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SCALED PHYSICAL MODEL STUDIES OF THE
STEAM DRIVE PROCESS

First Annual Report, September 1977 – September 1978

Work Performed for the Department of Energy
Under Contract DE-AT03-77ET12075

Date Published—December 1980

University of Southern California
Los Angeles, California

U. S. DEPARTMENT OF ENERGY

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STEAM DRIVE PROCESS

First Annual Report
For The Period
September 1977 – September 1978

T. M. Doscher, *Principal Investigator*
University of Southern California
Department of Petroleum Engineering
Los Angeles, California 90007
213/741-2066

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H. J. Lechtenberg, *Technical Project Officer*
San Francisco Operations Office
Fossil Energy Division
1333 Broadway
Oakland, California 94612
415/273-7951

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THE PROJECT

The project on the design, construction and operation of scaled physical models of the steam drive in viscous oil reservoirs was initiated in September 1977 at the University of Southern California. The project is sponsored by the Department of Energy, Oakland Operations Office, and has been carried out in the Department of Petroleum Engineering.

Scaled physical models of the steam drive operation have been used in some of the laboratories of the major oil operators, but have not been generally available to others. The goal of the project is to develop a system for scaling the steam drive that can provide insight to a wide audience of operators and engineers of the influence of the many parameters that control the oil recovery and the important economic criterion, the oil steam ratio.

During the first year of the program Messrs. Woontsing Wally Huang and Shie Wu Rhee, both candidates for the doctorate in petroleum engineering, were the senior students employed by the project under the supervision of Professor T.M. Doscher.

This report summarizes the work done during the first year of the project on the design and construction of the model. During the

second year, the model will be put into regular service to study the role of various parameters on the performance of the steam drive operation in a model whose parameters approach those of current steam drive operations in the Kern River Field, Kern County, California.

ABSTRACT

Scaling laws of the heat transport mechanism in steam displacement processes are developed based upon an integral energy balance equation. Unlike the differential approach adopted by previous workers, the above scaling laws do not necessitate the use of any empirical correction factor as has been done in previous scaling calculations. The results provide a complete and consistent scale-down of the energy transport behavior, which is the critical mechanism for the success of a steam injection process. In the course of the study, the scaling problems associated with relative permeability and capillary pressure are also discussed. A method which has often been used in scaling non-thermal displacement processes is applied to reduce errors due to scaling in relative permeability. Both dimensional and inspectional analyses are applied to illustrate their use in steam processes. Scale-up laws appeared in the literature and those used in this study are compared and numerical examples are given.

CHAPTER I

INTRODUCTION

Scaled physical models have been extensively used in engineering problems for many years to reproduce the behavior of an actual system on a laboratory scale. Their importance has been demonstrated especially for new processes whose mechanisms are not well understood or a mathematical description is difficult to formulate. A large number of bench-scale models have also been successfully developed to model multiphase fluid flow behavior, either isothermal or non-isothermal, in petroleum reservoirs.

Unscaled laboratory experiments may be misleading when applied to field operations because some variables in laboratory experiments may be unduly magnified; while others, equally important, may be suppressed. Although, in recent years, sophisticated numerical simulators, have been built with efficient and reliable computational schemes, and electronic computers with unprecedented high speed and huge storage capacity have become available; they still suffer from computer time and storage and it is not easy for numerical simulators to give detailed descriptions of a reasonably large field, containing heterogeneities, which, on the other hand, are relatively easy to

implement in physical models.

Thus, scaled models still play an indispensable role. They can be used to verify computational techniques, and to provide information of certain physical phenomena which are not properly formulated in numerical simulators; e.g., emulsion and multiphase flows at elevated temperatures in steam injection.

A properly scaled model in which all the pertinent variables are considered is a most effective tool for predicting reservoir performance. Examples are cited of the use of such models to (1) improve our understanding of relevant aspects of the process, (2) calibrate numerical models by ascertaining errors due to numerical instabilities, linearization, simplified approximation, dispersion or grid orientation, etc., (3) compare and optimize recovery processes, and (4) to predict field behavior by detailed reservoir description.

Like numerical simulators, physical models have practical limitations, and are developed based on certain approximations and simplifications. It is often difficult to find porous materials or laboratory fluids with physico-chemical properties such that pertinent scaling groups remain properly matched over wide operating ranges between prototype and model. Seldom can all scaling requirements be satisfied simultaneously. To compromise, engineering judgment is usually exercised to relax some parameters or phenomena believed to be significant. However in designing models, it is important to know the magnitude of errors introduced through relaxation of the scaling criteria. This problem has been very well recognized and

analyzed since the mid 1950's. For example, in isothermal immiscible displacement processes, the effects of the differences in relative permeability and capillary pressure relationships for laboratory unconsolidated bead packs with those of real reservoir have been illustrated prolifically.¹⁻⁶

The scaling of capillary pressure does not create too great a problem in constructing physical models of steam injection processes, because of the usually high crude oil viscosity in the prototype reservoir. The problem of relative permeability can be alleviated, if not eliminated, by introducing normalized (movable) saturations in scaling.⁵ In addition to the important transport processes in steam displacement, e.g., fluid flow and heat transfer, which have been properly scaled in previous physical models,⁷⁻¹³ there are many other relevant phenomena whose importance should be considered in scaling steam injection operations; e.g., rock-fluid interactions at elevated temperatures, temperature dependence of rock properties, emulsions, steam distillation, gas solubility, compressibility, and etc. These phenomena can be better simulated by high temperature, high pressure models employing actual reservoir fluids and rocks. However, hydrodynamic and heat transfer aspects of steam operations such as temperature and velocity distributions cannot be scaled as faithfully in high pressure as in low pressure models.^{9,10} Besides, it is relatively easy to construct and operate low pressure models, safer to use them and faster to turn them around. Since it is physically impossible to scale all aspects of steam displacement mechanisms,⁹ models should be built to meet a limited number of

objectives. In this study, emphasis was placed on heavy oil modelling, and no emphasis was placed on distillation, gas solution, rock-fluid interaction, etc.

In subsequent chapters, we will review briefly some relevant works in analytical or semi-analytical, and physical model studies of steam injection process in the literature (Chapter II). Results of some previous theoretical and laboratory investigations will be checked by the experimental observations in this study. Emphasis is placed on scaling of physical models of steam injection processes in which fluid flow and heat transport are involved (Chapter III). Scaling of heat transport is achieved by applying inspectional analysis on an integral energy balance describing the steam displacement process. An example procedure of scaling and some associated practical considerations are presented in Chapter IV. Model design, construction and experimental preparation and conduction are described in detail (Chapter V). Sensitivity studies of oil recovery to injection rate, steam quality, oil viscosity, permeability and existence of bottom water and basal permeable layer are made by using the scaled models operated in vacuum range (Chapter VI). Simple mass and energy balance solutions are used to interpret production and temperature data obtained in experimental runs. One of the runs was operated in conditions similar to the ten-pattern steam drive operation in Kern River Field, California^{62,63} to check the feasibility of the physical models used.

CHAPTER II

BACKGROUND INFORMATION

Theoretical and experimental studies on steam drive processes have appeared in the literature quite extensively. Previous studies relevant to this work will be reviewed briefly according to two categories: analytical or semi-analytical, and physical model studies. Although extensive reviews have appeared elsewhere, this review is included here for completeness.

Analytical or Semianalytical Studies of Steamflooding

Steam displacement processes involve complex problems of solving a few coupled non-linear partial differential equations for the energy and mass balances. Exact analytical solutions are very difficult, if not impossible, without a great deal of simplified assumptions or treating the problem semi-analytically and less vigorously.

The pioneer work by Lauwerier,¹⁵ in which thermal conductivities were assumed to be zero in the direction of flow and infinite over the longitudinal cross section of the reservoir, was used to predict the temperature distribution and thermal

efficiency of a hot, non-condensable, fluid injection process.

The temperature distribution derived by Lauwerier was not incorporated in the subsequent classic work of Marx and Langenheim¹⁶ who modeled a linear steam drive by assuming negligible heat transfer ahead of the steam zone. Remarkably, both studies led to the same thermal efficiency, although they differ in methodology and nature of the problem in terms of condensability. The Lauwerier approach was extended to include finite and non-zero thermal conductivities of the reservoir by some Russian workers as discussed in Spillette¹⁷ and Baker.¹⁴ Durie¹⁸ modified the boundary conditions of Lauwerier to describe the temperature distribution during steam injection. Ramey¹⁹ made use of Duhamel's superposition theorem to generalize the Marx-Langenheim solution to allow for variable-injection rates. The assumption of identical properties for the overburden and underburden used in Marx-Langenheim was relaxed by Faroug Ali²⁰ to allow use of different properties for the cap and base rocks. Mandl and Volek²¹ studied the effect of heat convection from the steam zone into oil/water region and found that the heat transfer occurred across the condensation front into the liquid zone changes from predominantly conductive to predominantly convective at a critical time, beyond which the Marx-Langenheim solution fails to describe the process accurately. They also showed that the heat flux into the hot condensate zone might be significant especially for large times.

The heat flux through the moving condensation front into the liquid zone was approximated by $M_s(T_s - T_i)(v_c - v)$, where $M_s(T_s - T_i)$ is the volumetric heat content of the steam zone, v_c is a critical

velocity, and v the frontal velocity at any time. As the frontal velocity continues to decrease with time in the steam injection process, more heat is transferred ahead of the steam zone. Similar observations were made in this study, as will be shown by the vertical temperature distributions discussed in a subsequent chapter. In addition to predicting steam zone growth when heat convection is important, Mandl and Volek's analysis can be used to determine water saturation at the downstream side of the steam front, given the fractional flow curves for different temperatures.

The thermal efficiency of steam injection processes can be determined with less restrictive assumptions if one neglects the detailed and transport behavior ahead of the steam front. Like Mandl and Volek, Prats,²² used Lauwerier's principal assumptions, to develop upper and lower bounds for the heat efficiency of steam drives. He found that heating by hot water is always more thermally efficient than heating by steam.

Myhill and Stegemeier²³ defined thermal efficiency as the ratio of the heat stored in the steam zone to the total heat injected, and obtained its upper and lower bounds by use of Mandl and Volek's solution. They also presented an equation for the cumulative oil steam ratio by assuming no contributions of the condensate zone to the produced oil.

The references cited up to this point provide approximate analytical solutions to steam zone growth and thermal efficiency by taking into account the heat losses to adjacent formations but

fail to accurately describe saturation distributions or mechanisms of fluid displacement. On the other hand, several investigators have applied a Buckley-Leverett approach to the non-isothermal displacements in thermal recovery. For example, Dougherty and Sheldon²⁴ applied multiple Buckley-Leverett fronts of cold and hot water in pattern injection without considering heat losses to adjacent formations. Detailed description of saturation profiles is possible with this method, and application to complex well patterns requires computer work. The predicted oil recovery is however too optimistic because of neglect of the heat losses.

