$21 000/acre. A disadvantage of the less expensive 45-m-radius shields

TABLE II.18
FORCES AND STRUCTURAL SUPPORT PARAMETERS FOR YIN-YANG SHIELD COILS

	Yin-Yang	Dipole Shields		Quadrupole Shields	
Radius (m)	11.23	20	45	20	45
Current (MA)	2 x 54.6	10.775	2.128	14.37	2.0
Ave. Magnetic Field (kG) in the Conductor		5.17	.529	6.7	.52
Total Force (10^6 N)	10680.	700.	31.8	1210.	29.4
(10^6 lbf)	2400.	157.4	7.16	272.	6.61
Vol. of Steel Support (m^3)	292.	68.5	6.96	118.7	6.43
Mass of Steel Support (Mg)	2274.	535.	54.3	926.	50.0

TABLE II.19
COMPONENT COSTS AND TOTAL COSTS FOR YIN-YANG MAGNETS AND SHIELD COILS

	Yin-Yang	Dipole Shields		Quadrupole Shields	
Radius (m)	11.23	20.	45.	20.	45.
Current (MA)	2 x 54.6	10.8	2.13	14.4	2.0
Conductor (MAm) Quantity	7564.	1354.	602.	1806.	565.
Cost (\$ 10^6)	19.67	3.52	1.565	4.696	1.469
Dewar Surface Area (m^2)	1506.	303.	681.	303.	681.
Cost (\$ 10^6)	0.452	0.091	0.204	0.091	0.204
Support Steel Mass (Mg)	2274.	535.	54.3	926.	50.1
Cost (\$ 10^6)	<u>27.3</u>	<u>6.41</u>	<u>0.652</u>	<u>11.1</u>	<u>0.601</u>
Total Cost (\$ 10^6)	47.42	10.02	2.42	15.89	2.27

TABLE II.20

RELATIVE COST COMPONENTS FOR YIN-YANG SHIELD COILS

Radius (m)	Dipole Shield Coil		Quadrupole Shield Coil	
	20.	45.	20.	45.
Current-Relative to Yin-Yang	9.9%	2.0%	13.2%	1.8%
Quantity of Conductor - Relative to Yin-Yang	17.9%	8.0%	23.9%	7.5%
Dewar Surface Area - Relative to Yin-Yang	20.1%	45.2%	20.1%	45.2%
Mass of Structural Support - Relative to Yin-Yang	23.5%	2.4%	40.7%	2.2%

TABLE II.21

COST COMPARISONS FOR YIN-YANG SHIELDS

Mode of Magnetic Field Control	Estimated Cost (\$10 ⁶)
land	
1 G enclosed	0.3
0.3 G enclosed	0.6
iron shield	
20-m radius	105.
45-m radius	47.
active shield coil (superconducting)	
dipole, 20-m radius	10.
dipole, 45-m radius	2.4
quadrupole, 20-m radius	15.9
quadrupole, 45-m radius	2.3

is that, being situated in the wall of the reactor building, they provide no shielding for the interior of the building.

The costs of controlling the magnetic field by the above methods are summarized in Table II.21.

H. Field-Reversed Mirror Reactor

1. Magnetic Fields. Plasma confinement in a field-reversed mirror reactor (FRMR) is achieved by the design of plasma "cells" in the interior of a large solenoidal magnet. The mirror field at the ends of the plasma cells can be relatively weak, less than 1% more than the central field, because the plasma is injected perpendicular to the solenoid axis and thus has low axial momentum. The active plasma volume in a cell does not extend far, less than 1 m, along the axis of the solenoid; therefore, design efficiency is improved by placing a number of cells along the axis.

Plasma confinement times and power output determine the design parameters of the plasma cells and solenoid. The design considered here is the Lawrence Livermore Laboratory FRMR preliminary design given in preprint UCRL-52170⁹ and subsequently modified.¹⁰ The modified design has 11 plasma cells in a primary solenoid 22 m long and 8.5 m in diameter. Figure II.26 is a schematic diagram of the active region of the reactor. The magnetic field strength inside the plasma cells is 4.1 T (41 000 G). Some FRMR designs provide the mirror fields by using small, relatively weak mirror coils spaced between the plasma cells. In the design we are considering, however, the mirroring results from judiciously spacing the coils in the primary solenoid (See Fig. II.26) to give a mirror ratio of magnetic field between the cells to magnetic field in the cells of 1.006.

We determined the solenoid coil current densities and the resulting peripheral magnetic field, using the plasma cell field strength of 4.1 T as input to the computer program CSYD, described in App. A. Figure II.27 shows the field strength as functions of axial and radial distance from the center. The field shows dipole behavior beyond 40 or 50 m. Figure II.28 gives the external fields around the reactor in the horizontal (occupation) plane. The field strengths are independent of azimuthal angle because the magnets have axial symmetry. The proposed reactor building wall⁹ is shown in Fig. II.28 as a dashed line; the 200-G contour and most of the 10-G contour is within the building. The magnetic field is above 1 G to distances of 190 m axially and 160 m radially, and above 0.3 G to distances of 285 m axially and 225 m radially.

2. Shielding. About 30 acres is required to enclose the 1-G field contour, and 64 acres is required to enclose the 0.3-G contour.

Fig. II.26

Cross section of a field-reversed mirror reactor.
Shaded regions are superconducting solenoid coils.

Fig. II.27

Graph of total magnetic field (B) as a function of distance
from the center of the field-reversed mirror reactor.

If an iron shield is placed just outside the ion-source apparatus, at 16 m, a 0.625-m thickness of iron is required to absorb all the magnetic field lines. If the iron is placed at a 20-m radius, the required radial thickness can be reduced to 0.4 m. The quantity of iron necessary to enclose the entire reactor is probably prohibitive: about 1570 m^3 at 16 m, and 1260 m^3 at 20 m.

Another shielding option is an active superconducting coil, which provides an equal and opposite magnetic dipole moment and cancels the distant magnetic fields of the primary solenoid. The program CSYD (see App. A) adjusts currents to produce desired magnetic field strengths; by requiring that the fields in the plasma cells remain at 4.1 T while fixing the level of external field at some desired point, both the currents in the primary solenoid and the current in the shield coil are computed. The external field is set to 0.3 G at an axial distance of 100 m from the shielded reactor center. Figure II.29 and Fig. II.30 give the resulting magnetic field strength contours of 200 G, 10 G, 1 G, and 0.3 G for the 16- and 20-m-radius shield coil, respectively. Results are similar for the two shields, except for an increase in radial bulging of the contours for the 20-m-radius coil. The 1-G contour is pulled in by both shield coils to about 80 m. The increase in the total current in the solenoid required to maintain the interior magnetic field is 3.3% for the 16-m-radius shield coil and 1.9% for the 20-m-radius shield coil. The 16-m-radius coil carries 4.8 MA and the 20-m-radius coil 3.04 MA of current. The superconducting coils are assumed to carry current densities of 5000 A/cm^2 , so their cross sections, given in Table II.22, are relatively small.

3. Forces. The force on a coil is proportional to the product of the magnetic field, mostly self-generated, and the electric current throughout the conductor (see Sec. I.D). As the magnetic field is in the axial direction, the force on the shield coil is radially outward. The computer program CSYD was again used to obtain the magnetic fields in the shield coils. The resulting average magnetic fields and forces on the coils are given in Table II.22. The cost of the structural support to contain these forces is a significant portion of the total shield coil cost. The force on the 16-m-radius shield coil is almost twice as large as that on the 20-m-radius shield coil ($2.42 \times 10^7 \text{ lbf}$ vs $1.6 \times 10^7 \text{ lbf}$). The force on the primary solenoid, $1.03 \times 10^9 \text{ lbf}$, does not change in the presence of a shield coil, because the

Fig. II.28

Contours of the total magnetic field near an unshielded field-reversed mirror reactor.

Fig. II.29

Contours of the total magnetic field near a field-reversed mirror reactor having a 16-m-radius shield coil.

Fig. II.30

Contours of the total magnetic field near a field-reversed mirror reactor having a 20-m-radius shield coil.

TABLE II.22

PARAMETERS AND COSTS FOR TWO SHIELD COILS FOR
FIELD-REVERSED MIRROR REACTOR

Shield Coil Radius (m)	16	20
Shield Coil Cross Section (m ²)	.40 x .24	.40 x .152
Shield Coil Current (MA)	4.799	3.042
Average Magnetic Field in Shield Coil (T)	.2234	.1237
Force on Shield Coil (10 ⁷ N)	10.8	4.72
(10 ⁷ lbf)	2.42	1.06
Total Quantity of Conductor Increase (MAm)	552.7	422.0
Cost (\$10 ⁶)	1.437	1.097
Dewar Surface Area for Shield Coil (m ²)	189	216
Cost (\$10 ⁶)	0.056	0.064
Mass of Stainless Steel Support (Mg)	65.0	35.6
Cost (\$10 ⁶)	<u>.780</u>	<u>.427</u>
Total Cost (\$10 ⁶)	2.273	1.589

effect of increased current in the primary solenoid is counterbalanced by the opposing magnetic field of the shield coil.

4. Costs. Based on \$5000/acre, enclosing the 1-G magnetic field contour within the site boundary costs about $\$1 \times 10^5$ and enclosing the 0.3-G contour costs about $\$2.7 \times 10^5$. These are above the cost of the minimum amount of land needed for the reactor site, taken as 10 acres.

The cost to completely encase the reactor with iron is about $\$37.1 \times 10^6$ at a 16-m radius, and is $\$29.7 \times 10^6$ at a 20-m radius.

The dimensions, currents, fields and cost components are tabulated and compared in Table II.22 for the two active shield coils. Costs are calculated on the following bases (see Sec. I.E): (1) fabricated superconductor costs \$2.60/kAm, (2) the price of the dewar is \$300/m² of surface area, (3) the working stress for structural stainless steel is 30 000 psi or 207 MPa, and

(4) fabricated stainless steel costs \$12/kg. To allow for changes in prices or technologies, the relative increases in quantity of superconductor, dewar surface areas, and quantity of structural stainless steel for the shielded FRMR, compared to the unshielded FRMR, are tabulated in Table II.23. Included in the additional quantity of superconductor is the increase in the FRMR currents required to counteract the effect of the shield coil on the reactor interior field. Active shielding is not cost effective unless the price of land is more than \$27 600/acre.

A summary of costs for controlling the FRMR magnetic fields by the three methods above is given in Table II.24.

I. Tandem Mirror Reactor

The tandem mirror reactor (TMR) is a linear fusion reactor designed to burn deuterium-tritium plasma within a long, axisymmetric, central solenoid.¹¹ The length of this central cell will determine the total electrical power output of the reactor. Large "plug" magnets, coaxial with the main solenoid, will create a dense plasma plug at each end of the central solenoid to confine the plasma in the central cell.

TABLE II.23
RELATIVE COST COMPONENTS OF SHIELD COILS FOR
FIELD-REVERSED MIRROR REACTOR

Shield Coil Radius (m)	16	20
Current Relative to Primary Solenoid	4.7%	3.0%
Increase in Current in the Primary Solenoid	3.3%	1.9%
Total Relative Increase in Conductor Quantity	26.0%	19.9%
Total Relative Increase in Structural Steel Support Mass	9.4%	5.1%
Total Relative Increase in Dewar Surface Area	19.8%	22.6%

TABLE II.24
COST COMPARISONS FOR FIELD-REVERSED MIRROR REACTOR SHIELDS

Mode of Magnetic Field Control	Estimated Cost (\$10 ⁶)
land	
1 G enclosed	0.1
0.3 G enclosed	0.3
iron shield	
16-m radius	37.
20-m radius	30.
active shield coil (superconducting)	
16-m radius	2.3
20-m radius	1.6

The TMR designed at Lawrence Livermore Laboratories (LLL) will have a total length between plug magnets of about 140 m, with the central-solenoid portion being 100 m of that length. The central cell is composed of 36 separate magnet modules. The individual magnet modules are each 2.78-m long and have a 5-m outside radius. The field strength within the solenoid will be about 2 T (20 000 G).

The plug magnets at each end of the reactor are more complicated in design. Each plug magnet will have a small Yin-Yang mirror coil within two 11-m diameter, 2.25-m long solenoid magnets. These magnets produce a field of about 16 T (160 000 G) in their interior.

1. Magnetic Fields. The peripheral fields of the TMR were computed by the numerical program, CSYD (see App. A), which was developed to find the magnetic fields of an axisymmetric, coaxial configuration of coils. The magnet current densities were calculated from the TMR interior fields and the proposed coil dimensions. The magnet parameters are given in Table II.25. The small Yin-Yang coils within the plug magnets were neglected, and the central coil was approximated by a single, 100-m-long solenoid.

The two plug magnets dominate the external reactor field. Figure II.31 gives the magnetic field contours of the unshielded reactor. The dimensions and structures of the proposed reactor site are included in the figure.¹² The 200-G contour lies generally within the evacuated building (a partial vacuum

FIG. II.31

Contours of the total magnetic field of the unshielded TMR (Tandem Mirror Reactor).

TABLE II.25
DESCRIPTION OF TMR MAGNETS AND SHIELD COILS

Coil	Mean Radius (m)	Axial Length (m)	Radial Thickness (m)	Mean Axial Position (m)	Current Density (A/cm ²)	Total Current (MA)
End plug magnets (4)	4.0	2.25	2.50	±70.0	1480.13	333.0
Central solenoid	4.75	100.0	1.60	0.0	99.16	158.7
80-m shield coil pair	25.0	.35	.26	±80.0	-5000.0	9.1
85-m shield coil pair	25.0	.33	.30	±85.0	-5000.0	9.9
90-m shield coil pair	25.0	.30	.25	±90.0	-5000.0	7.5
100-m shield coil pair	25.0	.35	.26	±100.0	-5000.0	9.1

will exist near the reactor) and bunker housing the reactor. This contour will be 15 m above ground level over the end mirror magnets. The 10-G contour overlaps much of the occupation area. The total field drops off to 0.3 G within 500 m.

2. Shielding. The necessity of shielding the TMR depends upon the magnetic field level deemed acceptable. The reactor field will be constant with time. For 10 G or greater, this device will need no shielding because the total magnetic field is below 10 G at the reactor site limits. Occupational field levels will be generally below 200 G, varying from about 200 G to around 1 G. If the maximum acceptable magnetic field is below 10 G then shielding may be required.

Several parameters govern shielding choices for the TMR. Because the end mirror magnets are responsible for most of the external magnetic field, the simplest choice of one central shield coil is not reasonable. Fields produced by a shield coil large enough to cancel the dipole moments of the end mirror magnets would have a severely disrupting effect on the central internal magnetic field and would impair the reactor's operation. The most efficient method of reducing the total magnetic field without affecting performance is by using separate shield coils for the end plugs.

Several reactor parameters govern the position and the size of the shield coils. The proposed elliptic direct converter size at the ends of the reactor requires at least 40-m-diameter shield coils.¹² The proposed reactor building will be about 40 m wide. However, because a small increase

in the width of the building will not appreciably affect the total reactor cost, shield coils to 50-m diameters are considered.

A trade-off between shielding efficiency and added cost exists for the active shielding option. Two shield coils over the two end mirror magnets, with equal and opposite dipole moments as the end magnets would give the most efficient shielding. The dipole moment varies directly as the current and as the square of the coil radius. In this case, however, our choice of shield coil radii is severely limited by other physical parameters. The same shield coil dipole moment as that of the end mirror coils requires large shield coil currents and correspondingly large interior fields, approximately 5 MA and 0.18 T. Thus, depending upon the shield coil placement, significant cost reductions are possible. Shielding effectiveness is little affected by moving the shield coils along the axis away from the reactor center.

