SLAC-PEP-NOTE--318

DE89 006347

ON-LINE ORBIT AND DISPERSION CORRECTION SCHEMES FOR PEP

A. Chao, M. Lee, E. Linstadt, N. Spencer
Stanford Linear Accelerator Center, Stanford, California

INTRODUCTION

FEB 02 1980

In the initial PEP operation, the orbit/dispersion correction will be done by a system of 96 monitors and 48 correctors (Ref. 1,2). Many schemes using this system have been investigated, and the schemes which have been selected to correct orbit/dispersion on-line are described in this note.

For closed orbit correction, the schemes are:

- ORBGLO - to minimize the rms orbit error globally as detected by the monitors using any number of correctors up to 48.
- ORBARC - to minimize orbit errors in an arc region (AR) which are caused by field errors in the same AR as well as to cancel their effects outside of this AR.
- ORBINS - to estimate the alignment errors of the Q1 and Q2 magnets within an insertion region (IR) as well as to cancel the orbit errors caused by field errors in this IR by using the correctors between the Q1 and Q2 magnets.
- ORBIP - to change the displacement and slope values of the orbit at any interaction point (IP) using the 4 correctors in the surrounding IR without affecting the orbit elsewhere.

The schemes for dispersion (eta) correction are:

- ETAGLO - to minimize the rms eta error globally as detected by the monitors using any number of correctors up to 48.
- ETAIP - to change eta and eta' at any interaction point using the 8 correctors within the sextant region (SR) between two symmetry points without affecting the orbit at the IP and the orbit/eta values outside of this SR.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

dx
MASTER

Two schemes for correcting the single pass orbit are PASINS and PASARC, which are the same as ORBINS and ORBARC, except that the orbit is that of a single pass beam rather than a stored beam. For simplicity, z will be used to denote either x or y .

ORBGL0

Let $\vec{\theta}$ be the vector whose elements are the corrector strengths in unit of kicking angles. The solution which minimizes the length of the orbit vector after correction, $\vec{z}_a$, is given by Ref. (1)

$$\vec{\theta} = -(\hat{C}\hat{C})^{-1} \hat{C}^T \vec{z}_b \quad (1)$$

with $\vec{z}_b$ the orbit vector before correction and $|\vec{z}_a| = |\vec{z}_b + \hat{C}\vec{\theta}| = \min$; the matrix C is defined by

$$C_{ij} = \frac{\sqrt{\beta_i} \beta_j}{2 \sin \pi \nu} \cos(\pi \nu - |\psi_i - \psi_j|) \quad (2)$$

where i and j denote the i th monitor and j th corrector; β , ψ and ν are the betatron function, betatron phase and tune, respectively; the transpose of the matrix C is $\hat{C}$.

The program ORBGL0 calculates up to 48 solutions using Eq.(1); each solution corresponds to one given value of n , the number of correctors desired. The scheme to find the correctors in these solutions is as follows: the value of $|\vec{z}_a|$ is calculated for all possible single correctors, and the corrector which produces the smallest value of $|\vec{z}_a|$ is selected as the first corrector, C_1 . Then the value of $|\vec{z}_a|$ is calculated for all possible corrector pairs, C_1 and C , and the corrector $C=C_2$ which produces the smallest $|\vec{z}_a|$ is selected for the $n=2$ case. The procedure is continued for $n=3, 4, \dots, 48$. The subroutines used in this scheme are extracted from the existing program MICADO (Ref. 3).

Note that, except for $n=1$ and 48, the solution obtained by this scheme for a given n may not be the one which yields the absolute

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

minimum value of $|\vec{z}_a^+|$ among all possible solutions containing n correctors. Note also that as n increases, the value of $|\vec{z}_a^+|$ decreases so that the smallest value of $|\vec{z}_a^+|$ is reached when $n=48$.

The accuracy of orbit correction using this scheme depends on the accuracy of our knowledge of the machine functions, β , ψ and ν , whose values are affected by magnet field errors such as the strength errors of the quadrupoles. A rough estimate (Ref. 4) shows that this will not be a serious problem for PEP provided that the tune error caused by the field errors is less than 0.03. If the tune is off by much more than .03, one could use the measured tune in the $\sin(\pi\nu)$ factor in the C-matrix elements, Eq. (2), to improve the accuracy of the orbit correction scheme.