Willman et al.²⁵ combined the Buckley-Leverett solution to the Marx-Langenheim equation for a radial steam drive. The resulting temperature and saturation distributions in the condensate zone were only rough approximations because heat losses were not treated rigorously. A more rigorous treatment was undertaken by Fournier.²⁶ Using a numerical approach, he coupled the Buckley-Leverett equation with the Lauwerier solution to predict saturation distribution during hot water injection. Shutler²⁷ used a constant average frontal velocity calculated from the Marx-Langenheim solution to develop a series of isothermal Buckley-Leverett fronts in linear steam displacement. His method treats fluid flow associated with steamflood more accurately, and it can be used to predict oil bank formation and saturation distributions throughout the reservoir.

Davis et al.²⁸ extended the Willman et al.²⁵ method to a five-spot pattern by approximating steamlines with straight line segments radiating from the injector and then converging to the

producer. They employed Ramey's generalization¹⁹ of the Marx-Langenheim solution instead to determine advances of steam front and total heated zone. In a similar and further development, Faroug Ali²⁹ provided a graphical prediction of oil recovery in a five-spot steamflood by combining Higgins and Leighton's stream tube technique³⁰ instead of the aforementioned convergent-divergent radial approximation.

The above studies are all limited to one-dimensional or two-dimensional areal steam displacements, without gravity override taken into account. No exact analytical solutions for steam displacement including gravity override, have so far appeared in the literature. However, a few approximate analytical solutions have been developed to describe the gravity effect on steam displacement in a reservoir. Van Lookeren³¹ used segregated flow principles based on the Dupuit approximation³² to calculate the approximate shape of the growing steam zone in linear and radial systems. The growth of the steam zone with tilted condensation front can be subsequently determined by integrating van Lookeren's solution according to either Marx-Langenheim's or Mandl-Volek's approaches. Rhee and Doscher³³ applied this approach to the Higgins-Leighton's areal model to describe three-dimensional steam displacement that also includes steam distillation.

Neuman³⁴ modelled a steam drive with injection occurring at the top of the injection interval. He formulated an analytical model by assuming that the vertical rate of growth could be

determined by the rate of steam condensation at the overburden and at the underlying steam interface. The rate of areal growth was determined by the thermal balance between the net heat injection as steam and that required for steam condensation to sustain vertical growth. He claimed a consistent check with the field observation in Kern River Field, despite the following assumptions:

(a) One-phase flow of water only below the condensation interface.

(b) Negligible heat carried vertically by the oil drained from the steam zone.

(c) Interface is treated as though it were horizontal.

(d) Negligible heat content increase in the areal direction.

(e) Oil saturation in the steam zone becomes its irreducible value in negligibly short period of time. His model predicts that the oil produced from the heated zone is directly proportional to the net heat injected as steam.

The basic approximate approach employed in most of the previous studies is to treat the conservation laws in an integral form^{16,21} over a control volume which extends through the whole steam zone. It has been widely used in problems exhibiting boundary layers characteristics.³⁶ Yortsos³⁵ extended this integral approach for the general steam process and developed a generalized Stefan problem, or moving boundary problem. Starting with integral forms of the water, oil, and steam mass and energy balances, he derived new expressions for upper and lower bounds, with some cases including the approximate

shape of the steam zone caused by gravity segregation, and asymptotic solutions at small and large times and approximate solutions of the steam displacement processes. He also treated in detail the heat transfer and fluid flow in the hot liquid zone. By incorporating the results in the integral balances, he developed and delineated the range of applicability of the previous works of Lauwerier, Marx-Langenheim, Mandl-Volek, Myhill-Stegemeier, and van Lookeren. The integral approach is effectively applicable to complex steam processes as long as the average volumetric heat capacity of the steam zone is essentially constant or only slowly varying with time.

There are some other integral approaches which have appeared in the literature of thermal recovery processes. Although they cannot furnish an exact analytical description of the process, they are of analytical nature in terms of methodology. Weinstein³⁷ applied variational methods^{38,39,40} to reduce the three-dimensional heat equation in the adjacent formations to a two-dimensional form. Application of this method to any finite difference thermal simulator will result in considerable saving in computer time and storage. He removed the z-dependence by minimizing a variational integral associated with the three-dimensional energy balance equation by approximating an overburden temperature functionality in the z-direction. Chase and O'Dell⁴¹ treated the same problem in a similar way except in their using a simpler approximate functionality in the cubic polynomial test functions.⁴²

Physical Model Studies of Steamflooding

During the past two decades laboratory models of steamflooding have evolved from simple qualitative observations to sophisticated scale models. Early experiments on steam displacement were usually restricted to basic observations, without scaling taken into account, on oil recovery mechanisms,²⁵ steam zone growth,⁴³ or sensitivity studies of recovery to operating parameters or reservoir characteristics.⁴³ When steam injection operations were commercially conducted, the necessity arose to design and construct properly scaled physical models to predict quantitatively field performances⁴⁴ and to calibrate and check the reliability of the thermal simulators.⁴⁵

The usefulness and reliability of a physical model depends upon what recovery mechanisms or phenomena are selected for scaling and upon how properly the scaling laws are followed. Not all relevant reservoir and operational parameters can be practically scaled simultaneously over wide operating ranges, e.g., scalings of capillary and viscous pressure gradients or proper match of oil viscosity, steam density, the pressure-temperature relation of steam throughout operating temperatures, or consistent relations of thermal properties in different parts of the reservoir-cap-base-rock systems between model and prototype.

The physical models are divided, for convenience of discussion, into three classes according to the sophistication of scaling and pressure level: unscaled, high pressure scaled, and low pressure scaled models. This review will focus on the latter two.

The scaled physical models that appeared in the literature

showed different degrees of scaling sophistication. Huygen's¹¹ half five-spot model considers only the heat flow in scaling calculations (similarly to the work of Sheinman, Malofeev and Sergeev⁴⁷). He studied the effects of oil viscosity, initial oil saturation, and distillation residue, on the oil recovery in a high pressure system (up to 843 psia) containing field crudes and crushed sandstone. Pursley⁴⁷ adopted Pujol and Boberg's⁷ scaling rules in his 1/8 five-spot model that simulated the reservoir at Cold Lake, Alberta. His model was constructed with identical materials to those occurring in the actual reservoir. By using high pressure steam, he investigated the effects of bottom water, gas cap, permeability, tight streaks, pattern size, steam quality and steam additives. Ehrlich¹², also using Pujol and Boberg's scaling, simulated the Wabasca heavy oil sand in a quarter five-spot model. He also determined two-phase relative permeabilities of steam and water from core flood experiments. High pressure steam at a temperature of 400°F was injected into the model constructed with bottom water and horizontal barriers to simulate the actual reservoir conditions. He observed the gravity override of the steam zone to reach a steady state condition after some long time. It is interesting to note that he defined an "apparent steam viscosity" to explain steam frontal stability and an "apparent steam relative permeability" to simplify the two-phase flow problem of saturated steam in steam zone. Lo⁴⁸ constructed an intermediate pressure (15 psig) model of 1/12 of a seven-spot. He scaled the model based on Pujol and Boberg's analysis except considering

mobility instead of permeability in scaling. Singhal¹⁰ matched steam quality by including the ratio of enthalpies of vapor to liquid water in the above scaling calculation. He made a thorough analysis of temperature, pressure, and production data in a model simulating a Lloydminster type oil sand by material and energy balances. The bead pack container of his model is constructed by stainless steel which may result in excessively high heat losses at early times of steam injection and in a distortion of the temperature distribution around the metal frames, where short circuit or channeling may occur for the heavy oil. Although the scaling of mobility rather than separate scaling of permeability and oil viscosity permits flexibility in selecting model materials, it fails to scale oil viscosity, as done by Stegemeier, Volek and Laumbach.⁹ The details will be discussed in the subsequent chapter on scaling.

Simulation of field steam operations by means of models operated in vacuum ranges was first described by Stegemeier et al.⁹ The previous restrictive assumption requiring identical physical and chemical properties for the model and prototype materials, was relaxed. Actual materials from the field need not be necessarily used for the model construction. The vacuum system was found more representative of the prototype, because the Clausius-Clapeyron relation for thermodynamic equilibrium and the change of oil viscosity with temperature were matched more properly. Two multiple well model studies of the Mt. Poso and the Midway Sunset Fields were presented as examples to illustrate what insight could be gained from the physical models. Prats⁴⁹ discussed another

example of the application of Stegemeier et al.'s scaling laws to the Peace River tar sand. Vacuum models appear most suitable for simulating the vaporization phenomena associated with pressure blowdowns in the pressure cycling process. This operational policy was found to prevent inefficient use of heat in the Peace River field where bottom water existed. Prats investigated the effect of the thickness of the bottom water sand permeability, aquifer, repeated pattern versus isolated pattern, heat production curtail, number of pressure buildup-depressuring cycles, etc. His observations indicate that the two pressure cycle process accelerates oil production, at least for relatively thick oil sands, and improves oil steam ratio compared to the single cycle process. Controlling the heat production by reducing injection rate, injection pressure, production rate or pressure during the injection period can result in higher oil steam ratio than steam drives with constant preset injection and production conditions.

CHAPTER III

SCALING OF PHYSICAL MODELS

Scaling procedures will be first reviewed in this chapter, and then followed by scaling of (nonthermal) multiphase, immiscible, and finally, thermal displacement processes. Development of scaling rules for the vacuum model of interest to this work will be discussed in detail at the end of the chapter.

Scaling Procedure

The application of scaling laws in designing of physical models is based on the concept of dimensional similarity. A perfectly scaled model requires physical, dynamic, and geometric similarity at each corresponding (homologous)⁵⁵ point, between model and prototype. Moreover, in a perfectly scaled model, the values of all relevant physical and geometrical variables are proportional for any scaled systems, at homologous points and times.⁵¹ In addition to the governing equations initial and boundary conditions must satisfy the similarity criteria which are expressed in terms of dimensionless parameters. There are two general approaches available for the derivation of the similarity groups: dimensional analysis, and inspectional analysis. The merits and drawbacks of each approach

are discussed in many references, for example, Geertsma, Croes, and Schwarz,¹ Loomis and Crowell⁶, and will not be repeated here.

Since the governing equations for macroscopic fluid flow and heat transfer phenomena in steamflooding are well established, inspectional analysis is preferred because significant physical insight can be gained by order of magnitude analysis of the dimensionless similarity groups. The details of the dimensional analysis calculations for isothermal multiphase displacements, both immiscible and miscible are presented in Greenkorn,¹³ and Loomis and Crowell.⁶ The former also gives an example of the calculation procedure for hot miscible displacement process.