For a set of coaxial shield coils placed beyond the end mirror magnets, cancelling the dipole moments of the end magnets no longer gives the most effective shielding. The magnetic field in the axial direction^a beyond the shield coils is greater than necessary for this configuration because the shield coils are closer to the axial point in question than the end plug magnets. The magnetic field in the radial direction is also not minimized because the end mirror magnets are closer to the point at which the fields are being evaluated than the shield coils. A shield coil placed beyond the last reactor mirror magnet is more effective in reducing the field axially beyond the reactor with less current; however, it is less effective in reducing the fields in the radial direction. This is not a hardship for the TMR, because the unshielded fields are smaller in the radial direction than in the axial direction because of the reactor geometry. The effect of such shield coils is to produce a total field that has more nearly circular 0.3- and 10-G contours than the original field. This shape lends itself well to fencing, etc.

^aThe term axial direction here means a position along the axis of the solenoidal magnets and not the axial component of the field. Similarly radial direction means at a radial distance from the coil, not the radial component of the field.

Shield coils of 20- and 25-m radii were studied for four distances from the reactor center; 80, 85, 90, and 100 m. These distances correspond to about 10, 15, 20, and 30 m, respectively, beyond the reactor ends. The 20-m-radius shields are more expensive than the 25-m-radius shields. Only the 25-m-radius coils are considered here.

The 80-m shield coils are quite close to the end plugs, and of the four designs they give the most effective shielding. Figure II.32 gives the 200-, 10-, 1-, and 0.3-G magnetic field contours of the shielded reactor. The 200-G contour is approximately the same as for the unshielded reactor, with the contour lying almost entirely within the reactor building and the earth bunker. The 200-G contour again extends above ground over the reactor end magnets. The 10-G contour has been brought in slightly, but still intersects several buildings on the site. At about 290 m along the reactor's axis and 210 m along its radius, the residual magnetic field has reached 0.3 G. The current required for these shield coils is 4.55 MA and the effect upon the interior magnetic field of the reactor is about 0.57%, requiring an increase in the reactor magnet currents of that amount.

The 85-m coils give approximately circular magnetic field contours. Figure II.33 gives the magnetic field contours relative to the proposed site structures. The contours are qualitatively the same as those of the 80-m shield case, except that by bringing in the contours in the axial direction, we have sacrificed shielding in the radial direction. These shield coils each require 4.95 MA. The shield coils have a 0.50% effect upon the internal fields of the end mirror magnets.

The 90-m shield coils gave less effective shielding than either the 80- or 85-m coils. Both the 200- and 10-G magnetic field contours shown in Fig. II.34 are about the same as those of the unshielded reactor. Within 340 m axially and 240 m radially the total field drops to 0.3 G. This shield requires the least current of the four designs, 3.75 MA, and has a 0.28% effect upon the interior magnetic fields of the reactor's end magnets.

The 100-m shield coils provide the least shielding of the four shield coil designs. The magnetic field contours of the 100-m shielded reactor are given in Fig. II.35. All of the contours are about 15 m farther from the reactor than in Fig. II.34. This set of coils also requires more current, 4.55 MA, to give less shielding than the 90-m shield case. It is the least desirable of all the shield coils tested. Its only good point is that it has

Fig. II.32

Contours of the total magnetic field of the TMR (Tandem Mirror Reactor) with the 80-m shield coils.

Fig. II.33

Contours of the total magnetic field of the TMR (Tandem Mirror Reactor) with the 85-m shield coils.

Fig. II.34

Contours of the total magnetic field of the TMR (Tandem Mirror Reactor) during operation with the 90-m shield coils.

Fig. II.35

Contours of the total magnetic field of the TMR (Tandem Mirror Reactor) with the 100-m shield coils.

slightly less effect upon the reactor's interior fields than the other shield coils, about a 0.21% effect upon the end mirror fields.

Passive shielding methods are also available for the TMR. The first method is to buy and fence in enough land so that the public can be put at an acceptable distance. The maximum amount of land needed, defined by the boundaries where the field falls below 0.3 G, is 0.19 km^2 , or about 46 acres.

The second mode of passive shielding is to put an iron shield around the reactor. To cancel the field beyond a distance of 25 m, a 24-cm-thick iron shell is necessary. A cylindrical iron shell running the reactor's length would require about 5300 m^3 of material, a prohibitive amount.

3. Forces. The forces on the TMR shield coils are calculated using the magnetic fields just outside the coils. Two perpendicular forces are acting upon the shield coils: a radially outward force generated by the coil itself, and an axial force generated by the field of the TMR, tending to push the coil away from the reactor.

The forces on the reactor are calculated for comparison with the shield coils. The central solenoid experiences primarily a radial hoop stress. The unshielded central solenoid experiences a total force over its entire length of $4.7 \times 10^9 \text{ N}$ ($1.06 \times 10^9 \text{ lbf}$). This value does not change appreciably for the shielded cases. The end plug magnets experience both a radial, tensional force and an axial force from the magnetic field of the solenoid. The total radial force, which doesn't change appreciably for the shielded cases, is $4.5 \times 10^{10} \text{ N}$ ($1.02 \times 10^{10} \text{ lbf}$). The axial force in the unshielded case is $29 \times 10^6 \text{ N}$ ($6.59 \times 10^6 \text{ lbf}$), and increases by a maximum factor of 2.5 in the shielded cases. This is still less than 1% of the radial force, and so is not taken into account in cost estimates.

The forces on the 80-m shield coils are appreciable in both the axial and radial directions. The radial force is $21 \times 10^7 \text{ N}$ ($4.66 \times 10^7 \text{ lbf}$), and the axial force is $7.2 \times 10^7 \text{ N}$ ($1.63 \times 10^7 \text{ lbf}$), about 35% of the radial force. The forces on the 85-m shield coils are $21 \times 10^7 \text{ N}$ ($4.8 \times 10^7 \text{ lbf}$) radially and $7.8 \times 10^7 \text{ N}$ ($1.75 \times 10^7 \text{ lbf}$) axially, slightly larger than the 80-m shield coil values. The forces acting on the 90-m shield coil are the least, $11.2 \times 10^7 \text{ N}$ ($2.53 \times 10^7 \text{ lbf}$) radially and $4.8 \times 10^7 \text{ N}$ ($1.09 \times 10^7 \text{ lbf}$) axially. The forces on the 100-m shield coil are $15.7 \times 10^7 \text{ N}$ ($3.53 \times 10^7 \text{ lbf}$) radially and $33 \times 10^6 \text{ N}$ ($7.40 \times 10^6 \text{ lbf}$) axially. Table II.26 gives the forces on the TMR magnets and shield coils.

4. Costs. The shielding cost estimates are computed by the method given in Sec. I.E. Table II.27 gives the conductor length, volume of stainless steel, and dewar surface area parameters used to compute the active shielding costs. The cost of confining the coils against the forces above are computed by assuming the use of stainless steel. This is unsuitable for cases in which the permeability of steel will significantly interfere with magnetic fields. However, the relative cost of confining the shield coils vs the cost of confining the unshielded device will be independent of the material used.

The costs of passive shielding are computed. The cost of an iron cylinder shield the length of the device is $\$1.2 \times 10^8$, far more than the cost of active shields.

In most areas, the 46 acres required to enclose the field to 0.3 G costs much less than the active shielding. The land would have to cost more than \$109 000/acre in order to make other shielding cost effective assuming that the land is available.

TABLE II.26
MAXIMUM TOTAL FORCE ON TMR COILS AND SHIELD COILS

Coils	Maximum Total Force	
	(10^6 N)	(10^6 lbf)
Central solenoid	4715	1060
End plug magnets (4)	45372	10200
80-m shield coils (2)	218	49
85-m shield coils (2)	227	51
90-m shield coils (2)	125	28
100-m shield coils (2)	160	36

Table II.28 gives the active shielding costs for the TMR. Costs are computed in 1977 dollars (see Sect. I.E). The most useful numbers in the table are the cost increases of each item for the different shields. These numbers are independent of inflation and of a change in construction materials. Costs of the active shields vary from 3.3% to 4.5% of the reactor magnet cost. Active and passive shielding costs are summarized in Table II.29.

TABLE II.27

PARAMETERS FOR TANDEM MIRROR REACTOR ACTIVE SHIELDING COSTS

<u>Coils</u>	<u>Total Conductor Length (MAm)</u>	<u>Dewar Outer Surface Area (10²m²)</u>	<u>Volume of Stainless Steel (m³)</u>
Unshielded End Magnets (4)	8369.2	10.16	1160
Unshielded Central Solenoid	4736.4	60.81	120
80-m shield coils (2)	1429.4	5.74	34.3
85-m shield coils (2)	1555.0	5.88	35.7
90-m shield coils (2)	1178.2	5.38	19.7
100-m shield coils (2)	1429.4	5.74	23.3
Increased Reactor Materials ^a (80-m shield)	64.4	0.15	-
Increased Reactor Materials ^a (85-m shield)	59.3	0.19	-
Increased Reactor Materials ^a (90-m shield)	37.7	0.13	-
Increased Reactor Materials ^a (100-m shield)	26.7	0.11	-

^a Increase in the conductor length, and dewar surface area required for the primary TMR magnets to compensate for the reduced interior fields resulting from the shield coils.

TABLE II.28

TMR ACTIVE SHIELDING COSTS

	<u>Coil System Cost^a (\$10⁶)</u>	<u>% Relative to TMR Magnet Cost</u>
<u>Unshielded Case</u>		
Conductor costs	34.9	
Dewar costs	2.1	
Confining costs	119.0	
Total	156.0	100
<u>80-m Shield</u>		
Conductor costs	3.9	11.2
Dewar costs	0.2	8.5
Confining costs	3.0	2.5
Total	7.1	4.5
<u>85-m Shield</u>		
Conductor costs	4.3	12.3
Dewar costs	0.2	9.4
Confining costs	3.0	2.5
Total	7.5	5.0
<u>90-m Shield</u>		
Conductor costs	3.2	9.2
Dewar costs	0.2	8.5
Confining costs	2.0	1.7
Total	5.4	3.3
<u>100-m Shield</u>		
Conductor costs	3.8	10.9
Dewar costs	0.2	8.5
Confining costs	2.0	1.7
Total	6.0	3.8

^aCost computed in 1977 dollars

TABLE II.29
COMPARISON OF ACTIVE AND PASSIVE SHIELDING
COSTS FOR THE TMR

Shielding	Computed Cost (\$10 ⁶)
80-m shield coil	7.1
85-m shield coil	7.5
90-m shield coil	5.4
100-m shield coil	6.0
Iron shield ^a	120
Land ^b	.23

^a Computed from a fabricated cost of \$3/kg for iron.

^b Computed from a cost of \$5000/acre for land.

III. ENERGY-RELATED TECHNOLOGIES REQUIRING HIGH MAGNETIC FIELDS: NONNUCLEAR

Magnetic fields with the largest extent yet envisioned are those associated with superconducting magnetic energy storage (SMES) devices, intended to supply electrical energy to large metropolitan areas. Because of its size, perhaps 300 m in diameter, a SMES device would significantly affect the ambient magnetic field for distances of one or two kilometers. The SMES system is discussed in Sec. A below.

Section B treats projected magnetohydrodynamic (MHD) generator magnets, which are much smaller in size than a SMES device but have high field strengths.

Magnetically levitated trains are discussed in Sec. C. Unlike other supermagnet applications, the levitated trains are proposed as public transportation, so magnetic field exposure cannot be controlled by a fence line.

Section D examines the fields of generators using superconducting coils. Rotating electrical ac generators using superconducting magnet windings will not have external magnetic fields greater than comparable conventional generators, which have very low external fields, demanded by design efficiency.

An application of large-scale superconductor technology where a magnetic field is not desired is underground superconducting transmission lines for electrical power. Section E gives a cursory discussion of their effects. Large currents will be flowing in these transmission lines, which raises the possibility of associated large magnetic fields (see App. C) but, since in general the net current is zero for both the dc lines, which are coaxial, and the ac lines, which have alternating currents and also alternating magnetic fields, the time-averaged associated magnetic fields will be zero.

A. Superconducting Magnetic Energy Storage

1. General Description. Superconducting magnetic energy storage (SMES) has been considered as a means of damping the short-term load fluctuations experienced by electric utilities since the discovery of high-field superconductors in 1961.¹³ Several SMES systems are being developed to store and deliver electrical energy for the purposes of load leveling, peak shaving, and stabilization of electric utility networks.¹⁴

Several institutions are developing large superconducting solenoid SMES devices for eventual use by the utilities. These large solenoids will have extensive external magnetic fields associated with them, larger than other SMES magnet designs, such as the torus.

2. Magnetic Fields. For reference purposes, we consider a 10-GWh SMES solenoid coil of 150-m radius, 100-m height and midplane buried 150 m below ground level. The 10-GWh SMES coil and active shield options are considered in detail in Ref. 15. Figure III.1 gives the magnetic field from the unshielded SMES coil as a function of horizontal distance from the magnet winding axis at four different elevations above ground level. Along the axis of the winding, the field is 0.86 T at ground level, falling to 0.006 T at an elevation of 900 m. At ground level, the total field falls to 200 G within 0.6 km of the axis and to 10 G within 1.6 km.

TABLE III.1^aRADIUS AND AREA REQUIREMENTS TO ATTAIN VARIOUS FIELDS LEVELS AT GROUND LEVEL
10 GWh MAGNETMidplane Buried at a Depth Z_0 Below Ground

Z_0 (m)	B(G)	R(m)	A(km ²)
150	200	580	1.06
	10	1550	7.50
	0.3	4980	78
450	200	480	0.72
	10	1540	7.45
	0.3	4960	77
750	200	0(B<166 G) 0	
	10	1500	7.07
	0.3	4930	76
1050	10	1390	6.07
	0.3	4870	74

^aFrom Ref. 15

3. Shield Options. A general class of environmental constraints would define the magnetic field over a given boundary, with access inside the boundary limited to plant employees. The appropriate action to satisfy these constraints is qualitatively different depending upon the value of the field and the boundary specified. The field values we have taken are 200, 10, and 0.3 G. It will make a great difference as to whether these field limitations apply:

- 1) everywhere at ground level;
- 2) at ground level beyond some boundary; or
- 3) beyond some height above ground level, as well. (There is some evidence that bird and insect navigation may be affected by relatively large magnetic fields).

Combining the three field levels with 1), 2), and 3) results in nine different sets of environmental constraints. These constraints should be met at minimum cost, with the only significant capital items being the shield coil and the land required for the facility. To limit the field along a ground level boundary, the easiest solution is to buy enough land, provided it is available. Table III.1 gives the radius and area requirements needed to meet constraints a), b) and c) for various burial depths of the 10 GWh reference magnet midplane. Numerical results were obtained for the SMES fields using the computer program CSYD (see App. A). Shield options will be discussed in the remainder of this section.

Because the shield coil cost is required to be only a modest fraction of the main coil cost, a shield coil with radius much larger than the radius of the main coil is required. Such a shield coil will have very little effect upon the total field in its interior.

Table III.1 reveals the limited utility of burying the magnet deeper. The magnetic field, $\vec{B}$, will change direction rapidly but will decrease in magnitude very slowly for very large changes in burial depth. Equivalently, the field above ground decreases only slowly with altitude for a magnet at fixed depth. The incremental cost of changing the magnet burial depth is unknown, but is probably not large over the depth range in Table III.1.

