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A RELIABILITY BASED INVESTIGATION OF DESIGN FACTORS

Topical Report

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Agbabian Associates
El Segundo, California

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Date Submitted — 30 June 1978

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ABSTRACT

In this report second-moment probabilistic techniques are used to formulate structural resistances and loads and to derive reliability-based safety, load, and strength factors for design. Existing concepts of the second-moment reliability theory have been extended to the practical case of multiple load combinations. This development consistently includes the stochastic character of loads and the unpredictable nature of their combinations. For this purpose, a new technique for the evaluation of load combinations is presented, whereby the moments of the extreme of combined loads is obtained in terms of the moments of individual loads and parameters describing their random fluctuations in time.

Reliability-based safety, load, and strength factors are derived in terms of the acceptable level of risk, the coefficients of variation of the loads and resistance, uncertainties associated with errors in modeling and estimation, and a set of parameters describing the stochastic nature of loads and their combinations. Two distinct formulations of these factors are presented. A numerical example is provided wherein the values of the factors for a number of specific assumptions are determined and the significance of various design variables on the reliability-based factors are examined.

In the light of the theoretical formulations in the report, the current deterministic design procedures are critically examined. Specific inconsistencies and inadequacies in these procedures are determined. It is shown how the proposed theory can be used to improve or further refine the present design methods.

FOREWORD

This report was prepared by Agbabian Associates under Contract No. EY-76-C-03-1011. The objective of studies under this contract is to develop criteria for the design of future nuclear facilities. This report is concerned with one aspect of this overall objective, namely, an investigation of improved procedures for considering loads and load combinations in the design of nuclear power plant facilities. Armen Der-Kiureghian was principal investigator for this subtask. Technical monitor for the Division of Reactor Research and Technology of the Department of Energy was C. Chester Bigelow. Project manager for Agbabian Associates was George A. Young. Author of the report was:

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CONTENTS

<u>Chapter</u>		<u>Page</u>
1	INTRODUCTION AND SUMMARY	1
	1.1 Introductory Remarks	1
	1.2 Report Summary	5
	1.3 Conclusions	8
	1.4 Direction of Future Study	9
2	BACKGROUND INFORMATION	11
	2.1 Deterministic Design Procedures	11
	2.2 Criteria for Safety and Serviceability	17
3	SECOND-MOMENT PROBABILISTIC DESIGN	23
	3.1 Evaluation of Risk	23
	3.2 Probabilistic Design Formats	30
	3.3 Evaluation of Statistics of S and R	33
	3.4 Practical Design Formats	41
4	LOADS AND THEIR COMBINATIONS	55
	4.1 Stochastic Evaluation of Load Effects	55
	4.2 Categorization of Loads	59
	4.3 Evaluation of Statistics of Extreme Loads	66
	4.4 Discussion	95
5	DESIGN FOR MULTIPLE LOAD COMBINATIONS	97
	5.1 Evaluation of Risk under Multiple Load Combinations	97
	5.2 Design Factors under Multiple Load Combinations	101
	5.3 Variables Affecting Reliability-Based Design Factors	105
	5.4 Illustrative Example	110

CONTENTS (CONCLUDED)

<u>Chapter</u>		<u>Page</u>
6	ON INVESTIGATION AND DEVELOPMENT OF CURRENT DESIGN PROCEDURES	130
6.1	Examination of Current Design Factors . . .	131
6.2	Design Factors for Unconventional Structures	139
6.3	Development of Probabilistic Design Procedures	140
	REFERENCES	141
<u>APPENDIXES</u>		
A	DERIVATION OF p AND q FUNCTIONS FOR GAUSSIAN PROCESS	A-1
B	EXTREME VALUE DISTRIBUTION FOR FILTERED POISSON PROCESS	B-1
C	DERIVATION OF EQUATION 4-32	C-1

ILLUSTRATIONS

<u>Figure</u>		<u>Page</u>
2-1	Types of Structural Systems	19
3-1	Probability Density Functions for Load and Resistance	25
3-2	Variation of α_R with r	43
4-1	Time-Independent Load, Category 1	61
4-2	Time-Dependent Continuous Load, Category 2	62
4-3	Time-Dependent Piecewise Continuous Load, Category 3	63
4-4	Occasional Short-Duration Load, Category 4	65
4-5	Sample of Stationary Gaussian Process	70
4-6	p and q Functions for a Gaussian Process	70
4-7	Sample of Poisson Square Wave Process	73
4-8	Sample of Filtered Poisson Process	73
4-9	p and q Functions for Poisson Square Wave and Filtered Poisson Processes with Normal Distribution	77
4-10	p and q Functions for Poisson Square Wave and Filtered Poisson Processes with Lognormal Distribution	78
4-11	p and q Functions for Poisson Square Wave and Filtered Poisson Processes with Various Distributions	79
4-12	Sample of Filtered Poisson Process with Overlapping	81
4-13	Significance of Overlapping in Filtered Poisson Processes with Normal Intensity Distribution	83

ILLUSTRATIONS (CONCLUDED)

<u>Figure</u>		<u>Page</u>
4-14	Sample Combination of Three Load Effects	85
4-15	Computed and Simulated Results for Combination of Poisson Square Wave Loads; S_1 -- Normal, S_2 -- Lognormal	89
4-16	Computed and Simulated Results for Combination of Filtered Poisson Loads; S_1 -- Normal, S_2 -- Lognormal	92
4-17	Computed and Simulated Results for Combination of Poisson Square Wave and Filtered Poisson Loads; S_1 -- Normal, S_2 -- Lognormal	94

<u>Table</u>		<u>Page</u>
3-1	Values of β_1 and β_2 Assuming Normal Z_1 and Lognormal Z_2	32
3-2	Decoupling Factor α_s of Equation 3-43	48
3-3	Assumed Values of the Decoupling Factor α_s . .	49
3-4	Decoupling Factor α_s of Equation 3-54	52
5-1	Assumed Load Parameters for Illustrative Example	111
5-2	Computed Results for Combinations of Loads . . .	113
5-3	Reliability Design Based on $Z_1 = R - S_T$	120
5-4	Reliability Design Based on $Z_2 = R/S_T$	122
5-5	Reliability-Based Safety Factors $\bar{\gamma}^*$ Comparable with Current Procedures	124
5-6	Reliability-Based Strength and Load Factors Comparable with Current Procedures	126

CHAPTER 1

INTRODUCTION AND SUMMARY

This report develops a comprehensive theoretical basis for evaluating and specifying reliability-consistent safety and load factors for design. It also critically examines current design procedures to determine whether there are major inadequacies so that a framework can be established for future improvements, if needed. This chapter provides introductory remarks and a brief summary of the results of the investigation.

1.1 INTRODUCTORY REMARKS

The basic objectives in design are to assure the safety and serviceability of the proposed facilities. A nuclear power plant, for example, should safely withstand any set of loads that may reasonably be expected to occur during its service life and should be capable of economically performing its operational functions under all normal load conditions. Ideally, for safe design, the designer must evaluate the effect of the most severe combination of potential loads, including loads due to accidents or natural hazards, and select structure,* component, and equipment proportions so that these loads can be safely resisted. Also, to maintain serviceability, the designer must evaluate the effect of loads that occur frequently and make certain that a satisfactory operating performance is possible during the occurrence of these loads.

Future loads on a structure, however, are uncertain and act in unpredictable combinations. Also, structural material properties are random and structural member behaviors are not fully understood. In addition, there are always errors in estimation of design variables, structural analysis, and construction techniques. Under these conditions of uncertainty the *absolute*

*The material in this report has equal applicability to the structural design of nuclear power plant structures, components, and systems. However, for simplicity, reference will hereafter be made only to a structure.

assurance of safety and serviceability, i.e., zero probability of failure, for most structures becomes impossible. Specifically, the absolute assurance of safety of a structure, which would require consideration of extremely rare phenomena such as loads due to catastrophic events, would result in such a huge and expensive design that its construction would be economically prohibitive.

For the above reason, the safety and serviceability of a structure may only be assured within acceptable levels of risk, i.e., small probabilities of structural failure or unserviceability are tolerated. The *acceptable level of risk* for a proposed structure should ideally be obtained from a cost-optimization analysis which considers the initial cost of the design as well as the expected socioeconomic losses due to its potential failure or unserviceability. This type of analysis indicates that structures whose functions are of high socioeconomic value, as well as structures whose failures may be associated with large socioeconomic loss, should have higher levels of reliability (i.e., lower levels of acceptable risk) than ordinary structures. For example, the acceptable risk for a hospital building, which has a critical function for an entire community, or a nuclear power plant, whose failure may have catastrophic consequences for large populations, should be smaller than the risk accepted for a residential building.

The uncertainties that are present in structural design, require that a rational structural design procedure be based on concepts of the probability theory. This has long been recognized by many investigators (see, for example, Freudenthal, 1947; Bolotin, 1969) who have studied various aspects of probabilistic structural analysis and design. In short, the probabilistic design of a structure would require (1) probabilistic assessment of loads, structural properties, and other design variables, and (2) probabilistic assessment of structural response to applied loads. The latter would involve the evaluation of risks associated with unfavorable structural response or failure. Hence, a design would be accepted if the calculated risk is smaller than the tolerable value of risk. This type of

rigorous probabilistic design procedure, however, has seldom been practically applied for reasons that will be indicated shortly.

The structural design procedures currently applied, on the other hand, are entirely based on deterministic principles. These procedures do not recognize the randomness inherent in future loads, structural properties, or other design variables. Also, they do not attempt to measure structural safety or serviceability in terms of the involved risks. Instead, these procedures assume *nominal* values of loads, material properties, etc., and on the basis of these values attempt to evaluate structural safety or serviceability in terms of arbitrary criteria which are not necessarily realistic measures of safety or serviceability. In this regard, it is often incorrectly implied that a structure which has been designed in accordance to these procedures has zero risk of failure or unserviceability. In view of the condition of uncertainty prevailing for most structural designs, this implication is baseless and erroneous. In fact, the major disadvantage of the deterministic design method is that the true reliability of a structure designed with these procedures remains unknown and in many cases it may be different from the desired reliability, for example, the structure may be too conservative (i.e., from an economic point of view) or it may be unconservative (i.e., associated with undesirably high levels of risk). This disadvantage is of particular concern in designing costly structures which require very high levels of reliability such as nuclear power plants.

There are several reasons why probabilistic design is not yet practically applicable. These may be summarized as follows: (1) probabilistic design procedures are increasingly complicated relative to the deterministic procedures; for instance in many cases closed form solutions are not possible and extensive additional computations are necessary, (2) the required statistical data, which must be the basis for probabilistic assessment of design variables, are not adequately available, and (3) the engineering community is not generally familiar with probability theory and has often rejected its use in structural design. In fact, many engineers reject the notion of acceptable risk and believe that "absolutely safe" structures may be designed with deterministic procedures.

In recent years, interest in probabilistic methods in engineering has greatly increased and considerable research in the areas of structural reliability and probabilistic design have been reported. Due to the complexity and impracticality of accurate analysis as well as the lack of complete probabilistic information, various approximate methods have been studied. Among these, the probabilistic structural analysis and design based on second moment theories has become the most promising (e.g., Cornell, 1969; Lind, 1971; Ang et al., 1974). In this method only means, variances, and covariances (i.e., first and second moments) of variables, rather than their full probability distributions, are used. The failure probabilities are then evaluated on the basis of reasonable distributions assigned to specified performance functions. In this method, uncertainties due to various errors or approximations may also be included in the analysis. Clearly, the failure probability evaluated with such an approximate procedure will not be accurate. In particular, for low probabilities of failure, which are of most practical interest, the evaluated probabilities are found to be sensitive to the distribution assigned to the performance function (Ang, 1973). As a result, failure probabilities which are evaluated on the basis of second moment theories may only be regarded as *relative* measures of the true risks. However, such relative measures of the true risk are all that are necessary for most practical applications. This is particularly so when the acceptable risk itself is only a relative value, i.e., when one structure is required to be a certain order of magnitude safer than another structure.

A particular advantage of the second moment probabilistic design procedure is that it can be developed in formats which are similar and comparable to deterministic design formats currently in use. This similarity will undoubtedly make the adoption of such a probabilistic design format into building codes much more feasible. In addition, this similarity in format allows direct comparison of probabilistic and deterministic formats thus providing a theoretical basis for critical investigation of design criteria currently used in practice. Such an investigation will be attempted during this study, specifically directed towards assessing the rationality and

appropriateness of various design parameters, such as safety factors or load and strength factors, as they are presently applied to design equations of various critical load combinations.

1.2 REPORT SUMMARY

The results of this investigation are presented in the five remaining chapters of this report. Chapter 2 provides background information, including a brief description of current deterministic design procedures, and some of the basic concepts of probabilistic design and analysis methods. Chapter 3 presents a development of the second moment probabilistic design method. In this chapter, the basic concept of this theory as applied to a single combination of loads is discussed, and practical formats parallel to the formats used in deterministic methods are developed. Also, in this chapter, the evaluation of statistics of load effects and strengths, as well as the analysis of various uncertainties associated with a design process are discussed.

Chapter 4 presents a new approach to the evaluation of stochastic load combination. A second moment approach is developed whereby the moments of the lifetime extreme of combined loads is obtained in terms of the moments of individual loads as well as their other descriptor parameters. This approach is ideal for use in conjunction with the second moment reliability approach.

In Chapter 5, the developments in the two previous chapters are combined to analytically derive safety, strength, and load factors for the case of multiple load combinations. Various design variables that may significantly affect these factors are discussed. A numerical example is provided at the end of this chapter, wherein reliability-based values of the design factors are evaluated for a number of load combinations.

Chapter 6 presents a critical qualitative examination of current procedures and discusses means through which these procedures can be improved and further developed. The basis for direct comparison of analytically evaluated factors with current factors is also discussed in this chapter.

The numerical example provided in Chapter 5 is of particular interest although it is only hypothetically designed to demonstrate the application of the theory. In this model a structural component was considered which could experience dead load, D , a sustained live load, L_s , an extraordinary transient live load, L_e , a sustained temperature load, T_o , and an earthquake load E . The following four load combinations were considered:

1. $D + L_s + T_o$
2. $D + L_s + T_o + L_e$
3. $D + L_s + T_o + E$
4. $D + L_s + T_o + L_e + E$

Results for service lives of 25 and 50 years and acceptable risk levels of 10^{-4} and 10^{-5} were compared using normal and lognormal assumed distributions for performance functions $Z_1 = R - S$ and $Z_2 = R/S$, respectively, where R and S denote the member resistance and load effect. Important results observed in this numerical example were the following.

Significance of Distribution. It was observed that the results for the two formulations based on Z_1 and Z_2 were generally different. This indicates that for the levels of risk considered (i.e., 10^{-4} and 10^{-5}), the reliability-based safety, load or strength factors are not insensitive to the distributions of R and S_T . In particular, the results based on Z_1 would be accurate if the underlying distributions were normal, whereas the results based on Z_2 would be accurate if they were lognormal. It is generally believed, however, that the lognormal distribution is more realistic for loads and resistances (Ang and Cornell, 1974.)* Based on this assumption,

*The normal distribution is not realistic since it allows both positive and negative values of the variate, whereas loads and resistances can only have positive values. The safety and strength factors are particularly influenced by this inconsistency.

the values corresponding to the formulation for Z_2 are considered to be more realistic. It is important to realize that regardless of the sensitivity to distribution, either of the formulations developed in this report can be useful and realistic when interpreted and used in a *relative* sense.

Acceptable Risk. The significance of the acceptable risk level was observed by comparing the results for risk levels of 10^{-4} and 10^{-5} . It was observed that this order of magnitude reduction in the risk level results in larger safety and load factors and in smaller strength factors. These variations due to risk are found to be quite significant. As an example, an increase of about 14% in the safety factors based on Z_2 result from the reduction in the acceptable risk.

Structure Life. The significance of structure lifetime was observed by comparing the load factors, which refer to the same values of loads (i.e., the instantaneous values), for $T = 25$ and 50 years. It appears that the structure lifetime has an appreciable influence on the factors for stochastic loads, whereas it has practically no significance on dead load factors. The increase in stochastic load factors for an increase in service life from 25 to 50 years is estimated to be about 3% to 4% for the formulation based on Z_2 .

Combinations of Loads. It was observed that safety and load factors for various combinations were generally different. These variations are related to the number of loads that are combined, and to the stochastic nature of loads and their combinations. It is apparent that variations in safety or load factors from one combination to another can be significant, particularly when transient loads are combined. The reduction in safety and load factors in going from the first three load combinations to the fourth combination, which includes the extraordinary live load, L_e , as well as the earthquake load, E , is of interest. It was found that with respect to the first or the third load combination, the reduction in the safety or the total load factor is about 23%, whereas with respect to the second combination this reduction is about 15%. Current design practice would normally allow a fixed 25% reduction in these factors.

The reduction in individual load factors varies from one load to another. This variation is due to the characteristics of each load as well as the assumed procedure for separating the effects of the loads. Reduction in factors of stochastic loads were observed to range from 10% to 27%. A significant observation is that the dead load factor exhibits only a reduction of less than 5%. This reduction can only be attributed to the increased number of loads being combined. These observations contradict the current practice which permits a fixed reduction of 25% in factors of all loads including the dead load.

1.3 CONCLUSIONS

Conclusions that can be drawn from the overall study can be summarized as follows:

- a. On the basis of the second-moment probability theory, it is possible to theoretically derive expressions for reliability-based safety load, and strength factors which consistently include the significance of various design variables.
- b. Variables which may significantly affect the reliability-based safety, load, and strength factors are determined. These are found to be the acceptable risk, the coefficients of variation of loads and the resistances, the useful life of the structure, and parameters describing stochastic fluctuations of loads and their combinations.
- c. Qualitative examinations of current deterministic design procedures reveal that safety, load, and strength factors specified in these procedures do not include the significance of the above variables in a consistent manner. As a result of this, structures designed with these procedures may have different reliabilities than desired. The inconsistency could be most critical for unconventional structures such as nuclear power plants which require unprecedented levels of reliability.

- d. The theoretical formulations developed in this report can be used as a framework for improving the existing deterministic procedures and for refining them so that they may include the effects of significant design variables. These formulations can also be used as a guideline for specification of design criteria for structures which require unprecedented levels of reliability. Specific suggestions in these regards are provided in Chapter 6.
- e. The techniques developed in this report, including the second-moment specification of stochastic loads and their combinations, and the evaluation of failure probability in terms of the first two moments of loads and the resistances, have considerable practical feasibility. These techniques can, therefore, form the framework for development of practical probabilistic design codes.

1.4 DIRECTION OF FUTURE STUDY

The problem of determining an improved set of safety or load factors for current deterministic procedures should not be viewed as a simple problem. It is believed that careful and comprehensive studies are required before any qualitative modifications of current values can be recommended. Towards that end, the following directions for future studies are recommended.

- a. Improve and further develop the theoretical models of loads and the formulation of reliability-based design formats. Carefully examine various assumptions that are involved and determine their ranges of validity.
- b. Perform sensitivity analysis of reliability-based design factors in order to investigate the relative significance of various design variables influencing them.

- c. Obtain statistical data of loads and resistances and evaluate nominal loads and resistances used in current practice relative to their respective actual means and standard deviations. Such information is necessary if direct comparison of current factors with reliability-based factors are required.
- d. Consider typical categories of structures and carry out comprehensive parallel studies with the current deterministic procedures as well as the probabilistic procedure developed in this report. By comparison of results on a relative basis, determine sets of safety or load factors that are consistent with desired levels of reliability. This study should involve both the theoretical formulations of the probabilistic approach and the valuable engineering experience inherent in the current design practice as applied to conventional structures.

CHAPTER 2

BACKGROUND INFORMATION

This chapter presents background information relative to various design philosophies. Section 2.1 briefly describes the current deterministic design methods and discusses their desirability from the viewpoint of a rational structural design procedure. Section 2.2 describes the basic criteria for safety and serviceability as applied in a probabilistic procedure and discusses the concepts of system reliability versus element reliability and their relevance in the second moment probabilistic design.

2.1 DETERMINISTIC DESIGN PROCEDURES

There are two design methods currently in use in the United States. These are the working stress design (WSD) and the load factor design (LFD) methods (the latter is also known as the ultimate strength design, USD, method). Both of these methods are basically deterministic in the sense that they do not include the randomness and uncertainty associated with loads, material properties, structural behavior, etc., or the probabilities that are associated with the occurrence of various loads or their possible combinations. Moreover, safety and serviceability criteria in these methods are not connected to any rationally defined risk level.

Both methods are based on a set of design loads. These loads supposedly represent upper bounds to "service" loads, i.e., loads that may occur under normal service conditions. These are either estimated by the designer (usually according to code provisions) or obtained from appropriate building codes. The service loads do not include unusual overloads which may occur under very rare circumstances. Such overloads are to be supported by providing additional capacity than that required under service loads. It must be understood that since future loads on a structure are uncertain, any given set of loads, such as the "service" loads, are necessarily associated with certain probabilities and thus the term "upper bound service loads"

is misleading. In fact the same design may be obtained if larger service loads are assumed and instead the required safety margin for the structure is reduced. Therefore, service loads used in current design methods are arbitrary. For this reason these loads will be defined herein as *nominal loads*.

In order to assure safety against rare overloads or unpredicted adverse structural behavior, both methods require structural capacities in excess of those required for nominal loads. This additional capacity is usually defined as the safety margin. The basic difference between the WSD and LFD methods lies in their respective philosophies for providing this safety margin.

The WSD method requires that under nominal (service) load conditions, stresses in a structure remain below an "allowable stress" level as defined by codes (usually about 50 to 60% of yield or ultimate strength of the material used). Thus safety against unusual overloads or unpredictable behavior is provided by maintaining reserve strength, in terms of maximum stress value, in structural components. The safety margin or safety factor is expressed in terms of the ratio of ultimate or yield stress to the allowable stress. For the purpose of the present study, the design equation for any combination of loads in this method may be expressed as

$$R' \geq \gamma \sum f_i X_i' \quad (2-1)$$

where R' = the nominal value of structural strength at a critical point in terms of *yield or ultimate stress*, X_i' = the nominal value of a load (e.g., dead, live, or earthquake load), f_i = the influence coefficient defining the stress at the point due to a unit value of the applied load, and γ = the safety factor, i.e., the ratio of R' to the allowable stress, as specified by the code. The summation is over all loads included in the combination.