Inspectional analysis for isothermal immiscible displacement is illustrated in Geertsma et al.¹ Rapoport,⁴ Perkins and Collins,⁵ and Loomis and Crowell.⁶ In all of these studies, scaling was achieved by inspecting a most general governing equation for fluid flow which accounts for viscous, gravitational and capillary forces. In Geertsma et al.¹ Baker,⁵⁷ and Stegemeier,⁹ inspection of the energy balance equation was the basis for scaling nonisothermal displacement. Because the steam injection process involves both fluid flow and heat transfer as driving mechanisms, it is beneficial to examine first the isothermal case before discussing steamflood modelling.

Incorporating Darcy's law in the two-dimensional form of the continuity equation, the following equation is obtained:

$$\phi \frac{\partial S_w}{\partial t} + \left(V_x \frac{\partial S_w}{\partial x} + V_z \frac{\partial S_w}{\partial z} \right) \frac{d}{dS_w} \left(\frac{1}{1+M'} \right) + \frac{\partial}{\partial x} \left(\frac{\lambda_o}{1+M'} \frac{dP_c}{dS_w} \frac{\partial S_w}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\lambda_o}{1+M'} \frac{dP_c}{dS_w} \frac{\partial S_w}{\partial z} \right) - \frac{\partial}{\partial z} \left(\frac{\lambda_o \Delta \rho g}{1+M'} \right) = 0$$

The more general three-dimensional equation can be easily generalized from the above two-dimensional equation in the Cartesian coordinate. Here M is the reciprocal of the mobility ratio, i.e., $k_{ro}\mu_w/k_{rw}\mu_o$, λ_o is the oil mobility (Kk_{ro}/μ_o). Note that the term $1/(1+M')$ is equal to the fractional flow of water in the absence of capillary and gravity terms. The above equation is applicable to immiscible displacement of a nonwetting phase by a wetting phase. The equation can be made dimensionless by introducing the dimensionless independent variables $x_d = x/L$, $z_d = z/L$ and scaling the capillary pressure by the Leverett J functions, $J = P_c \sqrt{K/\phi/\sigma \cos \theta}$. To relax somewhat the restriction on relative permeability, we can introduce the normalized saturation:

$$S_{wd} = (S_w - S_{wi}) / (1 - S_{or} - S_{wi})$$

and normalized relative permeabilities $k_{rod} = k_{ro}/k_{rowi}$ and $k_{rwd} = k_{rw}/k_{rwor}$,⁵ where k_{rowi} is the relative permeability to oil at irreducible water saturation and k_{rwor} is the relative permeability to water at residual oil saturation. Substituting in the above equation, we get

$$\begin{aligned}
& \frac{\phi L(1 - S_{or} - S_{wi})}{v} \frac{\partial S_{wd}}{\partial t} + \left(\frac{V_x}{v} \frac{\partial S_{wd}}{\partial x_d} + \frac{V_z}{v} \frac{\partial S_{wd}}{\partial z_d} \right) \frac{d}{dS_{wd}} \left(\frac{1}{1 + M'} \right) \\
& + \frac{Kk_{rowi} \sigma \cos \theta}{v \mu_o L} \sqrt{\frac{\phi}{k}} \left[\frac{\partial}{\partial x_d} \frac{k_{rod}}{1 + M'} \frac{dJ}{dS_{wd}} \frac{\partial S_{wd}}{\partial x_d} + \frac{\partial}{\partial z_d} \left(\frac{k_{rod}}{1 + M'} \frac{dJ}{dS_{wd}} \frac{\partial S_{wd}}{\partial z_d} \right) \right] \\
& - \frac{Kk_{rowi} \Delta \rho g}{v \mu_o} \frac{\partial}{\partial z_d} \left(\frac{k_{rod}}{1 + M'} \right) = 0
\end{aligned}$$

The above dimensionless equation together with the appropriate dimensionless initial and boundary conditions provide complete dimensionless description of the two-phase immiscible displacement. The behavior of the prototype and the model are similar, provided the following dimensionless parameters are identical:

$$\frac{\phi L(1 - S_{or} - S_{wi})}{vt}, \quad \frac{k_{owi} \sigma \cos \theta}{v \mu_o L} \sqrt{\frac{\phi}{K}}, \quad \frac{k_{owi} \Delta \rho g}{v \mu_o}, \quad \frac{k_{rowi} k_{rod} \mu_w}{k_{rwor} k_{rwd} \mu_o},$$

$$k_{rod}(S_{wd}), \quad J(S_{wd})$$

The first parameter represents the scaling of time based on a mass balance of the incompressible fluid flow. The second is the ratio of capillary to viscous forces, the third is the ratio of gravity to viscous forces and the fourth is related to the mobility ratio between the two phases. Complete scaling requires that the prototype and model have identical relative permeabilities and J-functions. In practice, the reservoir rocks are most often simulated, by unconsolidated sand or glass beads. Their relative permeability

and capillary pressure curves are quite different both with respect to end points and shapes of the curves. However, by introducing the above normalized permeability curves as a function of a normalized saturation we would be able to better match the multiphase flow problem between model and prototype, at least around the end points of the curves.

In the scaling of steam processes, in addition to the above mass balance, it is necessary for the model and the prototype to also satisfy the same dimensionless energy balance.

The inspectional analysis of the steam process is given in detail in Stegemeier et al.⁹ To complement the aforementioned inspectional approach, the steam process will be examined here by means of dimensional analysis. The primary dimensions involved are mass M , length L , time t (time when used as a dummy variable is designated τ) and temperature T . The dependent variables relevant to the steam process are velocity $V = [L/\tau]$, density $\rho = [M/L^3]$, viscosity $\mu = [M/L\tau]$, pressure $P = [M/L\tau^2]$, permeability $K = [L^2]$, gravity $g = [L/\tau^2]$, thermal conductivity $K_h = [ML/\tau^3T]$, heat capacity $C_p = [L^2/\tau^2T]$ and latent heat $L_v = [L^2/\tau^2]$. Therefore, we have four primary dimensions and twelve independent physical quantities including L , t , and T , the three independent variables. According to Buckingham's pi theorem,⁵¹ the process is governed by $12 - 4 = 8$ independent dimensionless similarity groups. A representative set can be obtained as follows: Select four variables which include all the primary dimensions, e.g., ρ , V , C_p , and L , as a set of bases. The dimensionless groups (π 's) are found by requiring that the products of the above variables each raised to an unknown power to each of the other

variables is dimensionless. According to this procedure, the following groups arise:

$$\pi_1 = \rho^a v^b C_p^c L^d t = (M/L^3)^a (L/\tau)^b (L^2/\tau^2 T)^c L^d \tau = vt/L$$

$$\pi_2 = \rho^e v^f C_p^g L^h T = (M/L^3)^e (L/\tau)^f (L^2/\tau^2 T)^g L^h T = D_p T/v^2$$

$$\pi_3 = \rho^i v^j C_p^k L^\ell \mu = (M/L^3)^i (L/\tau)^j (L^2/\tau^2 T)^k L^\ell \mu = \mu/\rho L v$$

$$\pi_4 = \rho^m v^n C_p^p L^q L_v = (M/L^3)^m (L/\tau)^n (L^2/\tau^2 T)^p L^q L_v = L v/v^2$$

$$\pi_5 = \rho^r v^s C_p^t L^u p = (M/L^3)^r (L/\tau)^s (L^2/\tau^2 T)^t L^u p = p/pv^2$$

$$\pi_6 = \rho^v v^w C_p^x L^y K = (M/L^3)^v (L/\tau)^w (L^2/\tau^2 T)^x L^y K = K/L^2$$

$$\pi_7 = \rho^A v^B C_p^C L^D g = (M/L^3)^A (L/\tau)^B (L^2/\tau^2 T)^C L^D g = Lg/v^2$$

$$\pi_8 = \rho^E v^F C_p^G L^H K_h = (M/L^3)^E (L/\tau)^F (L^2/\tau^2 T)^G L^H K_h = K_h/\rho C_p v L$$

The same calculation applies to the remaining variables. The unknown exponents in the above equations can be found by rendering each group dimensionless. We obtain the following dimensionless groups:

$$\frac{vt}{L}, \frac{C_p T}{v^2}, \frac{\mu}{\rho v L}, \frac{p}{\rho v^2}, \frac{K}{L^2}, \frac{Lg}{v^2}, \frac{K_h}{\rho C_p v L}, \frac{L_v}{v^2}$$

This set may be replaced by any other set obtained from the products of the groups above, with any other group raised to any power. Not all of the above groups are important (i.e., of order one or higher) in a steam injection process. For example v^2 is related to inertia forces which are negligible in porous media. Thus, the Reynold's number $\rho v \sqrt{K/\mu}$ and the ratio K/L^2 are also significant. Hence the above

set can be reduced to a set of four groups which are considered significant for the macroscopic description of the steam process:

$$\frac{vt}{L}, \frac{p}{\rho g L}, \frac{C_p (T_s - T_r)}{L_v}, \frac{\rho C_p v L}{K_h}$$

The first two groups above are related to the volumetric flow balance and the ratio of pressure to gravity forces as shown in the aforementioned isothermal modelling. On the other hand, the last two terms are the heat transfer groups referred to as Jacob's number and the Peclet number respectively (also derived by the inspectional analysis).⁹

Notice that no group containing either capillary pressure or surface tension appears in the above set since no such variables were included in the dimensional analysis. It can be shown that these two variables lead to dimensionless groups of $p_c/\rho g L$ and $\sigma \cos \theta / v \mu$ which will be discussed in more detail below.