ORBARC AND PASARC

Let the monitors in an arc be numbered from 1 to $m-1$; let the correctors be numbered from 1 to n , and let the monitors just outside of the arc be denoted by 0 and m . The orbit offset and slope at the monitor m due to the unknown field errors in the arc are given by

$$\begin{pmatrix} \Delta z_m \\ \Delta z'_m \end{pmatrix}_b = \begin{pmatrix} z_m \\ z'_m \end{pmatrix}_b - T(m|0) \begin{pmatrix} z_o \\ z'_o \end{pmatrix}_b \quad (3)$$

with

$$T(m|0) = \begin{bmatrix} \sqrt{\frac{\beta_m}{\beta_o}} (\cos \psi_{mo} + \alpha_o \sin \psi_{mo}) & \sqrt{\beta_m \beta_o} \sin \psi_{mo} \\ -\frac{(1 + \alpha_m \alpha_o) \sin \psi_{mo} + (\alpha_m - \alpha_o) \cos \psi_{mo}}{\sqrt{\beta_m \beta_o}} & \sqrt{\frac{\beta_o}{\beta_m}} (\cos \psi_{mo} - \alpha_m \sin \psi_{mo}) \end{bmatrix}$$

where subscripts b mean before correction, $\psi_{mo} = \psi_m - \psi_o$, and $T(m|0)$ is the transport matrix from monitor 0 to monitor m . The slopes z'_{ob} and z'_{mb} are calculated from the measured orbit at the monitors -1 and $m+1$, respectively. The strength of the correctors in the arc are chosen such

that

$$\begin{pmatrix} \Delta z_m \\ \Delta z'_m \end{pmatrix}_a = \begin{pmatrix} \Delta z_m \\ \Delta z'_m \end{pmatrix}_b + \sum_{j=1}^n T(m|j) \begin{pmatrix} 0 \\ \theta_j \end{pmatrix} = 0 \quad (4)$$

where $T(m|j)$ is the transport matrix from the j -th corrector to the monitor m . In addition, we require that the rms value of the orbit error in the monitors $\Delta \vec{z}_a$ (note: not the orbit $\vec{z}_a$) in the arc be minimized after correction:

$$|\Delta \vec{z}_a|^2 = \sum_{i=1}^{m-1} \left(\Delta z_{ib} + \sum_{j=1}^n M_{ij} \theta_j \right)^2 = \min. , \quad (5)$$

where Δz_{ib} is obtained from the z and z' values at the monitor 0 using an expression the same as Eq.(3) but with m replaced by i ; and the matrix M is defined such that the (i,j) element of M is equal to the $(1,2)$ element of the matrix $T(i|j)$, i.e.,

$$M_{ij} = \begin{cases} \sqrt{\beta_i \beta_j} \sin(\psi_i - \psi_j) & \text{if } \psi_i > \psi_j \\ 0 & \text{if } \psi_i < \psi_j \end{cases} \quad (6)$$

It can be shown that the solution which satisfies the above conditions also minimizes the sum

$$|\Delta \vec{z}_a|^2 + \lambda_1 \Delta z_{ma} + \lambda_2 \Delta z'_{ma} , \quad (7)$$

where λ_1 and λ_2 are the Lagrange multipliers. The solution is obtained from

$$\begin{bmatrix} \tilde{M} & M \\ M_{m1} \dots M_{mn} & \begin{matrix} \frac{M_{m1}}{2} & \frac{M'_{m1}}{2} \\ \vdots & \vdots \\ \frac{M_{mn}}{2} & \frac{M'_{mn}}{2} \end{matrix} \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \vdots \\ \Theta_n \end{bmatrix} = - \begin{bmatrix} \tilde{M} & 0 \\ 0 & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{bmatrix} \begin{bmatrix} \Delta z_1 \\ \vdots \\ \Delta z_{m-1} \\ \Delta z_n \\ \Delta z'_m \end{bmatrix} \quad (8)$$

where

$$M'_{mj} = \sqrt{\frac{\beta_j}{\beta_m}} \left[\cos(\psi_m - \psi_j) - \alpha_m \sin(\psi_m - \psi_j) \right] \quad (9)$$

$$\alpha_m = - \frac{1}{2} \beta'_m .$$

Since the values of M_{ij} depend upon the relative position of the monitors and correctors, in order to obtain a unique solution, the e^+ orbit will be treated the same as the e^- orbit, i.e., the transformation T is always clockwise.

ORBINS AND PASINS

Let the monitors in an IR be identified by 1,2, . . . and 6 as shown in Fig. 1. From the orbit measured, the values of z' can be computed at these monitors assuming that there are no field errors in the regions between monitors 1 and 2, 3 and 4, and 5 and 6. It is then possible to estimate the field errors resulting from the misalignment of the interaction region quadrupole magnets Q1 and Q2. It is also possible to calculate the strength of the correctors between Q1 and Q2 needed to cancel the orbit effect due to the Q1 and Q2 misalignments. These calculations are done in ORBINS (or PASINS for a single pass case), using a method similar to ORBARC.