It must be realized that in most structures the maximum stress in a component is not a good measure of the safety of the component. This is especially true for statically indeterminate structures where under the action of large loads inelastic behavior causes redistribution of stresses from highly stressed sections to other sections. For this reason in recent years the WSD method has been regarded as undesirable for safety analysis of structures and the LFD method has been predominantly recommended for this purpose.

In order to assure safety against rare overloads or unpredictable structural behavior, the LFD method attempts to directly include these effects into the design process. Thus, it requires that the "dependable" strength of each structural component, including the adverse effect of variabilities in material properties and behavior, be greater than the "largest" load effect resulting from the most critical combination of loads, including the effect of rare overloads. In order to arrive at an estimate of possible overloads, each nominal load value is multiplied by a *load factor* (usually greater than unity) which depends on the variability of the load. Thus a highly unpredictable load (e.g., earthquake load) has a larger load factor than a load which has small variability (e.g., dead load). The evaluation of the strength of structural components is based on approximate analysis techniques and nominal values of design variables, e.g., material properties or member dimensions. Thus the estimated strength represents a nominal value. To include the adverse effect of uncertainties in these factors, the nominal strength is multiplied by a *strength factor* which is smaller than unity. The design equation for any combination of loads in this method may be expressed as

$$\phi R^I \geq \sum_i \delta_i f_i X_i^I \quad (2-2)$$

where R' = the nominal value of strength of a particular component,
 X'_i = the nominal value of the load, f_i = the influence coefficient defining the load effect (bending moment, shear, etc.) at the component due to a unit value of the applied load, δ_i = the load factor (usually greater than or equal to unity), and ϕ = the strength factor (smaller than or equal to unity). The summation is over all loads included in a combination.

It must be realized that although load and strength factors are to include the effects of uncertainties in loads and resistances, they have not been evaluated on a probabilistic basis. Their values, as specified by codes, commonly remain invariant irrespective of the type of structure, its function, or its location, and regardless of the planned useful life of a structure. The determination of these values have mainly been subjective and on the basis of calibration with other methods of design (e.g., the WSD method).

The safety margin in the LFD method is not well defined, however, the ratio of the average load factor to the strength factor, or the ratio of the strength to the nominal (service) load effect is often used as a measure of the safety margin.

In designing for a specific load combination, both methods simply add the effects of various loads. With the exception of combinations involving rare loads (i.e., high intensity earthquake ground motions or high velocity winds), no modifications in the safety or load and strength factors are provided. *Thus, in these cases, no consideration is given to probabilities and correlations associated with the simultaneous occurrence of various loads. For the case of combinations involving rare loads usually a 25% reduction in safety or load factors is permitted. These modifications are to reflect the low probabilities that are associated with the occurrence of, for example, a major earthquake at the same time as the occurrence of extreme live loads. Although these modifications are reasonable, they are arbitrary because they are not related to the rate of occurrence of extreme loads, e.g., major earthquakes in the region of the site, or occurrence of large live loads for the particular type of structure under consideration.*

From the viewpoint of an ideal design approach the WSD and LFD methods have several shortcomings. These will be demonstrated clearly during the course of this study, however, on the basis of the above discussion they may be summarized as follows.

- a. Design loads and resistances are not clearly and consistently defined. Since these are random variables, a statistical definition, such as the number of standard deviations above or below mean values, is more desirable.
- b. In the WSD method, the safety criteria which is defined in terms of stress is not a realistic criteria.
- c. In the WSD method the uncertainties associated with loads, material properties, and structural behavior are not consistently included in the design process. A single safety factor, which remains constant for all types of structures and site locations, and for all load combinations cannot consistently account for the combined effect of these uncertainties.
- d. In the LFD method the variabilities in loads and resistances are roughly included in the design through the load and strength factors. However, these factors are not directly related to the variabilities of loads and resistances. In particular, a constant set of factors for all structures and site locations is not reasonable since variabilities of loads and resistances may change significantly from one structure to another or from one location to another.
- e. Neither method is able to consistently account for the uncertainties that may arise from approximations in structural analysis and errors in construction.

- f. In both methods, various loads are combined with no regard to the probabilities and correlations associated with their simultaneous occurrence. The constant reduction of safety and load factors, for combinations involving earthquake and wind loads is arbitrary. Rationally, these modifications must be related to the occurrence rate of earthquakes or winds in the region of the site as well as the occurrence of other loads.
- g. The safety margins, as defined in the WSD and LFD methods are not consistent measures of structural reliability. As a result, structures designed on the basis of either method would generally not have a consistent level of safety; i.e., some may be too conservative (i.e., more than desirable from economic viewpoint) while others may be unconservative (i.e., associated with undesirably high levels of risk). Moreover, if a structure with an unprecedented level of reliability is to be designed, there is no rational basis for specifying appropriate safety or load and strength factors that must be used. In this regard, it must be realized that the current values of the design factors have evolved through many years of engineering experience with conventional structures. Due to many trials, errors, and modifications in the course of this evolution, it could well be that these factors reasonably produce consistent designs of such structures. However, there is no reason to believe that these same sets of safety or load and strength factors are appropriate for designing an unconventional structure, i.e., one that requires a much higher level of reliability than conventional structures.

The latter point is particularly significant in developing design criteria for nuclear power plants. Due to the high economic value of these structures, and the catastrophic consequences that may result from their failures, exceptionally high levels of reliability are demanded. As a result, in addition to regular loads, loads due to extremely rare events (e.g., extreme intensity ground motions, tornado wind velocities, aircraft crash, and potential accidents during a natural hazard), which are commonly neglected in structural design, become significant and must be included in the design process. Since no theoretical bases exist for the evaluation of load factors, it is not clear what the appropriate factors should be for combinations involving these loads. In addition, due to the high level of required reliability, it is not reasonable to use the same design factors as in conventional designs (for example, it is not reasonable to use the same wind load factor for both a residential building and the containment structure of a nuclear power plant). In the absence of more appropriate load factors, the current practice (see USNRC Standard Review Plan, Sec. 3.7.1) is to use values that are determined on a subjected basis, or are arbitrarily derived from load factors used in conventional design. As a result, the true reliability of a nuclear power plant designed according to current load factors may not be consistent with the desired levels of safety. A conclusive solution of this problem is only possible if a theoretical basis is developed for the specification of design factors. As demonstrated in this report, such a theoretical basis can be developed using the second moment probability theory.

2.2 CRITERIA FOR SAFETY AND SERVICEABILITY

As discussed in Section 1.1, structural design is invariably performed under conditions of uncertainty that are associated with applied loads and structural resistances. As a consequence of this, the probability of an adverse event, such as excessive deformations or collapse of structural components or an entire structure, is virtually unavoidable. This probability, which is generally known as the "risk" or "probability of failure," can appropriately be the basis for the specification of criteria for safety and serviceability.

The safety and integrity of a structure is clearly of concern during the entire time of its operation. For this reason, the *useful life* of a structure is of primary interest in its safety analysis. In particular, the evaluation of the risk of structural damage or collapse should generally consider the following two modes:

- a. Structural damage or collapse under lifetime maximum load
- b. Structural damage or collapse under repeated (operational) loads which may result from fatigue phenomenon

For Case *a*, clearly, the lifetime maximum load would generally be a function of the structure's life. Also, for Case *b* the number of applications of the load also depends on this lifetime. It is apparent that a rational criterion for structural safety may be specified* in terms of an *acceptable level of risk during the structure's useful life*.

Besides being safe, a structure must also be able to perform its intended function economically. Excessive deformations, large amplitude vibrations, wide cracks, or frequent repairable damages (due to local yielding) could interfere with structural performance. Therefore, the risk of such events must be retained to acceptably small values, such that their infrequent occurrences are tolerable. It appears that the useful life of a structure in this case is of no interest; instead it is the percentage of time that a structure fails to operate that is important. Therefore, a rational criterion for serviceability may be specified* in terms of an *acceptable level of risk during a unit period of time*.

In specifying acceptable risk levels, one must distinguish component and system reliability (Moses, 1974). For a brief discussion of this subject consider the systems shown in Figure 2-1. The system in

*This specification can be in an indirect manner with no reference to risk, as discussed in Chapter 3.

(a) "Weakest-link" system

(b) "Fail-safe" system

FIGURE 2-1. TYPES OF STRUCTURAL SYSTEMS

Figure 2-1a is composed of N components connected as a chain. This type of a system, commonly known as a "weakest-link" system, would fail if any one of its components fails. Therefore, the risk or probability of failure of the system will in general be equal to or greater than the risk for the weakest component. Practical examples of such systems are statically determinate structures in which the failure of any member constitutes structural collapse. The system in Figure 2-1b is composed of N components connected in a parallel manner. This type of a system, commonly known as a "fail-safe" system, would only collapse if all of its components failed; that is, the failure of a component does not necessarily cause system collapse, although it constitutes damage of the system. Practical examples of "fail-safe" systems are statically indeterminate structures in which the failure of certain components does not result in structural collapse; this is particularly true for structures composed of ductile components. In these types of structures, the risk for structural collapse would in general be different from the risk for the weakest component, and in certain cases may be smaller than this value. The risk for structural damage, however, would in general be equal to, or greater than, the risk for the weakest component (Ang, 1977).

An accurate evaluation of system reliability in terms of component reliabilities is extremely complicated except for certain simple types of structures. However, in most cases, bounds to this reliability may be obtained in terms of component reliabilities. Also, for certain cases an approximate evaluation of system reliability based on the type and number of components and their reliabilities is possible (Moses, 1974; Ang, 1977).

For a probabilistic structural design, ideally, the reliabilities of various components as well as the structure as a whole must be specified. However, due to the difficulties in evaluation of structural collapse modes and total system reliability, practical design is formulated only on the basis of component reliabilities. The reliability of the entire structure may then be checked with some approximate means or by evaluation of its bounds. Thus, the development of the second-moment probabilistic design in the subsequent chapters only considers the specified values of acceptable risk for structural components.

Another important fact that must be realized is that the values of acceptable risks for components or systems are generally very small (i.e., of the order of 10^{-3} or smaller). Since such small values of probabilities may be sensitive to assumed distributions or various approximations in an analysis, the calculated values of risk must only be regarded as consistent relative measures of the underlying risks. Thus, for example, a component designed for a risk of 10^{-6} is regarded to be two orders of magnitude safer than a component that is designed (with the same procedure) for a risk of 10^{-4} , although each of these values may not correctly represent the absolute values of the risks that are involved.

CHAPTER 3

SECOND-MOMENT PROBABILISTIC DESIGN

This chapter presents a development of the second-moment probabilistic design concepts. Section 3.1 discusses the evaluation of risk for a structural component in terms of the first and second moment statistics (i.e., means and variances) of the load and resistance. Section 3.2 derives reliability-based design formats consistent with the second-moment statistics. Section 3.3 discusses the evaluation of the required statistics of loads and resistances which include the effects of the uncertainties that are involved. Finally, in Section 3.4 on the basis of the previous formulation, risk-consistent design factors, i.e., safety, and load and strength factors corresponding to given levels of risk, are derived. In the formulations of this chapter, a single combination of loads is considered; the case of multiple load combinations as well as the stochastic nature of loads are discussed in the subsequent chapter.

3.1 EVALUATION OF RISK

The evaluation of risk or failure probability is central to the development of a probabilistic design procedure. As discussed in Section 2.2, the assessment of this probability for each structural component is required. The component risk values as well as the information regarding the way components are assembled should then lead to an estimate of the risk for the entire structure.

Consider a component of a structure which is affected by a specific combination of loads. It is desired to evaluate the probability of failure of the component in a specific mode. Let S denote the combined load effect in a critical section of the component and R denote the corresponding resistance. As an example, in evaluating the probability of failure of a beam in flexural mode, S would correspond to the applied moment in the critical section whereas R would describe the flexural capacity of the section. Due to the uncertainties in loads and resistances, clearly S

and R are random variables. Let $f_S(s)$ and $f_R(r)$ describe the probability density functions (PDF) of S and R respectively. A typical illustration of these functions may appear as in Figure 3-1.

The failure of the component is defined as the event $\{R < S\}$, i.e., the event that the applied load effect exceeds the capacity of the component. The corresponding probability of failure, denoted by p_f , is then obtained as,

$$\begin{aligned} p_f &= P(R < S) \\ &= \int_{-\infty}^{\infty} P(R < S | S = s) f_S(s) ds \end{aligned}$$

which assuming statistically independent R and S becomes,

$$\begin{aligned} p_f &= \int_{-\infty}^{\infty} P(R < s) f_S(s) ds \\ &= \int_{-\infty}^{\infty} F_R(s) f_S(s) ds \end{aligned} \tag{3-1}$$

where $F_R(s) = \int_0^s f_R(r) dr$ is the cumulative distribution function (CDF) of R evaluated at $r = s$. The statistical independence of R and S is a reasonable assumption for most practical cases where the variabilities in load effects are mainly due to variabilities in loads and the resistances are independent of applied loads.

The evaluation of p_f from the above formulation requires a knowledge of probability distributions for both R and S . For most practical situations, where only limited amounts of statistical data are available, such complete information on R and S are impossible to obtain. However, by prescribing physically reasonable distributions which are consistent with the available data one may approximately evaluate the failure probability. For this purpose usually normal or lognormal distributions may be assumed which result in relatively simple closed form solutions of Equation 3-1.

FIGURE 3-1. PROBABILITY DENSITY FUNCTIONS FOR
LOAD AND RESISTANCE

In the above formulation of p_f , clearly, two separate distributions must be prescribed, i.e., one for R and one for S . This, however, is not necessary if the failure event is defined in terms of a performance function of R and S in which case only one distribution for the performance function must be prescribed. Two performance functions which give useful results for practical purposes are considered here.

First consider the performance function $Z_1 = R - S$. Obviously failure is defined by the event $\{Z_1 < 0\}$. Therefore the probability of failure may be obtained as

$$\begin{aligned} p_f &= P(Z_1 < 0) \\ &= F_{Z_1}(0) \end{aligned} \tag{3-2}$$

where $F_{Z_1}(0)$ is the CDF of Z_1 evaluated at $z_1 = 0$. Clearly, the distribution of Z_1 is all that is necessary to evaluate p_f . A second performance function which is of interest may be defined as $Z_2 = R/S$. In this case failure is defined by the event $\{Z_2 < 1.0\}$, and the probability of failure may be obtained as

$$\begin{aligned} p_f &= P(Z_2 < 1.0) \\ &= F_{Z_2}(1.0) \end{aligned} \tag{3-3}$$

where $F_{Z_2}(1.0)$ is the CDF of Z_2 evaluated at $z_2 = 1.0$. Again, the distribution of Z_2 must be prescribed in order to evaluate p_f .

In prescribing distributions for Z_1 and Z_2 the following must be considered.

- a. The distributions must be physically consistent, i.e., since Z_1 may have both positive and negative values $f_{Z_1}(z_1)$ must range over both positive and negative values of z_1 ; also, since Z_2 is always positive, $f_{Z_2}(z_2)$ must only exist over positive values of z_2 .
- b. The assumed distributions must be consistent with the supporting statistical data of R and S that is available. In this respect it is noted that if the available data suggest normal distributions for both R and S , the function $R - S$ would also be normally distributed, and hence a normal distribution for Z_1 would be appropriate. Also, if the available data suggest lognormal distributions for both R and S , the function R/S would also be lognormally distributed and therefore a lognormal distribution for Z_2 would be consistent with the data.
- c. For practical purposes it is desirable that the assumed distributions give simple analytical results. In this respect, a normal distribution for Z_1 and a lognormal distribution for Z_2 give simple formulations for the probability of failure, which, as will be shown in this chapter, can lead to very useful design formats. These two cases are discussed subsequently.

Normally Distributed Z_1 . The normal distribution of a random variable is fully defined by the mean and standard deviation of the variable. The mean and standard deviation of $Z_1 = R - S$ may be obtained as

$$\mu_{Z_1} = \mu_R - \mu_S; \text{ and } \sigma_{Z_1} = \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3-4)$$

where μ_{Z_1} , μ_R , and μ_S are the means and σ_{Z_1} , σ_R , and σ_S are the standard deviations of Z_1 , R , and S respectively. With these values the probability of failure is evaluated from Equation 3-2 as

$$p_f = 1 - \phi \left(\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \right) \quad (3-5)$$

where $\phi(\)$ is the standard normal probability tabulated widely (see Ang and Tang, 1975). It appears that in this case p_f may be evaluated as a function of the means and variances (i.e., first and second moments) of R and S . It is noted that the above result may also be obtained from Equation 3-1 if both R and S are assumed to have normal distributions.

Lognormally Distributed Z_2 . A lognormal distribution for Z_2 would imply a normal distribution for $\ln Z_2 = \ln R/S$. The mean and standard deviation of the latter may be obtained from first-order approximations as

$$\mu_{\ln Z_2} \approx \ln \frac{\mu_R}{\mu_S}; \text{ and } \sigma_{\ln Z_2} \approx \sqrt{\Omega_R^2 + \Omega_S^2} \quad (3-6)$$

where Ω_R and Ω_S are the coefficients of variation of R and S respectively. The probability of failure may then be evaluated as

$$p_f = P(\ln Z_2 < 0) \\ \approx 1 - \phi \left(\frac{\ln \frac{\mu_R}{\mu_S}}{\sqrt{\Omega_R^2 + \Omega_S^2}} \right) \quad (3-7)$$

where $\phi(\)$ is as defined above. It is observed again that p_f is evaluated as a function of the first and second moments of R and S . Also, note that for Ω_R and $\Omega_S \leq 0.30$, the above result could approximately be obtained from Equation 3-1 if both R and S were assumed to be lognormal.

For other distributions that may be assumed for Z_1 or Z_2 , the Equation 3-2 or 3-3, respectively, must be directly evaluated. In these cases, p_f is not necessarily a function of the first and second moments and could in general involve the higher moments of R and S . In most cases the available data is not adequate to justify the use of distributions other than those discussed above and thus the direct evaluation of Equations 3-2 and 3-3 is not of practical interest.

It is obvious that the values of p_f as evaluated from Equations 3-5 and 3-7 are approximate. This is especially true for small values of probability, i.e., $p_f \leq 10^{-3}$, for which the far tails of assumed distribution are significant. Thus, such calculated values of p_f , may not be regarded as absolute measures of the true risk. Nevertheless, Equations 3-5 and 3-7 represent consistent formulations of the risk and therefore it is logical to regard the calculated value of p_f from either of these equations as a relative measure of the underlying risk. Indeed, in most practical design applications, a relative measure of the safety or serviceability is all that is necessary. Thus, either of these equations can be assumed as the basis for development of a consistent design procedure.

In the following section both of these formulations will be pursued, which then lead to alternative formulations of the probabilistic design format.

3.2 PROBABILISTIC DESIGN FORMATS

As noted before, a probabilistic design format would seek to proportion structural members such that a prescribed level of reliability is assured, i.e., such that the failure probability is less than the specified acceptable level. Clearly, the reliability of each structural component, as well as the reliability of the entire structure as a whole must be assured. A practical design format, however, is conveniently developed on the basis of component reliabilities alone. Thus, structural dimensions are determined on the basis of specified acceptable risks for various components, whereas the reliability of the structure as a whole may then be examined for the assumed dimensions.

Suppose that the acceptable risk p_f is specified for a particular mode of failure of a component. Solving for the mean resistance μ_R in either of Equations 3-5 and 3-7, the minimum required strength may be determined. Thus, on the basis of Equation 3-5, a design format may be derived as

$$\mu_R \geq \mu_S + \phi^{-1}(1 - p_f) \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3-8)$$

and on the basis of Equation 3-7 an alternative format may be derived as

$$\mu_R \geq \mu_S \exp \left[\phi^{-1}(1 - p_f) \sqrt{\Omega_R^2 + \Omega_S^2} \right] \quad (3-9)$$

where $\phi^{-1}(1 - p_f)$ is the value of the standard normal variate at the probability level $1 - p_f$ (i.e., the required reliability). As in deterministic design procedures, the actual proportioning of components would require iterative calculations; that is, first a set of dimensions should be assumed leading to an estimate of the right-hand side of the design inequality (note that in general μ_S , σ_S , and Ω_S would depend on the assumed dimensions); if this inequality is not satisfied, a new set of dimensions must be assumed and the process must be repeated until an acceptable design is obtained.

The above formats were based on normal distributions for $R - S$ and $\ln(R/S)$, respectively. For any other viable distribution, the above formats may be generalized as

$$\mu_R \geq \mu_S + \beta_1 \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3-10)$$

$$\mu_R \geq \mu_S \exp \left(\beta_2 \sqrt{\Omega_R^2 + \Omega_S^2} \right) \quad (3-11)$$

where β_1 and β_2 would now depend on the assumed distributions for $R - S$ and $\ln(R/S)$, respectively, as well as on the required level of reliability. Ang and Ellingwood (1971) have shown that when p_f is in the order of 10^{-3} or larger, the values of β_1 and β_2 are not very much sensitive to the distributions assumed for $R - S$ and $\ln(R/S)$, respectively. However, for very small values of p_f in the order of 10^{-5} or smaller, they have shown that β_1 and β_2 are highly dependent on the assumed distributions. Since in practical design of important structures, such as nuclear power plants, risk levels smaller than 10^{-5} are usually required, in these cases an accurate specification of β_1 or β_2 will not be possible. Nevertheless, a value of β_1 or β_2 based on a prescribed distribution may be used as a *relative* measure of the required reliability. This, of course, presumes that a constant value of β_1 or β_2 will provide designs which have more or less consistent levels of reliability.

Cornell (1969), Lind (1971), and Rosenblueth and Esteva (1972) suggested that β_1 or β_2 be used as a *safety index* with no explicit reference to the failure probability. This idea would be of interest in developing building codes where a direct reference to the risk is not desirable. It must be realized that β_1 and β_2 can only be relative measures of reliability. Their ultimate determination for various typical structures and components in a probabilistic design code must, therefore, be on the basis of analysis of existing structures and by calibration with

existing codes and practices. For the purpose of illustration, the values of β_1 and β_2 for normal Z_1 and lognormal Z_2 , respectively, are listed in Table 3-1 for various levels of p_f . Note that for the assumed distributions $\beta_1 = \beta_2 = \phi^{-1}(1 - p_f)$.