To illustrate the complication of completely scaling the steam injection process, the governing equations and the associated similarity groups which should be satisfied between the prototype and the model are listed in Table 1.⁹

Note that all the variables shown in the table are actually selected characteristic quantities used to make a match between the prototype and the model. Ideally, all these groups must be identical in both systems in order to secure a perfect match. However, there exist some inherent difficulties in trying to match all the groups listed in the table. The term $K_h t / \phi (S_p C) L^2$ may not be satisfied at

Table 1

Governing Equations and the Associated Similarity Groups

Governing Equation	Similarity Group
Mass balance of oil	: $\phi SL/vt$
Mass balance of water and steam	: $\phi SL/vt$
Momentum balance (Darcy's Law)	: $\mu vL/Kk_{rp}, \rho gL/P$
Energy balance in reservoir	: $(\rho C)_c/\phi(S\rho C)_\ell, Lv/CT, vt/\phi SL,$ $K_h t/\phi(S\rho C)_\ell L^2$ (plus all normalized dimensionless functions, e.g., k_r, k_w, k_s , etc.)
Energy balance in cap rock	: $K_h t/(\rho C)_c L^2$
Boundary and initial conditions	: $\dot{m}/\rho_w vL^2, f_s Lv/CT$

Subscripts ℓ and c denote liquid phase and cap rock respectively.

the same time as $K_h t / (\rho C)_c L^2$, unless the ratio $\phi(S\rho C)_\ell / (\rho C)_c$ happens to be the same between model and prototype. The same thing happens to the L_v/CT in the energy balance equation and the $f_s L_v/CT$ in the boundary condition. The inconsistencies are caused by the difficulty of selecting proper characteristic quantities which are involved in the multiphase flow in the reservoir-cap-rock system. In addition, the following groups should also be matched:

$$\frac{\rho_o C_o}{\rho_w C_w}, \frac{\rho_c C_c}{\rho_w C_w}, \frac{(1 - \phi) \rho_r C_r + \phi(\rho_o C_o S_o + \rho_w C_w S_w)}{\rho_w C_w}$$

All these problems arise because we required a scaling of the complex steam process on a differential level. This, although provides more detailed local information about the process, requires equivalently more data on "local" quantities to describe it. Since it is practically impossible to satisfy every element of all scaling requirements in the differential sense, it may be easier to satisfy scaling requirements in an integral sense. The following equation is an integral form of the thermal energy balance over the steam zone modified from Yortsos.³⁵

$$\Delta T \frac{d}{dt} \int_{V(t)} M_1 dV + \int_{A_c(t)} \left(-K_{h_c} \frac{\partial T}{\partial n} \right) dA + \int_{A_f(t)} \left(-K_h \frac{\partial T}{\partial n} \right) dA$$

$$+ \Delta T \left[\sum_{i=w,o} C_i \int_{A_f(t)} \rho_i (u_{in} - \phi S_i v_n) dA - (1 - \phi) C_r \int_{A_f(t)} \rho_r v_n dA \right] = \dot{m}_s \left[f_s L_v + C_w \Delta T \right]$$

Here the volumetric integral covers the steam zone volume $V(t)$, and the areal integrals are evaluated over the steam front area $A_f(t)$ and the steam zone area contacting the adjacent formations $A_c(t)$, $\partial T/\partial n$ is the temperature gradient normal to the steam front, u_i is the velocity vector of fluid i , v_n the steam front velocity, and M_1 is the volumetric heat capacity of the steam zone,

$$(1 - \phi) \rho_r C_r + \phi (C_w \rho_w S_w + C_o \rho_o S_o + C_s \rho_s S_s) + \phi L_v \rho_s S_s / \Delta T$$

The above integral equation can be physically interpreted as follows:

Rate of heat stored in the steam zone (S.Z.) + Conductive heat flux to adjacent formations + Conductive heat flux to the liquid zone + Convective heat flux to the liquid zone = Rate of heat injection.

Applying inspectional analysis to the above equation, and expressing any variable, β , in terms of a characteristic quantity β_R and dimensionless quantity β' ($\beta = \beta_R \cdot \beta'$) one further obtains

$$\begin{aligned} & \frac{d}{dt'} \int_{V'(t)} M'_1 dV' + \frac{(K_{hc})_R t_R}{\sqrt{\alpha} C_R t_R M_{1R} L} \int_{A'_c} \left(-K'_{hc} \frac{\partial T'}{\partial n'} \right) dA' \\ & + \frac{(K_{hc})_R t_R}{M_{1R} L^2} \int_{A'_f} \left(-K'_h \frac{\partial T'}{\partial n'} \right) dA' + \frac{V_R t_R}{L} \left[\sum_{i=w,o} C'_i \int_{A'_f} \rho'_i (u'_{in} - \phi S'_i v'_n) dA' \right. \\ & \left. - (1 - \phi) C'_r \int_{A'_f} \rho'_r v'_n dA' \right] = \frac{\dot{m}_s [f_s L_v + C_w \Delta T] t_R}{M_{1R} \Delta T \cdot L^3} \end{aligned}$$

Here, V_R is the characteristic velocity which is chosen due to its most important quantity in steam injection process, e.g., velocity associated with volumetric injection rate. Each dimensionless group above represents an independent similarity group

$$\frac{(K_h \rho C)_c t}{M_1^2 L^2}, \quad \frac{(\rho C)_c}{M_1}, \quad \frac{vt}{L}, \quad \frac{[f_s L_v + C_w \Delta T] \dot{m}_s t}{M_1 \Delta T L^3}$$

where the subscript R is removed for convenience. Like the differential approach, such dimensionless parameters as $(\rho C)_c/M_1$, $\rho_w C_w/M_1$, ϕS_i and so on should remain the same for the prototype and the model to be completely scaled, something which is difficult to achieve practically. Even though this problem persists, the similarity groups derived by the integral approach may show some advantage over those developed by Stegemeier et al.⁹ The use of the groups $(K_h \rho C)_c t/M_1^2 L^2$ and $[f_s L_v + C_w \Delta T] \dot{m}_s t/M_1 \Delta T L^3$ as scaling parameters removes the aforementioned difficulty arising from the requirement to simultaneously match all the dimensionless coefficients in all the differential equations.

Stegemeier et al.⁹ introduced a correction factor A for the scaling of time and steam quality so that when (1) reservoir heating or heat production predominates, they use $(K_h t / \phi (S \rho C)_g) L^2$ and $f_s L_v / C_w T$ to scale t and f_s , (2) cap and base rock heating predominates, they use $K_h t / (\rho C)_c L^2$, and $\phi S \rho_w (f_s L_v / T + C_w) / (\rho C)_c$ to scale t and f_s . In cases where large differences of ϕS or $\rho_w C_w / (\rho C)_c$ exist between model and prototype, the quantities for scaled time and steam quality

will differ proportionally at the two extreme conditions, something that rarely occurs practically. Therefore, a question is raised as to what interpolation or averaging may be applied to an actual system where the contributions of cap rock heating, heat production and reservoir heating are equally important. It must be kept in mind that the scaling laws derived by the integral approach may create somewhat distorted pictures of the saturation or temperature distributions in the reservoir, although it represents more correct integral behavior than those by the differential approach. Actually, the important parameters or phenomena such as oil recovery, oil/steam ratio, efficiency or override are of integral nature, with which the integral approach used here may be more consistent.

Since the steam process involves complex interactions of heat transfer, mass transfer, multiphase fluid flows and other physico-chemical phenomena, which are impossible and impractical to be scaled simultaneously, it is important to evaluate how significant each phenomenon or parameter is regarding the oil recovery, and to examine what mechanisms have been considered or left out in the scaling by previous workers.⁷⁻⁹ Table 2 lists and compares the scaling groups used by Pujol and Boberg,⁷ Niko and Troost,⁸ Stegemeier, Volek and Laumbach,⁹ and this study. Note that many original dimensionless groups are recombined or rearranged in the Table and may lose their original appearance. All studies treat the phenomenon of gravity override in the same way by scaling $\Delta p / \Delta \rho g L$, which is the ratio of viscous to gravity forces. The scaling of time in all studies is associated with heat transfer, but different mechanisms are emphasized by the various workers: Pujol and Boberg (PB) used the ratio of

Table 2: LIST AND COMPARISON OF DIMENSIONLESS GROUPS USED IN THE LITERATURE
(Original forms are rearranged to comparable bases)

Parameter Scaled	Pujol & Boberg	Niko & Troost	Stegemeier, Volek & Laumbach	This Work
p	$\frac{\Delta P}{\Delta \rho g L}$	$\frac{\Delta p}{\Delta \rho g L}$	$\frac{\Delta p}{\Delta \rho g L}$	$\frac{\Delta p}{\Delta \rho g L}$
t	$\frac{K_{ho} t}{\phi (\rho C)_o L^2}$	$\frac{K_{hc} t}{(\rho C)_c L^2}$	$\frac{K_{hc} t}{(\rho C)_c L^2} \sim \frac{K_{hc} t}{\phi \Delta S (\rho C)_w L^2}$	$\frac{K_{hc} (\rho C)_c t}{M_1^2 L^2}$
q	$\frac{q (\rho C)_o}{K_{ho} L}$	$\frac{\rho_s q_s C_w}{K_{hc} L}$	$\frac{q (\rho C)_c}{\phi \Delta S K_{hc} L} \sim \frac{q (\rho C)_w}{K_{hc} L}$	$\frac{q M_1^2}{\phi \Delta S K_{hc} L (\rho C)_c}$
f _s	$\frac{f_s L v^*}{C_w \Delta T}$	$\frac{f_s L v}{C_w \Delta T}$	$\frac{f_s L v}{C_w \Delta T} \sim \frac{\rho_w (f_s L v + C_w \Delta T) \phi \Delta S}{(\rho C)_c \Delta T}$	$\frac{\rho_w (f_s L v + C_w \Delta T) \phi \Delta S}{M_1 \Delta T}$
μ _o	—	$\frac{\mu_o \rho_s C_w}{\mu_s f_s (\rho C)_c}$	$\frac{\mu_o \rho_s}{f_s \mu_s \rho_o}$	$\frac{\mu_o \rho_s}{f_s \mu_s \rho_o}$
K	—	$\frac{K \Delta \rho g L (\rho C)_c}{\mu_o K_{hc}}$	$\frac{K \Delta \rho g L (\rho C)_c}{\mu_o K_{hc} \phi \Delta S} \sim \frac{K \Delta \rho g L (\rho C)_w}{\mu_o K_{hc}}$	$\frac{K \Delta \rho g L M_1^2}{\mu_o K_{hc} \phi \Delta S (\rho C)_c}$
$\frac{K}{\mu_o}$	$\frac{K \Delta \rho g L (\rho C)_o}{\mu_o K_{ho}}$ **	—	—	—

* Not used by original authors, but added by Singhal.¹⁰ ** Permeability scaling, viscosity unscaled.

conductive to convective transfer in the reservoir, while Niko and Troost (NT) considered the rate of heat transfer in the adjacent formations. Stegemeier and coworkers (SVL) applied each ratio to extreme conditions, predominately reservoir heating or cap rock heating. As shown before, the scaling group used in this study is based on the thermal energy balance integrated over the steam zone. The group $(K_h \rho C)_c t / M_1^2 L^2$, which is the ratio of heat transport in the adjacent formation to the heat stored in the steam zone, and the group $(f_s L_v + C_w \Delta T) \dot{m}_s t / M_1 \Delta T L^3$, which is the ratio of total heat injected to the reservoir to the heat stored in the steam zone, provide a consistent scaling of the heat distribution in the reservoir-adjacent-formations system.

The scaling of flow rate q is derived from that of time t and the group $qt / \phi S L^3$, or just qt / L^3 , which represents rate of injection. The energy distribution is taken into account in scaling by the group $f_s L_v / C_w \Delta T$ (NT, Singhal),¹⁰ the ratio of vapor enthalpy to liquid enthalpy, which also results in the consistent heat content of the steam zone compared to its fluid content. SVL used the same group for the case wherein reservoir heating predominates, and corrected it by the factor $\phi \Delta S (\rho C)_o / (\rho C)_c$ for the case where cap and base rock heating predominates. NT expressed concern about the displacement stability of the steam front, which was controlled by the ratio of the pressure gradient downstream to the pressure gradient upstream of the condensation front, that is, $v_o \mu_o / v_s \mu_s$. Note that this scaling does not consider the significant difference of the ratios of the relative permeability to oil downstream to that to steam upstream of

the condensation front. If the relative permeabilities are included, it should be expressed as $\mu_o u_{rs} K_{rs} / \mu_s u_{sro} K_{sro}$.

