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AN OPTICAL FIBER

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Polarization Modulated Solitary Waves in an Optical Fiber

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Abstract

A family of solitary waves is found which satisfy the coupled nonlinear Schrödinger equations governing optical pulse propagation in an elliptical core optical fiber. These are polarization modulated pulses which are bound states of two solitons which when decoupled have constant and uniform orthogonal linear polarizations. The formation of these bound states is possible only above a minimum energy threshold determined by the ellipticity of the fiber.

Keywords: optical fiber, solitons, polarization

1. Introduction

The nonlinear Schrödinger equation [1] describes pulses propagating in an optical fiber under the influence of group velocity dispersion and third order nonlinearity, but with constant and uniform polarization. It was shown to be integrable by the inverse scattering transform by Zakharov and Shabat [2]. In an elliptical core optical fiber taking into account both transverse polarizations and allowing them to evolve according to Maxwell's equations one obtains a set of coupled nonlinear Schrödinger equations. In the usual slowly varying amplitude and rotating wave approximations one obtains [3],[4]

$$\begin{aligned} i\frac{\partial E_x}{\partial t} + \frac{\partial^2 E_x}{\partial z^2} + \alpha^2 E_x + a(|E_x|^2 + |E_y|^2)E_x + b(E_x^2 + E_y^2)E_x^* &= 0, \\ i\frac{\partial E_y}{\partial t} + \frac{\partial^2 E_y}{\partial z^2} - \alpha^2 E_y + a(|E_x|^2 + |E_y|^2)E_y + b(E_x^2 + E_y^2)E_y^* &= 0. \end{aligned} \quad (1)$$

where E_x and E_y are the slowly varying electric field amplitudes in the transverse directions and z is the propagation axis. The coefficient α^2 describes the ellipticity of the optical fiber and is zero for a perfectly circular fiber. This ellipticity is essential for the existence of the polarization modulated solitary waves discussed here. The nonlinear coefficients a and b are real and positive and the intensity has been rescaled so that $a + b = 1$.

The special case $\alpha = 0, b = 0$ is also integrable by the inverse scattering transform [5]. However when $\alpha = 0$ and $b \neq 0$ they fail the Painlevé integrability test [6] and so equations (1) are probably nonintegrable. If either of the fields E_x or E_y vanishes the remaining equation is equivalent to the scalar nonlinear Schrödinger equation. Thus each field in the absence of the other exhibits all the N soliton solutions and multisoliton bound states of that equation. The single solitons corresponding to these decoupled cases are given by

$$\begin{aligned} E_x(z, t) &= \sqrt{2}A \exp\{i[(A^2 + \alpha^2 - V^2/4)t + Vz/2 + \psi_0]\} \operatorname{sech}[A(z - Vt - \xi_0)], \\ E_y(z, t) &= 0, \end{aligned} \quad (2a)$$

$$\begin{aligned} E_x(z, t) &= 0, \\ E_y(z, t) &= \sqrt{2}A \exp\{i[(A^2 - \alpha^2 - V^2/4)t + Vz/2 + \psi_0]\} \operatorname{sech}[A(z - Vt - \xi_0)], \end{aligned} \quad (2b)$$

where A is proportional to the integrated intensity, V is the velocity, ξ_0 is the location of the maximum, and ψ_0 is an arbitrary phase.

Numerical work on this system [3] has found that for some parameter domains one of the decoupled solitons (2) is unstable and after shedding radiation evolves into a nondispersive pulse in which the energy is exchanged between the two fields. In this paper we construct solutions of the coupled nonlinear Schrödinger equations (1) which are bound states of the two decoupled solitons (2), and which appear to be those that are numerically observed.

2. Hirota method

We begin by assuming that the two fields are phase locked as follows

$$E_x(z, t) = \exp[iA^2 t + iV(z - Vt/2)/2 + i\psi_0] E_x(z - Vt), \quad (3)$$

$$E_y(z, t) = \exp[iA^2 t + iV(z - Vt/2)/2 + i\psi_0] E_y(z - Vt),$$

where A is a real constant and $E_x(z - Vt)$ and $E_y(z - Vt)$ are real. Substituting this into equations (1) gives

$$\begin{aligned} \frac{\partial^2 E_x}{\partial \xi^2} + (E_x^2 + E_y^2)E_x &= A_1^2 E_x, \\ \frac{\partial^2 E_y}{\partial \xi^2} + (E_x^2 + E_y^2)E_y &= A_2^2 E_y, \end{aligned} \quad (4)$$

where we have defined

$$\xi \equiv z - Vt, \quad A_1^2 \equiv A^2 - \alpha^2, \quad A_2^2 \equiv A^2 + \alpha^2, \quad (5)$$

and we are assuming $A^2 \geq \alpha^2$ so that A_1 and A_2 are both real. This restriction on A imposes a minimum energy threshold for the formation of these bound states.