TABLE 3-1. VALUES OF β_1 AND β_2 ASSUMING
NORMAL Z_1 AND LOGNORMAL Z_2

p_f	β_1 or β_2
10^{-1}	1.28
10^{-2}	2.33
10^{-3}	3.09
10^{-4}	3.72
10^{-5}	4.25
10^{-6}	4.75
10^{-7}	5.20
10^{-8}	5.60

Equations 3-10 and 3-11 are the most common reliability-based design formats. Equation 3-10 is more appropriate when normal distributions for R and S are reasonable, whereas Equation 3-11 is preferable when lognormal distributions for R and S are more realistic. Since, in general, R and S cannot have negative values and commonly their respective density functions are skewed (these properties contradict with normal distribution), the second form presented in Equation 3-11 is preferred (Ang and Cornell, 1974). However, from a practical point of view, Equation 3-10 may be preferable due to its convenient form. In the present study, both forms will be used for the investigation of design parameters. In this way, the influence of assumed format on the final results of the investigation may be studied.

3.3 EVALUATION OF STATISTICS OF S AND R

In the above derivation, S was defined as the combined load effect in a critical section and R was defined as the resistance of the component. For example, in designing for the tension reinforcement of a reinforced concrete beam, S would represent the acting moment at the critical section, while R would represent the capacity of the section. Both of these random variables are generally functions of other variables. For example, S is a function of each applied load (e.g., dead load, live load, earthquake load) as well as the corresponding influence coefficients for the critical section; also R is a function of such variables as material strength and member dimensions. In practice, the statistics of S and R are usually not directly available, and one has to obtain them on the basis of the statistics of the related function variables. This procedure is discussed in this section.

For the purpose of generality, consider the random variable Z as a function of random variables $X_1, \dots, X_n$,

$$Z = g(X_1, X_2, \dots, X_n) \quad (3-12)$$

where Z may represent R or S , and X_i may represent such variables as loads, dimensions, material strength, etc., and $g(\cdot)$ describes the relation of Z to X_i . Let the random variables X_i have means μ_{X_i} , standard deviations σ_{X_i} , and coefficients of variation (cov) $\Omega_{X_i} = \sigma_{X_i}/\mu_{X_i}$. On the basis of first-order approximations (Benjamin and Cornell, 1970, or Ang and Tang, 1975), the mean value of Z approximately is

$$\mu_Z \approx g(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n}) \quad (3-13)$$

and the variance is

$$\sigma_Z^2 \approx \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial g}{\partial X_i} \right)_0 \left(\frac{\partial g}{\partial X_j} \right)_0 \rho_{ij} \sigma_{X_i} \sigma_{X_j} \quad (3-14)$$

in which $(\partial g / \partial X_i)_0$ indicate the derivatives evaluated at the mean values, and ρ_{ij} indicates the coefficient of correlation between X_i and X_j . For uncorrelated X_i and X_j the above equation simplifies to

$$\sigma_Z^2 \approx \sum_{i=1}^n \left(\frac{\partial g}{\partial X_i} \right)_0^2 \sigma_{X_i}^2 \quad (3-15)$$

With mean and variance of Z known, its cov value may then be obtained from

$$\Omega_Z = \frac{\sigma_Z}{\mu_Z} \quad (3-16)$$

The above derivation assumes that the mean and variance of X_i are available; moreover, it assumes that the function g correctly describes the relation of Z to X_i . These, however, are seldom true for engineering applications. In particular, the statistics of X_i usually are obtained on the basis of incomplete data, and the function g is often a mathematical idealization of a complicated phenomenon, and therefore is imperfect. It appears, therefore, that besides the basic variabilities of X_i , additional uncertainties exist as a result of errors in modeling and statistical estimation. In order to include these additional uncertainties, Ang (1973) introduced the extended reliability model. According to this procedure, each of the true variates X_i is modeled as

$$X_i = N_{X_i} \hat{X}_i \quad (3-17)$$

where $\hat{X}_i$ is an estimator of X_i represented by the available data and has the mean $\bar{x}_i$ and cov δ_{X_i} (estimated from the available data which may be incomplete), whereas N_{X_i} is a random correction factor with mean $\bar{N}_{X_i}$ and cov Δ_{X_i} representing the bias and the error in the estimation, respectively. For an unbiased estimator, $\bar{N}_{X_i}$ is approximately equal to one; also as a first approximation Δ_{X_i} is taken equal to the error in the estimated mean $\bar{x}_i$ (see Ang, 1973). With this, then, the mean and cov of X_i are evaluated as

$$\mu_i \approx \bar{x}_i \quad (3-18)$$

and

$$\Omega_{X_i} \approx \sqrt{\delta_{X_i}^2 + \Delta_{X_i}^2} \quad (3-19)$$

where $\sigma_{X_i} = \Omega_{X_i} \mu_{X_i} \approx \Omega_{X_i} \bar{x}_i$.

In a similar manner, the true function g is modeled as

$$g = N_g \hat{g}(X_1, X_2, \dots, X_n) \quad (3-20)$$

where $\hat{g}$ is the assumed function which may be imperfect, whereas N_g is a random correction factor representing the error in the assumed function.

Here, two kinds of variabilities should be considered. First, consider the variability in function $\hat{g}$ about its mean value, for a given set of data (for known X_i); this is represented by the cov value δ_g . Second, consider the error due to limited amount of data which is represented by the cov of N_g denoted by Δ_g . Thus the total uncertainty of the function g is obtained as

$$\Omega_g \approx \sqrt{\delta_g^2 + \Delta_g^2} \quad (3-21)$$

Finally, assuming that $\hat{g}$ is an unbiased estimator of g (i.e., the mean of N_g is equal to 1.0), the mean of Z is obtained from Equations 3-13, 3-18, and 3-20 as

$$\mu_z \approx \bar{Z} \approx \hat{g}(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) \quad (3-22)$$

and the total cov of Z is obtained from Equations 3-14, 3-19, and 3-21 as

$$\Omega_z \approx \sqrt{\Omega_g^2 + \frac{1}{\bar{Z}^2} \left[\sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial g}{\partial X_i} \right)_o \left(\frac{\partial g}{\partial X_j} \right)_o \rho_{ij} \sigma_{X_i} \sigma_{X_j} \right]} \quad (3-23)$$

which for uncorrelated X_i becomes

$$\Omega_z \approx \sqrt{\Omega_g^2 + \frac{1}{\bar{Z}^2} \left[\sum_{i=1}^n \left(\frac{\partial g}{\partial X_i} \right)_o^2 \sigma_{X_i}^2 \right]} \quad (3-24)$$

The total standard deviation of Z is then obtained as

$$\sigma_z = \bar{Z} \Omega_z \quad (3-25)$$

Equations 3-18 and 3-19 and Equations 3-21 to 3-25 form the basis for the evaluation of the first and second moments of X_i and Z , respectively. Particular applications of these equations to the evaluation of statistics for S and R are discussed below.

3.3.1 STATISTICAL EVALUATION OF LOAD EFFECTS

The critical design load effect, S , in general results from a combination of several loads. Let X_i describe the random intensity of the i th load (i.e., the intensity of a distributed load, the intensity of an earthquake, the pressure resulting from wind). On the basis of the principle of superposition, the load effect S at a particular point in a structure is obtained as

$$S = f_1 X_1 + f_2 X_2 + \dots + f_n X_n = \sum_{i=1}^n f_i X_i \quad (3-26)$$

where f_i are influence coefficients which depend on the point under consideration as well as the loading pattern. Assuming that the principle of superposition is valid, and that the influence coefficients are deterministic, the mean and cov of the load effect S for uncorrelated loads may be obtained as

$$\mu_S \approx \bar{S} = \sum_{i=1}^n f_i \bar{X}_i \quad (3-27)$$

and

$$\Omega_S = \frac{1}{\bar{S}} \sqrt{\sum_{i=1}^n f_i^2 \bar{X}_i^{-2} \Omega_{X_i}^2} \quad (3-28)$$

where the cov of each load X_i must be obtained according to Equations 3-18 and 3-19, including the effect of errors of estimation, ΔX_i , as well as the basic variability of the load, δX_i , giving a total variability of $\Omega_{X_i} = \sqrt{\delta X_i^2 + \Delta X_i^2}$. As an example, consider the evaluation of the statistics for the earthquake induced forces. The seismic risk analysis required for such an evaluation is usually based on incomplete historical data of earthquakes and certain imperfect idealizations (Der Kiureghian and Ang, 1975). Accordingly, these errors may be incorporated

through Δ_{X_i} , while δ_{X_i} would describe the variability in ground motion intensities as estimated on the basis of the available data and idealized models.

In design for safety, limit states of structural behavior are of interest where the response usually is no longer linear. As a result, the principle of superposition does not hold and the linear relation assumed in Equation 3-26 would not be perfect. Thus, for such cases an additional uncertainty term due to the error in the assumed function may be included and Equation 3-28 then becomes

$$\Omega_s = \sqrt{\Omega_g^2 + \frac{1}{\bar{S}^2} \sum_{i=1}^n \bar{f}_i^2 \bar{x}_i^2 \Omega_{X_i}^2} \quad (3-29)$$

where Ω_g represents the error due to the superposition assumption. The value of Ω_g may be assigned subjectively on the basis of the expected variability of the true structural response from the assumed linear response.

A further refinement may be made by observing that the influence coefficients are in fact random variables. This results from the fact that in practical structural analysis, invariably approximations are employed, and thus the true influence coefficients remain unknown. Examples may include the replacement of complicated loading patterns with simple idealization (e.g., in the case of live loads), the replacement of dynamic loads with "equivalent" static loads (e.g., in the case of wind or earthquake loads), and other approximations such as neglecting the effect of certain forces or using idealized dimensions. The effect of these errors could directly be included in Ω_{X_i} (as shown in Sec. 4.1) of Equation 3-29; however, if an explicit formulation is required, this equation may be written as

$$\Omega_s = \sqrt{\Omega_g^2 + \frac{1}{\bar{S}^2} \sum_{i=1}^n \bar{f}_i^2 \bar{x}_i^2 (\Omega_{f_i}^2 + \Omega_{X_i}^2)} \quad (3-30)$$

where $\bar{f}_i$ is the mean value of the influence coefficient and Ω_{f_i} is its estimated variability.

It appears from the above discussion that many of the uncertainties resulting from approximations in structural analysis may consistently be incorporated into a probabilistic design process. In particular, in the above equation Ω_g would reflect errors due to simple superpositioning of load effects, whereas Ω_{f_i} would reflect errors due to approximations assumed in analyzing the effect of each load. Such consistent treatment of these uncertainties in the deterministic design procedures, of course, is not possible.

3.3.2 STATISTICAL EVALUATION OF RESISTANCE

The resistance R of a structural component for a particular load effect, such as the flexural capacity of a beam against applied bending moments, may be described in a functional form as

$$R = g(X_1, X_2, \dots, X_n) \quad (3-31)$$

where X_i are random variates describing material properties, member dimensions, etc. In the context of the extended reliability concepts, the basic variabilities of material properties, etc., as supported by the available data, may be introduced through δX_i terms, whereas any errors in the estimation (e.g., due to insufficiency of data) may be introduced through ΔX_i terms. Also any errors in the assumed function g may be introduced through Ω_g . Clearly the statistics of R in each particular case would depend on the form of the function g .

As an example consider the flexural capacity of a reinforced concrete beam with width b , effective depth d , tension steel area A_s , steel yield strength f_y , and concrete strength f_c . Assuming the reinforcement ratio is well below the balanced ratio (to ensure zero probability of brittle failure), the flexural capacity may be obtained as

$$R = A_s f_y d \left(1 - \eta \frac{A_s}{bd} \frac{f_y}{f_c} \right) \quad (3-32)$$

where η is the concrete stress block parameter (ACI, 1971). On the basis of Equations 3-22 and 3-24, the mean and coefficient of variation of R for uncorrelated parameters are

$$\mu_R \approx \bar{R} = \bar{A}_s \bar{f}_y \bar{d} \left(1 - \eta \frac{\bar{A}_s}{\bar{b}\bar{d}} \frac{\bar{f}_y}{\bar{f}_c} \right) \quad (3-33)$$

and

$$\begin{aligned} \Omega_R^2 \approx \Omega_g^2 + \left(\frac{1 - 2 \bar{\eta}\bar{q}}{1 - \bar{\eta}\bar{q}} \right)^2 \left(\Omega_{f_y}^2 + \Omega_{A_s}^2 \right) + \left(\frac{\bar{\eta}\bar{q}}{1 - \bar{\eta}\bar{q}} \right)^2 \left(\Omega_{f_c}^2 + \Omega_b^2 + \Omega_\eta^2 \right) \\ + \left(\frac{1}{1 - \bar{\eta}\bar{q}} \right)^2 \Omega_d^2 \end{aligned} \quad (3-34)$$

where $\bar{q} = \left(\bar{f}_y / \bar{f}_c \right) \bar{A}_s / (\bar{b}\bar{d})$ and a bar over a symbol connotes the mean value. The term Ω_η in the above equation reflects the error in the idealization of the nonlinear concrete stress block by a simple rectangular (or triangular) stress block. Also the term Ω_g reflects the error in the assumed function for R ; this may result as the effect of tension stresses in uncracked concrete, the effect of shear stresses, etc., which are not included in the derivation of Equation 3-32. Each of the cov terms $\Omega_g, \dots, \Omega_d$ on the right-hand side is composed of a basic variability term and an error term. For example the term Ω_d is obtained as $\Omega_d = \sqrt{\delta_d^2 + \Delta_d^2}$, where δ_d is the coefficient of variation of the effective depth, as observed from data on actual construction practices, whereas Δ_d is the error in estimation arising from insufficiency of such data.

It is observed from the above example that the total cov of R , Ω_R , includes all uncertainties which are associated with the design and construction processes of a structure, i.e., variabilities in material properties (e.g., Ω_{f_y} and Ω_{f_c}), construction and manufacturing errors (e.g., Ω_d, Ω_b , and Ω_{A_s}), and errors in the assumed mathematical model of the resistance (e.g., Ω_η and Ω_g). Such a consistent treatment of

uncertainties is essential for a rational design procedure. The above discussed resistance function as well as other resistance functions for reinforced concrete members are studied in detail by Ellingwood and Ang (1972, 1974). Similar studies for metal structures may be found in a publication by ASCE (1972).

3.4 PRACTICAL DESIGN FORMATS

For practical purposes it is desirable to develop probabilistic design formats which are similar and comparable to the deterministic formats currently in application. Clearly, such a format would be more readily received by the engineering community than a format which is drastically different from previous ones. More important for the present time, however, is the fact that such a format could be used as a theoretical basis for a critical examination of the current design criteria. The latter is of course the main objective of the present study.

In order to develop formats that are comparable to the deterministic formats of the WSD and LFD methods (i.e., Equations 2-1 and 2-2, respectively), it is necessary that (1) the terms corresponding to the resistance and the load-effect be separated, as they are in the WSD and LFD formats, (2) relationships between nominal and statistical values of the resistance and loads be specified, and (3) factors comparable to safety or load and strength factors be derived. The factors in the probabilistic format should of course be in functional forms rather than being constants such that a desired level of reliability can be assumed.

First consider the probabilistic format of Equation 3-10. With the means of R and S obtained from (unbiased) statistical data, this equation becomes

$$\bar{R} \geq \bar{S} + \beta_1 \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3-35)$$

In order to separate the terms corresponding to the resistance and load effect, the relation

$$\sqrt{\sigma_R^2 + \sigma_S^2} = \alpha_R(\sigma_R + \sigma_S) \quad (3-36)$$

is assumed where α_R is a decoupling factor equal to $\sqrt{\sigma_R^2 + \sigma_S^2}/(\sigma_R + \sigma_S)$. Clearly the value of α_R depends on the ratio of the two standard deviations. If we let r be the ratio of the smaller standard deviation to the larger one, then, α_R may be evaluated from

$$\alpha_R = \frac{\sqrt{1 + r^2}}{1 + r} \quad (3-37)$$

where $0 \leq r \leq 1.0$. The value of α_R from the above equation is seen to range from 0.707 (i.e., when $r = 1$) to 1.00 (i.e., when $r = 0$). Figure 3-2 shows the values of α_R as a function of r . It is observed in this figure that when the standard deviations of the resistance and the load effect are comparable, i.e., when r is close to 1.0, the value of α_R is not sensitive to r . In fact for $0.5 < r < 1.0$, α_R only varies from 0.71 to 0.75. For most practical cases σ_R/σ_S is expected to be in this range, therefore, it has been suggested that $\alpha_R = 0.75$ be assumed (Lind, 1971; Ang and Cornell, 1974). When the ratio of standard deviations is small, say $r < 0.4$, as it may happen for certain metallic structures whose resistances are calculated with high certainty, the value $\alpha_R = 0.75$ may be unconservative. In such cases, a higher value may be specified or the exact value may be calculated directly from Equation 3-37. For very small values of r , i.e., when σ_R is much smaller than σ_S , the uncertainty in resistance may be entirely neglected. This is justified by the fact that for very small r , say $r < 0.2$, $\sqrt{1 + r^2} \approx 1.0$ and we may write

$$\begin{aligned} \sqrt{\sigma_R^2 + \sigma_S^2} &= \sigma_S \sqrt{1 + r^2} \\ &\approx \sigma_S \end{aligned} \quad (3-38)$$

FIGURE 3-2. VARIATION OF α_R WITH r

The function $\sqrt{1 + r^2}$ is also plotted in Figure 3-2 showing that for $r < 0.2$ the above approximation is reasonable. Thus for such cases $\sigma_R = 0$ and $\alpha_R = 1.0$ may be assumed.

Substitution of Equation 3-36 in Equation 3-35 gives

$$\bar{R} \geq \bar{S} + \beta_1 \alpha_R (\sigma_R + \sigma_S)$$

which may be written as

$$\bar{R} - \beta_1 \alpha_R \sigma_R \geq \bar{S} + \beta_1 \alpha_R \sigma_S \quad (3-35a)$$

or

$$(1 - \beta_1 \alpha_R \Omega_R) \bar{R} \geq (1 + \beta_1 \alpha_R \Omega_S) \bar{S} \quad (3-39)$$

The above probabilistic design format is seen to be similar to the WSD format of Equation 2-1, where the safety factor with respect to the mean resistance and mean load effect is,

$$\bar{\gamma} = \frac{1 + \beta_1 \alpha_R \Omega_S}{1 - \beta_1 \alpha_R \Omega_R} \quad (3-40)$$

This safety factor is known as the *central safety factor*. Its relationship with the conventional safety factor, i.e., γ in Equation 2-1, is explored in Chapter 5.

It must be realized that in the above equation the coefficient of variation Ω_S , which is to be obtained from Equation 3-29, would in general involve the influence coefficients, f_i . This means that the value of Ω_S , and therefore $\bar{\gamma}$ depend on the component or the location of the critical section considered. Therefore, a risk-consistent design would require a

different safety factor at each critical section. This may be explained by noting that since different sections are affected differently under a given load, and since various loads may have different degrees of variability, it follows then that a section which is highly affected by loads with large variabilities must have a larger safety factor than a section which is not very much affected by such loads. For example, it is known that in a frame structure, lateral loads, such as those due to an earthquake, may develop large bending moments at ends of members but no effects at midspans. Therefore, if the lateral loads happen to have larger variabilities than gravity loads, one should expect that the end sections would require larger safety factors than the midspan sections.

Change in safety factor from section to section may not be desirable for practical purposes. Obviously, a reasonable solution of this problem would be to separate the terms for each load. This should, of course, lead us to a format similar to that of the LFD method. For this purpose, the value of Ω_S from Equation 3-29 is first substituted into Equation 3-39, resulting in

$$\left(1 - \beta_1 \alpha_R \Omega_R\right) \bar{R} \geq \left(1 + \beta_1 \alpha_R \sqrt{\Omega_g^2 + \frac{1}{\bar{S}^2} \sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2}\right) \bar{S} \quad (3-41)$$

In order to separate various terms on the right-hand side decoupling factors

$$\alpha_g = \frac{\sqrt{\Omega_g^2 + \frac{1}{\bar{S}^2} \sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2}}{\Omega_g + \frac{1}{\bar{S}} \sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2}} \quad (3-42)$$

and

$$\alpha_s = \frac{\sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2}}{\sum_{i=1}^n f_i \bar{x}_i \Omega_{X_i}} \quad (3-43)$$

are assumed and Equation 3-41 is rearranged to read

$$\left(1 - \beta_1 \alpha_R \Omega_R\right) \bar{R} \geq \sum_{i=1}^n \left[1 + \beta_1 \alpha_R \alpha_g \left(\Omega_g + \alpha_s \Omega_{X_i}\right)\right] f_i \bar{x}_i \quad (3-44)$$

where the equality $\bar{S} = \sum_{i=1}^n f_i \bar{x}_i$ is used.

The above probabilistic format appears to be similar to the LFD format of Equation 2-2, where relative to the mean resistance, $\bar{R}$, the *central strength factor* is

$$\bar{\phi} = 1 - \beta_1 \alpha_R \Omega_R \quad (3-45)$$

and relative to the mean value of each load the *central load factor* is

$$\bar{\delta}_i = 1 + \beta_1 \alpha_R \alpha_g \left(\Omega_g + \alpha_s \Omega_{X_i}\right) \quad (3-46)$$

The relationships between the above central factors and the conventional factors, i.e., ϕ and δ_i of Equation 2-2, are discussed in Chapter 6.

The decoupling factor α_g as defined in Equation 3-42 is of the same type as α_R ; therefore, it may be assumed to be 0.75 or 1.0 depending on the ratio of terms Ω_g and $\frac{1}{S} \sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2}$. The latter term is the coefficient of variation of the combined load effect, whereas the former is the error due to linear superposition of loads. For most cases the error due to linear superposition may be assumed to be much smaller than the variability in combined load effect; in that case, Ω_g may be neglected and a value of $\alpha_g = 1.00$ may be used. If the ratio of the above cov values is expected to be in the order of 0.4 or greater, then, Ω_g should not be neglected and a value $\alpha_g = 0.75$ would be reasonable (see Fig. 3-2).