Many previous laboratory experiments^{10, 58, 12} as well as this study, in which the front shape was not purposely scaled, always observed the existence of a stable steam front. This probably indicates that its scaling is not critically important.

A process parameter that plays a significant role in fluid flow, especially after the steam breakthrough, is steam density, ρ_s , for which there are large differences between the low pressure models and the corresponding prototypes. To prevent the inconsistency of steam density, P. van Meurs⁹ introduced the kinematic viscosity ratio $(f_s \mu_s / \rho_s) / (\mu_o / \rho_o)$. This is the ratio of the mass flux of oil to the mass flux of steam at a fixed pressure gradient between the producer and the injector; it presumes fully segregated flow occurs under reservoir and model conditions. The mass fluxes of the vapor phase and oil phase are expressed by the Darcy law:

$$\frac{f_s \dot{m}_s}{A_s} = \frac{k_{rs} K_{rs} \rho_s}{\mu_s} \nabla p$$

$$\frac{\dot{m}_o}{A_o} = \frac{k_{ro} K_{ro} \rho_o}{\mu_o} \nabla p$$

By dividing, we obtain

$$\frac{\dot{m}_o / A_o}{\dot{m}_s / A_s} = \frac{f_s \mu_s \rho_o k_{ro}}{\mu_o \rho_s k_{rs}}$$

where A_o , A_s are areas normal to the flows of oil and steam respectively. If k_{ro} in the liquid zone and k_{rs} in the steam zone approach unity, this ratio becomes the group given in SVL.

The scaling of permeability or mobility is achieved by combining $\Delta p / \Delta p_g L$ and an expression relating pressure and flow rate through Darcy's law, $KL\Delta p / \mu q$ exactly as in Singhal or PB. This allows the scaling of viscous force alone, and completes the consistent scaling laws that cover all major recovery mechanisms.

The effect of capillary pressure and relative permeabilities to model scaling were discussed extensively for isothermal displacements¹⁻⁶ and to a lesser degree for thermal recovery.^{7,9} It has been believed that the capillary forces in the usually very permeable model or prototype heavy oil reservoir are insignificant compared to the viscous or gravitational forces. SVL used the normalized saturation, i.e., conventional saturation divided by the recoverable saturation ($1 - S_{or} - S_{wi}$), to relax the matching of end points in the relative permeability curves. The use of effective permeability to oil in the steam zone instead of the absolute permeability in the group $KL\Delta p / \mu_o q$ (used for scaling permeability) may improve its matching; however the scarcity of this data in the literature and difficulty of its measurement preclude this use. As a consequence of representing saturation in terms of a normalized quantity, the experimental data on oil recovery should be corrected in order to be interpreted as prototype recoveries. Denoting cumulative oil recovered as Q_o and oil production rate as q_o , the following relation shows the conversion from model to prototype:

$$(Q_o \text{ or } q_o)_p = (Q_o \text{ or } q_o)_m \cdot \frac{(1 - S_{or} - S_{wi})_p}{(1 - S_{or} - S_{wi})_m} = (Q_o \text{ or } q_o)_m \cdot \frac{(S_{oi} - S_{or})_p}{(S_{oi} - S_{or})_m}$$

Similarly, the initial model oil saturation should be corrected by the same factor to determine the prototype initial conditions.

An examination of Table 2 indicates that these four scaling laws with the exception of viscosity scaling, may be essentially identical, if the ratio among $(\rho C)_c$, $(\rho C)_w$, $(\rho C)_o$, M_1 and the parameter $\phi \Delta S$ are matched between prototype and model (for example by using actual reservoir fluids and rocks). The simplified scaling rules with assumptions of identical properties between both systems are in fact given in PB and NT instead of the somewhat complicated forms as shown in Table 2.

Other physico-chemical phenomena not taken into account in the above scaling laws for steamfloods include:

1. steam distillation for less viscous oils,
2. in-situ emulsification of oil and water caused by rigorous agitation when steam is condensing,
3. effect of asphaltenes or carbon residue left behind at elevated temperatures,
4. thermal expansion and compressibility of the fluids and the porous medium,
5. capillary pressure or wettability.

These phenomena can be matched more adequately with high pressure models. However, their effects are believed secondary for the heavy

oils subjected to steam drive operations. Since these phenomena are not included in scaling calculations, one may question the advantages of using physical models over numerical simulators because they are not considered in both approaches. However, in scaled physical models, the above phenomena exist in reality during experiments, although not quantitatively predictable. In numerical simulators, on the other hand, they may not be fully accounted for.

An example procedure of scale-up from model parameters to corresponding prototype values by using the above developed laws is given in the next chapter. Some practical considerations in scaling calculation are also discussed briefly.

CHAPTER IV

EXAMPLE OF SCALING PROCEDURE

The dimensionless groups presented in Table 2 are rearranged for comparison purpose. Written in more familiar and original forms, the groups used in this study are:

$$\frac{\Delta P}{\Delta \rho g L}, \frac{(K_h \rho C)_c t}{M_1^2 L^2}, \frac{\dot{m} t}{\rho_w \phi (1 - S_{or} - S_{wi}) L^3},$$

$$\frac{(f_s L_v + C_w \Delta T) \dot{m}_s t}{M_1 \Delta T L^3}, \frac{\mu_o}{f_s^v \rho_o}, \frac{K \Delta \rho g t}{\mu_o L \phi (1 - S_{or} - S_{wi})}$$

where $\Delta \rho = \rho_o - \rho_s$, and the characteristic quantities ΔP , ΔT and ΔS are chosen as $P_s - P_{res}$, $T_s - T_{res}$ and $1 - S_{or} - S_{wi}$ respectively. All parameters should refer to their characteristic quantities. For example, the dependent variables pressure, temperature, and saturation are related by:

$$\left(\frac{P(x,y,z,t) - P_{res}}{P_s - P_{res}} \right)_m = \left(\frac{P(x,y,z,t) - P_{res}}{P_s - P_{res}} \right)_p$$

$$\left(\frac{T(x,y,z,t) - T_{res}}{T_s - T_{res}} \right)_m = \left(\frac{T(x,y,z,t) - T_{res}}{T_s - T_{res}} \right)_p$$

$$\left(\frac{S(x,y,z,t) - S_{irred}}{1 - S_{or} - S_{wi}} \right)_m = \left(\frac{S(x,y,z,t) - S_{irred}}{1 - S_{or} - S_{wi}} \right)_p \quad \text{for oil and water.}$$

where S_{irred} represents irreducible or residual saturation, either for oil or water. The initial conditions for these variables in model and prototype should meet the above requirements too. The boundary conditions of both systems ought to show the geometric similarity, both at the wellbores and at no-flow boundaries. The scale-down of the wellbore size creates some practical problems as will be discussed in the next chapter. In order to complete the scaling problem, we must also require matching of relevant rock or fluid properties, in dimensionless form, in terms of their characteristic quantities. For example:

$$\rho_{id} = \rho_{id}(P_d, T_d) \quad i = o, w, s$$

$$\mu_{id} = \mu_{id}(T_d)$$

$$k_{rid} = k_{rid}(S_d)$$

$$T_{sd} = T_{sd}(P_{sd}), \text{ and etc.}$$

where subscript d denotes dimensionless. The dimensionless form, in turn, is used in the scaling calculation. The following procedure is not attempted for design calculation, but for converting the experimental results of the model to the corresponding prototype values, after model materials have been selected and their properties are known. The scaling procedure for designing a model is presented in SVL.

(1) Scaling of pressure by $(P - P_p)/(\rho_o - \rho_s) gL$

Given: Scaling factor of geometrical dimensions $\gamma(L) = 191.68$, $(P_p)_p = 30$ psia, $(P_p)_m = 3.06$ psia, ρ_o can be calculated at reservoir temperatures, $(T_r)_p = 80^\circ\text{F}$, $(T_r)_m = 37^\circ\text{F}$, $(\rho_o)_p = 0.966$ gm/ml, $(\rho_o)_m = 0.984$ gm/ml. The steam density is much smaller than that of oil. It can be neglected with an error of less than 1%.

$$(P)_p = (P_p)_p + (P - P_p)_m \frac{\rho_{op}}{\rho_{om}} \gamma(L) = 188.174 (P)_m - 545.811$$

As an example, model steam pressure of 4.043 psia corresponds to its prototype value of 215 psia.

(2) Scaling of temperature of $(T - T_r)/(T_s - T_r)$

Given: $(T_r)_p = 80^\circ\text{F}$, $(T_r)_m = 37^\circ\text{F}$, $(T_s)_p = 388^\circ\text{F}$, and $(T_s)_m = 153.4^\circ\text{F}$ corresponds to $(P_s)_p = 215$ psia and $(P_s)_m = 4.043$ psia respectively.

$$(T)_p = (T_r)_p + (T - T_r)_m \frac{(T_s - T_r)_p}{(T_s - T_r)_m} = 2.646 (T)_m - 17.904$$

To improve the matching of the dimensionless pressure-temperature relationship of saturated steam. Stegemeier et al. use the temperature corresponding to the mid-value between the maximum injection pressure and the producing pressure instead of the above temperature. This indeed allows better matching in the higher pressure range, but results in inconsistency with the characteristic quantity

$T = T_{st} - T_{res}$. Figure 1 shows the match of the pressure-temperature relation of steam in dimensionless form for a model producing at pressure of 1, 3 and 6 psia respectively with a corresponding prototype of 215 psia. It indicates a less satisfactory fit by using the maximum pressure as "match point" rather than the mid-value. The lower the model pressure, the closer the fit. However, as will be shown below, the oil viscosity-temperature relation matches less satisfactorily for the lower pressure case.