Let θ_{q1} and θ_{q2} be the kicks due to the effective alignment errors of Q1 and Q2. For the errors in the left half of the insertion, θ_{q1} and θ_{q2} are obtained from the solution to the following equation:

$$\begin{pmatrix} \Delta z_3 \\ \Delta z'_{3b} \end{pmatrix} + \sum_{j=q2}^{q1} T(3|j) \begin{pmatrix} 0 \\ \theta_j \end{pmatrix} = 0 \quad (10)$$

where Δz_{3b} and $\Delta z'_{3b}$ are given by

$$\begin{pmatrix} \Delta z_3 \\ \Delta z'_{3b} \end{pmatrix} = \begin{pmatrix} z_3 \\ z'_{3b} \end{pmatrix} - T(3|2) \begin{pmatrix} z_2 \\ z'_{2b} \end{pmatrix}$$

Similar solutions are given for these kicks in the right half of the insertion if we replace indices 2 and 3 by 4 and 5, respectively.

The results of this calculation serve as a rough estimate of the Q1 and Q2 misalignments. Due to the fact that the betatron phase between Q1 and Q2 is small, these values may contain large errors. The combined effect of a Q1-Q2 pair, however, should be more accurate as considered below.

Let θ_l and θ_r be the strength of the correctors in the left and right half of the insertion, respectively. By replacing the indices Q1, Q2 and 3 with l , r and 5 in Eq.(10), respectively, we obtain the corrector kicks that are needed to cancel outside of the insertion.

After applying ORBARC and ORBINS to all of the PEP arcs and insertion regions, the closed orbit will be separated into independent segments which have been individually and locally corrected. This offers an alternative to the ORBGLO scheme mentioned before. A comparable scheme has been considered for LEP (Ref. 5).

ORBIP

Let the four correctors to be used for changing the values of z and z' at the interaction point be denoted by $j=1, \dots, 4$. Let $i=1$ and 2 denote the interaction point and an observation point at the exit of corrector 4 . The solution which introduces changes of Δz_1 and $\Delta z'_1$ at the interaction point, while not affecting the orbit outside of the region between correctors 1 and 4 , is given by

$$\begin{pmatrix} C \\ - \\ C' \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} \Delta z_1 \\ \Delta z_2 \\ \Delta z'_1 \\ \Delta z'_2 \end{pmatrix} \quad (11)$$

with $\Delta z_2 = \Delta z'_2 = 0$. Matrix C has been defined in Eq.(2);

$$C'_{ij} = - \frac{1}{2 \sin \pi \nu} \sqrt{\frac{\beta_j}{\beta_i}} \left\{ \alpha_i \cos (\pi \nu - |\psi_i - \psi_j|) \right. \\ \left. - \text{sign} (\psi_i - \psi_j) \sin (\pi \nu - |\psi_i - \psi_j|) \right\} \quad (12)$$

ETAGLO

The same equations and selection scheme as ORBGLO are used in ETAGLO to find the 48 solutions which minimize the length of the eta vector, $\vec{\eta}$, at the monitors. The orbit vectors, $\vec{z}_b$ and $\vec{z}_a$, are replaced by $\vec{\eta}_b$ and $\vec{\eta}_a$, the eta vectors before and after eta correction; the matrix C is replaced by the matrix D (Refs. 1, 6, 7)

$$\begin{aligned}
 D_{ij} = & \frac{\sqrt{B_4 B_1}}{2 \sin \pi \nu} \left\{ - \cos(\pi \nu - |\psi_1 - \psi_j|) \right. \\
 & + \sum_q \frac{B_q}{2 \sin \pi \nu} \left(\frac{g\lambda}{B\rho} \right)_q \cos(\pi \nu - |\psi_1 - \psi_q|) \cos(\pi \nu - |\psi_j - \psi_q|) \\
 & \left. + \sum_s \frac{\eta_s B_s}{2 \sin \pi \nu} \left(\frac{2\lambda\lambda}{B\rho} \right)_s \cos(\pi \nu - |\psi_1 - \psi_s|) \cos(\pi \nu - |\psi_j - \psi_s|) \right\}
 \end{aligned} \tag{13}$$

where the strengths of the quadrupole and sextupole magnets are given by $g\lambda/B\rho$ and $\lambda\lambda/B\rho$ with $g = \partial B_y / \partial x$, $2\lambda = \partial^2 B_y / \partial x^2$ and $B\rho$ = particle rigidity. The upper signs in Eq.(13) are for horizontal eta corrections, and the lower signs are for the vertical eta corrections.