The factor α_s of Equation 3-43 is used to separate the effects of various loads. It is seen in this equation that the value of α_s depends on the magnitude and variability of each load as well as the influence coefficients for the component or section under consideration. Therefore, it should vary from one component to another or from one critical section to another. However, these variations of α_s would in general be moderate and, therefore, for practical purposes constant values of α_s may be specified. In order to investigate possible ranges of α_s , we divide the numerator and denominator of the right-hand side of Equation 3-43 by the largest of the terms $f_i \bar{x}_i \Omega_{x_i}$ (note that these terms are in fact the standard deviations of load effects at the section). This would then give

$$\alpha_s = \frac{\sqrt{1 + \sum_{i=1}^{n-1} r_i^2}}{1 + \sum_{i=1}^{n-1} r_i} \quad (3-47)$$

where $0.0 < r_i < 1.0$, $i = 1, \dots, n - 1$. The value of α_s from the above equation is seen to range from 1.00 (i.e., when $r_i = 0$; $i = 1, \dots, n - 1$) to $\frac{1}{\sqrt{n}}$ (i.e., when $r_i = 1.0$; $i = 1, \dots, n - 1$). These two limits correspond, respectively, to (a) the case where the standard deviation of one load effect is much greater than that of the other load effects and (b) the case where the standard deviations of all load effects happen to be equal. Table 3-2 shows the values of α_s for a number of selective cases. In this table the cases of three, four, and five loads in a combination (i.e., $n = 3, 4$, and 5) are considered and for assumed values of r_i the factor α_s is evaluated from the above equation. An examination of these values indicates that the decoupling factor α_s remains fairly constant over wide ranges of variations of r_i ; it is only when one or more of the r_i 's is smaller than about 0.20 that α_s shows significant variations. This corresponds to cases where the standard deviations of some load effects happen to be far greater than the standard deviations of other load effects. In such cases, the load effects

TABLE 3-2. DECOUPLING FACTOR α_s OF EQUATION 3-43*

Case	n = 3			n = 4				n = 5				
	r_1	r_2	α_s	r_1	r_2	r_3	α_s	r_1	r_2	r_3	r_4	α_s
1	1.0	1.0	0.58	1.0	1.0	1.0	0.50	1.0	1.0	1.0	1.0	0.45
2	0.8	0.8	0.58	0.8	0.8	0.8	0.50	0.8	0.8	0.8	0.8	0.45
3	0.6	0.6	0.60	0.6	0.6	0.6	0.52	0.6	0.6	0.6	0.6	0.46
4	0.4	0.4	0.64	0.4	0.4	0.4	0.55	0.4	0.4	0.4	0.4	0.49
5	1.0	0.3	0.63	1.0	0.4	0.2	0.57	1.0	1.0	0.2	0.2	0.52
6	1.0	0.2	0.65	1.0	0.2	0.2	0.60	1.0	0.4	0.2	0.1	0.55
7	1.0	0.1	0.68	1.0	0.1	0.1	0.65	1.0	0.4	0.1	0.1	0.57
8	0.3	0.3	0.68	0.3	0.3	0.3	0.59	0.3	0.3	0.3	0.3	0.53
9	0.2	0.2	0.74	0.2	0.2	0.2	0.66	0.2	0.2	0.2	0.2	0.60
10	0.1	0.1	0.84	0.1	0.1	0.1	0.78	0.1	0.1	0.1	0.1	0.73

* n = Number of loads in combination

r_i = Ratio of standard deviations of load effects

with small standard deviations may be treated deterministically and thus an approximate value of α_s corresponding to a smaller number of load effects (i.e., a smaller n) would be appropriate.

It must be understood that in every practical case, knowing the values of f_i , $\bar{x}_i$, and Ω_{x_i} , the actual value of α_s can directly be calculated from Equation 3-43. However, for the purpose of convenience, a set of constant values appropriate for various design situations may be specified. For the purpose of the present study a set of values of α_s , obtained by investigation of Table 3-2, are given in Table 3-3. These factors are appropriate when the standard deviations of various load effects are more or less of the same order, the smallest ones not being less than 20 or 30% of the largest ones.

TABLE 3-3. ASSUMED VALUES OF THE DECOUPLING FACTOR α_s

Number of Loads, n	α_s
1	1.00
2	0.75
3	0.62
4	0.55
5	0.50
6	0.45

Considering the second probabilistic design format given in Equation 3-11, for R and S obtained from (unbiased) statistical data, the design equation becomes

$$\bar{R} \geq \bar{S} \exp\left(\beta_2 \sqrt{\Omega_R^2 + \Omega_S^2}\right) \quad (3-48)$$

This format is similar to the WSD format of Equation 2-1; thus the *central safety factor* is

$$\bar{\gamma} = \exp \left(\beta_2 \sqrt{\Omega_R^2 + \Omega_S^2} \right) \quad (3-49)$$

Since Ω_S is a function of the influence coefficients, the central factor is again seen to depend on the location of the section or the component in the structure under consideration. This fact was justified previously and will not be further expanded at this time.

In order to obtain a format similar to the LFD format, it is necessary that the strength and load terms be separated. Using the decoupling factor α_R of Equation 3-37, Equation 3-48 may be written as

$$\bar{R} \geq \bar{S} \exp \left[\beta_2 \alpha_R (\Omega_R + \Omega_S) \right] \quad (3-50)$$

or

$$\exp \left(-\beta_2 \alpha_R \Omega_R \right) \bar{R} \geq \exp \left(\beta_2 \alpha_R \Omega_S \right) \bar{S} \quad (3-51)$$

which with the value of Ω_S substituted from Equation 3-29 becomes

$$\exp \left(-\beta_2 \alpha_R \Omega_R \right) \bar{R} \geq \exp \left(\beta_2 \alpha_R \sqrt{\Omega_g^2 + \frac{1}{\bar{S}^2} \sum_{i=1}^n f_i^2 x_i^2 \Omega_{X_i}^2} \right) \bar{S} \quad (3-52)$$

Application of the decoupling factor α_g in the above equation yields

$$\exp \left(-\beta_2 \alpha_R \Omega_R \right) \bar{R} \geq \exp \left[\beta_2 \alpha_R \alpha_g \left(\Omega_g + \frac{1}{\bar{S}} \sqrt{\sum_{i=1}^n f_i^2 x_i^2 \Omega_{X_i}^2} \right) \right] \bar{S}$$

or

$$\begin{aligned} & \exp \left(-\beta_2 \alpha_R \Omega_R \right) \bar{R} \\ & \geq \exp \left(\beta_2 \alpha_R \alpha_g \Omega_g \right) \exp \left(\beta_2 \alpha_R \alpha_g \frac{1}{\bar{S}} \sqrt{\sum_{i=1}^n f_i^2 x_i^2 \Omega_{X_i}^2} \right) \bar{S} \end{aligned} \quad (3-53)$$

Now introduce a new decoupling factor α_s such that

$$\exp \left(\beta_2 \alpha_R \alpha_g \frac{1}{\bar{S}} \sqrt{\sum_{i=1}^n f_i^2 \bar{x}_i^2 \Omega_{X_i}^2} \right) \bar{S} = \sum_{i=1}^n \exp \left(\beta_2 \alpha_R \alpha_g \alpha_s \Omega_{X_i} \right) f_i \bar{x}_i \quad (3-54)$$

Substituting the above equation in Equation 3-53 yields

$$\exp \left(-\beta_2 \alpha_R \Omega_R \right) \bar{R} \geq \sum_{i=1}^n \exp \left[\beta_2 \alpha_R \alpha_g \left(\Omega_g + \alpha_s \Omega_{X_i} \right) \right] f_i \bar{x}_i \quad (3-55)$$

It is observed that the central strength and load factors in this case are

$$\bar{\phi} = \exp \left(-\beta_2 \alpha_R \Omega_R \right) \quad (3-56)$$

and

$$\bar{\delta}_i = \exp \left[\beta_2 \alpha_R \alpha_g \left(\Omega_g + \alpha_s \Omega_{X_i} \right) \right] \quad (3-57)$$

respectively.

The decoupling factor α_s of Equation 3-54 depends on values of $f_i \bar{x}_i / \bar{S}$ and Ω_{X_i} as well as the factor $\beta_2 \alpha_R \alpha_g$. Investigations show, however, that the variations of α_s over a wide range of practical design situations remain fairly insignificant. For the purpose of illustration, the values of α_s for a number of selective design situations, including two, three, or four loads in a combination, are shown in Table 3-4. These values were obtained by numerically solving for α_s in Equation 3-54. An investigation of the values in this table reveals that in fact for most practical situations, the variations in α_s are moderate, i.e., less than $\pm 10\%$. These variations are especially small when the standard deviations of various loads (i.e., $f_i X_i \Omega_{X_i}$) are of comparable magnitudes.

TABLE 3-4. DECOUPLING FACTOR α_s OF EQUATION 3-54*

n	h_1	h_2	h_3	h_4	$\beta_2 \alpha_R \alpha_g = 2.00$					$\beta_2 \alpha_R \alpha_g = 2.50$				
					Ω_1	Ω_2	Ω_3	Ω_4	α_s	Ω_1	Ω_2	Ω_3	Ω_4	α_s
2	0.5	0.5			a	a			0.71	a	a			0.71
					0.5	0.3			0.72	0.5	0.3			0.72
					0.5	0.1			0.77	0.5	0.1			0.76
					0.3	0.1			0.76	0.3	0.1			0.76
	0.3	0.7			a	a			0.76	a	a			0.76
					0.5	0.1			0.68	0.5	0.1			0.66
					0.1	0.5			0.86	0.1	0.5			0.85
3	0.33	0.33	0.33		a	a	a		0.58	a	a	a		0.58
					0.5	0.3	0.1		0.62	0.5	0.3	0.1		0.62
					0.3	0.2	0.1		0.61	0.3	0.2	0.1		0.61
	0.20	0.30	0.50		a	a	a		0.62	a	a	a		0.62
					0.5	0.3	0.1		0.56	0.5	0.3	0.1		0.56
					0.1	0.3	0.5		0.71	0.1	0.3	0.5		0.70
4	0.25	0.25	0.25	0.25	a	a	a	a	0.50	a	a	a	a	0.50
					0.5	0.4	0.2	0.1	0.54	0.5	0.4	0.2	0.1	0.54
	0.10	0.20	0.30	0.40	a	a	a	a	0.55	a	a	a	a	0.55
					0.5	0.4	0.2	0.1	0.49	0.5	0.4	0.2	0.1	0.49
					0.1	0.2	0.4	0.5	0.62	0.1	0.2	0.4	0.5	0.62

*n = Number of loads in combination.

$$h_i = \frac{f_i \bar{X}_i}{\bar{S}} = \frac{f_i \bar{X}_i}{\sum f_i \bar{X}_i}$$

Ω_i = Coefficient of variation of i^{th} load.

a = Any value of the coefficient of variation.

A comparison of the values of α_s in Table 3-4, which is based on Equation 3-54, with corresponding values of α_s in Table 3-2, which is based on Equation 3-47, reveals that these two decoupling factors are approximately identical for most situations (when calculated for the same values of f_i, X_i, Ω_{X_i}). On this basis the assumed values of α_s in Table 3-3 may also be used for the decoupling factor defined in Equation 3-54. Note that the values from this table are only appropriate when the standard deviations of various load effects are more or less of the same order.

It must be realized that the safety, load, and strength factors derived in this chapter correspond to the mean values of loads and resistances. For this reason they are known as *central* factors. The relations of these factors with the factors used in practice are discussed in Chapter 6.

CHAPTER 4

LOADS AND THEIR COMBINATIONS

In Chapter 3 two different formulations of the probabilistic design were developed and central values of safety and load and strength factors were derived. These formats and the corresponding factors were obtained in terms of a safety index and the first and second moments of the resistance and the loads. In this derivation it was assumed that a specific set of loads were acting on the structure. In an actual situation, however, a structure may be affected by several different combinations of loads during its useful life. Therefore it is necessary to investigate all such combinations of loads, and to determine those combinations which may be critical to the safety of the structure. In addition, the loads in each particular combination may vary with time. Clearly, under such conditions adding the peak values of loads would not be reasonable since the peaks in general would not occur at the same time. In such cases the peak value of the combined load effect is that which is critical and should be determined.

In Section 4.1 of this chapter a general formulation of the relationship between a load and its associated load effect at a critical point in a structure is developed. The stochastic nature of loads as well as their random distributions are included in this formulation. In order to study the combined effects of loads, in Section 4.2 loads are categorized with respect to the nature of their occurrence in time. Section 4.3 deals with the assessment of statistics for extreme value loads. Results for a number of examples as compared with Monte Carlo simulations are presented here.

4.1 STOCHASTIC EVALUATION OF LOAD EFFECTS

In evaluating load effects due to an applied load on a structure three types of uncertainties are encountered: (1) temporal and spatial randomness in load intensity and distribution, (2) uncertainties in influence

coefficients resulting from variabilities in structural dimensions and strengths, and (3) errors in methods of analysis and prediction.

As an example consider the evaluation of load effects due to wind forces on a structure. The applied loads in this case depend on the wind velocity profile. The amplitudes and the shape of this profile, however, randomly fluctuate with time. The resulting loads, therefore, would have both temporal and spatial randomness. On the other hand, for a given set of loads, the load effect at a critical component of a structure is obtained using influence coefficients. These coefficients, however, depend on dimensions and strengths of various structural components which themselves are random. Consequently a second uncertainty is introduced. Finally errors associated with imperfect modeling and idealizations (e.g., linear superposition, frame analysis) and those associated with estimation (e.g., due to inaccuracy and inadequacy of available data) introduce another source of uncertainty.

Each of the above uncertainties should ideally be included in the estimation of the mean and variance of load effects. It is expected, however, that uncertainties resulting from the load itself (i.e., due to temporal and spatial randomness in load intensities) have the dominant effect. For this reason more attention is given to this source of uncertainties.

Some aspects of this problem have been looked upon by a number of investigators (e.g., see Peir and Cornell, 1973, and McGuire and Cornell, 1974 for treatments of live loads; and Davenport, 1967, for analysis of wind loads), however, much remains to be done. In this section an attempt is made to merely formulate the problem and to suggest a practical approach for its solution. Additional investigations are needed in this area in order to achieve useful results.

Consider a case where a load is applied on the Area A of a structure. Let $w(x,y,t)$ denote the random intensity of the load (i.e., load per unit area) at the location described by the coordinates x and y and at time t . For a particular load effect under consideration let $l(x,y)$ denote the influence coefficients, i.e., the load effect due to a unit load at x and y . The required load effect, then, is

$$S(t) = \int_A w(x,y,t) l(x,y) dA \quad (4-1)$$

As it is explicitly shown in the above equation, the resulting load effect is a function of time, i.e., it is a stochastic process. The evaluation of the statistics of $S(t)$ from those of $w(x,y,t)$ in the above equation would, in general, be very complicated. In order to simplify the problem, it is often reasonable to assume that the unknown shape of load distribution remains unchanged with time. For example, in the case of wind loads, this assumption would imply a constant shape for velocity profiles of all winds; although this constant shape itself could remain random.

With this assumption, then, the load intensity can be described as

$$w(x,y,t) = w_0(t) \psi(x,y) \quad (4-2)$$

where $\psi(x,y)$, a random process, is the load distribution shape function such that

$$\int_A \psi(x,y) dA = A \quad (4-3)$$

and $w_0(t)$ is a stochastic process denoting the load intensity when it is uniformly distributed. Note that

$$\begin{aligned} \int_A w(x,y,t) dA &= w_0(t) \int_A \psi(x,y) dA \\ &= w_0(t) A \end{aligned} \quad (4-4)$$

Using Equation 4-2 in Equation 4-1 one obtains

$$\begin{aligned} S(t) &= \int_A w_o(t) \psi(x,y) I(x,y) dA \\ &= w_o(t) \int_A \psi(x,y) I(x,y) dA \end{aligned} \quad (4-5)$$

In the above equation the stochastic nature of load effects appears through $w_o(t)$, whereas uncertainties due to the spatial randomness of the load and due to variabilities in influence coefficients are reflected through $\psi(x,y)$ and $I(x,y)$, respectively.

For practical analysis and design purposes it is convenient to assume simple deterministic shape functions and use influence coefficients based on mean structural strengths and dimensions. Let $\psi^*(x,y)$ and $I^*(x,y)$ denote such values. To include the uncertainties from these sources, a random correction factor H may be included and Equation 4-5 may be written as

$$S(t) = w_o(t) H \int_A \psi^*(x,y) I^*(x,y) dA \quad (4-6)$$

where H is such that

$$H \int_A \psi^*(x,y) I^*(x,y) dA = \int_A \psi(x,y) I(x,y) dA \quad (4-7)$$

The variabilities of H for various types of structures and loads may be estimated through measurements and observations in existing structures as well as through analytical techniques. A number of structural types for each type of loading will have to be considered to cover most practical situations. Note that separate analysis must be made for each typical load effect. With the descriptions of $w_o(t)$ and H known, the moments of the load effect may be estimated from Equation 4-6. The discussion of such analyses is not pursued any further in this report; however, further studies in this direction should prove to be productive and useful.

It is noted that in the discussions of this report the load effect relationship has often been described (see Sec. 3.3.1) as

$$S = fX \quad (4-8)$$

where f is a deterministic influence coefficient and X is the random value of load intensity. With the discussion presented above, it should be clear that

$$X = w_0(t)H \quad (4-9)$$

and

$$f = \int_A \psi^*(x,y) I^*(x,y) dA \quad (4-10)$$

Clearly, with f a deterministic value, the stochastic characteristics of S are obtainable from those of X from Equation 4-8.

4.2 CATEGORIZATION OF LOADS

For the purpose of the present study loads affecting a structure are categorized, with respect to the nature of their occurrence in time, into the following five categories:

1. Time-independent loads
2. Time-dependent sustained loads
3. Time-dependent transient loads
4. Occasional short-duration loads
5. Consequential loads

The first category includes loads which do not vary with time, although they may have random intensity or distribution. Dead load is a principal example of this category of loads. The load value, X , (as well as the load effect at a critical section of a structure, S) in this category

remains constant in time. Therefore a plot of such a load as a function of time would appear as a horizontal line with a random amplitude, as shown in Figure 4-1a. Clearly, a single probability density function, as shown in Figure 4-1b, would fully describe the probabilistic character of a load from this category.

The second category includes loads that may act at all times and whose intensity as well as distribution may vary with time. Sustained live load is a common example of this category of loads. The load intensity X , as well as the load effect at a critical section of a structure due to this category of loads, clearly constitute stochastic processes. A typical sample function of such a process may appear as in Figure 4-2a.

With respect to the safety analysis of a structure, two values of the load in this category are of interest. These are the value of the load at any instant of time, designated by the *instantaneous load*, and the peak value of the load during a period of T years, designated by the *T -year extreme load*. The statistics of both of these values are necessary for a reliability based design. For the process with the sample function as in Figure 4-2a, the probability density functions of the instantaneous and T -year extreme values are shown schematically in Figure 4-2b. The evaluation of the characteristics of these functions is described in Section 4.3.

The third category includes loads whose intensities as well as distributions may vary with time, but they act during certain intervals of time only. Seasonal loads, such as snow loads; operational loads, such as temperature loads that act during the operation of a plant; loads due to natural hazards such as earthquakes and high winds, and all other transient loads are included in this group. A load (or the corresponding load effects) from this category would constitute a stochastic process, a sample function of which may appear as in Figure 4-3a. For this category of loads, a parameter that will be of interest is the duration of load at each occurrence. This parameter would in general be a random variable. For the purpose of subsequent

(a) Sample function

(b) Probability density function

FIGURE 4-1. TIME-INDEPENDENT LOAD, CATEGORY 1

(a) Sample function

(b) Probability density functions for instantaneous and extreme loads

FIGURE 4-2. TIME-DEPENDENT CONTINUOUS LOAD, CATEGORY 2

(a) Sample function

(b) Probability density functions for instantaneous and extreme loads

FIGURE 4-3. TIME-DEPENDENT PIECEWISE CONTINUOUS LOAD, CATEGORY 3

discussions we let τ represent this random variable. For this group of loads, we again define instantaneous and T-year extreme values. The probability density functions corresponding to these values are shown schematically in Figure 4-3b.

The fourth category includes loads that act occasionally but for very short periods of time. The durations of these category of loads are so short that their joint occurrences (i.e., simultaneous occurrences of two or more loads from this category) are unlikely. Therefore, in combining loads only one load at a time from this category should be included in any combination. Examples for this category of loads may include loads resulting from certain natural hazards, such as tornados and short duration ground shocks, and loads due to certain accidents, such as aircraft crash or explosions. This category of loads which occur at random times and with random magnitudes may be modeled as point processes. A typical sample function for such a process is illustrated as in Figure 4-4a, where each vertical bar indicates an occurrence with the height of the bar showing the random magnitude of the load. Again in this case two different distributions may be identified. One corresponding to the load at *any occurrence* and the other corresponding to the *peak* load during a period of T years. The probability density functions for these loads are illustrated schematically in Figure 4-4b.

The last category includes loads that may act as a consequence of other loads, especially occasional severe loads. Examples for this category may include loads which are due to an explosion or the failure of certain structural or nonstructural components and that happen as a result of the occurrence of a major earthquake or tornado. A key parameter to consider for this group of loads is the probability of occurrence of the event given a major earthquake or tornado. For the purpose of later discussions we designate this probability by P_e . Since this probability in general is very small, consideration of this category of loads may only be required for important structures for which very high levels of reliability are required. A practical example where consequential loads must be considered is the

(a) Sample function

(b) Probability density functions for any occurrence, annual peak, and extreme loads

FIGURE 4-4. OCCASIONAL SHORT-DURATION LOAD, CATEGORY 4

containment structures of nuclear power plants in which possible explosions or the breakage of highly pressured pipes during and after the occurrence of a major earthquake may cause additional loads which may endanger the safety of the structure. Since statistical data on these types of events are in general unavailable, in practice both the event and its probability of occurrence, P_e , will have to be prescribed subjectively.

This category of loads and their associated load effects would constitute point processes as portrayed in Figure 4-4 for Category 4 loads. The number of occurrences of these loads in unit time, however, would depend on the probability P_e .

Further analysis of above load categories is presented in the following section.

4.3 EVALUATION OF STATISTICS OF EXTREME LOADS

As was mentioned in the previous section there are two values of a stochastic load that are of interest in structural reliability. These are the instantaneous value at an arbitrary point in time and the peak or extreme value over a period of time, say T years.