(3) Scaling of time by $(K_h \rho C)_c \Delta t / M_1^2 L^2$

The data are given in Table 3. The $(K_{hc})_p$ value is for dry sandstone⁵⁴. Rock and fluid properties can be found in references 55 and 56. M_1 is equal to $(1 - \phi)(\rho C)_r + \phi(S_w \rho_w C_w + S_o \rho_o C_o)$, where steam enthalpy being very small has been neglected. The water saturation is estimated from Ehrlich's¹² core measurement of relative permeability for given steam quality, viz.,

$$f_s = \frac{1}{\frac{k_{rw}}{k_{rg}} \frac{\mu_g}{\mu_w} \frac{\rho_w}{\rho_g} + 1}$$

Table 3
COMPARISON OF RESERVOIR AND FLUID PARAMETERS
BETWEEN MODEL AND PROTOTYPE

Parameter	Prototype	Model	Parameter	Prototype	Model
Producing Pressure (psia)	30	3.06	Residual Oil Saturation to Steam	0.08	0.13
Reservoir (°F) Temperature	80	37	Irreducible Water Saturation	0.23	0.23
Steam (°F) Temperature	388	153.4	Thermal Conductivity of Cap Rock (Btu/ft hr °F)	0.98	1.05
Initial Oil Saturation	0.886	0.90	Specific Heat of Cap Rock (Btu/cf°F)	32	30
Oil Viscosity (cSt)			Specific Heat of Steam Zone (Btu/cf°F)	31.8	30.5
@ Res. Temp.	1210	8389			
@ Steam Temp.	3.19	22.15			
Permeability (D)	1.4	2000	Steam Quality (%)	66	17.6
Porosity	0.34	0.35	Flow Rate (P-B/D, M-ml/min)	660	385
Sand Thickness (ft)	73.5	0.383	Time (P-yr, M-min)	1	13.11

FIG. 1 — DIMENSIONLESS PRESSURE-TEMPERATURE RELATIONS OF SATURATED STEAM FOR PROTOTYPE AND DIFFERENT MODEL PRESSURES

If we use Ehrlich's relation, the low steam quality used in the vacuum model would result in very high water saturation in the steam zone.

$$(t)_p = (t)_m \frac{(K_h \rho C)_{cm} M_{lp}^2 [\gamma(L)]^2}{(K_h \rho C)_{cp} M_{lm}^2} = 13449.19 (t)_m$$

$$= (1/39.11 \text{ yr/min}) \cdot (t)_m$$

The above value is obtained by using the volumetric heat capacity of the steam zone based on Ehrlich's relation. In actual calculation of this study, another value based upon irreducible water saturation in the steam zone is also calculated for comparison (Table 4).

(4) Scaling of flow rate by $\dot{m}_t / \rho_w \phi (1 - S_{or} - S_{wi}) L^3$

The irreducible water saturation to steam S_{wi} is estimated from van Dijk's⁵⁷ core data and from Ehrlich's steam quality-water saturation correlation at 100% steam quality. Written in terms of volumetric flow rate or equivalent condensate volume, $q = \dot{m}_s / \rho_w$.

$$(q)_p = (q)_m \frac{1}{\gamma(t)} \frac{\phi_p (1 - S_{or} - S_{wi})_p [\gamma(L)]^3}{\phi_m (1 - S_{or} - S_{wi})_m} = 564.55 (q)_m$$

$$= (5.1137 \frac{B/D}{\text{ml/min}}) (q)_m$$

(5) Scaling of steam quality by $\rho_w (f_s L_v + C_w \Delta T) \phi (1 - S_{or} - S_{wi}) / M_1 \Delta T$

Since the scaling groups associated with heat transfer mechanisms have been developed based on an integral energy balance, the characteristic quantities used here should be averaged as well. This is, the

thermal parameters such as $f_s L_v$ and $C_w \Delta T$ are preferably referred to the injection condition. Therefore at 388°F, $(L_v)_p = 837.3$ Btu/lb, $(C_w \Delta T)_p = 314$ and at 153.4°F, $(L_v)_m = 1006.2$, $(C_w \Delta T)_m = 121.3$. $(\rho_w)_p = 53.81$ lb/CF, $(\rho_w)_m = 61.07$ lb/CF. For $(f_s)_m = 0.176$,

$$(f_s)_p = \left[\frac{\rho_w (f_s L_v + C_w \Delta T) \phi (1 - S_{or} - S_{wi})}{M_1 \Delta T} \right]_m \left(\frac{M_1 \Delta T}{\phi (1 - S_{or} - S_{wi}) \rho_w L_v} \right)_p - \left(\frac{C_w \Delta T}{L_v} \right)_p = 0.66$$

For $(f_s)_m = 0.230$, $(f_s)_p = 0.85$. It can be seen that the steam quality of the model and that of the prototype are not strictly linearly related. If vacuum pressure of 1 psia were used, $(f_s)_m$ of 0.176 would convert to prototype $(f_s)_p$ of 0.86.

(6) Scaling of viscosity by $\mu_o / f_s v_s \rho_o$. Oil viscosity is scaled with steam quality which, as mentioned above, is not linearly proportional between model and prototype. As the steam quality is changed between runs during experiments, the prototype oil viscosity should correspondingly be changed. Since, however, the same model oil is used, care should be taken to double check the corresponding prototype oil viscosity whenever the steam quality is varied. The scaling requirement dictates:

$$(\mu_o / \rho_o)_p = (\mu_o / \rho_o)_m (f_s v_s)_p / (f_s v_s)_m = 0.14423 (\mu_o / \rho_o)_m$$

where v_s denotes kinematic steam viscosity. The oil viscosity-temperature relation is given by Braden:⁶³

$$\log (v_2 + 0.6) = \left(\frac{T_1 + 460}{T_2 + 460} \right)^A \log (v_1 + 0.6)$$

which related kinematic viscosity v_2 at temperature T_2 to v_1 at temperature T_1 . Exponent A is to be determined by two known values of viscosity. Viscosity of the Kern River crude is scaled down to the model value (Figure 2) and compared with the model oil Dutrex 419. Figure 2 shows fairly good match between the scaled-down viscosity and that of the model oil used over wide temperature range. Excellent match is observed at the injected steam temperature at which the oil recovery is believed important. For comparison, the viscosity-temperature relation for the case of model producing pressure of 1.0 psia is also shown. It indicates that the lower pressure level in the model results in less satisfactory fit for the oil viscosity especially around the steam temperature.

(7) Scaling of permeability by $K\Delta\rho g t / \mu_o L \phi (1 - S_{or} - S_{wi})$

$$(K)_p = (K)_m \frac{(\Delta\rho)_m \gamma(v_o) (\rho_o)_p (\phi\Delta S)_p}{\gamma(t)(\Delta\rho)_p (\rho_o)_m (\phi\Delta S)_m} \cdot \gamma(L) = (K)_m / 1437.4$$

where $(\rho_o)_p / (\rho_o)_m = 0.886 / 0.963$, densities are evaluated at the average temperature $(3 T_s + T_r) / 4$. The model permeability of 2000 Darcys represents the prototype value of 1.39 Darcys.

Ideally, the fraction of the condensate equivalent of the superheated steam generated from the steam generator is not the steam quality at the injectors. The desired steam quality can be

FIG. 2 — VISCOSITIES OF FIELD
PROTOTYPE AND MODEL OILS

calculated from the energy balance over the injection lines:

$$f_b h_{ss} + (1 - f_b) h_l = (f_s L_v + C_w T)_{inj} + h_{loss}$$

where f_b and h_{ss} denote respectively the mass fraction and enthalpy of the superheated steam in the total input steam, h_l liquid water enthalpy at room temperature, L_v and $C_w T$ are latent and sensible heat respectively of the saturated steam at the injector condition referred to 32°F, h_{loss} is heat loss from the steam line in Btu/hr/lb/hr). The rate of heat loss can be calculated by considering heat transfer through a composite cylindrical system of the tubing wall of the steam line and the insulation tape around it, with the film resistance between steam and tube wall so small that it can be safely ignored. This value was estimated³⁶ to be 165 Btu/hr average which was approximately 4.6% of the boiler heat output at the rate of 95.9 cc/min. The enthalpy loss per unit mass injected is, however, lower at higher injection rate.

Tables 4, 5 and 6 present example numerical values of prototype parameters converted from their model counterparts based on the scaling groups of this study, Stegemeier et al.⁹ and Niko-Troost⁸ respectively. Sensitivities of length scale and water saturation in the steam zone S_{wst} are illustrated in the tables. Steam displacement results observed in a model experiment can be interpreted in terms of completely different sets of prototype reservoir and operational parameters such as steam pressure, quality, injection rate, permeability, time scale and viscosity, if different length scales are applied to our scaling laws (Table 4) - irreducible value (IRRED) of

Table 4 - Example of Prototype Parameters Converted from the Model Counterparts

Run 28

(Dutrex 419 / 8 - 12 M Beads)

Model Parameters

Steam Res. Init.	Inject.	Steam	Reserv	Permea-	Visco.	Visco.	Boiler
Temp. Temp Oil	Steam	Rate	Press.	bility	of Oil	Oil @	Steam
(F) (F) Satur	Qual.	/Min)	(psia)	(Darcy)	Temp.	Temp.	Qual.
=====	=====	=====	=====	=====	=====	=====	=====
155.7 37.0 0.945	0.176	95.90	4.32	3.06	2000.	8389.	20.86 0.229

Prototype Parameters And Their Effects Caused By Varying Scale & Initial Oil Saturation

Scale	Five Spot Acre	St.Z. Water Satu.	Steam Press. (psia)	Inject. Rate (EWPd)	Steam Qual.	Perme- bility (Darcy)	Time Factor (yr/hr)	Steam Temp. (F)	Res. Temp. (C.ST)	Viscos. of Oil @	Visco. Oil @	Kern. River Crude @	Kern. River Crude @
=====	=====	=====	=====	=====	=====	=====	=====	=====	=====	=====	=====	=====	=====
191.68	5.993	F(FS)	252.3	817.83	0.477	1.152	3.6800	401.7	746.3	1.86	3768.39	2.987	
100.00	1.632	IRRED	148.1	257.67	0.605	2.631	1.6585	357.4	1479.8	3.68	3768.39	4.241	
100.00	1.632	F(FS)	148.1	427.61	0.395	2.849	0.9994	357.4	965.5	2.40	3768.39	4.241	
140.0	3.200	IRRED	194.0	360.39	0.670	1.662	3.2537	379.2	1307.5	3.25	3768.39	3.535	
140.0	3.200	F(FS)	194.0	598.10	0.435	1.792	1.9606	379.2	849.4	2.11	3768.39	3.535	
180.00	5.289	IRRED	239.5	462.92	0.724	1.171	5.3838	397.2	1133.5	2.94	3768.39	3.086	
180.00	5.289	F(FS)	239.5	768.22	0.469	1.258	3.2442	397.2	765.9	1.90	3768.39	3.535	
220.00	7.901	IRRED	284.6	565.26	0.771	0.882	8.0500	412.6	1088.7	2.71	3768.39	2.772	
220.00	7.901	F(FS)	284.6	937.98	0.497	0.944	4.8512	412.6	702.1	1.75	3768.39	2.772	

NOTE - F(FS) means "Function of Steam Quality", IRRED means "Irreducible Value"

Table 5 - Example of Prototype Parameters Converted from the Model Counterparts Based on Stegemeier et al. Scaling Laws

Scaling Rules Based on Stegemeier et al

Run 28

(Dutrex 419' / 8 - 12 M Beads)

Model Parameters

								Visco.	Visco.	
								of Oil	Oil @	
Steam	Res.	Init.		Inject.	Steam	Reserv	Permea-	@ Res.	Steam	Boiler
Temp.	Temp	Oil	Steam	Rate (ML	Press.	Press.	bility	Temp.	Temp.	Steam
(F)	(F)	Satur	Qual.	/Min)	(psia)	(psia)	(Darcy)	(C.ST)	(C.ST)	Qual.
=====	=====	=====	=====	=====	=====	=====	=====	=====	=====	=====
155.7	37.0	0.945	0.176	95.90	4.32	3.06	2000.	8389.	20.86	0.229