The above discussion brings out the qualitative difference that environmental restrictions will make. If the field everywhere above ground were restricted to less than 200 G, the problem could be solved without a shield coil by burying the magnet deeper. This response would be useless if the 0.3 G criterion were to apply.

We will treat first the constraint combinations whose solutions can be quickly discussed.

a) Field everywhere at ground level less than a specified value.

This is the most severe restriction because of the great difficulty in reducing the field directly above the main coil. Deeper and deeper burial is required as the allowed field is decreased. Table III.2 gives the on-axis field at ground level as a function of the burial depth of the magnet midplane for an unshielded coil. Depths become unreasonable at a field level on the order of $1-2 \times 10^{-3}$ T. A shield coil for this region would have roughly the same radius as the main coil and would be roughly as expensive. It would have to be well separated from the main coil, so as not to affect the stored energy greatly.

In summary, the field everywhere above ground level may be reduced below 200 G, but probably not below 10 G, by simply burying the magnet. Still lower field standards would be very expensive to meet.

b) Field limited to less than 200 G within some boundary on and above ground. The 200 G point occurs close enough to the magnet so that a shield coil is relatively ineffective in determining its location. Table III.1 shows that the land area required to meet the criterion is very modest. Table III.2 shows that a vertical separation of 705 m is required directly above the magnet. Thus an unshielded coil 400 m below the surface would produce a field of 2×10^{-2} T or less everywhere 305 m above ground level. Again, more restrictive conditions directly above the magnet can be dealt with only by deeper burial, compensated by a smaller ground area requirement.

TABLE III.2^a
FIELD AT R=0, GROUND LEVEL, AS A FUNCTION OF MAGNET DEPTH, Z_0

$B(T)$	$Z_0(m)$
0.805	150
2×10^{-2}	705
2×10^{-3}	1390
1×10^{-3}	1750
3×10^{-5}	5420

^aFrom Ref 15

The field gradient at 200 G is very small. For a magnet buried 150 m below ground, the gradient at the surface is about 10 G/m, directed downward and inward. The estimated force on a piece of iron is only 2% of its weight, much too small to create a safety hazard in this respect.

c) The remaining cases, in which the boundary field is limited to values between perhaps 50 G and 0.3 G are more interesting, since they involve the detailed design of a shield coil. We shall present below detailed calculations on particular shield configurations that are designed to reduce the ground level field to satisfy certain criteria. These shields also greatly reduce the field above ground at positions beyond the shield coil radius. As stated above, however, there is little beyond deeper burial that can be done about the field directly above the main coil.

For reference purposes, we consider a main coil buried at $Z_0 = 150$ m below the surface. The coil has 500 MAT at a mean radius of 150 m. Consider for the moment a shield coil designed using the dipole moment condition with a major radius $a_s = 600$ m, thus requiring $(NI)_s = 31.3$ MAT. This shield must also be buried at $Z_0 = 150$ m. The field at ground level will be below 0.3 G at $R = 2050$ m, a considerable improvement over the value shown in Table III.1. The corresponding values for 10 G and 200 G are 1000 m and 690 m, respectively. The 200 G location is affected by the high field in the immediate vicinity of the shield winding.

This, however, is not the best design available. For instance, suppose the shield is designed to cancel B_z at ground level and $R = 2000$ m. This requires $(NI)_s = 29.2$ MAT, or 7% less material than the matched dipole. The axial field beyond 2000 m will be reversed in sign, and the field behavior with distance will be altered slightly, but the field will now fall below 0.3 G at ground level beyond $R = 1750$ m, a significant reduction.

Continuing this process, consider the same shield designed to cancel B_z at ground level and $R = 1100$ m (the reason for this choice will become clear later). This requires $NI = 23.9$ MAT, a 24% saving in material. The field now does not drop below 0.3 G until $R = 2800$ m, but it drops below 10 G at 980 m and is between 1-2 G over a very large region at ground level. Thus this coil would be a far better choice than the preceeding two if the field level criterion were to be set at 10 G.

The above examples illustrate that significant savings in both land and shield coil costs are possible over a matched dipole moment design, but that the exact design of the shield coil depends critically upon the environmental constraints to be met.

There is an unattractive feature to the buried shield coil. The magnet and dewar require a cavity only two to three meters on a side. It may be very difficult to excavate such a small tunnel and to wind the coil inside it. Excavating a larger tunnel for construction and then backfilling it to produce proper support for the magnet introduces extra costs of an unknown magnitude. We must leave these considerations as a possible problem connected with buried shields.

We turn instead to a detailed consideration of what can be accomplished with a shield coil located at ground level. Such a shield coil can affect only the z component of the magnetic field at ground level. Thus the fence

line can be moved in only to the point at which B_R alone reaches the allowed value. For $B_R = 10$ G, $R = 1100$ m is not much greater than the value that can be attained with a buried shield. For $B_R = 0.3$ G, $R = 3020$ m, but there will be a large region in which the field is 1-3 G, which may be an acceptable situation.

4. Reference Shield Design. For reference we have chosen a shield at ground level with $a_s = 850$ m, $(NI)_s = 5.48$ MAt, which exactly cancels B_z at $R = 1100$ m. Field profiles for the 10 GWh SMES with this shield coil are shown in Fig. III.2. For comparison purposes we have also considered a ground level shield with $a_s = 600$ m, $(NI)_s = 20.2$ MAt, which again cancels B_z at $R = 1100$ m. The latter is a much more expensive magnet. The fence line for 10 G is at the same location. For $B = 0.3$ G we find $R = 4030$ m for $a_s = 850$ m and $R = 3540$ m for $a_s = 600$ m. The 0.3 G fence line could be moved inward to a minimum value $R = 3020$ m, at the expense of moving the 10-G contour outward and building a magnet with significantly more ampere-turns, in fact, approaching the dipole moment limit.

Figure III.2 shows that the net field is almost constant above the earth's surface at fixed R greater than the shield radius, although the R component of the unshielded field increases above the ground. The shield coil thus also reduces a significant fraction of the radial component above the surface.

Burying the magnet deeper now has an interesting, counter-productive effect. Deeper burial raises B_R at a particular surface point while decreasing B_z . Since only B_z can be cancelled with a ground-level shield coil, the net field will actually increase and the fence lines will move outward for either 10 G or 0.3 G. For instance, burying the main magnet at 300 m rather than 150 m moves the 10 G point out to at least $R = 1300$ m, and the 0.3 G point out to at least 3760 m. The situation worsens with deeper burial down to at least $Z_0 = 1350$ m. Again, we find that severe restrictions directly over the main coil, which necessitate deeper burial, create a difficult situation - in this case, the need for using a buried shield coil. It seems likely from results such as Fig. III.2, however, that significant reductions in B_R at ground level can be obtained with a shield coil buried much closer to the surface than the main coil.

The parameters of the reference shield coil are presented in Table III.3. We have assumed the use of the same 50 kA conductor as in the main

FIG. III.1

The total magnetic field of the unshielded 10 GWh SMES coil, with midplane buried 150 m below ground, as a function of distance from the winding axis at four elevations above ground level.

(BOTH FIGURES ARE FROM REFERENCE 15)

FIG. III.2

Total magnetic field of 10 GWh SMES coil with reference shield coil as a function of distance from the winding axis at four elevations above ground level.

TABLE III.3^a
SHIELD COIL SPECIFICATIONS

Z_s , vertical location	ground level
a_s , mean radius	850 m
b_s , height	423 mm
c_s , radial thickness	260 mm
N_s , number of turns	110
L_s , conductor length	5.87×10^5 m
I_s , max	50,000 amp
L_s , self inductance	113 h
M , mutual inductance to main coil	55 h
ΔE (MWh)	-1.04
T_o	1.85 K

^aFrom Ref. 15

coil, with an overall current density of $J = 50 \text{ MA/m}^2$ that allows room for spacers and structure.¹⁵ A loop of circular cross section would have a surface field of 5.9 T, larger than is desirable. The rectangular configuration chosen is 5 turns thick radially and 22 turns high axially. It will have a maximum field of 5.5 T. A more highly elongated conductor arrangement will have a still lower field. There may be joint problems with this winding configuration. At any rate, the conductors may be arranged in almost any fashion desired without affecting the shielding characteristics.

5. Forces. In calculating the forces, it is most convenient to consider the coil a circular ring. The magnetic force on the shield coil may be broken into three contributions, caused by:

- a) The main coil axial and radial fields producing forces that are outward and upward on the shield;
- b) The large circumferential field of the conductors, which acts to crush the conductors along the minor radius. The resulting pressure, of the order of 10^7 MPa (1400 psi) is easily tolerated by the conductor. In a long straight wire only this contribution would be present. The fact that the shield coil is bent into a larger loop, however, creates an additional contribution;

- c) The axial field due to remote parts of the shield, producing an outward force. Direct calculation shows that the field is $B_z = 116$ G, $B_r = 31$ G of which $B_z = 63$ G is due to contribution (c). The resulting force components are given by

$$\overline{dF/d\ell} = \overline{NI} \times \overline{B}$$

$$F_z = 1.69 \times 10^4 \text{ N/m, upward}$$

$$F_r = 6.37 \times 10^4 \text{ N/m, outward.}$$

The upward force should be compared to a conductor weight of 1.26×10^3 N/m.

These forces are several times larger than could be taken in tension directly by the conductor, so that struts are required. (Cold structural material is shown in the next section to be far more expensive than struts.) The corresponding pressures averaged over the conductor surface are very modest, however.

$$G_z = 65 \text{ kPa (9.4 psi)}$$

$$G_r = 151 \text{ kPa (21.8 psi)}$$

The pressure on the ground at the outside dewar wall will be several times smaller than this.

The upward pressure is easily balanced by several feet of earth over the magnet dewar. The outward pressure should be readily supportable by compacted soil. Rock excavation is therefore unnecessary and foundation work can be held to a minimum.

6. Costs. The dewar cost, a major expense item, is very sensitive to the minor radius. The minor radius, in turn, depends upon the strut length. The struts are a minor cost item, however, because of the small forces involved. The strut area required may be estimated by considering a total "scalar force," $|F_z| + |F_r| = 8.06 \times 10^4$ N/m, or 4.31×10^8 N on the whole ring. If this is supported by strut material with a working stress of 3.45×10^8 N/m² (50,000 psi), at cryogenic temperatures with an additional 30% material allowance for decreased strength at room temperature, the

required strut area is only 1.62 m^2 . It is thus possible to use a strut design that will sharply reduce the dewar wall area while increasing the strut cost. One possibility would be to use an adaptation of the "Heim column" in which the support element is a set of connected, nested tubes. We assume that support elements could be designed that occupy 0.2 m above and below the magnet, resulting in a dewar minor diameter of 0.85 m. The heat leak is calculated assuming these struts have an effective length of 1.3 m and have one cooling station at 26°K .

An optimization of the magnet cross-sectional shape will eventually be possible. As noted above, the shape affects the following:

- a) the field at the conductor, and hence the superconductor cost; and
- b) the dewar minor radius, and hence the dewar cost.

The structural costs are figured assuming the use of stainless steel for the support structure for comparison purposes and also the struts called for in the design. Stainless steel support results in structural support costs for the main coil and the shield coil that are several orders of magnitude higher than the cost using struts.

The parameters used in calculating relative costs with stainless steel are given in Table III.4. The costs of the main SMES coil and the shield coil, and the percentage increase in cost for active shielding, using stainless steel, is given in Table III.5. The cost of the SMES magnet using struts is given in Table III.6 from Ref. 15. Table III.7 gives the relative costs of the SMES coil and the reference shield coil based upon the actual design specifications.

The cost of an iron shield for the device is prohibitive, approximately $\$3.5 \times 10^{10}$, or at least a factor of 10^3 more expensive than the active shield.

About 194 acres is required to enclose field intensities greater than 200 G. About 2000 acres is required to enclose the field to a level of 10 G, and 14 050 acres is required to enclose the field to a level of 0.3 G. Thus land may not be the cheapest way of shielding the SMES. At about \$5000 per acre, the land required would cost about $\$7.0 \times 10^7$ maximum, considerably more than the active shield alone. However, some land would be required with the active shield to meet the most stringent standard (0.3 G). An additional 6100 acres would be required to meet the 0.3-G standard at an additional cost of $\$3.0 \times 10^7$ dollars. Then the total cost of shielding to 0.3 G would be

TABLE III.4

PARAMETERS FOR SMES ACTIVE SHIELDING COST ESTIMATE

	Total Conductor Length (MAm)	Dewar Outer Surface Area (m ²)	Volume of Stainless Steel (m ³)
SMES coil	4.7×10^5	188,800	1.07×10^6
Shield coil	2.9×10^4	14,500	1733

TABLE III.5

COSTS OF REFERENCE SHIELD COIL FOR THE 10 GWh SMES
COIL USING COST METHODS DESCRIBED IN SECTION I.E

	Coil Cost (\$10 ⁶)	% Relative to costs of 10 GWh SMES coil
<u>10 GWh SMES magnet</u>		
Conductor costs ^a	1222	
Dewar costs	57	
Structural support cost ^b	100 150	
Total Cost	101 429	100
<u>Reference shield coil</u>		
Conductor costs ^a	75	6.1
Dewar costs	4	7.0
Structural support cost ^b	162	0.2
Total Cost	241	0.2

^a Conductor cost has been figured using copper stabilized conductor (see Sec. I.E) rather than the aluminum stabilized conductor called for in the design. The aluminum conductor is less expensive than the copper stabilized conductor (see Table III.6).

^b The structural cost has been figured using stainless steel rather than the struts called for in the design to make the numbers comparable with those of the rest of the study. The use of struts to support the coils will result in substantial savings in the structural support costs. Table III.7 gives the costs for the reference design.

TABLE III.6

COST OF 10 GWh SMES COIL

10^4 MWh ENERGY EXCHANGE; 1.3×10^4 MWh GROSS STORAGE CAPACITY

89% EFFICIENCY FOR 24 HR. CYCLE: 1.8° 30 STRAND Al STABILIZE,

$\rho = 1 \times 10^{-10} \Omega\text{-m}$ Al 5T.