In many instances it is reasonable to assume that probabilistic characteristics of loads do not change with time; i.e., they can be modeled as stationary processes. As a result of this, the mean, the variance, and the distribution of instantaneous values of loads may be obtained from statistical data collected through measurements at arbitrary points in time. Usually, measurements at various typical buildings or locations may be combined in order to increase the sample size. These types of measurements are in general practically feasible; in fact for certain loads considerable statistical data has already been collected which can be used to derive statistics of their instantaneous values.

The evaluation of the distribution and the statistics of extreme loads, on the other hand, is a more difficult problem. There are basically two possible approaches. The most direct approach for such an evaluation is through collecting statistical data over long periods of time. In this approach, in order to evaluate the statistics of the T-year extreme load, one would have to collect the values of the load over many T-year intervals. By plotting the histogram of the peak values of the load in the various intervals, one could obtain the desired statistical information. In view of the long observation period that is required, this approach clearly is not useful for many practical situations.

A second approach is that the statistics of the T-year extreme load are evaluated on the basis of theoretical models and statistical data that are obtained for a relatively short period of time. A number of classical studies on this topic are available. For example, according to Gumbel (1958), if various occurrences of a load are statistically independent and identically distributed, then the distribution of the extreme value after a number of occurrences will asymptotically approach to one of three standard distributions, known as Type 1, Type 2, and Type 3 Extreme Value Distributions. The selection of one of these distributions for a load depends on the shape of the far tail of the distribution of its instantaneous value. For the case of continuous loads modeled as stochastic processes, the evaluation of the distribution of the extreme value is more involved. Solutions for a number of cases are available in literature, see for example Lin (1967), Parzen (1967), and Bosshard (1975). These solutions, however, are often too complicated for practical use.

It must be realized that for use in the second moment reliability approach only the mean and the variance of extreme loads, rather than entire distributions are required. Most investigators, on the other hand, have been concerned with deriving the distribution of extreme loads (e.g., Bosshard, 1975; Grigoriu, 1975; Wen, 1977a, 1977b). By proper integrations the mean and the variance can, of course, be evaluated from these distributions, which can then be used in accordance with the concepts discussed in Chapter 3. This

approach, however, has several disadvantages. First, it is noted that in order to derive distributions one is bound to make simplifying assumptions and idealizations, which may not be realistic. Secondly, the solutions that are obtained are often very complicated, and considerable additional computations are required in order to evaluate the mean and the variance. Finally, since only the mean and the variance are utilized, the additional information that is contained in the distribution remains unused.

Realizing above facts, Der Kiureghian (1977) has recently developed a concept for obtaining the mean and the variance of extremes which avoids dependence on the corresponding distributions. In this approach the mean and the variance of the extreme load are evaluated as a function of the mean and the variance of the instantaneous load and a set of parameters describing the stochastic variations of the load. As will be shown subsequently, this approach has a number of advantages, including (1) there is less need for idealizations and simplifying assumptions; (2) the mean and the variance of extreme loads can be obtained with straightforward calculations using simple design charts, (3) the method can be further developed using analytical as well as Monte-Carlo simulation techniques, and (4) it can be extended to the case of multiple loads (see Sec. 4.3.2). With this approach useful results can also be obtained for cases where no analytical solutions for the distribution of the extreme exist.

In the following sections the development of the above approach for cases involving one load or a combination of loads is presented. This discussion will be limited to stationary and statistically independent loads only.

4.3.1 ONE LOAD CASE

Let S denote the instantaneous value of a stationary stochastic load effect and S_T denote its extreme value over a period of T years, i.e.,

$$S_T = \max_{0 \rightarrow T}(S) \quad (4-11)$$

Define the mean and the standard deviation of S by $\bar{s}$ and σ_S , respectively. These values are assumed to be known. Also, denote the mean and the standard deviation of S_T by $\bar{s}_T$ and σ_{S_T} , respectively.

One would clearly expect that the value of $\bar{s}_T$ increase with increasing values of $\bar{s}$, σ_S , and T . Also, it is expected that σ_{S_T} be closely related to σ_S . With this in mind, the following formats are assumed for the evaluation of $\bar{s}_T$ and σ_{S_T} :

$$\bar{s}_T = \bar{s} + p\sigma_S \quad (4-12)$$

$$\sigma_{S_T} = q\sigma_S \quad (4-13)$$

In the above equation p and q are yet unknown functions of T and other parameters which may describe the stochastic variations of S . It is noted that the parameter p must necessarily be a monotonically increasing function of T .

The above formats were originally obtained for Gaussian processes for which p and q are found to be functions of only one parameter. This idea was then extended to other processes for which it was found that p and q may depend on more than one parameter. The application of these formats to several processes which are useful in modeling stochastic loads are described below.

Stationary Gaussian Process: The stationary Gaussian process (Fig. 4-5) has often been used to model continuous loads which are of stochastic nature. This process can be used to model certain loads of Categories 2 and 3.

FIGURE 4-5. SAMPLE OF STATIONARY GAUSSIAN PROCESS

FIGURE 4-6. p AND q FUNCTIONS FOR A GAUSSIAN PROCESS

In Appendix A it is shown that if the upcrossings of a stationary Gaussian process from a level s are assumed to constitute a Poisson process, then, the functions p and q can be evaluated from

$$p = \int_{-\infty}^{\infty} u f_U(u) du \quad (4-14)$$

and

$$q = \left[\int_{-\infty}^{\infty} u^2 f_U(u) du - p^2 \right]^{1/2} \quad (4-15)$$

where $f_U(u)$ is density function of the normalized variate $U = (S_T - \bar{s})/\sigma_S$ and is shown to be

$$f_U(u) = \left[\frac{1}{\sqrt{2\pi}} + \nu T u \phi(u) \right] \exp \left[-\frac{u^2}{2} - \nu T \exp \left(-\frac{u^2}{2} \right) \right] \quad (4-16)$$

where $\phi(u)$ is the standard normal probability and ν is the mean rate of upcrossings of level $\bar{s}$ (i.e., mean number of upcrossings per unit time) of the process. The parameter ν describes the stochastic fluctuations of the process; a large ν would indicate rapid variations with time whereas a small ν would mean slow variations. This parameter is obtained from the statistical data.

It can be observed, in Equation 4-16, that $f_U(u)$ is only a function of the product νT . The functions p and q which are only dependent on $f_U(u)$ (see Eqs. 4-14 and 4-15) could, therefore, be evaluated in terms of νT . This product describes the mean number of upcrossing of level $\bar{s}$ over the time period T . Using Equations 4-14 and 4-15, the values of p and q are evaluated numerically and are shown in Figure 4-6. Thus, with the mean rate of upcrossings of a Gaussian process known, the mean and variance of the extreme can be obtained in terms of the instantaneous mean and variance by using the values of p and q from Figure 4-6 in Equations 4-12 and 4-13, respectively.

It is worthwhile to observe in Figure 4-6 that as νT becomes smaller (i.e., as the mean number of upcrossings decrease) the stochastic parameters p and q approach to 0.0 and 1.0, respectively. Equations 4-14 and 4-15 would, then, give $\bar{s}_T = \bar{s}$ and $\sigma_{S_T} = \sigma_S$. This clearly indicates that as the mean number of upcrossings diminishes the extreme value of the process approaches to its instantaneous value. This result is of course expected. On the other hand, it is observed in Figure 4-6 that p monotonically increases with νT , indicating larger values of $\bar{s}_T$ as the time interval or the upcrossing rate increase. The smaller values of q for larger νT indicate that there is a smaller uncertainty in the extreme value of a Gaussian process as the mean number of upcrossings increases. The latter behavior is a characteristic of the Gaussian distribution at its far tail, and, therefore, this behavior may not be generally expected of other processes.

Poisson Square Wave Process: This process has often been used to model sustained loads (Bosshard, 1975, Wen, 1977a). It is assumed in this model that the load intensity changes at random points in time following a simple Poisson process. It is also assumed that the intensities within intervals are independent and identically distributed random variables. A sample function of this process is shown in Figure 4-7.

If ν denotes the mean rate of load changes, and $F_S(s)$ describes the common distribution of load intensities, it is simple to show that the occurrence of $\{S > s\}$ following load changes is a Poisson process with the mean occurrence rate $\nu[1 - F_S(s)]$. Therefore, the cumulative distribution of the extreme value may be obtained as

$$\begin{aligned} F_{S_T}(s) &= P(S < s)_{t=0} P(\text{no } S > s \text{ following load changes})_{0 \rightarrow T} \\ &= F_S(s) \exp \{-\nu T[1 - F_S(s)]\} \end{aligned} \quad (4-17)$$

FIGURE 4-7. SAMPLE OF POISSON SQUARE WAVE PROCESS

FIGURE 4-8. SAMPLE OF FILTERED POISSON PROCESS

which upon differentiating with respect to s gives the density function

$$f_{S_T}(s) = f_S(s)[1 + \nu T F_S(s)] \exp \{-\nu T[1 - F_S(s)]\} \quad (4-18)$$

where $f_S(s)$ is the probability density function of S .

For any arbitrary distribution of S , the above density function may be used to evaluate p and q as functions of νT from the relations

$$p = \frac{\int_{-\infty}^{\infty} s f_{S_T}(s) ds - \bar{s}}{\sigma_S} \quad (4-19)$$

and

$$q = \frac{\left[\int_{-\infty}^{\infty} s^2 f_{S_T}(s) ds - (\bar{s} + p\sigma_S)^2 \right]^{1/2}}{\sigma_S} \quad (4-20)$$

where $\bar{s}$ and σ_S are the known mean and standard deviation of S .

Values of p and q as given in Equations 4-19 and 4-20 are evaluated for a number of useful distributions of S . These values are in general functions of νT , i.e., the mean number of load changes. In the case of certain distributions they are also related to the coefficient of variation of S . These results, together with those of the Filtered Poisson process introduced below, will be presented subsequently.

Filtered Poisson Process: This process is often used to describe transient loads defined herein as Category 3 loads (Bosshard, 1975; Grigoriu, 1975; Wen, 1977a, 1977b). It is assumed for this process that each occurrence has a random intensity as well as a random duration, and that occurrence times follow a simple Poisson process. It is assumed that at each occurrence the intensity and the duration are independent of one another. Also, intensities and durations at various occurrences are assumed to be independent and identically distributed. A sample function of this process is shown in Figure 4-8.

In Appendix B it is shown that with ν denoting the mean rate of occurrences, and $F_S(s)$ denoting the distribution of intensity at an arbitrary occurrence, the cumulative distribution of S_T is obtained as

$$F_{S_T}(s) = \exp\left\{-\nu T[1 - F_S(s)]\right\} + [H(s) - 1]\exp(-\nu T) \quad (4-21)$$

where $H(s)$ is the unit step function, i.e.,

$$H(s) = \begin{cases} 1.0 & \text{for } s > 0 \\ 0.0 & \text{for } s < 0 \end{cases} \quad (4-22)$$

Upon differentiating the distribution function of Equation 4-21, the density function of S_T is obtained as

$$f_{S_T}(s) = \nu T f_S(s) \exp\left\{-\nu T[1 - F_S(s)]\right\} + \delta(s) \exp(-\nu T) \quad (4-23)$$

where $\delta(s)$ is the Dirac delta function representing a spike of unit area at $s = 0$.

With the density function known, p and q are then evaluated from Equations 4-19 and 4-20. These values for this process are in general dependent on the coefficient of variation of S as well as the parameter νT representing mean number of occurrences over the period of time T .

It must be realized that in the above derivation (see App. B) the overlapping of occurrences has been neglected. This is quite reasonable for most applications. When the effect of overlapping is included, however, the problem is much more difficult. To the author's knowledge no analytical solutions for the distribution of S_T exist in that case. With the present method, however, it is possible to obtain the p and q functions using Monte Carlo simulation techniques. Some such results, taken from Der Kiureghian (1978), are discussed at the end of this section.

Figures 4-9 to 4-11 show the values of p and q as functions of νT for the Poisson square wave and the filtered Poisson processes and for a number of distributions of S . It must be noted that the parameter νT is defined differently for the two processes. More specifically, it denotes the mean number of intensity changes for the Poisson square wave process as opposed to the mean number of occurrences for the filtered Poisson process. These should be compared with the definition of νT for the stationary Gaussian process as the mean number of upcrossings of the mean level $\bar{S}$.

Figures 4-9 and 4-10 show the results when the distribution of S is normal and lognormal, respectively. Figure 4-11 compares the p and q functions for a number of distributions of S . The following general observations are possible in these figures.

It is generally observed that the values of p and q for the two processes are identical for large values of νT , i.e., greater than 10. This is due to the fact that for large νT the density functions in Equations 4-18 and 4-23 both approach to $\nu T f_S(s) \exp\{-\nu T [1 - F_S(s)]\}$. On the other hand, for small νT the two processes possess markedly different behaviors. It is observed that for the Poisson square wave process, as with the stationary

(a) p functions

(b) q functions

FIGURE 4-9. p AND q FUNCTIONS FOR POISSON SQUARE WAVE AND FILTERED POISSON PROCESSES WITH NORMAL DISTRIBUTION

(a) p functions

(b) q functions

FIGURE 4-10. p AND q FUNCTIONS FOR POISSON SQUARE WAVE AND FILTERED POISSON PROCESSES WITH LOGNORMAL DISTRIBUTION

(a) p functions

(b) q functions

FIGURE 4-11. p AND q FUNCTIONS FOR POISSON SQUARE WAVE AND FILTERED POISSON PROCESSES WITH VARIOUS DISTRIBUTIONS

Gaussian process, the values of p and q respectively approach to 0.0 and 1.0, as νT approaches to zero. This indicates that with diminishing mean number of intensity changes; i.e., when most probably intensity remains constant over the time T , the extreme value S_T approaches to the instantaneous value S . As with the filtered Poisson process it is observed that at small νT the functions p and q are highly dependent on the coefficient of variation. It can be shown that as νT approaches to zero, i.e., as the possibility of no occurrence increases, the values of p and q approach to $(\nu T - 1)\frac{1}{\Omega_S}$ and $\sqrt{\nu T \left[1 + (1 - \nu T)\frac{1}{\Omega_S^2} \right]}$, respectively. These predict $\bar{s}_T = \sigma_{S_T} = 0.0$ for νT approaching to 0.0, which is consistent with the expected behavior.

It is obvious that similar results could be obtained for other stationary load processes. In this respect it is worthwhile to note that Monte-Carlo simulation techniques could be used to generate the p and q functions for any type of model which is too complicated to be solved analytically. More important, however, is the fact that such results could be obtained from the statistical data of a load, if such data were available. For this purpose once the occurrence rate ν has specifically been defined, the values of p and q for any T could be evaluated using the mean and standard deviation of the instantaneous and the peak values of the load, obtained respectively by sampling at random points and at peaks over periods T . Thus, without a need for idealizations and assumptions, which are often unrealistic, one could develop the above useful functions to relate statistics of the instantaneous and extreme values of a load.

As an example of a situation where analytical solutions do not exist, consider the filtered Poisson model including the effect of overlapping. A typical sample function of such a process may appear as in Figure 4-12. A key parameter in this case is the random duration at each occurrence, which is denoted by τ in Figure 4-12.

FIGURE 4-12. SAMPLE OF FILTERED POISSON PROCESS WITH OVERLAPPING

Using Monte Carlo simulation techniques, Der Kiureghian (1978) obtained the p and q functions for a filtered Poisson process with normal intensity distribution which includes the effect of overlapping. The duration at each occurrence was assumed to be uniformly distributed with a coefficient of variations, Ω_{τ} , of 30%. The results for mean durations of 0.001, 0.01, 0.05, and 0.10 time units are shown in Figure 4-13 together with the corresponding results of the filtered Poisson process which neglect the effect of overlapping.

It is observed in Figure 4-13 that the effect of overlapping on the p and q functions and, therefore, on the mean and variance of the extreme can be significant. More specifically, it is found that as the possibility for overlapping increases, which is either due to longer durations (i.e., larger $\bar{\tau}$) or more frequent occurrences (i.e., larger νT), the p and q functions predict larger values of the mean and the variance of the extreme.

It must be obvious that once results of the type shown above are available for typical loads, the statistics of the extreme over any given period of time can be evaluated with extremely simple and straightforward calculations. This ease in computation is particularly significant for practical applications of the second-moment reliability theory.

4.3.2 MULTIPLE LOAD CASE

Consider a combination of n stationary and stochastically independent loads, giving load effects $S_1, S_2, \dots, S_n$ at a particular component. If this combination includes transient loads of Categories 3 and 4, clearly, the concurrent action of these load effects would only occur during a portion of time period T . We define S as the combined load effect due to this particular combination of loads such that

$$S = \begin{cases} S_1 + S_2 + \dots + S_n & ; \text{ when all } S_i \text{ are acting} \\ 0.0 & ; \text{ otherwise} \end{cases} \quad (4-24)$$

(a) p functions

(b) q functions

FIGURE 4-13. SIGNIFICANCE OF OVERLAPPING IN FILTERED POISSON PROCESSES WITH NORMAL INTENSITY DISTRIBUTION

Note that with the above definition S has a zero value when any of the loads in the combination are not acting. A sample of such a combination for three loads is schematically illustrated in Figure 4-14.

The intention here is to determine the peak value of S over the time T as defined by

$$S_T = \max_{0 \rightarrow T} (S) \quad (4-25)$$

It should be realized that according to the above definition S_T is the peak load effect under the specific combination $S_1 + S_2 + \dots + S_n$. It is not necessarily the peak load effect resulting from the acting loads, since the largest peak may occur under another combination. All possible combinations will, therefore, have to be analyzed in order to account for the largest peak (see Sec. 5.1).

The mean and standard deviation of S at an arbitrary occurrence are obtained from Equation 4-24 as

$$\bar{s} = \sum_i \bar{s}_i \quad (4-26)$$

and

$$\sigma_S = \sqrt{\sum_i \sigma_{S_i}^2} \quad (4-27)$$

where $\bar{s}_i$ and σ_{S_i} are the mean and standard deviation of S_i . Using these values and the formats in Equations 4-12 and 4-13, the mean and standard deviation of S_T can be obtained as

$$\bar{s}_T = \sum_i \bar{s}_i + p \sqrt{\sum_i \sigma_{S_i}^2} \quad (4-28)$$

FIGURE 4-14. SAMPLE COMBINATION OF THREE LOAD EFFECTS

and

$$\sigma_{S_T} = q \sqrt{\sum_i \sigma_{S_i}^2} \quad (4-29)$$

where p and q are yet unknown functions of T for the process S .

It must be realized that on the basis of the limited information that is used, exact evaluations of p and q will generally be impossible. Approximate relationships will, therefore, have to be developed. For this purpose, suppose first that it is possible to obtain for each load effect S_i a set of parameters p_i and q_i defining its peak S_{iT} during the time of combination. Now observe that if the variance of one load effect, say S_j , is far greater than the variances of other load effects, then, the peak combined load effect, S_T , would occur at the same time as S_{jT} . Using this fact, it can be shown (Der Kiureghian, 1978) that p and q in such a case would respectively approach p_j and q_j .

When variances are comparable, exact analytical evaluations of p and q , on the basis of the first two moments of S_i is (with the exception of sum of Gaussian processes) impossible. On the basis of the above argument, however, it is found that fairly good approximations of p and q in this case are obtained as the averages of p_i 's and q_i 's of the individual loads weighted by the corresponding variances. That is,

$$p = \frac{\sum_i p_i \sigma_{S_i}^2}{\sum_i \sigma_{S_i}^2} \quad (4-30)$$

and

$$q = \frac{\sum_i q_i \sigma_{S_i}^2}{\sum_i \sigma_{S_i}^2} \quad (4-31)$$

It must be realized that formats other than those shown above may also be found, which may give good results. In selecting such formats, the convenience in application and the accuracy as checked with correct analytical or simulation results, should be the determining factors. For a number of load combinations discussed below, the above formats are found to be satisfactory.

In order to demonstrate the applicability of the above procedures and to examine the accuracy of results, a number of combinations are studied below.

Sum of Stationary Gaussian Processes: In this particular case for Category 2 loads, it is possible to obtain exact values of p and q in terms of the first two moments of the individual loads. It is well known that the sum of Gaussian processes is itself a Gaussian process. In Appendix C it is shown that for independent processes, the mean rate of upcrossings of the combined process above its mean level can be obtained as

$$v = \sqrt{\frac{\sum_i v_i^2 \sigma_{S_i}^2}{\sum_i \sigma_{S_i}^2}} \quad (4-32)$$

where v_i is the mean upcrossing rate of S_i from its mean level. With the value of v known, p and q can be determined from Figure 4-6 for the combined loads.

Sum of Poisson Square Wave Processes: Consider n Category 2 load effects modeled as Poisson square wave processes. Let ν_i denote the mean rate of intensity changes for S_i . Depending on the type of distribution of the load intensity (as inferred from statistical data of the load or as prescribed subjectively on the basis of experience and judgment), p_i and q_i for each load effect S_i are obtained from Figures 4-9, 4-10, or 4-11. These values are read from these figures against the mean number of intensity changes, i.e., $\nu_i T$, and for the instantaneous coefficient of variation of the load. Equations 4-30 and 4-31 are then used to evaluate p and q .

To check the method, computed mean and standard deviation values for combinations of two loads were compared with results obtained from Monte Carlo simulations. The distributions of intensities for the two loads were assumed to be normal and lognormal.

Figure 4-15a compares computed and simulated results for $\nu_1 = \nu_2$, representing cases where mean rates of intensity changes are comparable. Results for four sets of standard deviations are given. A remarkable agreement is observed in this figure over a wide range of the mean rate of intensity changes.

To examine combinations of loads when mean rates of intensity changes are different, Figure 4-15b compares computed and simulated results for $\nu_2 T = 1000$ with $\nu_1 T$ varying from 10 to 1000. Although the agreement seems to deteriorate with increasing difference between the mean rates of intensity changes, a fair approximation is obtained even for ν_2/ν_1 approaching 100.

When the ratio ν_2/ν_1 is very large, a better approximation is possible by first finding the peak of S_2 during the time $1/\nu_1$, and then combining this peak with S_1 , using the mean rate of intensity change ν_1 . For this purpose, it is necessary to use an appropriate extreme value distribution for the peak of S_2 during the time $1/\nu_1$. The dashed curves in Figure 4-15b were obtained following this procedure.