Prototype Parameters And Their Effects Caused By Varying Scale & Initial Oil Saturation

Scale	Five Spot Acre	St.S. Water Satu.	Steam Press. (psia)	Inject. Rate (BWPD)	Steam Qual.	Permeability (Darcy)	Time Factor (yr/hr)	Steam Temp. (F)	Res. Temp. (C.ST)	Viscos. of Oil @ Res. Temp. (C.ST)	Kern River Crude @ Steam Temp. (C.ST)	Kern River Crude @ Steam Temp. (C.ST)
191.68	5.993	F(FS)	252.3	726.01	0.621	1.331	4.1454	401.7	971.7	2.42	3768.39	2.987
100.00	3.200	IRRED	148.1	378.76	0.498	3.184	1.1283	357.4	1218.4	3.03	3768.39	4.241
100.00	1.632	F(FS)	148.1	378.76	0.498	3.184	1.1283	357.4	1218.4	3.03	3768.39	4.241
140.00	3.200	IRRED	194.0	530.26	0.558	2.035	2.2114	379.2	1088.3	2.71	3768.39	3.535
140.00	3.200	F(FS)	194.0	530.26	0.558	2.035	2.2114	379.2	1088.3	2.71	3768.39	3.535
180.00	5.289	IRRED	239.5	681.77	0.608	1.448	3.6556	397.2	993.9	2.47	3768.39	3.086
180.00	5.289	F(FS)	239.5	681.77	0.608	1.448	3.6556	397.2	993.9	2.47	3768.39	3.086
220.00	7.901	IRRED	284.6	833.27	0.652	1.100	5.4608	412.6	921.3	2.29	3768.39	2.772
220.00	7.901	F(FS)	284.6	833.27	0.652	1.100	5.4608	412.6	921.3	2.29	3768.39	2.772

NOTE - F(FS) means "Function of Steam Quality", IRRED means "Irreducible Value"

Table 6 - Example of Prototype Parameters Converted from the Model Counterparts Based on Niko-Troost's Scaling Laws

Scaling Rules Based on Niko-Troost

Run 28

(Dutrex 419 / 8 - 12 M Beads)

Model Parameters

Steam Temp. (F)	Res. Temp (F)	Init. Oil Satur	Steam Qual.	Inject. Rate (ML /Min)	Steam Press. (psia)	Reserv bility (psia)	Permea- bility (Darcy)	Visco. of Oil @			Boiler Steam Qual.
								@ Res. Temp. (C.ST)	Visco. Temp. (C.ST)	Steam Temp. (C.ST)	
155.7	37.0	0.945	0.176	95.90	4.32	3.06	2000.	8389.	20.86		0.229

Prototype Parameters And Their Effects Caused By Varying Scale & Initial Oil Saturation

Scale	Five Spot Acre	St.S. Water Satu.	Steam Press. (psia)	Inject. Rate (BWPD)	Steam Qual.	Permea- bility (Darcy)	Time Factor (yr/hr)	Steam Temp. (F)	Viscos. of Oil @			Kern River Crude @ Temp. (C.ST)	Kern River Crude @ Temp. (C.ST)
									Res. Temp. (C.ST)	Steam Temp. (C.ST)	Steam Temp. (C.ST)		
191.68	5.998	F (FS)	252.3	654.96	0.637	1.253	4.1454	401.7	996.2	2.48		3768.39	2.987
100.00	1.632	IRRED	148.1	352.45	0.498	2.937	1.1283	357.4	1218.6	3.03		3768.39	4.241
100.00	1.632	F (FS)	148.1	352.45	0.498	2.937	1.1283	357.4	1218.6	3.03		3768.39	4.241
140.00	3.200	IRRED	194.0	486.29	0.564	1.895	2.2114	379.2	1100.6	2.74		3768.39	3.535
140.00	3.200	F (FS)	194.0	486.29	0.564	1.895	2.2114	379.2	1100.6	2.74		3768.39	3.535
180.00	5.289	IRRED	239.5	617.16	0.622	1.361	3.6556	397.2	1016.0	2.53		3768.39	3.086
180.00	5.289	F (FS)	239.5	617.16	0.622	1.361	3.6556	397.2	1016.0	2.53		3768.39	3.086
220.00	7.901	IRRED	284.6	745.46	0.673	1.043	5.4608	412.6	951.5	2.37		3768.39	2.772
220.00	7.901	F (FS)	284.6	745.46	0.673	1.043	5.4608	412.6	951.5	2.37		3768.39	2.772

NOTE - F(FS) means "Function of Steam Quality", IRRED means "Irreducible Value"

0.23 is used for both model and prototype, and different values are determined from the steam qualities of model and prototype as represented by $F(FS)$. For example, prototype value of S_{wst} 0.317 and model value 0.620 can be determined from Ehrlich's S_{wst} vs f_s correlation.¹²

The relative magnitudes of prototype parameters calculated by the scaling laws of Stegemeier et al., Niko-Troost or this study do not show general trends of one being always larger than the others. It depends on the thermal and flow properties of the materials used in the model. To determine which scaling laws result in most accurate representation of the field performance calls for identical reservoir and operating conditions between prototype and field. No definite conclusion can be made here because different injection schemes are involved in the Kern River Steam project which is scaled in this study. The difficulty of precisely matching field production history with scaled models arises from the fact that steam soaking and steam drive operations are usually undertaken simultaneously in field projects. Also, in many projects the production wells are "old" wells which are multi-zone completions and the resulting production response due to the steam drive is not well isolated.

CHAPTER V

MODEL CONSTRUCTION AND EXPERIMENTAL PROCEDURE

The construction details of the experimental apparatus used for modeling field steamflood was improved several times during the development period. Two identical models were used for most of the experimental runs. However, the physical model consists basically of three components: a refrigerated vacuum bead pack, and the associated injection and production facilities. A schematic diagram of the apparatus layout is shown in Figure 3.

Bead Pack and Its Controlling Equipment

A simple, basic element of symmetry was initially used for the model geometry, in order to develop a working construction technique. A quarter of five-spot pattern was thought slightly easier to construct and handle, and it was chosen as the model geometry. Two injectors and one producer were adopted for an inverted pattern. This configuration was considered convenient for saturating the model with oil during experimental operations. To simulate one-eighth of a five-spot on a different scale, one well can be shut in. Reservoir parameters similar to those of Kern River Field, California were taken as prototype data in the scaling

FIG. 3 — SCHEMATIC DIAGRAM OF
EXPERIMENTAL APPARATUS

calculations.

The frames of the right-angle triangular bead pack are made by machine-workable fiberglass boards of 3/8 inch thickness. The model reservoir has thickness of 4.6 inches and well spacing of 22.69 inches. The bead sizes vary from 8 x 12 mesh to 12 x 20 mesh depending on prototype permeability. The fiberglass frames, which serve as no-flow boundaries of the pattern, are cemented together by high temperature epoxy. Each model well, fabricated from fiberglass blocks, has a vertical cylindrical chamber of 1/4 inch diameter. A slit was cut along the chamber to allow fluid flow between the bead pack and the injection line or production line. Normally, the slit width, w , can be scaled down directly from the prototype wellbore circumference by the proper geometrical scaling factor. However, this may result in a slit width comparable in magnitude to the average bead size r_{bead} , whereas the ideal case requires the bead size to be much smaller than the slit width or wellbore size. The slit width was estimated by interpolating w/r_w (r_w effective radius) between 2π for the ideal case and 1.0 for the most undesirable case by the assumed linear relationship between w/r_w and $\log(r_{\text{bead}})$. A value of 2.59 was chosen for w/r_w , corresponding to $w \approx 2 r_{\text{bead}}$ in our case.

Each model well is completed at the bottom half. The cylindrical chamber size was chosen to reduce gravity segregation of vapor and water at the low flow rates of the injected steam. A thermocouple and a pressure transducer are connected to each well to monitor and control the steam pressure and quality. A differential pressure transducer was connected between the injector and the

producer to measure the pressure drop and an absolute transducer was used to measure the production pressure.

A thin fiberglass board was cemented to the bottom of the model frame to reduce leakage problems and to promote model handling. The use of a fiberglass bottom will change the thermal conductivity of the base rock in the scaling calculation. This was taken into account in scaling by considering the base rock as a composite system of fiberglass board and the sandstone, which was used to simulate the base rock.

After the model was packed with glass beads, a bondable 0.005 inch TFE teflon film was carefully cemented to the top flange with high temperature epoxy. The bead pack was placed on a triangular sandstone base rock of 9 inches thickness. On top of the pack and under the overlying sandstone cap rock, a thin neoprene sheet was placed to improve thermal contact between them. Since the thermal conductivity of neoprene is close to that of fiberglass, the presence of the neoprene sheet makes the conductivity of cap rock closer to that of the model base rock and improves the simulation of heat losses. It also serves as a gasket for the vacuum system of the model. The finite thickness (9 inches) of the cap rock does not preclude reliable simulation of the actual heat losses to infinite adjacent formations, provided that the process does not last longer than some limited period of time. This time can be calculated from the following relationship:⁶⁴

$$2 \sqrt{\pi} \sum_{m=1}^{\infty} \text{ierfc} \frac{mh}{\sqrt{\alpha_c t}} = \epsilon$$

where ϵ is the maximum error allowed (say 1%). The root t of the nonlinear equation is solved by applying Newton's iterative method, as shown in the Appendix. In our case, it was found that 8.4 hours of operation would produce a maximum error of less than 1% while 10.24 hours an error of less than 2%. (Example calculations are shown in the Appendix.)

Foam glass of 3 inch thickness was glued to the sidewalls of the model with silicone rubber cement to provide insulation for the no heat flow boundaries of the quarter five-spot model. The maximum heat flux through the heated sidewall and the insulation was approximated³⁶ to 15.8 Btu/hr-ft². An upper bound of the heat loss through the insulated area was estimated³⁶ to be 39 Btu/hr which is 1.1% of the heat input to the model. Any exposed metal surfaces and thermocouples were covered by glass fiber insulation.

A pyrex glass window was cut into one wall to allow observation of any gravity override and the vertical sweep in the steam displacement process. By removing the cap rock, visual observation can be made through the transparent teflon film to check areal sweep and any undesirable channelling of steam through the least resistant route along the joint between the film and the frame.

Twenty three copper constantan thermocouples, 1/8 inch in diameter, were installed throughout the pack. Some thermocouples were so aligned so as to provide vertical profiles of temperature data in one of the two models. One thermocouple was installed at the output end of the steam generator to aid in monitoring steam quality. A 24-channel strip chart recorder was used to record the temperature.

data. In addition to the two pressure transducers connected to the wells, as described before, there was one attached to measure the superheated steam pressure at the steam generator. A three-channel digital indicator displayed pressure readings.