MATERIAL COSTS

Conductor Al Stabilizer 7.4×10^6 kg @ \$2/kg	\$ 14.8×10^6
Conductor Hi-Strength Al 1.5×10^6 kg @ \$2/kg	3.0
Conductor NbTi 0.78×10^6 kg @ \$45/kg	35.6
Strut Epoxy Fiberglass 11.3×10^6 kg @ \$2/kg	22.6
Therm. Insul. $3.8 \times 10^6 \text{ m}^2$ @ \$2/m ²	7.6
Dewar Hi-Strength Al 5.9×10^6 kg @ \$1.40/kg	8.3
TOTAL MATERIAL	\$ 91.9

FABRICATION COSTS

Conductor Al $(7.4 + 1.5) \times 10^6$ kg @ 4.40° /kg	\$ 40.9
NbTi 0.79×10^6 kg @ \$3.30/kg	2.6
Strut 11.3×10^6 kg @ \$2/kg (100% of Material Costs)	22.6
TOTAL FABRICATION	\$ 66.1

ASSEMBLY COSTS

Magnet	\$ 10.0
Struts (100% of Material Costs)	22.6
Therm, Insul. (100% of Material Costs)	7.6
Dewar 5.9×10^6 kg @ \$2.80/kg (200% of Material Costs)	16.9
TOTAL (Including 30% for Supervision)	\$ 74.2
Rock Excavation $1.9 \times 10^5 \text{ m}^3$ @ \$50/m ³	9.4
Helium	9.0
Refrigerator	20.6
BALANCE OF PLANT	\$ 39.0
TOTAL COST	\$271.2
ENGINEERING @ 12%	\$ 32.5
TOTAL	\$303.7 x 10^6

Net Output = 10,000 MWh $[1 - .02 - .03] = 9,500 \text{ MWh}$

$$\text{Storage Cost} = \frac{303.7 \times 10^6}{9.5 \times 10^6 \text{ kWh}} = \$32/\text{kWh}$$

TABLE III.7

RELATIVE COST OF 10 GWh SMES MAGNET AND REFERENCE SHIELD COIL
BASED UPON GIVEN DESIGN SPECIFICATIONS

	Coil Cost (\$10 ⁶)	% Relative to Costs of 10 GWh SMES Coil
<u>10 GWh SMES Magnet</u>		
Conductor costs ^a	109.9	
Dewar costs ^b	75.1	
Structural support costs ^c	74.6	
Total	<u>259.6</u>	100
<u>Reference Shield Coil</u>		
Conductor costs ^a	6.8	6.2
Dewar costs	4.0	5.3
Structural support costs ^b	0.4	.5
Total cost	<u>11.2</u>	<u>4.3</u>

^aCost based on aluminum stabilized conductor specified in design.¹⁵

^bDewar costs from Reference 15.

^cStructural support costs based on struts specified in design.

\$41 x 10⁶, still considerably cheaper than just buying the land at the price of \$5000/acre. Land must be less than \$1860/acre to be cost-effective.

Table III.8 gives a comparison of the costs of active and passive shielding for the 10⁴ MWh SMES unit.

B. Magnetohydrodynamic Power Generation

1. General Description. Power generation by means of magnetohydrodynamic (MHD) separation of charge is being considered primarily as a "topping cycle" in gas- or coal-fired power generating facilities to increase power output. Gaseous products of combustion in a conventional power plant are to be forced through a channel perpendicular to a strong magnetic field, thereby separating positive and negative charged particles (see App. C). The power in watts produced is the product of the rate of charge separation in coulombs per second (amperes), and the potential difference, in volts, across the separated

TABLE III.8

COMPARISON OF ACTIVE AND PASSIVE SHIELDING COSTS FOR THE
10 GWh SMES MAGNET

<u>Shielding</u>	<u>Computed Cost (\$10⁶)</u>
850-m shield coil ^a	11.2
Iron shield ^b	35 000
Land ^c	70

^aComputed cost based on cost estimate given in Table III.7.

^bComputed from a fabricated cost of \$3/kg for iron.

^cComputed from a cost of \$5000/acre for land, for the maximum area of land necessary.

charge. When multiplied by the efficiency for transforming to on-line electrical power, the result gives the net increase in power output.

The U. S. Department of Energy (DOE) has specified design criteria for the MHD magnet system, including channel size and field strengths,¹⁶ which we use for consideration of the magnetic fields and shielding procedures. The "warm bore" of the MHD generator, where the charge separation takes place, is a nozzle-shaped chamber, of circular or square cross section, 16 m long and tapering from about 1.5 m in diameter at the inlet end to about 3.5 m at the outlet end. The windings required to produce a magnetic field at right angles to the channel axis must be outside the core tube. The most efficient winding configuration is two saddle-shaped winding modules placed longitudinally along opposite sides of the core tube, illustrated in Fig. III.3. The windings lie along the core tube for a 20-m distance, about 2 m from the axis at the inlet end and about 3.5 m from the axis at the outlet end. The DOE specifications require a magnetic field strength of 6 T at the inlet and 3.5 T at the outlet on the channel axis; to produce fields of this size each winding module must carry about 36 MA of current.

2. Magnetic Fields. Rather than considering the details of a magnet design that is still in the conceptual stage, we examine the fields at distances that are large compared with the scale of a generalized magnet system conforming to the DOE specifications. At large distances, greater than

FIG. III.3

Winding configurations for the magnet system of an MHD generator.

20 m, the magnetic field strength is dominated by the magnetic dipole of the central magnets. To meet the DOE specifications, the magnetic dipole moment, M , the product of the current flow and the area enclosed by that current, of an MHD magnet system is roughly $72 \text{ MA} \times 20 \text{ m} \times 5.5 \text{ m}$ or about 8000 MA m^2 . The magnetic field strength is approximately 2 CM/r^3 at a distance r from the center in the direction of the magnetic field at the center, and CM/r^3 for r in a direction perpendicular to the central field. The constant C is $0.1 \text{ T} \cdot \text{m/MA}$ (see App. C). Therefore, if there is no appreciable amount of ferromagnetic material around the device, the magnetic field strength would fall below 1 G between 200 m and 250 m, and below 0.3 G between 300 m and 375 m from the magnets.

3. Shielding. From the above discussion, the land area required to enclose the region where the magnetic field strength is greater than some value B around a magnetic dipole of moment M is $4(2C^2 M^2/B^2)^{1/3}$. Using the values of C , M , and B given above, we find that to enclose the 1 G field contour would require $2 \times 10^5 \text{ m}^2$ or about 50 acres, and to enclose the 0.3 G contour would require $4.5 \times 10^5 \text{ m}^2$, or 112 acres. These requirements can be reduced by about 20%, a factor of $2^{1/3}$, by orienting the magnet winding modules so that the magnetic field in the core is vertical and charge separation takes place horizontally. Then the strongest magnetic fields would be in the vertical direction and away from the horizontal occupation plane.

Ferromagnetic shielding would require an iron shell, with a height comparable to that of the device, about 8 m. The shell does not have to be circular, but for studying relative practicability it suffices to assume that it is. Then the thickness of the shield is given in Sec. I.E, $t = \text{CM}(B_{\text{sat}} R^2)$, and hence the total volume of iron required for the shield is approximately $V = 2 \pi R t \ell = 2 \pi C (M/B_{\text{sat}})(\ell/R)$, where ℓ is the height, 8 m, and R the distance of the shield from the center of the device. For an arbitrary shield radius of 40 m, the required thickness t would be about 0.23 m, and the total volume of iron required would be 234 m^3 , with a mass of 1.84 Gg. Although the amount of iron required decreases as the shield radius increases, construction problems preclude a radius larger than 40 to 50 meters.

To calculate the parameters for active shielding costs of the MHD magnets, we have recourse to the discussion of the three component cost factors

given in Sec. I.E, for shielding a magnet dipole moment M . Using the expressions there with $M = 8000 \text{ MA}\cdot\text{m}^2$ we find that for a shield coil of radius R in meters, the quantity of conductor needed is $16/R \text{ GA}\cdot\text{m}^2$, the dewar surface area is $(179 + 3.82R)\text{m}^2$, and the volume of stainless steel structural support is approximately $1.86 \times 10^5 R^{-3} \text{ m}^3$.

4. Forces. Because no definite geometry can be specified now for the shield coil, we will continue to confine our discussion to the general consequences of the DOE specifications for the MHD magnet system.¹⁶ We assume that the primary dipole moment is vertical and that the shield coil is in the horizontal plane.

In Sec. I.D an approximate expression is presented for the magnetic force F in the radial direction on a shield coil carrying a current I : $F \approx 6\pi^2 CI^2$. One-third of this force is produced by the external magnetic field and about two-thirds, depending upon the geometry, is self-induced. For shield radius R and magnetic moment $M = \pi R^2 I$, the radial force is $F \approx 6CM^2/R^4$, with $C = 10^5 \text{ N}/(\text{MA})^2$. For an $8000\text{-MA}\cdot\text{m}^2$ magnetic moment and a 40-m radius, the radial force is $1.5 \times 10^7 \text{ N}$. The force decreases as $1/R^4$, but (see Sec. I.E), the total mass m of structural steel required to support this force decreases as: $m = 6C\rho_w M^2/\sigma_w R^3$, where ρ is the density of stainless steel, $7.8 \text{ Mg}/\text{m}^3$, and σ_w is the working stress, 207 MPa. Using the figures above, we find that the mass of the stainless steel support for a 40-m-radius shield coil would be about 22.6 Mg.

5. Costs. If the magnetic field is vertical in the central magnet system, the total land area within the 1 G contour is 40 acres, and the area within the 0.3 G contour is 89 acres. Assuming that the facility area is 10 acres, and that the price of land is \$5000/acre, the costs of the additional land needed to enclose the 1 G and 0.3 G magnetic field contours would be \$150 000 and \$395 000 respectively.

The price of the 40-m-radius iron shield discussed above is about $\$5.5 \times 10^6$, based on a fabricated cost of \$3/kg for iron. Though the cost decreases with increasing shield radius, iron shielding still appears to be relatively expensive.

The shield coil cost varies with its size. The three cost factors are given in Sec. 3 above as a function of shield coil radius R for an $8000 \text{ MA}\cdot\text{m}^2$ MHD magnetic dipole moment. Using costs of \$2.60/kA of conductor, \$300/m² of dewar surface, and \$12/kg of stainless steel, we find the cost components

Fig. III.4

Cost components and total cost for active shielding of an MHD generator as a function of shield coil radius.

as a function of shield coil radius. Figure III.4 gives the costs as a function of R. For R of 22 m or less, the stainless steel support cost is the largest component, but for greater R the conductor cost dominates. The total cost of a 40-m-radius shield coil would be $\$1.4 \times 10^6$. In addition, the amount of current in the primary magnet system would have to be increased by about one-third of one percent to counteract the shield coil effects in the MHD channel. The relative distribution of current in the winding modules may have to be altered slightly to maintain the specified magnetic field profile within the warm bore.

A summary of costs for the three methods of magnetic field control is given in Table III.9.

C. Magnetically Levitated Vehicles

1. General Description. Several countries are presently doing research and development on magnetically-levitated tracked vehicles. All of the programs are designing contactless vehicle support and guidance, and may be able to offer high-speed (200-400 km/hr) transportation without technical problems associated with conventional railway systems operated at high speeds, such as safety, noise, maintenance, and pollution.

TABLE III.9
COST COMPARISONS FOR MHD POWER GENERATION SHIELDS

<u>Mode of Magnetic Field Control</u>	<u>Estimated Cost ($\\$10^6$)</u>
land	
1 G enclosed	0.15
0.3 G enclosed	0.4
passive ferromagnetic shield	5.5
active shield coil (superconducting)	
20 m	4.3
30 m	2.1
40 m	1.4
50 m	1.1

The research and development programs are now designing and testing magnets and scale-model vehicles. Several countries have developed tentative designs for full-scale magnetically levitated vehicles. These designs employ superconducting magnets on-board the vehicle carriage for levitation and guidance, and some designs utilize on-board superconducting coils for vehicle propulsion. Other modes of propulsion, such as rockets, have also been proposed.

2. Magnetic Fields. A problem associated with magnetically levitated vehicles is the relatively large dc magnetic fields that will be present in the vehicle passenger compartment because of the on-board magnets. The fields are calculated in the passenger area of several magnetically levitated vehicle designs using the computer code, P8MAFCO (see App. B).

The United States' magneplane design for a full-scale vehicle is examined briefly. The vehicle design is similar to a modern commercial aircraft fuselage, such as a Boeing 707, from aerodynamic considerations.¹⁷ The levitating magnets are elliptic pancake coils wrapped into the bottom 120° of arc of the vehicle (see Fig. III.5). This unshielded magnet design would result in fields of 0.13 T at the floor of the passenger compartment, 0.1 T at seat level, and 0.06 T at head level (see Fig. III.6).

The Canadian design for a full-scale Maglev vehicle was also studied.¹⁸ This vehicle looks more like a conventional bus or train than the Magneplane. It has a flat base and an approximately square cross section (see Fig. III.7). The vehicle magnets are flat, rectangular coils, which produce a maximum field of 0.05 T at floor level, and 0.006 T at eye level within the passenger compartment.

Several researchers have examined aspects of shielding levitated vehicles. Researchers in the Tracked Magnetically Levitated Vehicle Program (TMLV)¹⁹ examined the fields for their design. They concluded that the TMLV would have an unshielded field of 0.09 T at floor level and 0.04 T at seat level. The TMLV levitation coils are flat, rectangular coils.

From the discussion above, magnetically levitated vehicles being researched will have unshielded magnetic fields in the vehicle passenger sections of about 0.05 to 0.1 T. The magnet parameters used to compute the magnetic fields are listed in Table III.10.

3. Shielding. There are several unique problems to consider in shielding magnetically levitated vehicles. The first is that obviously we

FIG. III.5

Cross section of the magneplane design showing coil position relative to the passenger compartment.

FIG. III.6

Graph of the total magnetic field of the unshielded magneplane with height above the magnets.

FIG. III.7

Cross section of Maglev Vehicle showing coil position relative to the passenger compartment.

TABLE III-10

LEVITATED VEHICLE ON-BOARD MAGNETS

Magneplane Vehicle Levitation Coils

<u>Major Outside Diameter (cm)</u>	<u>Major Inside Diameter (cm)</u>	<u>Minor Outside Diameter (cm)</u>	<u>Minor Inside Diameter (cm)</u>	<u>Thickness (cm)</u>	<u>Total Current (MA)</u>
330	150	240	60	2.5	1.25

Coil Specifications for MAGLEV and TMLV

<u>Vehicle</u>	<u>Coil</u>	<u>Length (cm)</u>	<u>Width (cm)</u>	<u>Current (kAt)</u>
Canadian Maglev	Shield Coil	150	40	-145
	Levitation coil	150	40	500
TMLV	Shield coil	150	50	-125
	Levitation coil	150	50	500

have no option merely to increase the distance between the magnets and the public. The second problem is that vehicle weight is extremely important. We cannot increase the vehicle weight to the point that the magnets are unable to generate enough lift at operating velocities to get the vehicle off the ground.

Active shielding for the magneplane design is complicated by the large quadrupole field near the magnets. This field component is from the vertical current component of the pancake magnet coils. The magneplane design has incorporated smaller compensation coils, placed above the main coils and carrying several percent of the current in the main coils, to reduce the field within the passenger compartment to a maximum of 0.02 T near the floor.¹⁷ Magneplane shielding costs are not computed because the design specifications incorporate shield coils.

The thickness of iron required to shield the magneplane passenger section is about 8 cm over the entire floor. The added weight of the iron shield,

about 10⁷ kg, is several times the loaded vehicle weight with active shielding. Clearly in this case, iron would be unacceptable.

Shielding is also studied for the Canadian Maglev vehicle. Shield coils of the same configuration as the main coils are placed 45 cm above the main coils. Shield coils carrying about 30% of the main coil current decrease the maximum magnetic field within the passenger compartment from 0.05 T to 0.005 T.

About 10100 kg of iron is required to shield the passenger compartment, which is about one-third of the total loaded vehicle weight. Again, the weight penalty in using iron makes it unacceptable.

Researchers in the TMLV program examined shielding for their vehicle design.¹⁹ About 10500 kg of iron is required to shield the interior of the TMLV, or about a fourth of the vehicle weight. This was felt to be unacceptable.