To solve equations (4) we use the Hirota method by first making a change of dependent variables from the electric field components $E_x(\xi)$ and $E_y(\xi)$ to the functions $F(\xi)$, $G(\xi)$, and $H(\xi)$, where

$$E_x(\xi) = \frac{G(\xi)}{F(\xi)}, \quad E_y(\xi) = \frac{H(\xi)}{F(\xi)}, \quad (6)$$

and $F(\xi)$ satisfies the relation

$$E_x^2(\xi) + E_y^2(\xi) = 2 \frac{\partial^2 \ln F(\xi)}{\partial \xi^2}. \quad (7)$$

From (4), (6), and (7) we obtain the following coupled equations for $F(\xi)$, $G(\xi)$, and $H(\xi)$

$$\begin{aligned} F \frac{\partial^2 G}{\partial \xi^2} - 2 \frac{\partial F}{\partial \xi} \frac{\partial G}{\partial \xi} + G \frac{\partial^2 F}{\partial \xi^2} &= A_1^2 F G, \\ F \frac{\partial^2 H}{\partial \xi^2} - 2 \frac{\partial F}{\partial \xi} \frac{\partial H}{\partial \xi} + H \frac{\partial^2 F}{\partial \xi^2} &= A_2^2 F H, \\ G^2 + H^2 &= 2 F^2 \frac{\partial^2 \ln F}{\partial \xi^2}. \end{aligned} \quad (8)$$

and then we look for solutions in the form of power series in a parameter ϵ ,

$$F = 1 + \sum_{n=1}^{\infty} \epsilon^n F^{(n)}, \quad G = \sum_{n=1}^{\infty} \epsilon^n G^{(n)}, \quad H = \sum_{n=1}^{\infty} \epsilon^n H^{(n)}. \quad (9)$$

Substituting (9) into (8) we deduce relations connecting the different $F^{(n)}$, $G^{(n)}$, and $H^{(n)}$ at each power of ϵ . At first order we obtain

$$\frac{\partial^2 F^{(1)}}{\partial \xi^2} = 0, \quad \frac{\partial^2 G^{(1)}}{\partial \xi^2} = A_1^2 G^{(1)}, \quad \frac{\partial^2 H^{(1)}}{\partial \xi^2} = A_2^2 H^{(1)}, \quad (10)$$

for which we consider the following solution

$$F^{(1)} = 0, \quad G^{(1)} = 2\sqrt{2} A_1 \exp(\theta_1), \quad H^{(1)} = 2\sqrt{2} A_2 \exp(\theta_2), \quad (11)$$

where we have defined

$$\theta_1(z, t) \equiv A_1(z - Vt - \xi_{10}), \quad \theta_2(z, t) \equiv A_2(z - Vt - \xi_{20}), \quad (12)$$

with ξ_{10} and ξ_{20} arbitrary constants. To second order in ϵ , taking into account (11) we find

$$\begin{aligned} \frac{\partial^2 F^{(2)}}{\partial \xi^2} &= \frac{1}{2} G^{(1)} G^{(1)} + \frac{1}{2} H^{(1)} H^{(1)}, \\ \frac{\partial^2 G^{(2)}}{\partial \xi^2} &= A_1^2 G^{(2)}, \quad \frac{\partial^2 H^{(2)}}{\partial \xi^2} = A_2^2 H^{(2)}, \end{aligned} \quad (13)$$

which gives

$$F^{(2)} = \exp(2\theta_1) + \exp(2\theta_2), \quad G^{(2)} = 0, \quad H^{(2)} = 0. \quad (14)$$