(a) Equal occurrence rates

(b) Varied occurrence rates

FIGURE 4-15. COMPUTED AND SIMULATED RESULTS FOR COMBINATION OF POISSON SQUARE WAVE LOADS; s_1 -- NORMAL, s_2 -- LOGNORMAL

Sum of Filtered Poisson Processes: Consider first the combination of two independent filtered Poisson load effects S_1 and S_2 . Let ν_1 and ν_2 denote the mean occurrence rates and τ_1 and τ_2 denote the random durations. Neglect the effect of self-overlapping of the respective processes. As indicated by Wen (1977a), the combined load effect $S = S_1 + S_2$ would also have Poisson occurrences. The mean joint occurrence rate, denoted by ν_{12} , is approximately obtained as (Wen, 1977a)

$$\nu_{12} = \nu_1 \nu_2 (\bar{\tau}_1 + \bar{\tau}_2) \quad (4-32)$$

where $\bar{\tau}_1$ and $\bar{\tau}_2$ are the mean durations. The above mean rate of occurrence of the combination is used to obtain p_1, q_1 and p_2, q_2 for the two loads from Figures 4-9, 4-10, or 4-11, depending on the distribution of intensities. With these values known, p and q are obtained from Equations 4-30 and 4-31.

It can be shown (see Wen, 1977a) that the mean duration of the joint occurrence of two loads is

$$\bar{\tau}_{12} = \frac{\bar{\tau}_1 \bar{\tau}_2}{\bar{\tau}_1 + \bar{\tau}_2} \quad (4-33)$$

This result, together with Equation 4-32, can be used to obtain mean joint occurrence rates for combinations of more than two filtered Poisson processes. For example, in the case of three loads

$$\begin{aligned} \nu_{123} &= \nu_{12} \nu_{33} (\bar{\tau}_{12} + \bar{\tau}_3) \\ &= \nu_1 \nu_2 \nu_3 (\bar{\tau}_1 \bar{\tau}_2 + \bar{\tau}_2 \bar{\tau}_3 + \bar{\tau}_3 \bar{\tau}_1) \end{aligned} \quad (4-34)$$

Also

$$\begin{aligned}\bar{\tau}_{123} &= \frac{\bar{\tau}_{12}\bar{\tau}_3}{\bar{\tau}_{12} + \bar{\tau}_3} \\ &= \frac{\bar{\tau}_1\bar{\tau}_2\bar{\tau}_3}{\bar{\tau}_1\bar{\tau}_2 + \bar{\tau}_2\bar{\tau}_3 + \bar{\tau}_3\bar{\tau}_1}\end{aligned}\tag{4-35}$$

To examine the method, computed and simulated results for combinations of two filtered Poisson processes were compared. Durations of the two loads were assumed to be uniformly distributed with mean values of 0.01 years and coefficients of variation of 0.30. Normal and lognormal distributions were assumed for intensities of the two loads.

Figure 4-16a shows computed and simulated results for $\nu_1 = \nu_2$. The mean number of joint occurrences, $\nu_{12}T$, varies from 1 to 400. Very good agreement is obtained, except for $\nu_{12}T = 1$. The error in computed values at that point is attributed to the sharp sensitivity of the p and q functions to the instantaneous coefficient of variation and to the product νT at small values of this product; see Figures 4-9 to 4-11. This, however, is not alarming since such combinations (i.e., with very small mean number of occurrences) would most probably not be critical in most practical applications.

Figure 4-16b shows computed and simulated results for $\nu_2T = 1000$, with ν_1T varying from 50 to 1000. The mean number of joint occurrences, $\nu_{12}T$, varies from 20 to 400. A very good agreement with simulation results is again observed.

(a) Equal occurrence rates

(b) Varied occurrence rates

FIGURE 4-16. COMPUTED AND SIMULATED RESULTS FOR COMBINATION OF FILTERED POISSON LOADS; S_1 -- NORMAL, S_2 -- LOGNORMAL

Sum of Poisson Square Wave and Filtered Poisson Processes: Consider first the combination of a Poisson square wave process S_1 and a filtered Poisson process S_2 . Let ν_1 denote the mean rate of intensity changes of S_1 and ν_2 denote the mean rate of occurrences of S_2 . Assume that the duration of occurrence of S_2 , denoted by τ_2 , is much smaller than the time between intensity changes of S_1 . Consider the following two cases:

1. $\nu_1 \ll \nu_2$: In this case one would expect several occurrences of S_2 between consecutive changes of S_1 . The mean number of intensity changes of S_1 in the combination, therefore, is $\nu_1 T$. The values of p_1, q_1 and p_2, q_2 for the combination should, therefore, be obtained on the basis of $\nu_1 T$ and $\nu_2 T$, respectively, using the appropriate curves in Figures 4-9, 4-10, and 4-11.
2. $\nu_1 \gg \nu_2$: In this case multiple occurrences of S_2 between consecutive changes of S_1 would be unlikely. The mean number of intensity changes of S_1 would, therefore, be approximately equal to $\nu_2 T$. The values of p_1, q_1 as well as p_2, q_2 for the combination should, therefore, be obtained on the basis of $\nu_2 T$, using the appropriate curves in Figures 4-9, 4-10, and 4-11.

When ν_1 and ν_2 are comparable, the above procedures could still be used. Reasonably accurate results are expected due to the relatively slow variations of p and q with νT for the Poisson square wave process. With the values of p_1, q_1 and p_2, q_2 known, p and q for the combination are then obtained from Equations 4-30 and 4-31.

Figure 4-17 shows computed and simulated results for combinations of a Poisson square wave process and a filtered Poisson process. For simulation analysis, the duration of occurrence τ_2 was assumed to be uniformly distributed with mean of 0.01 years and coefficient of variation 0.30. The distributions of intensities for the two processes were respectively assumed

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FIGURE 4-17. COMPUTED AND SIMULATED RESULTS FOR COMBINATION OF POISSON SQUARE WAVE AND FILTERED POISSON LOADS; S_1 -- NORMAL, S_2 -- LOGNORMAL

to be normal and lognormal. Results are shown for $\nu_1 T = 50$ with $\nu_2 T$ varying from 5 to 500. A very good agreement between the two sets of results is observed in this figure.

The above procedure can be extended to combinations involving several Poisson square wave and filtered Poisson processes. For this purpose one should first determine the joint occurrence rate of the filtered Poisson processes. This occurrence rate is used to determine the p_i and q_i values for each filtered Poisson process. The values of p_i and q_i for each Poisson square wave process are then obtained from either the procedure 1 or 2, depending on the magnitude of its intensity change rate relative to the above occurrence rate.

4.4 DISCUSSION

In this chapter an attempt was made to develop a practically applicable method for evaluating first two moments of random loads. On the premise that spatial randomnesses of most loads are less significant than their temporal variability, it was suggested that the influence of the spatial randomness on various effects of a load be incorporated through a random correction factor (Sec. 4.1). In this way loads can be modeled as stochastic processes.

In Section 4.3 a set of functions, denoted by p and q , relating the first two moments of the extreme and the instantaneous values of certain stationary processes were introduced. It was shown that these functions can be used to evaluate the mean and the standard deviation of the extremes of stochastic load combinations. Fairly simple and straightforward calculations are necessary for this purpose.

The functions p and q may be regarded as second-moment descriptors of stationary processes. Such functions may be obtained for any stationary process through analytical procedures, simulation techniques, or statistical inference of existing data. In particular, such functions may be obtained for various types of loads that are of interest in practice. For example, sets of functions may be specified for live loads in various types of buildings, or sets of functions may be specified for earthquake and wind loads (on a macro-time scale) in various geographical regions. The significance of such a development with respect to the formulation of probabilistic structural design codes must be obvious.

An example is provided in the next chapter which serves to demonstrate some of the above facts.

CHAPTER 5

DESIGN FOR MULTIPLE LOAD COMBINATIONS

Structures are usually affected by loads resulting from several independent sources. For the safety analysis and design of a structure under these conditions, it is necessary to evaluate the risk of failure within the life of structures under all possible combinations of loads. This problem is addressed in the first section of this chapter. In the second section, reliability-based design factors corresponding to various combinations of loads are formulated on the basis of concepts developed in Chapters 3 and 4. An example application is provided in the last section of this chapter.

5.1 EVALUATION OF RISK UNDER MULTIPLE LOAD COMBINATIONS

Assume that a structure may be affected by a number of loads which may act in m different combinations during the time period T . Concentrating on a specific component of the structure, we define the events C_i , $i = 1, 2, \dots, m$, where

C_i = The component survives under the i th combination of loads during period T .

and

$\bar{C}_i$ = The complementary event.

In order to define C_i in more precise terms consider the following example. Assume that a structure may be affected by dead, live, wind, and earthquake loads. Denote the corresponding load effects by S_D , S_L , S_W , and S_E , respectively. Also assume that only three combinations, namely $S_D + S_L$, $S_D + S_L + S_W$, and $S_D + S_L + S_E$, denoted by the numbers 1, 2, and 3, respectively, are considered to be critical for this structure. Now imagine three *separate* experiments in which we expose the structure to each of the

load combinations separately. For example, in the first experiment for Combination 1 we isolate the structure from wind and earthquake loads and let the dead and live loads act for a period of T years. In the second experiment for Combination 2 we isolate the structure from any combination other than dead, live, and wind loads for the period of T years; that is we isolate the structure from earthquake loads and from dead and live loads when no wind loads are acting. Finally, in the third experiment we isolate the structure from all loads other than the concurrent action of dead, live, and earthquake loads. According to the above definition, the possible outcomes of these experiments, respectively, are $\{C_1, \bar{C}_1\}$, $\{C_2, \bar{C}_2\}$, and $\{C_3, \bar{C}_3\}$. That is, for example, if the component survived during the second experiment we say the event C_2 has occurred. Also if the component failed during the third experiment we conclude that the event $\bar{C}_3$ has occurred.

In order for the component to survive during the time period T, it must obviously survive under each of the m load combinations. Therefore the probability of survival of the component may be obtained as

$$P(\text{Survival}) = P(C_1 \cap C_2 \cap \dots \cap C_m) \quad (5-1)$$

where $\cap$ indicates intersection of events. By virtue of de Morgan's law, the failure probability of the component is then obtained as

$$\begin{aligned} P(\text{Failure}) &= 1 - P(\text{Survival}) \\ &= P(\bar{C}_1 \cup \bar{C}_2 \cup \dots \cup \bar{C}_m) \end{aligned} \quad (5-2)$$

where U indicates the union of events. Since the events C_i are in general correlated, it is not possible to evaluate the failure probability from the above equation; however bounds to this probability may be calculated as follows:

It is first noted that

$$\{C_1 \cap C_2 \cap \dots \cap C_m\} \subset C_i \quad \text{for any } i \quad (5-3)$$

where $\subset$ denotes a subset. Therefore

$$P(C_1 \cap C_2 \cap \dots \cap C_m) \leq P(C_i) \quad (5-4)$$

and in particular

$$P(C_1 \cap C_2 \cap \dots \cap C_m) \leq \min_i P(C_i) \quad (5-5)$$

On the other hand we may assume that any two events C_i and C_j are positively correlated; i.e.,

$$P(C_i | C_j) \geq P(C_i) \quad \text{for any } i \text{ and } j \quad (5-6)$$

The above assumption implies that the knowledge of survival under Combination j indicates an increased chance for survival under Combination i . Based on this assumption one may derive

$$\begin{aligned} P(C_i \cap C_j) &= P(C_i | C_j) P(C_j) \\ &\geq P(C_i) P(C_j) \end{aligned} \quad (5-7)$$

By induction in Equation 5-1, this gives the lower bound on the survival probability,

$$P(C_1 \cap C_2 \cap \dots \cap C_m) \geq P(C_1) P(C_2) \dots P(C_m) \quad (5-8)$$

From the above and Equation 5-5 the bounds on the failure probability may be calculated as

$$1 - \min_i P(C_i) \leq P(\text{Failure}) \leq 1 - \prod_{i=1}^m P(C_i) \quad (5-9)$$

The lower limit in the above expression is equivalent to

$$1 - \min_i P(C_i) = \max_i P(\bar{C}_i) \quad (5-10)$$

whereas the upper limit may be expressed as

$$\begin{aligned} 1 - \prod_{i=1}^m P(C_i) &= 1 - \prod_{i=1}^m [1 - P(\bar{C}_i)] \\ &\approx \sum_{i=1}^m P(\bar{C}_i) \end{aligned} \quad (5-11)$$

where higher order terms involving products of probabilities have been neglected. Thus

$$\max_i P(\bar{C}_i) \leq P(\text{Failure}) \leq \sum_{i=1}^m P(\bar{C}_i) \quad (5-12)$$

The above expression indicates that the actual failure probability of the component for the period T is greater than the failure probability under the most critical combination and is smaller than the sum of the failure probabilities under all combinations. If a particular combination is much more critical than other combinations (i.e., the probability of failure under that combination is much greater), then the failure probability would

clearly approach to the probability of failure under that particular combination. On the other hand if the failure probabilities under various combinations are of the same order, then the failure probability could conservatively be evaluated as the sum of those probabilities.

As was discussed in Chapter 3, in reliability-based design an acceptable risk for component failure, p_f , is specified. Under multiple load combinations it is necessary that this acceptable risk be "allocated" to the various load combinations that may be critical. The allocation can be done in any manner as long as it is assured that the actual probability of failure of the design remains smaller than p_f . Using the upper bound in Equation 5-12, this can be done by requiring that the sum of acceptable risks for all possible combinations be equal to or smaller than p_f .

For economical reasons it may be desirable to have various load combinations require more or less equal resistances. For this purpose one would have to allocate larger portions of p_f to combinations which are more critical (i.e., combinations which require larger resistances for the same risk level). On the other hand, if for any reason it is desired to maintain a high level of reliability under a specific combination, then one would have to allocate a smaller acceptable risk to that combination. This may, of course, result in that combination effectively determining the required resistances.

5.2 DESIGN FACTORS UNDER MULTIPLE LOAD COMBINATIONS

Consider the general case of a combination including time-independent (dead) load effects D_j and stochastic load effects S_i . Let p_f^* denote the acceptable risk allocated to the combination.

The peak combined load effect within the life of structure T is defined as

$$S_T = \sum_j D_j + \max_{0 \rightarrow T} \left(\sum_i S_i \right)$$

Using the method developed in Chapter 4, the mean and standard deviation of S_T is obtained as

$$\bar{s}_T = \sum_j \bar{d}_j + \sum_i \bar{s}_i + p \sqrt{\sum_i \sigma_{S_i}^2} \quad (5-13)$$

and

$$\sigma_{S_T} = \sqrt{\sum_j \sigma_{D_j}^2 + q^2 \sum_i \sigma_{S_i}^2} \quad (5-14)$$

p and q in the above equations are specific to the combination and are evaluated as discussed in Chapter 4.

With the coefficient of variation obtained as $\Omega_{S_T} = \sigma_{S_T}/\bar{s}_T$, the reliability-based central safety factor corresponding to the formulation on $Z_1 = R - S_T$ is obtained as (see Eq. 3-40)

$$\bar{\gamma} = \frac{1 + \beta_1 \alpha_R \Omega_{S_T}}{1 - \beta_1 \alpha_R \Omega_R} \quad (5-15)$$

The central safety factor corresponding to the formulation on $Z_2 = R/S_T$, is obtained as (see Eq. 3-49)

$$\bar{\gamma} = \exp \left(\beta_2 \sqrt{\Omega_R^2 + \Omega_{S_T}^2} \right) \quad (5-16)$$

β_1 and β_2 in the above expressions are the corresponding reliability indices obtained in terms of p_f^* . Note that these factors are to be applied on the mean value of S_T as obtained from Equation 5-13.

To obtain reliability-based load factors, it is necessary to separate the effects of loads in the combination. For this purpose, Equations 5-13 and 5-14 are written as

$$\bar{s}_T = \sum_j \bar{d}_j + \sum_i \bar{s}_i + \alpha_{s2}^p \sum_i \sigma_{S_i} \quad (5-17)$$

and

$$\sigma_{S_T} = \alpha_{s1} \left(\sum_j \sigma_{D_j} + q \sum_i \sigma_{S_i} \right) \quad (5-18)$$

where

$$\alpha_{s1} = \frac{\sqrt{\sum_j \sigma_{D_j}^2 + q^2 \sum_i \sigma_{S_i}^2}}{\sum_j \sigma_{D_j} + q \sum_i \sigma_{S_i}} \quad (5-19)$$

and

$$\alpha_{s2} = \frac{\sqrt{\sum_i \sigma_{S_i}^2}}{\sum_i \sigma_{S_i}} \quad (5-20)$$

Using above equations together with Equations 3-35 and 3-36 of Chapter 3, it is easily verified that for the formulation based on $Z_1 = R - S_T$, the central load factors for D_j and S_i , respectively, are

$$\bar{\delta}_{D_j} = 1 + \beta_1 \alpha_R \alpha_{s1}^{\Omega} \sigma_{D_j} \quad (5-21)$$

and

$$\bar{\delta}_{S_i} = 1 + \beta_1 \alpha_R \alpha_{s1}^{q\Omega} \sigma_{S_i} + \alpha_{s2}^{p\Omega} \sigma_{S_i} \quad (5-22)$$

These expressions do not include the effect of additional uncertainty Ω_g which is due to errors in formulation of loads. In a manner similar to that in Chapter 3, this may be included to give

$$\bar{\delta}_{D_j} = 1 + \beta_1 \alpha_R \alpha_g (\Omega_g + \alpha_{s1} \Omega_{D_j}) \quad (5-23)$$

and

$$\bar{\delta}_{S_i} = 1 + \beta_1 \alpha_R \alpha_g (\Omega_g + \alpha_{s1} q \Omega_{S_i}) + \alpha_{s2} p \Omega_{S_i} \quad (5-24)$$

The above factors are to be applied on the instantaneous mean values $\bar{d}_j$ and $\bar{s}_i$.

For the formulation based on $Z_2 = R/S_T$, it can be shown that central load factors for D_j and S_i may be obtained, respectively, as

$$\bar{\delta}_{D_j} = \exp \left[\beta_2 \alpha_R \alpha_g (\Omega_g + \alpha_{s1} \Omega_{D_j}) \right] \quad (5-25)$$

and

$$\bar{\delta}_{S_i} = \exp \left[\beta_2 \alpha_R \alpha_g \left(\frac{\Omega_g + \alpha_{s1} q \Omega_{S_i}}{1 + \alpha_{s2} p \Omega_{S_i}} \right) \right] (1 + \alpha_{s2} p \Omega_{S_i}) \quad (5-26)$$

In the above equations, α_R , α_g , and α_{s2} are as before, but α_{s1} must be obtained from a transcendental equation similar to Equation 3-54 of Chapter 3. It is felt, however, that Equation 5-19 could still be used to obtain good approximations of α_{s1} . Note that Table 3-3 of Chapter 3 may be used to obtain approximate values of α_{s1} and α_{s2} depending on the number of loads in a combination.

It is noted that strength factors corresponding to the two formulations remain as given in Chapter 3. They are given in Equations 3-45 and 3-56 of that chapter.

It should be noted that safety, load, and strength factors based on $Z_1 = R - S_T$, may be obtained by linearizing the corresponding expressions of the factors obtained for $Z_2 = R/S_T$. This can be observed by expanding the exponential terms in Equations 5-16, 5-25, and 5-26 in Taylor series and by deleting the higher order terms involving products of coefficients of variation.

5.3 VARIABLES AFFECTING RELIABILITY-BASED DESIGN FACTORS

Reliability-based design factors derived in this study were seen to be functions of various design variables. Some of these variables are significant, whereas others may be insignificant. It is believed that a sensitivity analysis of the design factor functionals will be very helpful in specifying those variables which are significant and in determining the effects of their variations within practical ranges. Such a sensitivity analysis is not performed at this time; however, a brief qualitative discussion on variables affecting design factors is presented below.

Central Safety Factor: From Equations 5-15 and 5-16, variables affecting the central load factor are seen to be the acceptable risk p_f^* allocated to the combination (this is through the reliability index β_1 or β_2 ; see Equations 3-8 and 3-9 and their subsequent discussion), and the coefficients of variation Ω_R and Ω_{S_T} of the resistance R , and of the peak combined load effect, S_T , respectively. The parameter α_R in Equation 5-15 is effectively a constant.

It is observed in Equations 5-15 and 5-16 together with Table 3-1 that the central load factor $\bar{\gamma}$ is larger for a larger β_1 or β_2 (i.e., a higher required reliability) or for a smaller p_f^* (i.e., a lower acceptable risk). This indicates a more conservative design when higher reliability is required.

While the coefficient of variation Ω_R remains unchanged from one combination to another, the coefficient of variation Ω_{S_T} of the peak combined load effect, obtained from Equations 5-13 and 5-14 as

$$\Omega_{S_T} = \frac{\sqrt{\sum_j \sigma_{D_j}^2 + q^2 \sum_i \sigma_{S_i}^2}}{\sum_j \bar{d}_j + \sum_i \bar{s}_i + p \sqrt{\sum_i \sigma_{S_i}^2}}, \quad (5-27)$$

is clearly dependent on the combination under consideration. This is not only due to the dependence of Ω_{S_T} on the (instantaneous) means and standard deviations of loads in combination, but also due to its dependence on the second-moment descriptors p and q . As was shown before, p and q depend on the stochastic behavior of each load and on the useful life, T , assumed for the structure.

Since p is a monotonically increasing function of T , and q is usually (but not always) a decreasing function of T , it appears that Ω_{S_T} and therefore, $\bar{\gamma}$ would decrease with increasing T . Although this result may not appear reasonable at first, it becomes logical when one notes that this safety factor is applied on $\bar{s}_T$ which itself monotonically increases with T .

Finally, it is recalled that the mean and standard deviation values of load effects are related to the mean and standard deviation values of loads through the influence coefficients. This means that for a given combination of loads, and for a specific T , the value of Ω_{S_T} and therefore, the safety factor $\bar{\gamma}$, would change from one component to another component in the structure.

Central Strength and Load Factors: The central strength factor, $\bar{\phi}$, given in Equations 3-45 and 3-56 of Chapter 3 is seen to be a function of the acceptable risk, p_f^* , through the reliability index β_1 or β_2 , and the

coefficient of variation Ω_R . For both formulations $\bar{\phi}$ is seen to decrease with increasing β_1 or β_2 or with decreasing p_f^* . The strength factor is also seen to decrease with increasing values of Ω_R . These indicate a more conservative estimate of strength when a high reliability is required or when a large uncertainty is associated with strength evaluation.