Active shielding of the TMLV vehicle, using a shield coil of the same configuration as the suspension coils and located 30 cm above them, would reduce the passenger compartment field from 0.09 T to 0.03 T at floor level, and from 0.04 T to 0.02 T at seat level. The shield coils carry 25% of the current in the suspension coils. Reitz et al. suggested that the active shielding could be improved by adding small amounts of iron in high field regions to reroute the magnetic flux without adding appreciable weight to the vehicle.²⁰

4. Forces. The forces on the Canadian Maglev vehicle shield coils and levitating coils are a self-generated radial tensional force and an axial body force caused by the magnetic field interaction of the two coils. The radial, axial and total forces on each coil are given in Table III.11. The total force on each shield coil is 5.9×10^4 N (1.3×10^4 lbf). The total force on each suspension coil is 4.7×10^5 N (1.1×10^5 lbf).

The forces on the shield coils and levitating coils of the TMLV vehicle are also given in Table III.11. The total force acting on each shield coil in the TMLV is 13.4×10^4 N (3.0×10^4 lbf). The total force acting on each suspension coil is 40.9×10^4 N (9.2×10^4 lbf).

5. Costs. The shielding costs are determined (see Sec. II.E) on a per-magnet basis. The magnet parameters for determining the costs are given in Table III.12. The costs are given in Table III.13 along with the shielding cost as a percentage of the cost of the unshielded levitation magnet. The active shielding cost is high, over 40% of the levitated magnet cost. The

TABLE III.11

FORCES ACTING ON LEVITATION COILS AND SHIELD COILS
Canadian Maglev Vehicle

Coil	Radial Force		Axial Force		Total Force		Weight of Stainless Steel (kg)
	(10 ³ N)	(10 ³ lbf)	(10 ³ N)	(10 ³ lbf)	(10 ³ N)	(10 ³ lbf)	
Shield coil	24.5	5.5	53.4	12	58.7	13.2	1.7
Levitation coil	467	105	53.4	12	470	105.7	11.9

TMLV

Coil	Radial Force		Axial Force		Total Force		Weight of Stainless Steel (kg)
	(10 ³ N)	(10 ³ lbf)	(10 ³ N)	(10 ³ lbf)	(10 ³ N)	(10 ³ lbf)	
Shield coil	58	13	120	27	133.5	30	4.3
Levitation coil	391.4	88	120	27	409	92	12.3

TABLE III.12

PARAMETERS USED IN FIGURING COSTS

Canadian Maglev Vehicle

	Total Conductor Length (kAm)	Dewar Outer Surface Area (m ²)	Weight of Stainless Steel (kg)
Shield coil	551	2.68	1.7
Levitation coil	1900	3.01	11.9

TMLV

	Total Conductor Length (kAm)	Dewar Outer Surface Area (m ²)	Weight of Stainless Steel (kg)
Shield coil	500	2.77	4.3
Levitation coil	2000	3.17	12.3

iron shield cost per levitation magnet is also listed in Table III.13, and is much lower, 10-13% of the levitation magnet cost. This, however, means little if the added weight of iron prevents efficient vehicle operation. Because of the necessary emphasis upon keeping the vehicle weight to a minimum, active shielding is probably the practical alternative in spite of the added cost.

TABLE III.13

COSTS OF SHIELDING THE CANADIAN MAGLEV VEHICLE

<u>Unshielded (Cost per levitation coil)</u>	<u>amount (\$10² 1977 dollars)</u>	
Conductor costs	49.4	
Dewar costs	9.0	
Structural support costs	1.4	
Total per-magnet cost	59.9	
<u>Active Shield</u>	<u>amount (\$10²)</u>	<u>% increase</u>
Conductor costs	66.7	35
Dewar costs	17.1	89
Structural support costs	1.6	14
Total per-magnet cost	85.4	43

Iron shield cost per magnet is \$606, or an increase in price of 10%.

COST OF SHIELDING THE TMLV

<u>Unshielded (Cost per levitation coil)</u>	<u>amount (\$10² 1977 dollars)</u>	
Conductor costs	52.0	
Dewar costs	9.5	
Structural support costs	1.5	
Total per-magnet cost	63.0	
<u>Active Shield</u>	<u>amount (\$10²)</u>	<u>% increase</u>
Conductor costs	69.7	34
Dewar cost	17.8	87
Structural support cost	2.0	35
Total per-magnet cost	89.5	42

Iron shield cost per magnet is \$790, or an increase in price of 13%.

D. Superconducting AC Generators

1. General Description. Generators are being designed to use superconducting magnets with high magnetic field strengths, 6 or 7 T. Complete interfacing with existing power generation technology is planned, the only change being in the rotor producing the magnetic field. Instead of ordinary conductor rotor windings, the new generators will have superconducting rotor windings, thereby saving on the system power input requirements. The superconducting windings require supercoolant and insulation, with additional equipment for the supercoolant. The rest of the generator and auxiliary equipment, however, will remain the same. This includes the use of iron for field enhancement and containment.

2. Magnetic Fields. One of the design objectives of both conventional and superconducting generators is the magnetic field containment within the stator region, where the changing magnetic field creates ac current, to optimize the current produced. Therefore, the magnetic field strength outside the stator region should be small, if the design is efficient; the amount of flux leakage, or external magnetic field, from a superconducting generator should be no more than from a conventional one.

To determine the flux leakage to be expected from a typical generator of about 5 MW output, field measurements were taken with a portable incremental gaussmeter around the conventional generators at the Los Alamos Scientific Laboratory power plant. The maximum rms ac magnetic field strengths were from 1 to 3 G at the ends of the generator housing near the axis. The field strength in the area of human accessibility was 0.03 G rms or less. Although the maximum interior field strength for these conventional generators is between 1.6 and 1.8 T, roughly one-quarter that of the large superconducting generators, the external fields are so small that even if the flux leakage scaled correspondingly, the external fields would still be less than the most conservative proposed environmental standards.

Thus design efficiency of the superconducting ac generators demands that the external magnetic field strengths be too low to require any additional shielding.

E. Underground Transmission Lines

1. General Description. Several underground transmission line designs are either in use or being developed. One dc current line is being

considered, a superconducting coaxial cable. All the other designs are for ac power transmission.

The ac power transmission lines are generally three-phase lines, consisting of three cables carrying alternating currents 120° out of phase (see Fig. III.8a). Cables made with normal conductor carry 1 kA maximum current. Cryoresistive three-phase lines can carry 4.5 kA,²¹ and a superconducting transmission line can carry up to 13 kA.

Some three-phase lines are constructed by separating the three lines and burying them in different parallel trenches. This results in three one-phase lines travelling parallel to each other under an area (see Fig. III.8b). Each of the single-phase lines carries one of the current components of the three-phase lines described above.

The dc superconducting line being considered is designed as a coaxial cable (see Fig. III.8c), an inner conductor surrounded by an outer conducting cylinder. The inner conductor carries a dc current in one direction while the outer cylinder carries an equal amount of current in the opposite direction.

2. Magnetic Fields. The magnetic fields outside underground transmission lines are found analytically using the formula for the magnetic field around an infinite line of charge (see App. C):

$$B(R) = \frac{I}{5R} \quad (1)$$

in (A,G,cm) units, where I is the current, R is the distance from the wire to the observation point, and $B(R)$ is the total field at that point.

We also use the theorem that the magnetic field outside of a cylindrically symmetric configuration of current is equal to the magnetic field caused by a line carrying the same net current situated along the cylinder axis. For the coaxial cable, the current in the interior conductor is equal and opposite to current in the outer cylinder. Thus, the net current in a coaxial cable is always zero. Since the net current is zero, the field outside the cable, from Eq. (1), is zero. This also holds for ac coaxial cables.

For the three-phase transmission lines, the currents in the three cables can be written:

3-PHASE TRANSMISSION LINE

FIG. III.8a

Cross section of three-phase underground transmission line.

3 ONE-PHASE TRANSMISSION LINES

FIG. III.8b

Cross section of three one-phase lines buried separately.

DC COAXIAL CABLE

FIG. III.8c

Cross section of dc coaxial cable.

$$I_1 = I_0 \sin \omega t ,$$

$$I_2 = I_0 \sin (\omega t - 2/3 \pi) , \text{ and} \quad (2)$$

$$I_3 = I_0 \sin (\omega t + 2/3 \pi) .$$

where I_0 is the amplitude of the ac wave, t is time, and ω is angular frequency. The total current through the cable at any time is then:

$$I_T = I_1 + I_2 + I_3 = 0 . \quad (3)$$

However, in this case there is not cylindrical symmetry, so the total field is:

$$B_T(R) = \frac{I_0}{5} \frac{\sin \omega t}{R + \Delta R_1} + \frac{\sin(\omega t - 2/3 \pi)}{R + \Delta R_2} + \frac{\sin(\omega t + 2/3 \pi)}{R + \Delta R_3} \quad (4)$$

where $\Delta R_1, \Delta R_2, \Delta R_3$ are the appropriate increments of distance due to the different positions of the three wires in the three-phase line (see Fig. III.8a). In the limiting case, where $\Delta R_1 \approx \Delta R_2 \approx \Delta R_3 \ll R$, then (see Eq. (3)):

$$B_T(R) \approx \frac{I_T}{5 R} = 0 . \quad (5)$$

A large, three-phase normal conducting transmission line might be 10 cm across, and carry a maximum current of about 1 kA. Its depth of burial is about 1.4 m. Such a transmission line would create a maximum magnetic field of about 0.6 G at a distance of 0.5 m from the line (in this case, this point is 0.9 m underground), and at ground level, the maximum field is less than 0.1 G.

A cryoresistive three-phase transmission line of the above dimensions will carry about 4.5 kA, and create a maximum field of 0.4 G at the surface.

The three-phase superconducting line now being constructed is composed of three ac coaxial cables. Each of the coaxial cables will have no net current in it, and thus no magnetic fields outside of the cable (see above discussion on coaxial cables).

In the case of the three-phase lines separated and buried in different trenches (see Fig. II.8b), the magnetic fields will be greater than those discussed above. These lines are now normal-conducting, and, as yet there are no plans for constructing cryoresistive cables. Equation (4) gives the field for this configuration, except that in this case, ΔR_1 , ΔR_2 , and ΔR_3 are much larger. Since the distances between the cables vary, only the magnetic field from a single-phase line is examined, as this represents the maximum field from such lines (the other two lines tend to cancel out the field). Assuming three single-phase lines buried in parallel trenches, the maximum field at ground level will be about 1.4 G, assuming a depth of burial²¹ of 75 cm. For a depth of burial of 1.4 m, the maximum field at ground level will be 0.45 G.

3. Shielding. The fields associated with underground transmission lines are too small to require shielding. This is especially true in the case of the coaxial cables and the three-phase transmission lines.

In the case of the three parallel single-phase lines, although an ac field of approximately 1 G will be at ground level right over the lines, the area over which the field will be greater than 0.3 G is small (.7 m on either side of the lines at ground level), and shielding is considered unnecessary.

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APPENDIX A
DOCUMENTATION OF PROGRAM CSYD (COIL SYSTEM DESIGN)

1. Description of Program. CSYD²² is a numerical program written for designing cylindrically symmetric solenoid systems. The program is in two parts. The first section will take a given set of coil restrictions, coil geometry variables, and current variables, and will fit desired magnetic field and coil parameter values within the specified coil limitations. The second section will solve for a desired magnetic field parameter at requested points using either the solution from the first section or a given set of coil geometries and currents.

2. Files Associated with CSYD. Five computer files are associated with CSYD. Two are program decks, one is the data input deck, and the last two are output files.

The two program files are COILSRC and COILRUN. COILSRC is the fortran source deck for CSYD, modified slightly to conform to terminal operation. COILRUN is the binary file created by the Fortran Extended (FTN) compiler from COILSRC.

INFILE is the data input file to CSYD. The file is assigned to logical unit 2 (TAPE 2 = INFILE), and is in card image format. The data and format for INFILE are described in the next section.

Two output files are produced during execution of a problem. OUT1, assigned to logical unit TAPE 3 on the program card, is written for the line printer, with 120 columns available for printout. OUT2, which is assigned to TAPE 4 on the program card, prints out an abridged version of OUT1 on an 80 column terminal printer. The output files are described in more detail below.

3. Input to CSYD -- INFILE. The first data card is designated the start card. The start card defines: 1) the number of coils or symmetric coil pairs in the coil system; 2) if dimensions are in inches or centimeters; 3) the type of magnetic quantity to be fitted; 4) the Cartesian grid of points at which the magnetic quantity is fitted.

Field points are located at every intersection in the grid defined on the start card by the parameters (see below) NZ, NA, DLZ, Z1, DLA, A1. The magnetic quantity fitted at the field points may be any one of six magnetic quantities which can be designated by NFLD (see below). The field grid must always be specified.

A second grid of points axially covering all or part of the field grid may be specified by NV and IV. The second grid is called the gradient grid, and is optional. If the option is used, NV * NA gradient points are defined beginning at the axial value IV on the field grid. The magnetic quantity fitted at the gradient points is designated by the parameter, NGRD.

i. The start card is in the format (A6,10I3,4F9.4)

Name, MK, IN, IP, ID, NGRD, NV, IV, NFLD, NZ, NA, DLZ, Z1, DLA, A1

Dimension MK ≤ 40 , (NZ + NV) * NA ≤ 80

<u>Columns</u>	<u>Description</u>
1-6	Name -- problem identification name
7-9	MK -- the number of coils or symmetrical coil pairs in the system
10-12	IN -- 1 if data in inches, or 0 if in centimeters
13-15	IP -- 0 to print matrices, or 1 not to print matrices
16-18	ID -- Blank (0) - not used
19-21	NGRD -- designates the magnetic quantity to be fitted at the gradient points (see NFLD). This may be blank.
22-24	NV -- the number of axial values in the gradient grid
25-27	IV -- the index of axial value on the field grid where the gradient grid begins
28-30	NFLD -- designates the magnetic quantity to be fitted at the field points.
	NFLD or NGRD = 1, magnetic quantity fitted = B axial (in gauss)
	NFLD or NGRD = 2, magnetic quantity fitted = B radial (gauss)
	NFLD or NGRD = 3, magnetic quantity fitted = vector potential, A (G-cm, G-in)
	NFLD or NGRD = 4, magnetic quantity fitted = B total (gauss)
	NFLD or NGRD = 5, magnetic quantity fitted = radius times vector potential, $r * A$ (G-cm ² , G-in ²)
	NFLD or NGRD = 6, magnetic quantity fitted = partial derivative of B radial with respect to z (G/cm, G/in)
31-33	NZ -- the number of axial values in the field grid
34-36	NA -- the number of radial values in the field grid

37-45 DLZ -- the axial field grid spacing. If the spacing is not uniform, leave this DLZ blank, and the axial grid spacing will be defined by DTZ(I) (see iii. below).

46-54 Z1 -- the axial distance to the first field grid row from the origin

55-63 DLA -- the radial grid spacing

64-72 A1 -- the radial distance to the first field grid column from the origin

Of the above parameters, NFLD, NZ, and NA must always be greater than zero. The others may be left blank.

ii. The second card group is in the format (A4, I2, I6, 2I3, 6F9.4). The total number of cards required in this group is equal to the parameter, MK, specified above.

Name, IC, KGV(N), NMP(N), MPI(N), ZO(K), AO(K), B(K), D(K), CD(K)

Dimension KGV(41), NMP(40), MPI(40), ZO(80), AO(80), B(80), D(80), CD(80)

Although ZO, AO, B, D, and CD are dimensioned (80) in CSYD, the maximum number of cards allowed in this second card group is 40. This allows for 40 coil pairs. Also, the total number of variables must be less than or equal to 40.