At third order in ϵ , taking into account (11) and (14) we obtain

$$\begin{aligned}\frac{\partial^2 F^{(3)}}{\partial \xi^2} &= 0, \\ \frac{\partial^2 G^{(3)}}{\partial \xi^2} + F^{(2)} \frac{\partial^2 G^{(1)}}{\partial \xi^2} - 2 \frac{\partial F^{(2)}}{\partial \xi} \frac{\partial G^{(1)}}{\partial \xi} + G^{(1)} \frac{\partial^2 F^{(2)}}{\partial \xi^2} &= A_1^2 \left(G^{(3)} + F^{(2)} G^{(1)} \right), \\ \frac{\partial^2 H^{(3)}}{\partial \xi^2} + F^{(2)} \frac{\partial^2 H^{(1)}}{\partial \xi^2} - 2 \frac{\partial F^{(2)}}{\partial \xi} \frac{\partial H^{(1)}}{\partial \xi} + H^{(1)} \frac{\partial^2 F^{(2)}}{\partial \xi^2} &= A_2^2 \left(H^{(3)} + F^{(2)} H^{(1)} \right),\end{aligned}\tag{15}$$

for which we take the solution

$$\begin{aligned}F^{(3)} &= 0, \\ G^{(3)} &= 2\sqrt{2} A_1 \left(\frac{A_1 - A_2}{A_1 + A_2} \right) \exp(\theta_1 + 2\theta_2), \\ H^{(3)} &= -2\sqrt{2} A_2 \left(\frac{A_1 - A_2}{A_1 + A_2} \right) \exp(\theta_2 + 2\theta_1),\end{aligned}\tag{16}$$

and in a similar manner at fourth order we have

$$\begin{aligned}\frac{\partial^2 F^{(4)}}{\partial \xi^2} + F^{(2)} \frac{\partial^2 F^{(2)}}{\partial \xi^2} - \frac{\partial F^{(2)}}{\partial \xi} \frac{\partial F^{(2)}}{\partial \xi} &= G^{(1)} G^{(3)} + H^{(1)} H^{(3)}, \\ \frac{\partial^2 G^{(4)}}{\partial \xi^2} &= A_1^2 G^{(4)}, \quad \frac{\partial^2 H^{(4)}}{\partial \xi^2} = A_2^2 H^{(4)},\end{aligned}\tag{17}$$

with the solution

$$F^{(4)} = \left(\frac{A_1 - A_2}{A_1 + A_2} \right)^2 \exp(2\theta_1 + \theta_2), \quad G^{(4)} = 0, \quad H^{(4)} = 0.\tag{18}$$

At this point we assume the series can be truncated, that is, we postulate a solution with all the higher orders set to zero. Then putting $\epsilon = 1$ we have

$$\begin{aligned}F(z) &= 1 + \exp(2\theta_1) + \exp(2\theta_2) + \left(\frac{A_1 - A_2}{A_1 + A_2} \right)^2 \exp(2\theta_1 + 2\theta_2), \\ G(z) &= 2\sqrt{2} A_1 \exp(\theta_1) \left[1 + \left(\frac{A_1 - A_2}{A_1 + A_2} \right) \exp(2\theta_2) \right], \\ H(z) &= 2\sqrt{2} A_2 \exp(\theta_2) \left[1 - \left(\frac{A_1 - A_2}{A_1 + A_2} \right) \exp(2\theta_1) \right].\end{aligned}\tag{19}$$

Substituting (19) into equations (8) verifies that this is indeed a solution, thus justifying our assumption. The electric field amplitudes according to (3) and (6) are then

$$E_x(z, t) = 2\sqrt{2}A_1 \exp\{iA^2t + iV(z - Vt/2)/2 + i\psi_o + \theta_1\} \left[1 + \left(\frac{A_1 - A_2}{A_1 + A_2} \right) \exp(2\theta_2) \right] \\ \times \left[1 + \exp(2\theta_1) + \exp(2\theta_2) + \left(\frac{A_1 - A_2}{A_1 + A_2} \right)^2 \exp(2\theta_1 + 2\theta_2) \right]^{-1}. \quad (20)$$

$$E_y(z, t) = 2\sqrt{2}A_2 \exp\{iA^2t + iV(z - Vt/2)/2 + i\psi_o + \theta_2\} \left[1 - \left(\frac{A_1 - A_2}{A_1 + A_2} \right) \exp(2\theta_1) \right] \\ \times \left[1 + \exp(2\theta_1) + \exp(2\theta_2) + \left(\frac{A_1 - A_2}{A_1 + A_2} \right)^2 \exp(2\theta_1 + 2\theta_2) \right]^{-1}.$$

which vanish as $\xi \rightarrow \pm\infty$. These are a five parameter ($\psi_o, V, A, \xi_{10}, \xi_{20}$) family of polarization modulated solitary waves. The meaning of the first two parameters is clear; the remaining three are interpreted by considering some special cases.