Variables affecting central load factors for dead loads, $\bar{\delta}_{Dj}$, are seen in Equations 5-23 and 5-25 to be the reliability index β_1 or β_2 (or the corresponding acceptable risk p_f^*), the coefficient of variation Ω_{Dj} , and the decoupling factor α_{s1} . The factors α_R and α_g may be regarded as constants. The coefficient of variation Ω_g is the same for all loads and combinations.

It is observed in Equations 5-23 and 5-25 that the central dead load factor increases with increasing β_1 or β_2 or with decreasing p_f^* . It is also seen to increase with increasing values of the coefficient of variation of the load. The parameter α_{s1} , as given in Equation 5-19, is dependent on the standard deviations of individual loads as well as the p and q values. On the basis of results in Tables 3-2 and 3-4, however, this parameter is seen to be primarily a function of the number of loads in a combination; see Table 3-3 for suggested values. It is concluded, therefore, that the dead load factor is dependent on the number of loads in combination; it decreases with increasing number of loads. The dependence of the factor on individual load characteristics and the values of p and q are expected to be insignificant. It is noted that the parameter α_{s1} and, therefore, the dead load factor would also be dependent on the influence coefficients, but this dependence is also expected to be insignificant. The dead load factors are, therefore, effectively independent of the location of a component in a structure.

Variables affecting central load factors for stochastic loads, $\bar{\delta}_{Si}$, are seen in Equations 5-24 and 5-26 to be the reliability index β_1 or β_2 (or the corresponding acceptable risk p_f^*); the instantaneous coefficient

of variation of load, Ω_{S_i} ; the second-moment descriptors p and q , and the decoupling factors α_{s1} and α_{s2} for each combination. The factors α_R and α_g , and the coefficient of variation Ω_g are the same for all loads and combinations.

It is observed in Equations 5-24 and 5-26 that the central load factor for a stochastic load increases with increasing β_1 or β_2 or with decreasing p_f^* . It is also observed to increase with increasing value of the instantaneous coefficient of variation Ω_{S_i} . The relation of the load factor to the stochastic behavior of loads appears through the p and q values for each combination. Since these values are dependent on the standard deviations of individual load effects and, therefore, on the influence coefficients, it is concluded that the load factor for a stochastic load in a specific combination would depend on the location of the component in the structure. The significance of this dependence is not clear at this time.

The load factor $\bar{\delta}_{S_i}$ is also dependent on the useful life of the structure, T , through the second-moment descriptors p and q . Based on the studied behaviors of p and q with respect to T , it is observed that the load factor increases with increasing value of T .

The decoupling factors α_{s1} and α_{s2} generally depend on the characteristics of loads in the combination. As mentioned before, however, these factors can effectively be determined in terms of the number of loads in the combination (see Table 3-3). Based on such an approximation, the load factor $\bar{\delta}_{S_i}$ is found to decrease with increasing number of loads in a combination.

In summary, variables which significantly affect the central design factors are:

1. Central safety factor (applied to the mean value of the peak combined load effect, $\bar{S}_T$):
 - a. The acceptable risk of the combination.
 - b. The coefficient of variation of the resistance.

- c. Means and standard deviations of individual load effects in the combination.
 - d. The second-moment descriptors p and q of the combination.
 - e. The useful life of the structure.
 - f. The location of the component in the structure.
2. Central strength factor (applied to the mean resistance, $\bar{R}$):
- a. The acceptable risk of the combination.
 - b. The coefficient of variation of the resistance.
3. Central load factor for dead loads (applied to the mean dead load, $\bar{d}_j$):
- a. The acceptable risk of the combination.
 - b. The coefficient of variation of the dead load.
 - c. The number of loads in the combination.
4. Central load factor for stochastic loads (applied to the instantaneous mean load, $\bar{s}_i$):
- a. The acceptable risk of the combination.
 - b. The instantaneous coefficient of variation of the stochastic load.
 - c. The second-moment descriptors p and q of the combination.
 - d. The useful life of the structure.

- e. The location of the component in the structure.
- f. The number of loads in the combination.

In the above section the effects of various variables on reliability-based design factors were discussed. It is expected that the effects of certain variables be insignificant, which may then be neglected in practical applications. A careful sensitivity analysis together with the application of the theory to typical structures will be necessary to determine the relative significance of the variables.

A note on the relative convenience and desirability of the safety and load factor approaches is in place. From the viewpoint of developing a probabilistic structural design procedure, the safety factor approach appears to be more direct and straightforward. There seems to be no reason for separating the effects of the individual loads. This was pointed out by Ang (1973). However, from the standpoint of developing a theoretical basis for the critical investigation of current deterministic procedures, and for the purpose of developing semiprobabilistic design procedures or improving the existing procedures, analysis of the load factor design approach is worthwhile.

5.4 ILLUSTRATIVE EXAMPLE

To demonstrate the application of the theory developed in this report, a hypothetical example is provided below. For convenience, the theoretical models discussed in Chapter 4 have been used to model the loads. Other load models can be developed and used in a similar manner.

Consider a structural component affected by a dead load effect, D , a sustained live load effect, L_s , extraordinary transient live load effects, L_e , a sustained temperature load effect, T_o , and earthquake load effects, E . The assumed models for these loads are described in Table 5-1.

TABLE 5-1. ASSUMED LOAD PARAMETERS FOR ILLUSTRATIVE EXAMPLE

Load	Load Model	Random-Point-in-Time Load Effect			ν_i 1/year	Mean Duration $\bar{\tau}_i$, years	Lifetime Peak Load Effect					
		Mean	St. Dev.	COV			T = 25 years			T = 50 years		
							Mean	St. Dev.	COV	Mean	St. Dev.	COV
D	Random Variable	100	20	0.20	--	--	100.0	20.0	0.20	100.0	20.0	0.20
L_s	Poisson Square Wave with Gamma Dist.	50	15	0.30	0.5	--	77.0	11.8	0.15	83.6	11.1	0.13
T_o	Stationary Gaussian	50	10	0.20	20.0	--	86.8	3.3	0.04	88.5	3.2	0.04
L_e	Filtered Poisson with Gamma Dist.	60	24	0.40	10.0	10^{-2}	150.7	17.8	0.12	160.3	17.2	0.11
E	Filtered Poisson with Exponential Dist.	50	30	0.60	0.5	10^{-6}	113.0	38.4	0.34	134.0	38.4	0.29

On the basis of stiffnesses assumed in a preliminary design, the random-point-in-time mean and standard deviation values of load effects in various components can be evaluated in terms of the influence coefficients and the random-point-in-time mean and standard deviation of the corresponding loads. For the component under consideration, assume these values are evaluated as shown in Table 5-1. The assumed values of the parameter v_i for each load (i.e., the mean intensity change rate of L_s ; the mean occurrence rates of L_e and E , and the mean upcrossing rate of T_o) and the mean durations, $\bar{\tau}_i$, for the transient loads, L_e and E , are also given in this table.

To investigate the effect of the structure lifetime T , the values of $T = 25$ and 50 years are assumed. The lifetime peak load effects of individual loads for the two values of T , obtained using Equations 4-12 and 4-13 and Figures 4-6 and 4-11, are given in the right-hand portion of Table 5-1.

All possible combinations of the loads are considered in Table 5-2. These include combination 1 involving the sustained loads D , L_s , and T_o ; combinations 2 and 3 involving the sustained loads plus the transient loads L_e and E , respectively; and combination 4 involving the sustained loads plus both of the transient loads. The last combination thus considers the possibility of coincidence of the two transient loads.

The second moment descriptors p and q for each combination are evaluated and are given in Table 5-2 for $T = 25$ and 50 years. The lifetime peak combined load effects for each combination are also evaluated and are given in this table. For the purpose of demonstration, three example calculations for $T = 50$ years are described below:

1. Combination $D + L_s + T_o$:

Load L_s :

$$vT = 0.5 \times 50 = 25 \quad (\text{Mean number of intensity changes})$$

$$p = 2.24, \quad q = 0.74 \quad (\text{Fig. 4-11})$$

TABLE 5-2. COMPUTED RESULTS FOR COMBINATIONS OF LOADS

Comb. No.	Load Combination	T = 25 years					T = 50 years				
		p	q	Lifetime Peak Combined Load Effect			p	q	Lifetime Peak Combined Load Effect		
				Mean	St. Dev.	COV			Mean	St. Dev.	COV
1.	$D + L_s + T_o$	2.38	0.65	242.9	23.2	0.10	2.74	0.61	249.4	22.8	0.09
2	$D + L_s + T_o + L_e$	3.18	0.71	355.5	29.2	0.08	3.59	0.68	367.6	28.7	0.08
3	$D + L_s + T_o + E$	2.01	1.13	320.4	45.0	0.14	2.63	1.13	342.0	45.0	0.13
4	$D + L_s + T_o + L_e + E$	0.11	1.28	314.7	57.9	0.18	0.67	1.24	338.4	56.0	0.17

Load T_O :

$$vT = 20 \times 50 = 1000 \quad (\text{Mean number of upcrossings})$$

$$p = 3.86, \quad q = 0.32 \quad (\text{Fig. 4-6})$$

For the Combination:

$$p = \frac{2.24(15^2) + 3.86(10^2)}{15^2 + 10^2} = 2.74 \quad (\text{Eq. 4-12})$$

$$q = \frac{0.74(15^2) + 0.32(10^2)}{15^2 + 10^2} = 0.61 \quad (\text{Eq. 4-13})$$

$$\bar{s}_T = 100 + 50 + 50 + 2.74 \sqrt{15^2 + 10^2} = 249.4 \quad (\text{Eq. 5-13})$$

$$\sigma_{s_T} = \sqrt{20^2 + 0.61^2(15^2 + 10^2)} = 22.8 \quad (\text{Eq. 5-14})$$

$$\Omega_{s_T} = 22.8/249.4 = 0.09$$

2. Combination $D + L_s + T_O + E$:

Load E :

$$vT = 0.5 \times 50 = 25 \quad (\text{Mean number of occurrences})$$

$$p = 2.80, \quad q = 1.28 \quad (\text{Fig. 4-11})$$

Load L_s :

$$vT = 25 \quad (\text{see page 87})$$

$$p = 2.24, \quad q = 0.74$$

Load T_o :

$$\nu T = 25^* \quad (\text{see footnote below})$$

$$p = 1.96, \quad q = 0.51 \quad (\text{Fig. 4-9})$$

For the Combination:

$$p = \frac{2.24(15^2) + 1.96(10^2) + 2.80(30^2)}{15^2 + 10^2 + 30^2} = 2.63 \quad (\text{Eq. 4-12})$$

$$q = \frac{0.74(15^2) + 0.51(10^2) + 1.28(30^2)}{15^2 + 10^2 + 30^2} = 1.13 \quad (\text{Eq. 4-13})$$

$$\bar{s}_T = 100 + 50 + 50 + 50 + 2.63\sqrt{15^2 + 10^2 + 30^2} = 342.0 \quad (\text{Eq. 5-13})$$

$$\sigma_{S_T} = \sqrt{20^2 + 1.13^2(15^2 + 10^2 + 30^2)} = 45.0 \quad (\text{Eq. 5-14})$$

$$\Omega_{S_T} = 45.0/342.0 = 0.13$$

3. Combination $D + L_s + T_o + L_e + E$:

Joint Occurrence Rate of L_e and E :

$$\nu = 10 \times 0.5(10^{-2} + 10^{-6}) = 0.05 \quad (\text{Eq. 4-32})$$

$$\nu T = 0.05 \times 50 = 2.5 \quad (\text{Mean number of joint occurrences})$$

* Since earthquakes occur infrequently relative to the variations in temperature (i.e., mean occurrence rate = $0.5 \ll$ mean upcrossing rate = 20), and since the mean duration of earthquakes is much smaller than the mean time between consecutive upcrossings ($= 1/\nu = 1/20$ year = 18.3 days), the effect of T_o in this combination can be approximated as a random selection of uncorrelated values of T_o , with the mean number of selections equal to the mean number of occurrences. Based on this, the p and q values for T_o are obtained from Figure 4-9 for Poisson square wave process with normal distribution.

Load L_e :

$$vT = 2.5$$

$$p = 0.41, \quad q = 1.34 \quad (\text{Fig. 4-11})$$

Load E :

$$vT = 2.5$$

$$p = 0.78, \quad q = 1.28 \quad (\text{Fig. 4-11})$$

Load L_s :

$$vT = 2.5$$

$$p = 0.85, \quad q = 0.95 \quad (\text{Fig. 4-11})$$

Load T_o :

$$vT = 2.5 \quad (\text{see footnote in previous page})$$

$$p = 0.81, \quad q = 0.84 \quad (\text{Fig. 4-9})$$

For the Combination:

$$p = \frac{0.85(15^2) + 0.41(24^2) + 0.81(10^2) + 0.78(30^2)}{15^2 + 24^2 + 10^2 + 30^2} = 0.67 \quad (\text{Eq. 4-12})$$

$$q = \frac{0.95(15^2) + 1.34(24^2) + 0.84(10^2) + 1.28(30^2)}{15^2 + 24^2 + 10^2 + 30^2} = 1.24 \quad (\text{Eq. 4-13})$$

$$\bar{s}_T = 100 + 50 + 50 + 60 + 50 + 0.67\sqrt{15^2 + 24^2 + 10^2 + 30^2} = 338.4 \quad (\text{Eq. 5-13})$$

$$\sigma_{S_T} = \sqrt{20^2 + 1.24^2(15^2 + 24^2 + 10^2 + 30^2)} = 55.7 \quad (\text{Eq. 5-14})$$

$$\Omega_{S_T} = 55.7/338.4 = 0.17$$

With the above values calculated, it is then possible to obtain the central safety and load factors for each combination. For this example it is assumed that the acceptable risk is equally allocated to the four combinations. To investigate the effect of the acceptable risk level, the cases $\beta_1 = \beta_2 = 3.72$ (i.e., $p^* \approx 10^{-4}$; see Table 3-1), and $\beta_1 = \beta_2 = 4.27$ (i.e., $p_f^* \approx 10^{-5}$) are considered. Assuming $\Omega_R = 0.20$, $\Omega_g = 0.0$, $\alpha_R = 0.75$, and $\alpha_g = 1.0$, the central safety, strength, and load factors corresponding to the formulations based on $Z_1 = R - S_T$ and $Z_2 = R/S_T$, for each combination are calculated and are shown in Tables 5-3 and 5-4, respectively. Example calculations for $\beta_1 = \beta_2 = 3.72$ and $T = 50$ years are described below for the combination $D + L_s + T_o + L_e + E$.

Formulation Based on $Z_1 = R - S_T$

1. Central Safety Factor

$$\bar{\gamma} = \frac{1 + 3.72 \times 0.75 \times 0.17}{1 - 3.72 \times 0.75 \times 0.20} = 3.31 \quad (\text{Eq. 5-15})$$

2. Central Strength and Load Factors

a. Strength factor calculation:

$$\bar{\phi} = 1 - 3.72 \times 0.75 \times 0.20 = 0.44 \quad (\text{Eq. 3-45})$$

b. Load factor calculation:

$$\alpha_{s1} = \frac{\sqrt{20^2 + 1.24^2(15^2 + 10^2 + 24^2 + 30^2)}}{20 + 1.24(15 + 10 + 24 + 30)} = 0.48$$

(Eq. 5-19)

$$\alpha_{s2} = \frac{\sqrt{15^2 + 10^2 + 24^2 + 30^2}}{15 + 10 + 24 + 30} = 0.54 \quad (\text{Eq. 5-20})$$

$$\bar{\delta}_D = 1 + 3.72 \times 0.75 \times 1.0(0.0 + 0.48 \times 0.20) = 1.27 \quad (\text{Eq. 5-23})$$

$$\begin{aligned} \bar{\delta}_{L_s} &= 1 + 3.72 \times 0.75 \times 1.0(0.0 + 0.48 \times 1.24 \times \Omega_{L_s}) \\ &\quad + 0.54 \times 0.67 \times \Omega_{L_s} \\ &= 1 + 2.01\Omega_{L_s} = 1 + 2.01 \times 0.30 = 1.60 \quad (\text{Eq. 5-24}) \end{aligned}$$

$$\bar{\delta}_{T_o} = 1 + 2.01 \times 0.20 = 1.40 \quad (\text{Eq. 5-24})$$

$$\bar{\delta}_{L_e} = 1 + 2.01 \times 0.40 = 1.80 \quad (\text{Eq. 5-24})$$

$$\bar{\delta}_E = 1 + 2.01 \times 0.60 = 2.21 \quad (\text{Eq. 5-24})$$

Formulation Based on $Z_2 = R/S_T$

1. Central Safety Factor

$$\bar{\gamma} = \exp\left(3.72 \sqrt{0.20^2 + 0.17^2}\right) = 2.63 \quad (\text{Eq. 5-16})$$

2. Central Strength and Load Factors

a. Strength factor calculation:

$$\bar{\phi} = \exp(3.72 \times 0.75 \times 0.20) = 0.57 \quad (\text{Eq. 3-56})$$

b. Load factor calculations:

$\alpha_{s1} = 0.46$ obtained from iteration, and $\alpha_{s2} = 0.54$ as above

$$\bar{\delta}_D = \exp[3.72 \times 0.75 \times 1.0(0.0 + 0.46 \times 0.20)] = 1.29 \quad (\text{Eq. 5-25})$$

$$\begin{aligned}
\bar{\delta}_{L_s} &= \exp \left[3.72 \times 0.75 \times 1.0 \left(\frac{0.0 + 0.46 \times 1.24 \times \Omega_{L_s}}{1 + 0.54 \times 0.67 \times \Omega_{L_s}} \right) \right] \\
&\quad \left(1 + 0.54 \times 0.67 \times \Omega_{L_s} \right) \\
&= \exp \left(\frac{1.58 \Omega_{L_s}}{1 + 0.36 \Omega_{L_s}} \right) \left(1 + 0.36 \Omega_{L_s} \right) \\
&= \exp \left(\frac{1.58 \times 0.30}{1 + 0.36 \times 0.30} \right) \left(1 + 0.36 \times 0.30 \right) = 1.70 \text{ (Eq. 5-26)} \\
\bar{\delta}_{T_o} &= \exp \left(\frac{1.58 \times 0.20}{1 + 0.36 \times 0.20} \right) \left(1 + 0.36 \times 0.20 \right) = 1.44 \text{ (Eq. 5-26)} \\
\bar{\delta}_{L_e} &= \exp \left(\frac{1.58 \times 0.40}{1 + 0.36 \times 0.40} \right) \left(1 + 0.36 \times 0.40 \right) = 1.99 \text{ (Eq. 5-26)} \\
\bar{\delta}_E &= \exp \left(\frac{1.58 \times 0.60}{1 + 0.36 \times 0.60} \right) \left(1 + 0.36 \times 0.60 \right) = 2.66 \text{ (Eq. 5-26)}
\end{aligned}$$

A discussion of the results shown in Tables 5-3 and 5-4 is provided in Section 5.4.2.

5.4.1 FACTORS COMPARABLE WITH CURRENT PROCEDURES

In order to make a proper comparison of safety and load factors it is necessary that they apply to a consistent set of loads and resistances. Values of loads and resistances used in current procedures, however, are unfortunately seldom well-defined with respect to their distributions. An accurate comparison, therefore, is difficult. As a reasonable approach, it is assumed here that design resistances and loads used in current procedures correspond to the *mean* values of resistance and lifetime peak loads, respectively. When these are different from the mean values, an appropriate adjustment must be made as discussed in Section 6.1.4.