<u>Columns</u>	<u>Description</u>
1-4	Name -- for coil identification
5-6	IC -- 1 if a single coil, 0 if a symmetrical coil pair, 2 if an opposed coil pair

Note: Do not mix single coils and coil pairs in the same problem.

7-12 KGV(N) -- designates the geometry variables for the coil

KGV(N) = 0, no geometry variables (can vary current only)

KGV(N) = 1, variable coil position

KGV(N) = 2, variable group position

KGV(N) = 3, variable position - axial field values move with coil

KGV(N) = 4, independently moving axial field values
 KGV(N) = 5, variable length
 KGV(N) = 6, variable length; left, or inner end
 KGV(N) = 7, variable length; right, or outer end
 KGV(N) = 8, variable radius
 KGV(N) = 9, variable radial thickness with mean radius fixed
 KGV(N) = 10, variable radial thickness, inner radius variable
 KGV(N) = 11, variable radial thickness, outer radius variable

A total of three variables may be specified for each coil; for example, KGV(N) = 1510 specifies variable position, variable length, and variable inner radius for coil (N). Two-digit variables are last. Several restrictions hold for these variables. The total of three variables for a coil includes current density, if set as a variable (see CD(K) below). Several of the above geometric variables may not be used in conjunction with each other. Variables 9, 10, and 11; 5, 6, and 7; and 1, 2, and 3, are mutually exclusive. If a gradient grid is specified, there can be no moving field values.

13-15	NMP(N) -- the number of moving axial values [KGV(N) = 3 or 4] or the number of coils moving in a group [KGV(N) = 2]
16-18	MPI(N) -- the index of the first moving axial value; or designates group current density number I, i.e., the current density in coils with MPI = -I is one variable
19-27	ZO(K) -- the mean axial position of the coil from the origin
28-36	AO(K) -- the mean coil radius
37-45	B(K) -- the axial length of the coil
46-54	D(K) -- the radial thickness of the coil
55-63	CD(K) - current density of the coil. If left blank (0.0), then coil current density is a variable in the program

The first coil in a group position block must not be the first coil in a group current block unless the two blocks are identical.

After reading this group of cards, the program returns to CSYD, which calls RWDATA. The RWDATA subroutine reads data card groups iii through v.

iii. The third card group is in format (10F8.4). If DLZ is positive on the start card (group i), then uniform axial spacing has been defined, and

these cards are blank (0.0). One card is required for the first 11 axial values, and for each additional 10 axial values. Then $DTZ(I)$, $I = 1, (NZ - 1)$, is dimensioned $DTZ(80)$.

$DTZ(I)$ -- the grid spacing between axial values (I) and $(I + 1)$

iv. The fourth card group is in format (10F8.4), and specifies the values of the magnetic quantities (see NFLD and NGRD) to be fitted at the field and gradient points. One card is required for each ten field points and for each ten gradient points. $BZS(I)$, $I = 1, [(NZ + NV)*NA]$, is dimensioned $BZS(80)$.

$BZS(1)$ through $BZS(NZ)$ -- the specified values of the magnetic quantity, fitted at the field points, designated by NFLD above, at axial values (1) through (NZ) at radius $A1$.

$BZS(NZ + 1)$ through $BZS(NZ + NV)$ -- the specified values of the magnetic quantity, fitted at the gradient points, designated by NGRD above, at axial values (IV) through $(IV + NV - 1)$ at radius $A1$.

$BZS(NZ + NV + 1)$ through $BZS(2\ NZ + NV)$ -- the specified values of the magnetic quantity fitted at the field points, at axial values (1) through (NZ) at radius $(A1 + D1A)$.

$BZS(2\ NZ + NV + 1)$ through $BZS(2\ NZ + 2\ NV)$ -- the specified values of the magnetic quantity fitted at the gradient points, at axial values (IV) through $(IV + NV - 1)$ at radius $(A1 + D1A)$.

This continues until $[(NZ + NV)*NA]$ points have been specified.

v. The fifth card group is in format (10F8.4). One card is required for each 10 field points and for each 10 gradient points. This card group designates the weight factors assigned to the values of the magnetic quantities given in the fourth card group. $W(I) = 1, [(NZ + NV)*NA]$ and is dimensioned $W(80)$.

W(1) through W(NZ) -- the weight factors for the specified values of the magnetic quantity being fitted at the field points with axial values (1) through (NZ) at radius (A1).

W(NZ + 1) through W(NZ + NV) -- the weight factors for the specified values of the magnetic quantity being fitted at the gradient points with axial values (IV) through (IV + NV - 1) at radius (A1).

W(NZ + NV + 1) through W(2 NZ + NV) -- the weight factors for the specified values of the magnetic quantity being fitted at the field points with axial values (1) through (NZ) at radius (A1 + DLA).

W(2 NZ + NV + 1) through W(2 NZ + 2 NV) -- the weight factors for the specified values of the magnetic quantity being fitted at the gradient points with axial values (IV) through (IV + NV - 1) at radius (A1 + DLA).

This continues until [(NZ + NV)*NA] points have been specified. If [(NZ + NV)*NA] equals the total number of variables, the W(I) values are to be blank (0.0) and the BZS(I) values are fitted exactly.

When [(NZ + NV)*NA] is larger than the total number of variables, the weight factors, W(I), may be specified as desired. Positive weight factors will specify a weighted least squares fit for the corresponding values of BZS(I). An option exists to fit selected values of BZS(I) exactly, by setting W(I) negative for the BZS(I) for which an exact fit is desired. When this option is used, the number of variables plus the number of exactly fitted points must be less than or equal to 40.

After reading the fifth card group, the program completes the optimization part of the calculations (see Sec. 1), except for the "delta" tables (see below). If optimization calculations are not wanted, that is, the coils have been completely specified, and the solution for the resulting fields, or other magnetic quantity, is desired, the first five card groups are given with the following values.

First card group:

NGRD = NV = IV = DLZ = Z1 = DLA = A1 = 0

NZ = NA = 1

Second card group:

KGV(N) = 0, CD(K) is not blank

NMP(N) = MPI(N) = 0

Third card group:

one blank card

Fourth card group:

one blank card

Fifth card group:

one card with 1.0 in columns 1-3

vi. The sixth card group is optional, and in format (A6, 10I3, 4F9.4). These are start cards used to specify new Cartesian grids for calculating the magnetic quantities of the final coil system, and/or delta tables of the magnetic quantities. The delta tables are calculated for inspection of deviations (as a function of Z and R) of a particular magnetic quantity from a selected normalizing value. Delta table (A) is normalized to a selected value (I) of BZS(I), a specified value of the fitted magnetic quantity. Delta table (B) is normalized to final calculated values of the magnetic quantity at a selected axial value on the new grid.

Name, MK, IN, IP, ID, NGRD, NV, IV, NFLD, NZ, NA, DLZ, Z1, DLA, A1

Dimension NZ $\leq$ (80), NA $\leq$ (20), or NZ $\leq$ (160), NA $\leq$ (10)

<u>Columns</u>	<u>Description</u>
1-6	Name -- problem identification
7-9	MK -- blank (0.0) no second group data cards are to be read
10-12	IN -- 1 (units in inches), or 0 (units in centimeters)
13-15	IP -- 0 to print matrices, or 1 not to print matrices
16-18	ID -- blank (0) -- not used
19-21	NGRD -- designates the magnetic quantity used to calculate delta table B

22-24 NV -- the index of the axial value on the new grid used to calculate delta table B. At the points located at this axial value, the calculated values of the magnetic quantity designated by NGRD are to be set as the zero value in delta table B. If NV = 0, delta table B is not calculated

25-27 IV -- the index (I) of the specified value BZS(I) (see fourth card group above) to be the zero value in delta table A. If IV = 0, delta table A is not calculated

28-30 NFLD -- designates the magnetic quantity used to calculate delta table A. NFLD must be the same as either NFLD or NGRD on the first data card

31-33 NZ -- the number of axial values in the new Cartesian grid

34-36 NA -- the number of radial values in the new Cartesian grid

37-45 DLZ -- the axial grid spacing

46-54 Z1 -- the axial distance to the grid from the origin

55-63 DLA -- the radial grid spacing

64-72 A1 -- the radial distance to the grid from the origin

Any number of cards may be in the sixth card group, each defining a new Cartesian grid on which the magnetic quantities for the final coil configuration are computed.

Any number of related or unrelated problems may be solved in succession. After completing the computations for one problem, the program returns to read a new set of data cards for the next problem. A start card with MK not equal to zero initiates a new problem. When no data cards remain, or the last card has the letters STOP in columns 1-4, the run is terminated.

4. Output. Two output files are created during a run; OUT1 and OUT2. This section describes the output in file OUT1. OUT2 output is described briefly in Sec. 5.

There are several types of problems for which CSYD may be used, and the output print depends on which problem type is being solved.

In one problem type, current densities are the only variable. The magnetic quantities are linear functions of the current densities. The output print for this problem type will have pages 1, 3 through 9, described below, and if the delta table option is used, page group 16.

In another problem type, current densities and geometry variables are used. The magnetic quantities are linear functions of the current densities, but nonlinear functions of geometry changes. The output print for this problem type will have pages 1 through 15, and if the delta table option is used, page group 16.

First Page	The input data on the first and second data card groups are printed.
Second Page	The coil system variables designated on the second data card group are printed.
Third Page	The axial grid spacing, specified by either DLZ on the first data card or DTZ(I) on the third data card group, and the Z coordinates at the axial values are printed.
Fourth Page	The specified values of the magnetic quantities on the fourth input card group, and the weight factors for the specified values on the fifth input card group, are printed.
Fifth Page	The coefficient matrix for determining the current density distribution in the coil system is printed. The coefficients in the first column of the matrix are the designated magnetic quantities produced by the first coil in the coil system with unit current density. Successive columns are calculated for each coil in turn, the order being determined by the order in which the cards are read (input card group 2).
Sixth Page	The matrix equation:

$$(1) \quad I(MTRAN * W * M) = B(MTRAN * W)$$

is solved for

$$(2) \quad I = (MTRAN * W * M)^{-1} * B(MTRAN * W)$$

where

M = the coefficient matrix
 $MTRAN$ = the transposed coefficient matrix
 W = the (diagonal) weight factor matrix
 B = the column vector of the specified values of the magnetic quantities
 I = the column vector of the current densities

The matrix $(MTRAN * W * M)^{-1} * (MTRAN * W)$ is printed on the sixth page.

Seventh Page

The current density distribution in the coil system (= the solution to the matrix equation = I) is printed. Two values for the current density in each coil are printed: The first values = I_1 , and the corrected values = I_2 . The corrected values are $I_2 = I_1 * EPS$, where I_1 is determined from equation (2), and EPS = the matrix inversion error = $(MTRAN * W * M)^{-1} * (B(MTRAN * W) - (MTRAN * W * M)I_1)$. If $IN = 1$ on the first data card, the unit of current density is (A/in^2) . If $IN = 0$ the unit of current density is (A/cm^2) . Also printed on this page are the gaps between the ends of the coils and the axial distances to the center of the gaps. Gap number 1 is the axial distance from the right end of coil number 1 to the left end of coil number 2.

Eighth Page

(Coils output printed) - The first line printed is the data on the start card. The next MK lines are the data on the coil cards (input group ii.) and the current density ($CD(K) = I_2$) for each coil or symmetrical coil pair. The values of the magnetic quantities produced by the coil system with current densities I_2 follow.

Ninth Page

The $DEL B$ vector is printed. The $DEL B$ vector equals the values of the fitted magnetic quantities minus the achieved values. Also the sum

of the weighted squares of the delta quantities $SUM = DELB(TRANPOSED) * (M) * DELB$ is printed. This is the last page of output (except for the delta tables) for problems in which the current densities in all the coils are variable and no geometry variables are used.

Tenth Page The coefficient matrix for determining changes in the current densities and geometry is printed.

Eleventh Page The matrix equation $DELI(MTRAN * W * M) = DELB(MTRAN * W)$ is solved for $DELI = (MTRAN * W * M)^{-1} * DELB(MTRAN * W)$ where DELI vector = the changes of current density and geometry. The matrix $(MTRAN * W * M)^{-1} * (MTRAN * W)$ is printed on this page.

Twelfth Page The relaxation factor and the DELI vector are printed. The DELI vector which is printed has been multiplied by the relaxation factor.

Thirteenth Page (Coils output print) - The updated coil system configuration, in which the current densities and geometry have been changed by the value of the DELI'S printed on page 12, and the values of the magnetic quantities for this coil system are printed.

Fourteenth Page The DELB vector and the sum (as explained in ninth page above) are printed. In addition, information related to the linearity test that controls the relaxation factor, and the anticipated final sum are printed. In the program, the iteration procedure now begins and the printout of pages 10 through 14 is repeated until the problem converges or is terminated.

Fifteenth Page The successive sums in the iteration procedure are printed.

The magnetic quantities for the final coil system on a new grid (see input sixth card group) and/or delta tables of the magnetic quantities are printed.

5. Example -- SMES I/O. This is an abbreviated I/O section for a typical application of CSYD to SMES problems. Generally, the coil variables for the SMES coils are known and the information of interest is the field that the coil configuration will produce in a grid area. The field values of interest are typically only B axial, B radial, and B total.

Input: The input procedure is to give all coil parameters and use the optional sixth input card group to determine the field in a specified grid. See Sec. 3 for details of input parameters.

- 1st group: A. Specify name and MK
 B. Specify IP = 1 (do not print matrices)
 C. The program requires that NFLD, NZ and NA be greater than zero so NFLD is usually set = 4, while NZ = NA = 1
 D. All other fields are 0.0

2nd group: For each coil or coil pair

- A. Name and IC are specified
 B. KGV, MPI and NMP are zero
 C. ZO, AO, B, D, CD are specified

3rd group: One blank card

4th group: No field or gradient points were specified so use one card for this group with 0.0 in cols. 1-3.

5th group: No field or gradient points were specified so use one card for this group with 1.0 in cols. 1-3

6th group: For each card (each grid where the field is to be calculated)

- A. Specify a name and IN
 B. Specify MK = 0
 C. Specify IC = 0 (to get coils output)
 D. Specify NFLD = 4
 E. Specify grid points and location (NZ, NA, DEZ, Z1, DLA, A1)

F. $NGRD = MV = IV = 0$

The program was modified so that the output relevant to the problems in this paper can be printed at a time-share terminal. A number of write statements were added to logical unit 4. This unit is associated with file, OUT2. The regular output of CSYD is printed on logical unit 3, which is associated with file OUT1.

Program CSYD (INFILE,OUT1,OUT2,TAPE2=INFILE,TAPE3=OUT1,TAPE4=OUT2)

The output on OUT2 consists of the information on the input cards 1 - 5 plus the value of B at one grid point ($NZ=NA=1$ on first card). Following this is the input on card 1 of group six, followed by B axial, B radial, and B total for each Z value at the first R value. These values are then printed out for the second and consecutive R values. The process is repeated for each card in group six. OUT1 is formatted for the 120 column line printer, while OUT2 is formatted for the 80 column terminal printer.

APPENDIX B

DOCUMENTATION OF PROGRAM MAFCO (MAGNETIC FIELD CODE)

1. Description of Program. MAFCO is designed to take a given set of current-carrying elements of arbitrary geometry, calculate the resulting magnetic field B at any point in space, and plot the magnetic field lines within a given space.²³

Because of limited computer memory and practical considerations of the problems of interest here, the number of loops the program would take was extended, while sacrificing the field line plots. Thus for the purposes of this paper a modified version, P8MAFCO, was used that would not give stereo plots of magnetic field lines.