3. Special and limit cases

Taking the limit $\xi_{20} \rightarrow \infty$ with ξ_{10} constant in (20) gives

$$E_x(z, t) = \sqrt{2}A_1 \exp\{i[(A_1^2 + \alpha^2 - V^2/4)t + Vz/2 + \psi_o]\} \text{sech}[A_1(z - Vt - \xi_{10})], \\ E_y(z, t) = 0, \quad (21)$$

which is just the first decoupled soliton (2a) with amplitude A_1 and position of maximum ξ_{10} . Taking the limit $\xi_{10} \rightarrow \infty$ with ξ_{20} constant we find

$$E_x(z, t) = 0, \\ E_y(z, t) = \sqrt{2}A_2 \exp\{i[(A_2^2 - \alpha^2 - V^2/4)t + Vz/2 + \psi_o]\} \text{sech}[A_2(z - Vt - \xi_{20})], \quad (22)$$

which is the other decoupled soliton (2b) with amplitude A_2 and position of maximum ξ_{20} .

In the limit $\xi_{20} \rightarrow -\infty$ with constant ξ_{10} we obtain

$$E_x(z, t) = \sqrt{2}A_1 \exp\{i[(A_1^2 + \alpha^2 - V^2/4)t + Vz/2 + \psi_o + \pi]\} \text{sech}[A_1(z - Vt - \xi_{10} - \Delta)], \\ E_y(z, t) = 0, \quad (23)$$

which is again (2a) but with a phase shift of π and a displacement of Δ in the position of the maximum, where

$$\Delta = -\frac{1}{A_1} \ln \left[\frac{A_2 - A_1}{A_2 + A_1} \right]. \quad (24)$$

Similarly in the limit $\xi_{10} \rightarrow -\infty$ with ξ_{20} constant we find

$$E_x(z, t) = 0, \quad (25)$$

$$E_y(z, t) = \sqrt{2} A_2 \exp\{i[(A_2^2 - \alpha^2 - V^2/4)t + Vz/2 + \psi_0]\} \text{sech}[A_2(z - Vt - \xi_{20} - \Delta')],$$

where

$$\Delta' = -\frac{1}{A_2} \ln \left[\frac{A_2 - A_1}{A_2 + A_1} \right], \quad (26)$$

which is again the other decoupled soliton (2b) with amplitude A_2 and phase shift Δ' . It is then clear that (20) describes a bound state of the two individual solitons (2). The parameters A_1 and A_2 are the amplitudes, and ξ_{10} and ξ_{20} are the positions, of the constituent solitons.

Let us consider some special cases. If we set

$$A^2 = 5\alpha^2/3, \quad \xi_{10} = -(3/2)^{\frac{1}{2}} \frac{1}{2\alpha} \ln 3 + \xi_0, \quad \xi_{20} = -(3/2)^{\frac{1}{2}} \frac{1}{4\alpha} \ln 3 + \xi_0, \quad (27)$$

we obtain the solution

$$\begin{aligned} E_x(z, t) = & -2\alpha \exp[i(5\alpha^2 t/3 - V^2 t/4 + Vz/2 + \psi_0)] \\ & \times \text{sech}[(2/3)^{\frac{1}{2}} \alpha(z - Vt - \xi_0)] \tanh[(2/3)^{\frac{1}{2}} \alpha(z - Vt - \xi_0)]. \end{aligned} \quad (28)$$

$$E_y(z, t) = 2\alpha \exp[i(5\alpha^2 t/3 - V^2 t/4 + Vz/2 + \psi_0)] \text{sech}^2[(2/3)^{\frac{1}{2}} \alpha(z - Vt - \xi_0)],$$

which surprisingly has a hyperbolic secant squared intensity profile

$$|E_x(z, t)|^2 + |E_y(z, t)|^2 = 4\alpha^2 \text{sech}^2[(2/3)^{\frac{1}{2}} \alpha(z - Vt - \xi_0)]. \quad (29)$$