Eq.(13) is an approximation since it takes into account only those terms first order in the corrector strengths. The accuracy of this linear approximation has been studied by comparing the results of Eq.(13) and the results of an exact calculation which is valid to all orders of corrector strength. It was found (Ref. 7) that the linear approximation is accurate (error $\lesssim 20\%$) provided that the error in η to be corrected is $\lesssim 0.8m$ for the vertical case and $\lesssim 0.2m$ for the horizontal case.

Similar to the global orbit correction scheme, ORBGLO, in case the measured tune $\bar{\nu}$ differs significantly from the predicted tune, ν , it may be helpful to use the measured tune in the $\sin(\pi \nu)$ factors in Eq.(13). But, if $\bar{\nu}$ differs from ν by more than ~ 0.2 , the correction scheme would fail even using $\bar{\nu}$ in the $\sin(\pi \nu)$ factors.

EATIP

Let the eight correctors in a sextant between two symmetry points be denoted by $j=1, \dots, 8$. Let 1=1 and 2 denote the interaction point

and the symmetry point at the right of the sextant, respectively. The solution which produces changes of Δz_1 , $\Delta z'_1$, $\Delta \eta_1$ and $\Delta \eta'_1$ but not affecting the values of orbit and eta outside of this sextant is given by

$$\begin{pmatrix} \frac{C}{D} \\ \frac{C'}{D'} \end{pmatrix} \begin{pmatrix} \Theta_1 \\ \vdots \\ \Theta_8 \end{pmatrix} = \begin{pmatrix} \frac{\Delta z_1}{\Delta z_2} \\ \frac{\Delta \eta_1}{\Delta \eta_2} \\ \frac{\Delta z'_1}{\Delta z'_2} \\ \frac{\Delta \eta'_1}{\Delta \eta'_2} \end{pmatrix} \quad (14)$$

with $\Delta z_2 = \Delta \eta_2 = \Delta \eta'_2 = \Delta z'_2 = 0$. Matrices C, C' and D have been defined before;

$$D'_{ij} = \frac{1}{2 \sin \pi \nu} \sqrt{\frac{\beta_j}{\beta_i}} \left[\alpha_i \cos(\pi \nu - |\psi_i - \psi_j|) - \text{sign}(\psi_i - \psi_j) \sin(\pi \nu - |\psi_i - \psi_j|) \right] \quad (15)$$

$$+ \sum_q \frac{\beta_q (g \ell / E \rho)}{2 \sin \pi \nu} \left[\alpha_i \cos(\pi \nu - |\psi_i - \psi_q|) - \text{sign}(\psi_i - \psi_q) \sin(\pi \nu - |\psi_i - \psi_q|) \right] \cos(\pi \nu - |\psi_j - \psi_q|)$$

$$+ \sum_p \frac{\eta_{ps} \beta_s (2 \lambda \ell)}{2 \sin \pi \nu} \left[\alpha_i \cos(\pi \nu - |\psi_i - \psi_s|) - \text{sign}(\psi_i - \psi_s) \sin(\pi \nu - |\psi_i - \psi_s|) \right] \cos(\pi \nu - |\psi_j - \psi_s|)$$

The convention of the upper and lower signs is the same as that in Eq.(13).

With a slight modification, ETAIP can be used to make changes in est and eta' at one of the rf cavity locations while keeping z, z' at the nearby IP and the orbit and dispersion outside of the sextant unchanged.

ACKNOWLEDGEMENTS

We would like to thank C.Y. Yao* for helping us on the Lagrange multipliers used in Eq.(8). R. Helm had also thought about a method for orbit correction similar to ORBINS.

REFERENCES

1. E. Close, M. Cornacchia, A. King and M.J. Lee, IEEE Trans. Nucl. Sci., NS-26, 3502 (1979).
2. H. Wiedemann, PTM-136R2 (1979).
3. B. Autin and Y. Marti, CERN/ISR-MA/73-17 (1973).
4. A. Chao and P. Morton, PTM-129 (1977).
5. H. Zyngier, LEP-70/105 (1978).
6. M. Lee, PTM-134 (1978).
7. Y.C. Yao and A. Chao, PTM-205 (1979).

* Visiting Scientist from University of Science and Technology, Hefei, China.

Fig. 1