The code implicitly assumes that currents are not time-varying, that current elements have infinitesimal cross-section, and that no magnetically permeable material is present.

P8MAFCO approximates the desired current configuration by a combination of the following: 1) circular loops with any designated spatial position and orientation 2) circular arcs with any designated spatial position and orientation 3) helices with axes along the z-axis with any designated pitch, starting point, and ending point; 4) straight lines with any designated starting and

ending points; 5) general elements described by a list of points that the code connects with straight lines.

For a given set of current configurations, P8MAFCO calculates the magnetic field $\vec{B}$ (B_R , B_θ , B_z , $|B|$) at points on rectangular grids in vertical planes (planes including the z-axis).

2. Files Associated with P8MAFCO. Eight files are associated with P8MAFCO. Two are program decks, one is an input deck, one is an output deck, two generate input decks for special cases, and the last two are data decks for the input-generating files.

The two program files are P8MAFCO and PBMAFCO. P8MAFCO is the modified fortran source deck, and PBMAFCO is the binary file generated from P8MAFCO by the FTN compiler.

The input file for the code is A. The file is in card image format and assigned to logical unit 10 in the program (TAPE 10=A). Section 3 describes the data and format for file A.

The output is printed on file, DUM. The file, DUM, is assigned to logical unit 9 (TAPE 9=DUM), and is formatted for printout on the 120 column line printer. The output file is described in Sec. 4.

Two input-generating files are available for special applications. The first, FATLOOP, takes circular coils of finite thickness and length, and generates a set of infinitesimal current loops approximating the coils given. The input data deck for FATLOOP is DATA1, and the output file is A. FATLOOP and DATA1 are described in Sec. 5. The second input generating file is FATARC. This file takes circular arcs of finite dimensions, and generates sets of infinitesimal current arcs approximating the arcs given. The input data deck for FATARC is DATA2, and the output file is A. These two data generating decks are independent of each other; however, they can be run on the same problem consecutively. FATARC and DATA2 are described in Sec. 6.

3. Input -- File A. The input file for MAFCO is file A. The units for the the data are centimeters, amperes, and degrees (for angles).

i. first card group (2 cards)

The first two cards are title cards for description of the set of problems, and are in format (12A6). Two cards must be included here.

ii. second card group (1 card)

This card gives the number of problems to be solved during the run. The format is (I5).

The above three cards are given once for each run. The cards described below are repeated for each problem (see second card group).

iii. third card group (3 cards)

This group includes two cards of problem description. The cards are in format (9A8). The third card gives the problem number, and is in format (A8).

iv. fourth card group (1 card)

NLOOP,NARC,NHELIX,NLINE,NGCL,MPNTS

Dimensions NLOOP(2000), NARC(200), NHELIX(50), NLINE(300), NGCL(100), MPNTS(101)

The card is in format (6I5).

<u>Columns</u>	<u>Description</u>
1-5	NLOOP -- total number of loops in problem
6-10	NARC -- total number of arcs
11-15	NHELIX -- total number of helices
16-20	NLINE -- total number of straight lines
21-25	NGCL -- total number of general current element groups
26-30	MPNTS -- total number of grids relative to the z-axis at which fields are to be calculated
v.	fifth card group (NLOOP number of cards)
	These cards give the parameters of the circular current loops, and are in format (4F12.4,2F8.4,E15.5).

<u>Columns</u>	<u>Description</u>
1-12	X
13-24	Y
25-36	Z
	} Coordinates of the loop center
37-48	A -- loop radius
49-56	ALPHA -- Euler angle, α . Angle through which x-axis is rotated (counterclockwise is positive) to new position, ξ . See Fig. B.1.
57-64	BETA -- Euler angle, β . Angle through which loop axis is rotated, about the ξ -axis, from the z-axis to give the new coordinate system (x_1, y_1, z_1) and the desired configuration with

FIG. B.1

Position of circular loop or arc. (a) The (ξ, η, ζ) system is obtained by rotation through the angle α about the z -axis. (b) The (x_1, y_1, z_1) coordinate system (in which the loop or arc lies in the xy -plane with center at the origin) is obtained by rotation through the angle β around the ξ -axis. The arrow I shows the direction for positive current.

respect to the original coordinate system. Positive BETA is counterclockwise looking down the new x-axis, x_1 . See Fig. B.1.

65-79 I -- loop current. Positive current is counterclockwise looking down the new positive loop axis (the z_1 axis).

vi. Sixth card group (2(NARC) number of cards)

These cards describe the arcs. The format of each card is (4F12.4, 2F8.4, E15.5).

Card 1

<u>Column</u>	<u>Description</u>
1-12	X } Y } Z } Coordinates of the arc center of curvature
13-24	
25-36	
37-48	A - arc radius
49-56	ALPHA -- Euler angle, α . Angle through which x-axis is rotated (counterclockwise is positive) to new position, ξ (see Fig. B.1).
57-64	BETA -- Euler angle, β . Angle through which arc axis is rotated about the ξ -axis. BETA is the angle measured from the z-axis to the arc axis. Positive BETA is counterclockwise looking down the new x-axis, x_1 . See Fig. B.1.
65-79	I -- arc current. Positive current is counterclockwise looking down the positive arc axis.

Card 2

1-12	ϕ_1 -- angle in the arc plane measured from the positive x_1 -axis (positive is counterclockwise looking down the arc axis) to the arc beginning (see Fig. B.1).
13-24	ϕ_2 -- angle in the arc plane measured from the positive x_1 -axis (positive is counterclockwise looking down the arc axis), to the end of the arc (see Fig. B.1).

vii. seventh card group (NHELIX number of cards)

These input cards describe the helices in the problem. The format for these cards is (2F12.4, 2F8.4, E15.5, F12.4).

<u>Columns</u>	<u>Description</u>
1-12	A -- helix radius
13-24	D -- distance parallel to the z-axis of 1/2 turn of the helix (see Fig. B.2)
25-32	ϕ_1 -- angle from x-axis to start of helix measured in xy plane (counterclockwise is positive).
33-40	ϕ_2 -- angle to end of helix measured from x-axis in xy plane (counterclockwise is positive) ($\phi_2 > \phi_1$). Note that often ϕ_2 will be much greater than 360° , because it will include all the helix rotations (see Fig. B.2).
41-55	I -- helix current. The current is positive if from ϕ_1 to ϕ_2 .
56-67	z_0 -- distance of the helix origin (start defined by ϕ_1) from the $z = 0$ plane.

viii. eighth card group (NLINE number of cards)

These cards describe the parameters for the straight lines in the problem. The cards are in format (6F11.4, E12.4).

<u>Columns</u>	<u>Description</u>
1-11	X_1
12-22	Y_1
23-33	Z_1
34-44	X_2
45-55	Y_2
56-66	Z_2
67-78	I -- line current [positive if from (X_1, Y_1, Z_1) to (X_2, Y_2, Z_2)].

ix. ninth card group [NGCL number of card groups (1+2)]

These cards describe the general current element groups in the problem. There can be a maximum of 100 general current element groups, containing a maximum total of 2400 points. The first card in each group gives the first point and the total number of points in that group, and is in format (3F12.4, E15.5, I5)

FIG. B.2

Helix concentric about the z-axis in cylindrical coordinate system.

<u>Columns</u>	<u>Description</u>
1-12	X_1 }
13-24	Y_1 } Coordinates of the first point in the general current
25-36	Z_1 } element group.
37-51	I -- current in the element. I is positive if the current is from the first point (X_1, Y_1, Z_1) to consecutively described points $(X_i, Y_i, Z_i); i=2, n$.
52-56	n -- the number of points in the general current element group.

The second card group describes consecutive points along the element, and is in format (3F12.4), for Card i where i goes from 2 to n.

<u>Column</u>	<u>Description</u>
1-12	X_i }
13-24	Y_i } Coordinates of the next point along the general current
25-36	Z_i } element.
x.	the tenth card group (MPNTS number of cards)
	The tenth card group gives the grids on which the fields are calculated. The grids are in planes that include the z-axis, and are specified in cylindrical coordinates (R, θ , Z). The maximum number of cards in this group is (100). The format for these cards is (3F12.4, F8.4, 3F12.4).

<u>Columns</u>	<u>Description</u>
1-12	$R_{\min}$ -- R minimum of grid
13-24	ΔR -- R increment
25-36	$R_{\max}$ -- R maximum of grid
37-44	θ -- angle of rotation of the grid plane from the x-axis in the xy plane (counterclockwise is positive).
45-56	$Z_{\min}$ -- Z minimum of grid
57-68	ΔZ -- Z increment
69-80	$Z_{\max}$ -- Z maximum of grid

If $Z_{\min} = Z_{\max}$, the code computes fields radially from the z-axis. Otherwise it computes fields parallel to the z-axis.

4. Output - File DUM. The output file for P8MAFCO is file DUM. It is formatted for printout on the 120 column line printer. After running a problem, the output file must be disposed to the proper output queues.

All input information in file A is reprinted and labeled to identify the problem. Then, if ΔZ (see tenth card group above) is not zero, columns are printed for "Fields along a line parallel to z-axis":

R θ Z B_Z B_R B_θ | B |.

If $\Delta Z = 0$, columns are printed for "Fields in a radial direction":

Z θ R B_Z B_R B_θ | B |.

For $\Delta Z \neq 0$, the field grids are printed out in tables. The first table gives the magnetic field values for $Z_{\min}$ to $Z_{\max}$ by increments of ΔZ at $R_{\min}$. This continues for consecutive increments of R, up to $R_{\max}$. If $Z_{\min} = Z_{\max}$, the magnetic field values are listed for R from $R_{\min}$ to $R_{\max}$ by increments of ΔR .

5. Data-Generating Files -- FATLOOP and DATA1. FATLOOP generates the file, A, for a problem that includes a circular coil of significant axial length and radial thickness. Other current configurations described above may also be in the problem. DATA1 is the input file for FATLOOP, and A is the output file. FATLOOP prints out the coil input parameters separately. FATLOOP divides the coils specified in DATA1 into a set of infinitesimal current loops, with the appropriate current, arranged in a three-dimensional grid occupying the same space as the original coil. The input coils may have any dimensions and any spatial orientation. The infinitesimal current loop array is then put into the output file, A, in input format for P8MAFCO.

DATA1, the input file for FATLOOP, has the same input format as file A (see Sec. 2 above) for card groups 1-4. The input data for the circular loops have been changed. The first card of the fifth card group gives the number of coils in the problem (NCS). The format for this card is I5.

For each coil there are two cards of description. The format of the first card is (4F15.5,2I5).

<u>Columns</u>	<u>Description</u>
1-15	RO -- inner coil radius in cm
16-30	L -- axial winding length in cm
31-45	H -- radial thickness in cm
46-60	J - current density in A/cm ²
61-65	NL - number of infinitesimal current loops desired in axial direction of coil
66-70	NR - number of infinitesimal current loops desired in radial direction of coil

Due to the dimension limitations in P8MAFCO, $\sum_{i=1}^{i=NCS} NL \cdot NR$

must be less than or equal to 2000.

The second coil input card is in format (5F15.5).

<u>Columns</u>	<u>Description</u>
1-15	ALPHA -- Euler angle, α . Angle through which x-axis is rotated (counterclockwise is positive) to new position, ξ (see Fig. B.1).
16-30	BETA -- Euler angle, β . Angle through which loop axis is rotated, around the ξ -axis, from the z-axis to give the new coordinate system (x_1, y_1, z_1), and the coil's desired configuration with respect to the original coordinate system. Positive BETA is counterclockwise looking down the new x-axis, x_1 (see Fig. B.1).
31-45	X } Coordinates of coil center (on midplane and centerline)
46-60	
61-75	

This section of cards, then, contains a total of $2NCS + 1$ cards. The rest of the card groups in DATA1 are identical in format to card groups 6-10 in File A.

6. Data-Generating Files -- FATARC and DATA2. FATARC generates the input file, A, for a problem that includes a circular arc of finite winding length and radial width. Other current configurations described above may also be in the problem. DATA2 is the input file for FATARC, and A is the output file. FATARC also prints out the arc input parameters in DATA2 separately.

FATARC is a modification of FATLOOP. It divides the arcs specified in DATA2 into arrays of infinitesimal current arcs, with appropriate currents, occupying the same space as the original arc segments. The arc segments may have any dimensions and any spatial orientation. The maximum total number of infinitesimal arcs must be 200 due to the arc dimensions in P8MAFCO. The array of infinitesimal current arcs is put into the output file, A, in the proper format for input to P8MAFCO.

DATA2 has the same input format as file A for card groups 1-5 (see Sec. 2). The sixth card group, which describes the current arcs, is different.

The first card of the sixth card group gives the number of arc segments (NAS) in the problem, and is in format (I5). After this card, two cards of description for each arc segment follow for a total of $(2NAS+1)$ cards in the sixth card group.

The first card of description for each arc segment is in format (7E10.4), and gives X,Y,Z,A,L,H, and J.

<u>Columns</u>	<u>Description</u>
1-10	X } Y } Z } coordinates of arc center (cm)
11-20	
21-30	
31-40	A -- arc inner radius (cm)
41-50	L -- axial winding length (cm)
51-60	H -- radial winding thickness (cm)
61-70	J -- arc current density (A/cm^2). Positive is counterclockwise

The second card of description for each arc segment is in format (4F10.4,2I5), and gives ALF,BET,PHI,PHI2,NL, and NR.

<u>Columns</u>	<u>Description</u>
1-10	ALF -- Euler angle, α . Angle through which x-axis is rotated (counterclockwise is positive) to new position, ξ (see Fig. B.1)
11-20	BET -- Euler angle, β . Angle through which axis of arc is rotated around the ξ -axis. BET is the angle measured from the z-axis to the axis of the arc. Positive BET is counterclockwise looking down the new x-axis, x_1 (see Fig. B.1)

- 21-30 PHI1 -- angle in the arc plane measured from the positive x_1 -axis (positive is counterclockwise looking down the arc axis), to the beginning of the circular arc (see Fig. B.1)
- 31-40 PHI2 -- angle in the arc plane, measured from the positive x_1 -axis (positive is counterclockwise looking down the arc axis), to the end of the circular arc (see Fig. B.1).
- 41-45 NL -- number of infinitesimal current arcs desired along the axial winding length (L)
- 46-50 NR -- number of infinitesimal current arcs desired along the radial winding thickness (H)

The rest of the cards in DATA2 are in the same format as in file A, card groups 7-10.

APPENDIX C

MAGNETIC FIELDS FROM ELECTRIC CURRENTS

The concept of magnetic fields is used to explain the experimentally observed fact that moving electric charges exert forces different from electrostatic forces on one another due to their motion.²⁴ The electric field at a point in space from an electric charge at another point is proportional to the amount of charge, inversely proportional to the distance from the charge to the observation point, and in the direction of the vector joining the two points. The electrostatic force on a second charge placed at the observation point is then the product of its charge and the electric field strength at the observation point caused by the first charge. The magnetic field, however, is proportional to both the magnitude and the velocity of the electric charge (or equivalently electric current) and is in a direction perpendicular to both the distance vector from the observation point to the moving charge and the velocity vector of the charge. If a second charge is introduced into this magnetic field, it experiences no magnetic force unless it is in motion. Then the magnetic force on it will be proportional to its charge and velocity and the magnetic field strength due to the first charge. The direction of the magnetic force on the second charge will be perpendicular to both its velocity and the direction of the magnetic field. All these relationships are governed

by the "right-hand" rule. If the first three digits of the right hand are held mutually perpendicular, and the index finger is assumed to point in the direction of motion of a positive charge with the middle finger pointing from that charge to a position in space, then the thumb will point in the direction of the magnetic field at that position. Further, to define the direction of magnetic force, if the thumb is pointing in the direction of the magnetic field and the middle finger is pointed in the direction of motion of a positively charged particle in that field, the index finger will point in the direction of the resulting magnetic force on that particle (see Fig. I.1 in text). As a consequence, two like charges, or currents, moving with equal (parallel) velocities, experience equal and opposite magnetic forces toward the other particle; if the sign of either the charge or the velocity of one of the particles is reversed, the magnetic forces will also reverse, and tend to separate the particles.