The bead pack, cap and base rock, were placed in a 4 foot square refrigerated box. A thermal controller was used to control the model temperature to the desired value which could be monitored on the temperature recorder.

Injection Equipment

This part of the system includes two reservoir tanks for storing distilled water and polyethylene glycol solution, two deaeration chambers, pumps, flowmeters, a steam generator and a thermal controller. Distilled water is converted to superheated steam in the steam generator and then mixed with the polyethylene glycol solution before entering the injectors. The quality of the saturated steam injected into the model reservoir can be controlled by maintaining constant mass flow rates of the superheated steam at a preset temperature and pressure, and of glycol solution at room temperature. These two streams are mixed at a known pressure to obtain the desired steam quality at the injector. Each of the two aforementioned streams is deaerated while flowing into a vacuum chamber via a needle valve. A high pressure drop across the boiler caused by boiling of water requires a pump to inject steam into the vacuum model. A tubing pump with multiple pump heads is used to provide relatively constant flow rates. A rotameter is used to measure the flow rate of each stream with an accuracy of about 1%.

It was found that the flowmeter response was sensitive to glycol concentration. Calibration was made before each experimental run.

The once-through steam generator consists of an 8 foot-1/8 inch stainless steel tubing which has been silver soldered to two pieces of heating elements side-by-side. The rod-like heating elements require a voltage of 225 volts and provide a total wattage of 6 Kw. The wattage is controlled by an autotransformer for the upstream element and a temperature controller for the downstream element. The non-uniform temperature distributions created in this type of boiler were alleviated by wrapping the tube-element assembly with copper wires or copper braids. The injection line and all heated fittings are insulated by glass fiber insulation tapes to reduce heat losses to ambient air. A tubing coil is shunted on the steam line at the lowest location to serve as a steam trap in order to remove condensate formed in the line, especially during start-up. A valve on this line placed before each injector was found to be very convenient during the process of saturating the model with fluids.

Production Equipment

The produced fluids pass through a "double pipe" heat exchanger before entering a collector. The heat exchanger was made by placing a 4 foot-3/8 inch tubing concentrically in a 3-1/2 inch x 3/4 inch aluminum tubing with each end sealed by a tubing tee and a reducer. Tap water flows counter-current in the annular space of the condenser to cool down any produced steam. Before steam breakthrough, the refrigerated produced oil may clog the production line due to its high viscosity. Hot water is run through the annulus to

heat up cold effluent and improve its mobility.

The fluid collector consists of three flasks positioned in three vacuum chambers between which a 4-way solenoid valve is used for switching to allow change of beakers and release of vacuum during experiment. Another solenoid valve is connected between chambers and a vacuum pump to serve the same purpose for the vacuum line.

The polycarbonate vacuum chambers are connected to the vacuum pump through a vacuum trap. A drierite column is placed in between to prevent any accidental overflow of steam into the vacuum pump.

Because of the large volumes involved and a high tendency for leakage, a vacuum pump of higher displacement capacity rather than of higher vacuum was preferably selected.

A bypass route is connected in the lines before the injectors and a point between the producer and the condenser. This bypass line is used in flowmeter calibration before each run.

Preparation Before Experiment

The following steps outline the basic procedure taken in the preparation stage of experiment:

- (1) Pack the model with glass beads, seal the model and let it cure.
- (2) Connect the bead pack to injection and production lines, connect thermocouples and pressure transducers and check that the recording devices are in working condition.
- (3) Saturate the model with water and oil, and refrigerate the model until it reaches the desired uniform reservoir temperature.

(4) Calibrate the flowmeters, preheat injection and production lines and preset flowmeter valves to establish a stabilized constant rate of steam injection.

The beads were packed by uniformly sprinkling them on the vibrating model frame into which water was added simultaneously. After the model was filled up, beads near the top flange of the frame were trimmed flush so that a teflon film could be properly cemented to the top flange. Care was taken to avoid any possibility of channeling through the less resistant route along the joint between the film and the sidewalls. A rubber gasket and a load were applied uniformly during the curing period for the epoxy. Heat was applied to the epoxy to speed up the operation.

After the seal was set, the pack was placed on the base rock which was located in the refrigerated box. Connections were made to the injection and production lines. The vacuum pump was turned on to check any leaks in the system. If the pressure level was satisfactory, a warm solution of polyethylene glycol was pumped into the model until it was about 25% saturated. Meanwhile, the model oil which was stored inside a heating mantle was warmed to about 130°F. The warm oil was injected into the pack by turning on the vacuum pump until no further water was produced. The average oil saturation was calculated from a material balance on the injected and produced mass of oil. Some care was taken to prevent plugging of the injection line by cold oil after refrigeration and during start-up. The feed line to the oil storage was closed at the end of the oil saturation phase, but the vacuum pump remained on, for a limited period of time,

determined empirically for each model. Then the pack was isolated from the rest of the system by closing the injection and production valves. The vacuum pump was turned off after production lines were drained free of oil. This improved the determination of the average oil saturation and prevented the lines from clogging.

Over the teflon film a uniform film of potting compound was placed followed by a thin neoprene sheet. The sandstone cap rock was lifted into position and set down using a hydraulic crane. The box was closed and refrigeration begun.

About a day was usually needed to reach a uniform temperature of about 36°F. The flowmeter for the polyethylene glycol solution was calibrated by checking the material balance of the effluent from the bypass line. The injection lines were heated to about 140°F by heating tapes to reduce excessive heat losses of the superheated steam during start-up. The desired rates of water and glycol solution were established by turning on the tubing pump and adjusting each metering valve. The boiler power was then switched on and the generated steam was led to the bypass line. As the flow of each stream and the temperature and pressure of steam stabilized at the desired values, the system was ready to be started. At this time, the condenser was used to condense the steam before entering the polycarbonate receivers.

Operation of Experiment

Before the experiment is started, the production valve is open to deplete the reservoir slightly and establish the vacuum. As flow rates, temperatures and pressures of the injected fluids

stabilize, the flow is switched from the bypass line to the model. The timer is set at zero as soon as the temperature and pressure of the injectors start to increase. By properly adjusting the thermostat, pump rates and vacuum regulator, steam of desired quality and temperature can be obtained. Before steam breakthrough, the production lines are maintained warm by heating the annular water in the heat exchanger with heating tapes. After breakthrough, it is necessary to run tap water at room temperature to the annulus to condense the produced steam. The produced fluids are usually emulsified, and the emulsions are broken by adding demulsifier or aromatic solvent in separatory funnels which contain the samples.

The experiment proceeds until the oil steam ratio falls to a sufficiently low value. This may take approximately eight hours or several pore volumes injected. After the run, the teflon film is torn. At this time, the oil saturation distribution can be estimated in-situ. The glass beads are then cleaned with solvent and dried.

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NOMENCLATURE

A	Area, sq. ft.
A_f	Steam front area, sq. ft.
A_c	Contact area between steam zone and adjacent formations, sw. ft.
C, C_p	Specific heat per unit mass, Btu/lb °F
f_s	Steam quality, fraction
g	Gravitational constant, ft/sec ²
h	Thickness of cap or base rock used in the model, ft
K	Permeability, darcy
K_h	Thermal conductivity, Btu/hr ft °F
k_r	Relative Permeability, fraction
L	Dimension of linear geometry, distance, ft
L_v	Latent heat, Btu/lb
$\dot{m}_s$	Mass injection rate of steam, Btu/hr
M_s, M_1	Volumetric specific heat of steam zone, Btu/cubic ft. °F
M'_s	Volumetric specific heat used in Neuman's method
M'	Reciprocal of mobility ratio, $k_{ro}\mu_w/k_{rw}\mu_o$
n	Normal direction of a cross sectional area
N_p	Cumulative oil production, ml or bbl
P	Pressure, psi
P_c	Capillary pressure, psi
q	Volumetric rate, ml/min or Bbl/day
Q	Cumulative production or injection, ml or Bbl

Q_h	Cumulative heat injection, Btu
S	Saturation, fraction
ΔS	$= 1 - S_{or} - S_{wi}$
t	Time, min or day
T	Temperature, °F
ΔT	$= T_s - T_r$
u, v	Velocity, ft/min or ft/day
v_c	Critical velocity
V	volume
W_p	Cumulative water production

Greek Symbols

α	Thermal diffusivity, ft ² /hr
γ	Scaling factor, dimensionless
θ	Contact angle
λ	Time, hr
λ_o	Oil mobility, Kk_{ro}/μ_o
μ	Dynamic viscosity, cp
ν	Kinematic viscosity, cp
ρ	Density, gm/cc or Lb/ft ³
$\Delta\rho$	$= \rho_o - \rho_s$
σ	Surface tension, Dyne/cm
ϕ	Porosity, fraction

Subscripts

av	Average
b	Boiler, steam
c	Cap or base rock
d	Dimensionless
hw	Hot water zone
i	Irreducible or initial
ℓ	Liquid
m	Model
ma	Rock matrix
o	Oil
P	Prototype
prod	at producer
r	Reservoir or residual
R	Reference or characteristic quantity
ref	At reference temperature
rm	At room temperature
s, st	Steam
w	Water
x,y,z	Coordinate

APPENDIX

ERROR ESTIMATE OF HEAT LOSS TO A CAP ROCK OF FINITE THICKNESS

The exact simulation of heat lost to adjacent formations requires infinite thickness of cap and base rocks. However, in practice, for bench size models, materials of finite thickness are used. The approximation of finite thickness is good at early time of injection but becomes poorer with increase in time. The accuracy of such an approximation can be estimated as a function of time by the following nonlinear equation which represents the differences between the solutions for the heat flux obtained for the one-dimensional diffusivity equation for a finite slab, and a semi-infinite medium subjected to a constant step increase in temperature at the boundary.⁶⁴

$$f(t) = \sum_{m=1}^{\infty} \text{ierfc} \frac{mh}{\sqrt{\alpha_c t}} - \frac{\epsilon}{2\sqrt{\pi}} = 0 \quad (\text{A-1})$$

Here ϵ is the error caused by the finite thickness, ierfc is the integral of complementary error function. The latter is given by⁶⁵

$$\text{ierfc}(x) = \frac{1}{\sqrt{\pi}} e^{-x^2} - x \text{erfc}(x)$$

The derivative of Equation (A-1) with respect to time is

$$f'(t) = \frac{h}{2\sqrt{\alpha_c t^3}} \sum_{m=1}^{\infty} m \text{erfc} \frac{m h}{\sqrt{\alpha_c t}}$$

The root t of Equation (A-1) can be calculated by the Newton-Raphson's iteration method⁶³ of the following form, at the iteration step i :

$$t_{i+1} = t_i - \frac{f(t_i)}{f'(t_i)} \quad i = 0, 1, 2, 3, \dots$$

Convergence rate is quadratic for this case. A computer program was written to solve the nonlinear problem. The results check very well with Whitten's solution.⁹

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