MASTER

LOCATING A BURIED MAGNETIC DIPOLE*

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Thurlow W. H. Caffey
TM Development Division 9421
Sandia Laboratories, Albuquerque, New Mexico, 87115

Consider a buried magnetic dipole oriented as shown in Figure 1. It is desired to find the location of the dipole in the surface coordinate frame as well as the angles (α, β) to facilitate recovery of the instrument package or person who deployed the dipole. The frequency of the dipole current is chosen so that the dipole field near the surface of the earth is virtually the same as a static dipole in free space.

Theoretical Outline

The most convenient coordinates to choose with respect to the buried dipole is a cylindrical (ρ, ϕ, z) system with the z -axis along the dipole axis (Figure 2). The unit vector in the z direction can be written immediately:

$$\hat{a}_z = \hat{a}_x \sin \alpha \cos \beta + \hat{a}_y \sin \alpha \sin \beta + \hat{a}_z \cos \alpha . \quad (1)$$

The total H-vector measured at surface position P_i is

$$\vec{H}_i = \hat{a}_x H_{xi} + \hat{a}_y H_{yi} + \hat{a}_z H_{zi} . \quad (2)$$

and the unit vector in the ϕ -direction is given by

$$\hat{a}_{\phi i} = (\hat{a}_z \times \vec{H}_i) / |\hat{a}_z \times \vec{H}_i| . \quad (3)$$

The unit vector $\hat{a}_{\rho i} = \hat{a}_z \times \hat{a}_{\phi i}$ can now be found and the observer's position, P_i , is located with respect to the dipole if α , β , ρ_i , and z_i are known.

Suppose measurements are made at several surface points and one point, P_j , is chosen as a reference. The horizontal

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radius vector drawn from P_j to another point P_i is given by

$$\vec{r}_i = \hat{a}_x (x_i - x_j) + \hat{a}_y (y_i - y_j) + \hat{a}_z (z_i - z_j) \quad (4)$$

and $\vec{r}_j \equiv 0$. To get from P_j to the dipole via P_i requires traversing any of "m" paths given by the dipole location vector

$$\vec{D}_i = \vec{r}_i - (\hat{a}_\rho \rho)_i - \hat{a}_\phi \phi_i \quad i = 1, 2, \dots, m. \quad (5)$$

By equating $\vec{D}_j$ to each of the other location vectors a set of $(m - 1)$ equations of this form can be written:

$$(\hat{a}_\rho \rho)_j + \hat{a}_\phi \phi_j = -\vec{r}_i + (\hat{a}_\rho \rho)_i + \hat{a}_\phi \phi_i \quad \begin{matrix} i = 1 \dots m. \\ i \neq j \end{matrix} \quad (6)$$

When each of these equations is dot multiplied by $\hat{a}_{\phi i}$, a set of $(m - 1)$ equations of this form result:

$$\begin{aligned} & K \left\{ A_i + B_i \cos \beta \tan \alpha + C_i \sin \beta \tan \alpha \right\} \\ & = D_i + E_i \cos \beta \tan \alpha + F_i \sin \beta \tan \alpha. \end{aligned} \quad (7)$$

The coefficients $(A \dots F)_i$ are functions only of surface position and measured H-components; the coefficient, $K = |\vec{H}_j|/\rho_j$, is eliminated in the solution process.

Computational Outline

The vertical magnetic dipole has been proposed for the rescue of trapped miners and will be treated first as an introductory case. Measurements are made at two surface points, denoted by subscripts 1 and 2. The cylindrical radius vector which partially locates P_1 with respect to the dipole is given by:

$$\rho_1 = \frac{H_{x2} (y_2 - y_1) - H_{y2} (x_2 - x_1)}{H_{x1} H_{y2} - H_{y1} H_{x2}} \left(H_{x1}^2 + H_{y1}^2 \right)^{1/2} \quad (8)$$

because $\alpha = 0$ and we are free to choose $\beta = 0$. The ratio of the H_1 -fields along $\hat{a}_z$ and $\hat{a}_{\rho 1}$ is denoted by γ_1 which can be found from measured values as

$$\gamma_1 = H_{z1} \left(H_{x1}^2 + H_{y1}^2 \right)^{-1/2} \quad (9)$$

In both Eqs (8) and (9), it is necessary to avoid locating P_1 directly over the VMD which would make $H_{x1} = H_{y1} = 0$. The depth may be obtained from the formula for the axial field [1],

$$z = \frac{3\gamma_1 \rho_1}{4} \left[1 \pm \left(1 + 8/9 \gamma_1^2 \right)^{1/2} \right] \quad (10)$$

One of the two solutions to Eq (10) may be eliminated by physical reasoning, and the computations can be checked by repeating with P_2 as a reference.