This may be expressed in terms of two current elements I_1 and I_2 with infinitesimal vector lengths $\bar{l}_1$ and $\bar{l}_2$, at positions $\bar{r}_1$ and $\bar{r}_2$, magnetic fields as a function of position r , $\bar{B}_1(r)$ and $\bar{B}_2(r)$ caused by I_1 and I_2 respectively, and the magnetic forces $\bar{F}_1$ and $\bar{F}_2$ on the two current elements:

$$\bar{B}_1(r) = CI_1 \frac{\bar{l}_1 \times (\bar{r} - \bar{r}_1)}{|\bar{r} - \bar{r}_1|^3}, \quad (C-1)$$

$$\bar{B}_2(r) = CI_2 \frac{\bar{l}_2 \times (\bar{r} - \bar{r}_2)}{|\bar{r} - \bar{r}_2|^3}, \quad (C-2)$$

where C is a unit conversion factor, 0.1 G-cm/A. The vector cross product of current and relative position vector is an expression of the right-hand rule stated above. The magnetic forces are:

$$\bar{\mathbf{F}}_1 = I_1 \bar{\boldsymbol{\ell}}_1 \times \bar{\mathbf{B}}_2(\mathbf{r}_1) = CI_1 I_2 \frac{\bar{\boldsymbol{\ell}}_1 \times [\bar{\boldsymbol{\ell}}_2 \times (\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2)]}{|\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2|^3} \quad (\text{C-3})$$

$$\bar{\mathbf{F}}_2 = I_2 \bar{\boldsymbol{\ell}}_2 \times \bar{\mathbf{B}}_1(\mathbf{r}_2) = CI_2 I_1 \frac{\bar{\boldsymbol{\ell}}_2 \times [\bar{\boldsymbol{\ell}}_1 \times (\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_1)]}{|\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_1|^3} \quad (\text{C-4})$$

The constant C is the same as above, but here is more conveniently expressed as 0.01 dyn/A^2 . In general, C varies with the medium; the value given is for vacuum or air. If we assume a separation of 1 cm and charges of 1 coulomb each moving at a speed of 1 cm/sec (equal to the product $I\ell$ of 1 A-cm) perpendicular to their separation, then the magnetic field strength at each charge due to the other is 0.1 G, and the mutual force of magnetic attraction is 0.01 dyn (in addition to the electrostatic force).

Because discrete moving charges produce changing electric fields that affect the magnetic fields, features of magnetic fields are more easily studied using steady currents. Then magnetic fields and forces are the sum, or integral, of these quantities over the length elements composing the current path, which can be assumed to be closed.

Thus

$$\bar{\mathbf{B}}_1(\mathbf{r}) = CI_1 \oint_1 \frac{d\bar{\boldsymbol{\ell}}_1 \times (\bar{\mathbf{r}} - \bar{\mathbf{r}}_1)}{|\bar{\mathbf{r}} - \bar{\mathbf{r}}_1|^3} \quad (\text{C-5})$$

and

$$\bar{\mathbf{F}}_2 = CI_1 I_2 \oint_2 \oint_1 \frac{d\bar{\boldsymbol{\ell}}_2 \times [d\bar{\boldsymbol{\ell}}_1 \times (\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_1)]}{|\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_1|^3} \quad (\text{C-6})$$

where the symbol $\oint_i$ indicates integration around the current path $i = 1$ or 2 , and now the incremental current length is denoted by $d\ell$. If the currents are not infinitesimally thin paths, but are described by a general distribution $\bar{J}_i(r_i)$ A/cm² ($i = 1$ or 2), then the appropriate integrals $I_i \oint_i d\ell_i$ in the above expressions must be replaced by volume integrals over the current distribution $\int \bar{J}_i(r_i) dv_i$.

These expressions are generally too complicated to solve, except by numerical integration, or for simple geometries. However, a useful relation can be derived from charge conservation in the form: $\text{div } \mathbf{J} = 0$. Taking the curl of the above expression for $\mathbf{B}$, with $I d\ell$ replaced by $\mathbf{J} dv$, we find that $\text{curl } \bar{\mathbf{B}} = 4\pi C \bar{\mathbf{J}}$. Integration over a surface S of the normal component of this equation, with the aid of Stokes' theorem,²⁴ gives:

$$\oint_c \bar{\mathbf{B}} \cdot d\bar{\ell} = 4\pi C \int_s \bar{\mathbf{J}} \cdot \bar{\mathbf{n}} da \quad (\text{C-7})$$

where c is the closed path around S and $\mathbf{n}$ is the unit vector normal to S at the area element da . This relation is known as Ampere's circuital law. It states that the total magnetic field along a closed path is $4\pi C$ times the total current flowing through the surface enclosed by that path. If there is symmetry, and the contour integral of $\mathbf{B}$ gives a straightforward geometrical factor, the magnetic field strength can be computed easily. Consider the following two examples.

The first example is a long straight cylindrical wire (not necessarily infinitesimally thin) carrying a total current I . Because of the azimuthal symmetry, the magnitude of the magnetic field at some radial distance r from the center of the wire is independent of the azimuthal angle θ . The magnetic field is always in the θ direction, circling the wire clockwise when viewed in the direction of current flow. Therefore, if we choose as a contour a circle of radius r from the center of the wire, the integral of $\bar{\mathbf{B}} \cdot d\bar{\ell}$ around the contour is $2\pi r B(r)$. The integral of $\bar{\mathbf{J}} \cdot \bar{\mathbf{n}}$ over the surface of the circle enclosed is the total current I , so we have $2\pi r B(r) = 4\pi C I$ from Ampere's circuital law; or

$$B(r) = \frac{2CI}{r} \quad (\text{C-8})$$

The second example is an infinitely long solenoid, wound with N turns per unit axial length of a wire carrying a current I . The magnetic field in the interior of the solenoid is in the axial direction and is constant. The field just outside the solenoid is zero. To find the field inside, we choose a contour along the axis and parallel to the field extending a distance L . The loop is closed by a radial contour extending a distance larger than the solenoid radius and then an axial contour outside the solenoid. Then the integral of $B \cdot d\mathbf{l}$ is just BL . The total current through a surface enclosed by the contour is INL , so Ampere's circuital law gives

$$B = 4\pi CIN \quad (C-9)$$

for the magnetic field inside a solenoid. This may be extended to find the field inside a torus.

In practice current-bearing circuits are seldom very long straight wires or solenoids, so the above results are at best approximations. The computation of the magnetic field resulting from a current distribution generally requires point-by-point numerical integration of the complicated basic integral expressions. However, for certain current configurations, the integrals can be simplified somewhat to facilitate the process. The configurations used in the computer programs (see Apps. A and B) in the present project are (1) straight lines of finite length, (2) circular loops, (3) circular solenoids of finite length and (4) circular arcs.

(1) For straight line current elements the expression for $B(r)$ is a relatively simple one-dimensional integral:

$$B(r) = CI \int_{-\ell/2}^{+\ell/2} \frac{\sin \psi}{d^2} dx \quad (C-10)$$

for a line of length ℓ , where $\sin \psi$ is the angle made by the vector from a line element dx to the point r and the current direction, and d is the magnitude of that vector. Then if $d_{\perp}$ denotes the perpendicular distance from r to the current element or its projection, we have

$$\sin \psi = \frac{d_{\perp}}{d} \quad (C-11)$$

and d is

$$d = \sqrt{d_{\perp}^2 + (r \cos \theta - x)^2} \quad (C-12)$$

where $r \cos \theta$ is the projection of r in the direction of the current element, measured from the center of the line segment (see Fig. C.1). The magnetic field integral thus is

$$B(r) = CI d_{\perp} \int_{-\ell/2}^{+\ell/2} \left[(x - r \cos \theta)^2 + d_{\perp}^2 \right]^{-3/2} dx \quad (C-13)$$

which can be solved exactly to give

$$B(r) = CI \frac{\cos \phi_1 - \cos \phi_2}{d_{\perp}} \quad (C-14)$$

ϕ_1 and ϕ_2 are the angles from the line direction to the vectors from the two ends of the line to r , as in Fig. C.1. This is one of the few cases where an exact algebraic expression for the magnetic field can be obtained.

(2) For a the circular current loop of radius a , the expression for the magnetic field at distance r from the loop center also is a one-dimensional integral, the most convenient parameter of integration being the azimuthal angle ϕ , as $d\ell = a d\phi$. The magnetic field components can be constructed most easily using Cartesian coordinates, the axis of the loop being the z axis and the x axis taken as the direction of the projection of the vector $\vec{r}$ on the loop plane. With θ as the polar angle from the z axis to r , the expressions used in computing the magnetic field B can be written in terms of $\vec{i}$, $\vec{j}$, and $\vec{k}$, the unit vectors in the x , y , and z directions respectively (see Fig. C.2).

$$d\vec{\ell} = -a \sin \phi \vec{i} + a \cos \phi \vec{j} \quad (C-15)$$

FIG. C.1

Parameters for calculating the magnetic field from a straight segment of current.

FIG. C.2

Parameters for computing the magnetic field of a circular loop of current.

$$\vec{d} = (r \sin \theta - a \cos \phi) \vec{i} - a \sin \phi \vec{j} + r \cos \theta \vec{k}. \quad (\text{C-16})$$

Therefore

$$d\vec{\ell} \times \vec{d} = \left[ar \cos \theta \cos \phi \vec{i} + ar \cos \theta \sin \phi \vec{j} + (a^2 - ar \sin \theta \cos \phi) \vec{k} \right] d\phi \quad (\text{C-17})$$

and

$$d = \sqrt{r^2 + a^2 - 2ra \sin \theta \cos \phi}. \quad (\text{C-18})$$

Thus the magnetic field component integrals are

$$B_x = 2 CI \int_0^\pi \frac{ar \cos \theta \cos \phi d\phi}{d^3} \quad (\text{C-19})$$

$$B_y = 0 \quad (\text{C-20})$$

$$B_z = 2 CI \int_0^\pi \frac{(a^2 - ar \sin \theta \cos \phi) d\phi}{d^3}. \quad (\text{C-21})$$

These can be expressed as complete elliptic integrals of the first and second kind,²⁵ and can be computed numerically.

(3) A three-dimensional solenoid of rectangular cross section can be handled by extending the expressions obtained for a circular current loop. If the length of the solenoid is ℓ and its radial thickness is Δa , the current I must be replaced by integrals over the current density:

$$I \rightarrow \frac{I}{\ell \Delta a} \int_{-\ell/2}^{+\ell/2} dz \int_{a - \Delta a/2}^{a + \Delta a/2} d\alpha \quad (\text{C-22})$$

In terms of the parameters used for the circular loop, the distance vector $\bar{d}$ from an element of current to the field point r is now

$$\bar{d} = (r \sin \theta - \alpha \cos \phi) \bar{i} - \alpha \sin \phi \bar{j} + (r \cos \theta - z) \bar{k} \quad (C-23)$$

so

$$\begin{aligned} d &= \sqrt{r^2 \sin^2 \theta + \alpha^2 - 2r\alpha \sin \theta \cos \phi + (r \cos \theta - z)^2} \\ &= \sqrt{r^2 \sin^2 \theta \sin^2 \phi + (\alpha - r \sin \theta \cos \phi)^2 + (z - r \cos \theta)^2} \end{aligned} \quad (C-24)$$

and the magnetic field components can be written as

$$B_x = C \frac{I}{\ell \Delta a} \int_{-\ell/2}^{+\ell/2} dz \int_{a - \Delta a/2}^{a + \Delta a/2} d\alpha \int_0^\pi d\phi \frac{\alpha(r \cos \theta - z) \cos \phi}{d^3} \quad (C-25)$$

$$B_y = 0 \quad (C-26)$$

$$B_z = C \frac{I}{\ell \Delta a} \int_{-\ell/2}^{+\ell/2} dz \int_{a - \Delta a/2}^{a + \Delta a/2} d\alpha \int_0^\pi d\phi \frac{(\alpha^2 - \alpha r \sin \theta \cos \phi)}{d^3} \quad (C-27)$$

The integrals over z and α can be done analytically, but the results are complicated. Integration over the angle ϕ again gives complete elliptic integrals of the first and second kind. The program CSYD (App. A) does the integrations for given sets of solenoid dimensions.

(4) The expressions for the magnetic field from a circular arc of current are similar to those for a circular loop, the difference being that the integration over the azimuthal angle is from ϕ_1 to ϕ_2 rather than from 0 to 2π , ϕ_1 representing the azimuthal angle from the projection of the vector $\bar{r}$ onto the plane of the arc to the start of the arc (current source) and ϕ_2 the angle to the end of the arc (current sink). Because there is now no cylindrical symmetry, B_y does not vanish; however, it can be calculated

analytically. The magnetic field component expressions for a circular arc of radius a can be written directly from the expressions for a circular loop:

$$B_x = CI \int_{\phi_1}^{\phi_2} \frac{ar \cos \theta \cos \phi d \phi}{d^3} \quad (C-28)$$

$$B_y = CI \int_{\phi_1}^{\phi_2} \frac{ar \cos \theta \sin \phi d \phi}{d^3} = CI \cot \theta \left(\frac{1}{d_1} - \frac{1}{d_2} \right) \quad (C-29)$$

$$B_z = CI \int_{\phi_1}^{\phi_2} \frac{(a^2 - ar \sin \theta \cos \phi) d \phi}{d^3} \quad (C-30)$$

where d_1 and d_2 are the distances from the field point r to the beginning and end of the arc, respectively. B_x and B_z can be expressed as incomplete elliptic integrals of the first and second kind. The program P8MAFCO (App. B) does these integrals for given sets of arc parameters.

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16. Abstract The peripheral magnetic fields of several energy-related technologies are calculated, and shielding options are studied for three field intensities as possible exposure levels: 200 G (interim exposure limit at research institutes), 10 G (field reported to affect pacemaker), and 0.3 G (average value of earth's magnetic field). Seven fusion reactor designs are studied. For a 200-G field level, shielding is not required. For the 10- and 0.3-G levels, land is the most economical shielding method, with shield coils an acceptable alternative at 0.3 G. Superconducting ac generators and underground transmission lines require no shielding The superconducting magnetic energy storage coil requires no shielding for 200 G. Both a shield coil and land are needed to meet 10 G or 0.3 G. The MHD generator needs no shielding to 200 G and 10 G. Land is the most economical means of meeting the 0.3-G level. Most of the magnetically levitated vehicles require no shielding to 200 G. The field on-board can be reduced to 200-25 G, depending upon the vehicle design, with shield coils. The use of iron, or another permeable material, is necessary to reduce the field to 10 G or 0.3 G. However, iron introduces too much added weight to allow efficient operation.			
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