At the threshold value $A^2 = \alpha^2$ we obtain a single decoupled soliton

$$E_x(z, t) = 0, \quad (30)$$

$$E_y(z, t) = 2\alpha \exp[i(\alpha^2 t - V^2 t/4 + Vz/2 + \psi_0)] \text{sech}[\sqrt{2}\alpha(z - Vt - \xi_{20})],$$

In the limit $\alpha \rightarrow 0$, $A_1 \rightarrow A_2 = A$, and we obtain

$$\mathbf{E}(z, t) = \sqrt{2} A \hat{\mathbf{e}} \exp[i(A^2 t - V^2 t/4 + Vz/2 + \psi_0)] \text{sech}[A(z - Vt - \xi_0)], \quad (31)$$

where $\hat{\mathbf{e}}$ is an arbitrary unit vector lying in the transverse plane. Apart from this unit vector (31) is simply a single soliton of the scalar nonlinear Schrödinger equation.

Returning to the general bound state solution (20) we deduce from (6), (8) and the first of (19)

$$\int_{-\infty}^{\infty} d\xi (|E_x|^2 + |E_y|^2) = 4(A_1 + A_2) = 4[(A^2 - \alpha^2)^{\frac{1}{2}} + (A^2 + \alpha^2)^{\frac{1}{2}}], \quad (32)$$

and so these are finite energy pulses, and since these solutions exist only for $A^2 \geq \alpha^2$ we find a minimum energy threshold

$$\int_{-\infty}^{\infty} d\xi (|E_x|^2 + |E_y|^2) \geq 4\sqrt{2}\alpha. \quad (33)$$

It is also interesting to note that, loosely speaking, (4) can be viewed as a quantum mechanical problem for a bounded potential $-(E_x^2 + E_y^2)$. In this interpretation $E_x(\xi)$ and $E_y(\xi)$ are energy eigenfunctions with eigenvalues $-A_1^2$ and $-A_2^2$, respectively. Then in the usual manner one can deduce the orthogonality relation

$$\alpha^2 \int_{-\infty}^{\infty} d\xi E_x(\xi) E_y(\xi) = 0, \quad (34)$$

and so this integral vanishes for $\alpha \neq 0$.

The intensity profile described by (20) can exhibit interesting forms, depending on the parameters, including a double peak structure. This is easily understood since these solutions are a superposition of two solitons with possibly different locations. At the trailing edge the polarization lies along the principal axis of the elliptical core optical fiber defined by $\hat{\mathbf{x}}$, and as one moves through the pulse the plane of polarization rotates until at some point it lies along the other principal axes defined by $\hat{\mathbf{y}}$. Proceeding through the pulse the rotation continues until at the leading edge the polarization is once again along the first principal axis. That is, the linear polarization rotates through one full cycle of 180° from one edge of the pulse to the other. In figures (a)–(d) are depicted intensity and

polarization profiles for $A_2 = 2A_1 = 2$ and $\xi_{10} = -\xi_{20} \equiv \delta$, with differing values for δ . The solid lines indicate the intensity profiles while the dotted lines the angle of rotation from the $\hat{x}$ principal axis. In figure (a), where $\delta = 3.00$, the two constituent solitons are widely separated. The polarization is essentially constant throughout each pulse but orthogonal for the two pulses, lying along each of the two principal axes. Only in the wings is the polarization modulated, and as discussed previously goes through a full cycle as ξ ranges from $-\infty$ to $+\infty$. In figure (b), on a different horizontal scale, the separation is decreased to $\delta = 0.90$. The constituent solitons begin to overlap and the polarization becomes modulated in this overlap region. In figure (c), with the separation $\delta = 0.30$, one still observes the double peak structure but a smoother polarization modulation. The last figure, with $\delta = -0.14$, which is one of the special cases discussed previously, exhibits a symmetric intensity profile and a kink-like polarization modulation.

4. Discussion

We have obtained a family of bound state solitary waves for the coupled nonlinear Schrödinger equations governing optical pulse propagation in an elliptical core optical fiber. These solutions are valid above a certain threshold value $A^2 \geq \alpha^2$. This suggests that a bifurcation takes place at $A^2 = \alpha^2$ and that below this value there exists a qualitatively different family of analytic solutions.

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Figure Caption

Solid lines are intensity profiles and dotted lines are the angle of rotation of the linear polarization. (a) $\delta = 3.00$, (b) $\delta = 0.90$, (c) $\delta = 0.30$, (d) $\delta = -0.14$.