In the general case, Eq (7) is solved by using a modified Gauss elimination method and sets of five surface points. Five points appear to be the minimum required to avoid extracting β -values as roots of inverse trigonometric quantities.

The uncertainty in each of the measured field quantities may be as great as 3%, which makes the solution depend upon the order in which the data are presented. Some solutions can be eliminated by dividing each of Eq (7) by its right-hand side and setting each result equal to the next result in turn. This eliminates K , and multiplication followed by collection of terms gives three equations of the form:

$$(GA + GB + GC + GD + GE + GF)_i = 0 \quad (11)$$

where the $(GA, \dots GF)_i$ are combinations of the coefficients $(A \dots F)$ and a pair of (β, α) solutions. When the measurement accuracy is unlimited,

$$\sum_{i=1}^3 (GA + GB + \dots GF)_i \equiv 0 \quad (12)$$

so that the amount by which Eq (12) is nonzero may be used to eliminate (β, α) solutions.

The radial distance ρ_i is found by dot-multiplying each side of Eq (6) by $\hat{a}_{\phi j}$ to obtain

$$\rho_i = \frac{-\hat{a}_{\phi j} \cdot \vec{r}_j}{\hat{a}_{\phi j} \cdot \hat{a}_{\rho i}} \quad (13)$$

because both $\hat{a}_{\phi j}$ and $\hat{a}_{\rho i}$ can be computed for each (β, α) pair. Finally, $\mathcal{J}$ is given by Eq (10). Depending upon the accuracy of the input data, a number of location vectors may be generated with the use of Eq (5). The best location is selected by computing the fields for each location vector and comparing those fields with the measured field values.

Conduct of a Field Test

A tiltable coil was mounted on the rear of a small electronics package and lowered into a vertical borehole within a three-layered earth near Edgewood, NM. The conductivities of the successive layers had been established as 0.024, 0.165, and 0.046 S/m with interfaces at depths of 9 and 49 meters. The operating frequency of 390 Hz was chosen by comparing the static free-space Hz-field with the exact values computed from Wait [2], assuming a uniform conductivity of 0.2 S/m and a vertical dipole. At a horizontal radius of 10 m and a depth of 20 m, the free-space approximation was in error by about 2 percent. A moment of 0.45 amp-m² was used which required 150 mW. The coil was tilted at 65° and lowered to 20 meters. The rotation angle was not controlled but was observed after the test. The surface fields were measured with a pair of tripod-mounted coils, and the origin of coordinates was established by finding a point where good SNR's could be obtained for all three components. The second coil was positioned about the first coil at 5 stepped-off intervals on the perimeter of a circle with a 10-meter radius (Figure 3). Six surface locations were used instead of the minimum of five and the data were processed as six sets of five points. The measured and computed values are compared below.

	<u>x-Offset</u> <u>(meters)</u>	<u>y-Offset</u> <u>(meters)</u>	<u>Depth</u> <u>(meters)</u>	<u>Tilt</u> <u>(degrees)</u>	<u>Rotation</u> <u>(degrees)</u>
Measured	11.4	0.10	-20.0	65	158
Computed	11.4	0.16	-20.6	67.4	157
Difference	0.0	0.06	- 0.6	2.4	1

The results are illustrative of 26 field tests conducted with 9 different combinations of location and orientation [3] which confirm the method outlined here.

References

1. R. Plonsey and R. E. Collin, Principles and Applications of Electromagnetic Fields, McGraw-Hill, New York, NY, 1961, p. 209.
2. J. R. Wait, "Electromagnetic Induction Technique for Locating a Buried Source," IEEE Trans. Geo Sci. Electr., April 1971, pp. 95-98.
3. T.W.H. Caffey, "Locating a Buried Earth Penetrator," Sandia Laboratories Report SAND77-0646, Nov. 1977. Available NTIS, Springfield, Va. 22151.

Figure 1. Cartesian Coordinate Frames

Figure 2. Dipole Location via Cylindrical Frame

Figure 3. Surface Layout and Block Diagram