TABLE 5-3. RELIABILITY DESIGN BASED ON $Z_1 = R - S_T$

$$(a) \beta_1 = 3.72 \quad (p_f^* \approx 10^{-4})$$

Load Combination	T = 25 years								T = 50 years							
	Safety Factor, $\bar{\gamma}$	Strength and Load Factors						Req'd Mean Resistance	Safety Factor, $\bar{\gamma}$	Strength and Load Factors						Req'd Mean Resistance
		$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}$	$\bar{\delta}_{T_o}$	$\bar{\delta}_{L_e}$	$\bar{\delta}_E$			$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}$	$\bar{\delta}_{T_o}$	$\bar{\delta}_{L_e}$	$\bar{\delta}_E$	
D + L _s + T _o	2.87	0.44	1.36	1.86	1.57	--	--	697	2.84	0.44	1.36	1.92	1.61	--	--	708
D + L _s + T _o + L _e	2.78	0.44	1.30	1.90	1.60	2.20	--	988	2.76	0.44	1.30	1.97	1.64	2.29	--	1014
D + L _s + T _o + E	3.15	0.44	1.30	1.89	1.60	--	2.79	1008	3.09	0.44	1.30	2.01	1.67	--	3.02	1055
D + L _s + T _o + L _e + E	3.42	0.44	1.27	1.53	1.35	1.71	2.06	1076	3.31	0.44	1.27	1.60	1.40	1.80	2.21	1120

- Note: 1. Safety factor is relative to the mean lifetime peak combined load effect.
 2. Strength factor is relative to the mean resistance.
 3. Load factors are relative to mean instantaneous loads.
 4. Required mean resistance is obtained as $\bar{\gamma}_{S_T}$ or as $\frac{1}{\bar{\phi}} \sum \bar{\delta}_i s_i$

TABLE 5-3. (CONCLUDED)

$$(b) \beta_1 = 4.27 \quad (p_f^* \approx 10^{-5})$$

Load Combination	T = 25 years								T = 50 years							
	Safety Factor, $\bar{\gamma}$	Strength and Load Factors						Req'd Mean Resistance	Safety Factor, $\bar{\gamma}$	Strength and Load Factors						Req'd Mean Resistance
		$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}$	$\bar{\delta}_{T_o}$	$\bar{\delta}_{L_e}$	$\bar{\delta}_E$			$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}$	$\bar{\delta}_{T_o}$	$\bar{\delta}_{L_e}$	$\bar{\delta}_E$	
D + L _s + T _o	3.63	0.36	1.41	1.91	1.61	--	--	882	3.60	0.36	1.41	1.97	1.65	--	--	897
D + L _s + T _o + L _e	3.51	0.36	1.34	1.95	1.63	2.26	--	1247	3.48	0.36	1.34	2.01	1.67	2.35	--	1279
D + L _s + T _o + E	4.03	0.36	1.35	1.97	1.65	--	2.94	1290	3.95	0.36	1.35	2.09	1.72	--	3.17	1349
D + L _s + T _o + L _e + E	4.42	0.36	1.31	1.61	1.40	1.81	2.21	1391	4.26	0.36	1.31	1.68	1.45	1.90	2.35	1441

- Note: 1. Safety factor is relative to the *mean lifetime peak combined* load effect.
 2. Strength factor is relative to the *mean* resistance.
 3. Load factors are relative to *mean instantaneous* loads.
 4. Required mean resistance is obtained as $\bar{\gamma}_{s_T}$ or as $\frac{1}{\bar{\phi}} \sum \bar{\delta}_i s_i$

TABLE 5-4. RELIABILITY DESIGN BASED ON $Z_2 = R/S_T$

$$(a) \beta_2 = 3.72 \quad (p_f^* \approx 10^{-4})$$

Load Combination	T = 25 years								T = 50 years							
	Safety Factor, $\bar{\gamma}$	Strength and Load Factors						Req'd Mean Resistance	Safety Factor, $\bar{\gamma}$	Strength and Load Factors						Req'd Mean Resistance
		$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}$	$\bar{\delta}_{T_o}$	$\bar{\delta}_{L_e}$	$\bar{\delta}_E$			$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}$	$\bar{\delta}_{T_o}$	$\bar{\delta}_{L_e}$	$\bar{\delta}_E$	
D + L _s + T _o	2.28	0.57	1.43	1.90	1.60	--	--	554	2.27	0.57	1.43	1.96	1.63	--	--	566
D + L _s + T _o + L _e	2.24	0.57	1.34	1.93	1.62	2.25	--	796	2.22	0.57	1.34	1.99	1.66	2.33	--	816
D + L _s + T _o + E	2.48	0.57	1.34	1.99	1.64	--	3.11	795	2.44	0.57	1.33	2.05	1.69	--	3.20	834
D + L _s + T _o + L _e + E	2.75	0.57	1.28	1.63	1.39	1.92	2.62	865	2.63	0.57	1.29	1.70	1.44	1.99	2.66	890

- Note: 1. Safety factor is relative to the mean lifetime peak combined load effect.
 2. Strength factor is relative to the mean resistance.
 3. Load factors are relative to mean instantaneous loads.
 4. Required mean resistance is obtained as $\bar{\gamma}_{s_T}$ or as $\frac{1}{\bar{\phi}} \sum \delta_i s_i$

TABLE 5-4. (CONCLUDED)

$$(b) \beta_2 = 4.27 \quad (p_f^* \approx 10^{-5})$$

Load Combination	T = 25 years								T = 50 years							
	Safety Factor, $\bar{\gamma}$	Strength and Load Factors						Req'd Mean Resistance	Safety Factor, $\bar{\gamma}$	Strength and Load Factors						Req'd Mean Resistance
		$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}$	$\bar{\delta}_{T_o}$	$\bar{\delta}_{L_e}$	$\bar{\delta}_E$			$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}$	$\bar{\delta}_{T_o}$	$\bar{\delta}_{L_e}$	$\bar{\delta}_E$	
D + L _s + T _o	2.58	0.53	1.51	1.97	1.64	--	--	627	2.56	0.53	1.51	2.02	1.67	--	--	638
D + L _s + T _o + L _e	2.52	0.53	1.40	1.99	1.65	2.33	--	896	2.50	0.53	1.41	2.05	1.69	2.41	--	919
D + L _s + T _o + E	2.84	0.53	1.40	2.09	1.70	--	3.38	910	2.78	0.53	1.41	2.20	1.78	--	3.56	951
D + L _s + T _o + L _e + E	3.19	0.53	1.33	1.75	1.45	2.09	2.98	1004	3.03	0.53	1.34	1.81	1.50	2.15	2.97	1025

- Note: 1. Safety factor is relative to the *mean lifetime peak combined load effect*.
 2. Strength factor is relative to the *mean resistance*.
 3. Load factors are relative to *mean instantaneous loads*.
 4. Required mean resistance is obtained as $\bar{\gamma}_{s_T}$ or as $\frac{1}{\bar{\phi}} \sum \delta_i s_i$

As indicated earlier, the safety factor formulated in Equation 5-15 or 5-16 refers to the mean value of lifetime peak *combined* load effect, $\bar{s}_T$. In current procedures this factor is applied on the *sum* of design loads, i.e., $\sum_j \bar{d}_j + \sum_i \bar{s}_{iT}$, where $\bar{s}_{iT}$ is the mean lifetime peak of S_i . The safety factor comparable with current procedures, denoted by $\bar{\gamma}^*$, may therefore be obtained as

$$\bar{\gamma}^* = \bar{\gamma} \frac{\bar{s}_T}{\sum_j \bar{d}_j + \sum_i \bar{s}_{iT}} \quad (5-28)$$

Values of $\bar{\gamma}^*$ evaluated for the example under consideration are tabulated in Table 5-5 for formulations based on Z_1 and Z_2 and for $T = 50$ years. These results are discussed in the next section.

TABLE 5-5. RELIABILITY-BASED SAFETY FACTORS $\bar{\gamma}^*$
COMPARABLE WITH CURRENT PROCEDURES

$T = 50$ years

Load Combination	$Z_1 = R - S_T$		$Z_2 = R/S_T$	
	$\beta_1 = 3.72$	$\beta_1 = 4.27$	$\beta_2 = 3.72$	$\beta_2 = 4.27$
$D + L_s + T_o$	2.60	3.30	2.08	2.35
$D + L_s + T_o + L_e$	2.35	2.96	1.89	2.13
$D + L_s + T_o + E$	2.60	3.33	2.05	2.34
$D + L_s + T_o + L_e + E$	1.98	2.55	1.57	1.81

Note: These safety factors are relative to the *sum of mean lifetime peak* load effects and the *mean* resistance.

It was also indicated before that load factors formulated in Equations 5-23 to 5-26 refer to the mean instantaneous load values. Load factors comparable with current procedures, denoted by $\bar{\delta}_i^*$, may therefore be obtained from

$$\bar{\delta}_i^* = \bar{\delta}_i \frac{\bar{s}_i}{\bar{s}_{iT}} \quad (5-29)$$

Results for this set of load factors for $T = 50$ years are shown in Table 5-6. A discussion of these results is provided in the next section.

Shown in Table 5-6 is also a *total load factor* for each combination. This factor, which has not been previously introduced in this report, relates to the total load in each combination. Consistent with the factors introduced earlier, the central total load factor $\bar{\zeta}$ is used in the design equation

$$\bar{\phi} \bar{R} \geq \bar{\zeta} \bar{s}_T \quad (5-30)$$

It is observed that

$$\bar{\zeta} = \bar{\phi} \bar{\gamma} \quad (5-31)$$

In particular, for comparison with current procedures, we obtain

$$\bar{\zeta}^* = \bar{\phi} \bar{\gamma}^* \quad (5-32)$$

The values of $\bar{\zeta}^*$ as evaluated for $\beta_1 = \beta_2 = 3.72$ and $T = 50$ years are shown in Table 5-6. These results are discussed in the next section.

TABLE 5-6. RELIABILITY-BASED STRENGTH AND LOAD FACTORS COMPARABLE WITH CURRENT PROCEDURES

$T = 50$ years

(a) $Z_1 = R - S_T$

Load Combination	$\beta_1 = 3.72$							$\beta_1 = 4.27$						
	$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}^*$	$\bar{\delta}_{T_o}^*$	$\bar{\delta}_{L_e}^*$	$\bar{\delta}_E^*$	$\bar{\zeta}^*$	$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}^*$	$\bar{\delta}_{T_o}^*$	$\bar{\delta}_{L_e}^*$	$\bar{\delta}_E^*$	$\bar{\zeta}^*$
$D + L_s + T_o$	0.44	1.36	1.15	0.91	--	--	1.14	0.36	1.41	1.18	0.93	--	--	1.19
$D + L_s + T_o + L_e$	0.44	1.30	1.18	0.93	0.86	--	1.03	0.36	1.34	1.20	0.94	0.88	--	1.07
$D + L_s + T_o + E$	0.44	1.30	1.20	0.94	--	1.13	1.14	0.36	1.35	1.25	0.97	--	1.18	1.20
$D + L_s + T_o + L_e + E$	0.44	1.27	0.96	0.79	0.67	0.82	0.87	0.36	1.31	1.00	0.82	0.71	0.88	0.92

(b) $Z_2 = R/S_T$

Load Combination	$\beta_2 = 3.72$							$\beta_2 = 4.27$						
	$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}^*$	$\bar{\delta}_{T_o}^*$	$\bar{\delta}_{L_e}^*$	$\bar{\delta}_E^*$	$\bar{\zeta}^*$	$\bar{\phi}$	$\bar{\delta}_D$	$\bar{\delta}_{L_s}^*$	$\bar{\delta}_{T_o}^*$	$\bar{\delta}_{L_e}^*$	$\bar{\delta}_E^*$	$\bar{\zeta}^*$
$D + L_s + T_o$	0.57	1.43	1.17	0.92	--	--	1.19	0.53	1.51	1.21	0.94	--	--	1.25
$D + L_s + T_o + L_e$	0.57	1.35	1.19	0.94	0.87	--	1.08	0.53	1.41	1.23	0.96	0.90	--	1.13
$D + L_s + T_o + E$	0.57	1.32	1.23	0.95	--	1.19	1.17	0.53	1.41	1.32	1.00	--	1.33	1.24
$D + L_s + T_o + L_e + E$	0.57	1.29	1.02	0.81	0.74	0.99	0.89	0.53	1.34	1.08	0.85	0.80	1.11	0.96

Note: 1. Load factors are relative to *mean lifetime peak loads*.

2. Strength factor is relative to *mean resistance*.

3. The total load factor $\bar{\zeta}^*$ is relative to the *sum of mean lifetime peak loads*.

5.4.2 DISCUSSION OF NUMERICAL RESULTS

It is worthwhile to note a number of observations in Tables 5-3 to 5-6. These observations will be discussed for a number of items that are significant to design.

Significance of Distribution. It is observed that the results for the two formulations based on Z_1 and Z_2 are generally different. This indicates that for the levels of risk considered (i.e., 10^{-4} and 10^{-5}), the reliability-based safety, load or strength factors are not insensitive to the distributions of R and S_T . In particular, the results based on Z_1 would be accurate if the underlying distributions were normal, whereas the results based on Z_2 would be accurate if they were lognormal. It is generally believed, however, that the lognormal distribution is more realistic for loads and resistances (Ang and Cornell, 1974).^{*} Based on this then, the values corresponding to the formulation on Z_2 are considered to be more realistic. It is important to realize that regardless of the sensitivity to distribution, either of the formulations developed in this report can be useful and realistic when interpreted and used in a *relative* sense.

Acceptable Risk. The significance of the acceptable risk level can be observed by comparing the results for risk levels of 10^{-4} and 10^{-5} . It is observed that an order of magnitude reduction in the risk level results in larger safety and load factors and in smaller strength factors. These variations due to risk are found to be quite significant. As an example, an increase of about 14% in the safety factors based on Z_2 result from the said reduction in the acceptable risk (see Table 5-5).

^{*}The normal distribution is not realistic since it allows both positive and negative values of the variate, whereas loads and resistances can only have positive values. The safety and strength factors are particularly influenced by this inconsistency.

Structure Life. The significance of structure lifetime can be observed by comparing the load factors in Tables 5-3 and 5-4, which refer to the same values of loads (i.e., the instantaneous values), for $T = 25$ and 50 years. It appears that the structure lifetime has an appreciable influence on the factors for stochastic loads, whereas it has practically no significance on dead load factors. The increase in stochastic load factors for going from $T = 25$ to 50 years is estimated to be about 3% to 4% for the formulation based on Z_2 .

Combination of Loads. It is observed that safety and load factors for various combinations are generally different. These variations are related to the number of loads that are combined, and to the stochastic nature of loads and their combinations. These variations are of particular interest for the factors shown in Tables 5-5 and 5-6, which are comparable with current procedures. It is apparent from these tables that variations in safety or load factors from one combination to another can be significant, particularly when transient loads are combined. As an item of special interest, observe the reduction in safety and load factors in going from the first three combinations to the last combination, which includes the extraordinary live load, L_e , as well as the earthquake load, E . It is found that with respect to the first or the third combination, the reduction in the safety or the total load factor is about 23%, whereas with respect to the second combination this reduction is about 15%. It is recalled that current procedures would normally allow a fixed 25% reduction in these factors.

The reduction in individual load factors varies from one load to another. This variation is due to the characteristics of each load as well as the assumed procedure for separating the effects of the loads through the factors α_s . Reduction in factors of stochastic loads are observed to range from 10% to 27%. A significant observation here is that the dead load factor exhibits only a reduction of less than 5%. This reduction can only be attributed to the increased number of loads, since the dead load factor as given in Equation 5-23 or 5-25 is virtually independent of p and q .

These observations contradict the current procedure which permits a fixed reduction of 25% in factors of all loads including the dead load.

Comparison with Current Factors. The results shown in Tables 5-5 and 5-6 appear to be significantly different from factors used in current procedures. To avoid possible misinterpretation of the results, it is necessary to emphasize that these factors refer to the *mean* values of the resistance and the lifetime peak load. It is believed that values of resistance estimated in current design are generally conservative and well below the mean. The values of design loads may also be different from the mean values. A proper adjustment of these factors through the method presented in Section 6.1.4 is expected to yield results which are in the neighborhood of current factors.

It should also be realized again that the results given in this report apply only to the specific example with the assumed value of $\Omega_R = 0.20$ and the acceptable risk levels of 10^{-4} and 10^{-5} . These assumptions, as well as those regarding the characteristics of individual loads, clearly influence the derived values of safety, strength, and load factors.

CHAPTER 6

ON INVESTIGATION AND DEVELOPMENT OF CURRENT DESIGN PROCEDURES

In Chapter 5 formulations of reliability-based safety, load, and strength factors were presented. It was observed that these formulations involve the effects of most significant design variables, including the required reliability, the structure life, the uncertainties associated with resistances and loads, and the variables describing stochastic behavior of loads and their combinations. The example application indicated that for the most part these formulations predict the type of variations of these factors with various design variables that one would rationally expect.

It is realized that due to the approximate nature of the underlying theory, these formulations may not give absolutely correct values of the design factors, especially at very low risk levels. In particular, the results obtained for these factors may significantly depend on the assumed form of the performance function. On a relative basis, however, these formulations are believed to give consistent results. On such grounds, therefore, they can be used as a theoretical basis for critical investigations of the current deterministic design procedures, and as a framework for the development of more rational procedures.

More specifically, the above formulations can be utilized to:

- a. Critically examine the current procedures and evaluate possible measures of improvement
- b. Systematically extrapolate design criteria for unconventional structures from the criteria established for conventional structures
- c. Develop probabilistic design procedures, including rational specification of design criteria

It must be understood that for a complete study and implementation of each of the above items, the theoretical developments in this report must be complemented by engineering experience and judgment. Such a thorough investigation of these items is not performed at this time. Continuation of studies such as the present one, however, should assist and lead the engineering profession to acquire the needed knowledge and experience in order to accomplish the above tasks.

The above three items are discussed in some detail below.

6.1 EXAMINATION OF CURRENT DESIGN FACTORS

In light of the theoretical formulation of design factors in this report, it is now possible to make a critical examination of the current procedures at least on a qualitative basis. A comparative study of design factors recommended by current methods with those formulated in this report reveals inconsistencies or deficiencies in the existing procedures that are associated with the following items.

6.1.1 RISK LEVEL

While various types of engineering structures undoubtedly require different levels of reliability, currently recommended values of the design factors (i.e., the safety or the load and strength factors) do not include any measure of the required reliability or the acceptable risk. To see the implications consider, for example, the design of a hospital versus a residential building. From a socioeconomic viewpoint, clearly the former would demand a higher level of reliability than the latter. Yet, the current practice recommends using the same set of factors for the two structures. This would obviously result in either the hospital not being reliable enough (i.e., from the society's viewpoint) or the residential building being overconservative (i.e., from economic viewpoint).

In the formulations developed in this report the required reliability was included through the reliability index β_1 or β_2 . It was found that for a higher reliability the safety and load factors should be increased while the strength factor should be decreased.

A future study aimed at improving the current procedures on this item may follow the following steps.

- a. Quantify relative measures of required reliabilities associated with various categories of structures.
- b. Using the formulations developed in this report, evaluate the variations in reliability-based design factors corresponding to the above variations in the risk level.
- c. If the variations in these factors are found to be significant, implement corresponding variations (on a relative basis) in the existing design factors for the various categories of structures.

6.1.2 STRUCTURE LIFETIME

It has been observed that because of the stochastic nature of loads acting on a structure, the lifetime T influences the reliability-based safety and load factors. It was shown that the variations in these factors are such that they require a larger resistance for the same reliability but for a longer lifetime.

Engineering structures are designed for various anticipated lifetimes. While a lifetime of 30 to 50 years is common, there are structures of temporary nature with lifetimes of as short as a few years (e.g., military structures) and structures with intended lifetimes of hundreds of years (e.g., bridges).

Current deterministic design procedures, on the other hand, do not explicitly include the significance of the lifetime. In most cases it is entirely neglected and loads, as well as, design factors are specified regardless of its anticipated value. For certain structures (e.g., dams) the lifetime is included in the evaluation of loads or other environmental effects. The design factors, however, are virtually always kept constant.

This clearly introduces another element of inconsistency in the current procedures. Through the example provided in Chapter 5, it was shown that even when the safety and load factors are applied on the lifetime peak loads (which are calculated with due consideration of the lifetime), these factors would still be affected by variations in the lifetime (see, for example, variations in the safety factors for $T = 25$ and 50 years in Table 5-3). These variations result from the stochastic nature of loads and their combinations.

A future study aimed at improving the current procedures on this item may follow the following steps:

- a. Determine anticipated lifetimes associated with various groups of structures.
- b. Using appropriate models of stochastic loads and the design factor functionals developed in this report, evaluate the variations in the reliability-based factors corresponding to the above variations in the structure lifetime.
- c. If variations in these factors are found to be significant, implement corresponding variations (on a relative basis) in the existing design factors for the various groups of structures.

6.1.3 VARIABILITIES IN LOADS AND RESISTANCES

The significance of variabilities in loads and resistances are invariably recognized. These variabilities, however, are included in existing deterministic procedures in very crude and subjective manner. Both the WSD and the LFT methods attempt to account for these variabilities through purposely overestimated loads and underestimated resistances (i.e., with respect to their expected or most probable values). In addition, a reserved resistance is provided through applying a safety factor (greater than one)

in the WSD method and a strength and load factors (the former is less than one and the latter are greater than one) in the LFD method. These factors have unfortunately been developed with no consistent consideration of the underlying actual variabilities in loads and resistances, and are essentially subjective.

On this particular item the WSD method is believed to be even less desirable than the LFD method, since it provides no means for accounting for differences in uncertainties of various loads. By using larger load factors for less predictable loads, the LFD method subjectively accounts for such differences. One should note, however, that the same results could be achieved in the WSD method by using more conservative design values for less predictable loads.

In order to improve the existing design procedures on this vital item, it is necessary that variabilities in loads and resistances be explicitly included in the design process. For convenience, the coefficients of variation of loads and resistances may be used as measures of the corresponding variabilities. The formulations developed in this report should form the framework for establishing appropriate relations of the design factors to the coefficients of variation of loads and resistances.

6.1.4 NOMINAL LOADS AND RESISTANCES

In the second-moment approach presented in this report, the *mean* values of loads and resistances were used to specify the design equation. The corresponding coefficients of variation, representing measures of the underlying variabilities, determined the safety or load factors such that a desired reliability was achieved.

The values of design loads and resistances used in current procedures, on the other hand, are not consistently defined. Generally, nominal values are used which are either specified by codes or are obtained following code specified procedures. The basis for these values, or their relation

with respect to the underlying distributions of actual loads and resistances, in most instances, remain indetermined.

While it is recognized that *any* appropriate set of design loads and resistances may be used (i.e., by appropriately adjusting the corresponding safety, strength, or load factors), it is essential that the assumed procedure remain consistent for all loads and resistances under all circumstances. Such a consistent procedure may be obtained by specifying nominal loads and resistances in terms of their mean values. Expressions that have often been proposed are (e.g., Ang and Cornell, 1974)

$$X' = \bar{X}(1 + k_X \Omega_X) \quad (6-1)$$

and

$$R' = \bar{R}(1 - k_R \Omega_R) \quad (6-2)$$

where X' and R' are the nominal load and resistance and $\bar{X}$, $\bar{R}$, and Ω_X , Ω_R are the corresponding mean and coefficient of variation values. The constants k_X and k_R denote the number of standard deviations between the nominal and the mean values. Through these constants it is hence possible to specify consistent measures of nominal loads and resistances.

Consistent with the above definition of nominal loads and resistances, a set of safety, load and strength factors may be introduced which are applicable on these values. These factors may be obtained in terms of the corresponding central factors and the constants k_R and k_X . For example, the strength and load factors for the nominal resistance, R' , and the nominal load, X' , are

$$\begin{aligned} \phi &= \bar{\phi} \frac{\bar{R}}{R'} \\ &= \frac{\bar{\phi}}{1 - k_R \Omega_R} \end{aligned} \quad (6-3)$$

and

$$\begin{aligned}\delta &= \bar{\delta} \frac{\bar{X}}{X^*} \\ &= \frac{\bar{\delta}}{1 + k_X \Omega_X}\end{aligned}\tag{6-4}$$

One should note that the selection of k_X and k_R , i.e., the selection of the nominal design load and resistance, should have no influence on the outcome of a design. Their selection should, therefore, be based on convenience only. In particular, a convenient choice might be $k_X = k_R = 0$; i.e., nominal loads and resistances equal to the mean values.

From the above discussion it should now be obvious that a direct comparison (i.e., on an absolute basis) of the existing factors with those obtained in this report could be achieved if the nominal loads and resistances in the current procedures are specified as shown in Equations 6-1 and 6-2.

6.1.5 COMBINATION OF LOADS

Variations of reliability based safety and load factors from one combination to another, as observed in the example of Chapter 5, is attributed to two items:

1. Number of loads in a combination
2. Stochastic fluctuations of loads

It was observed that with increasing number of loads in a combination, the individual load factors tend to decrease. This was apparent through the decoupling factors α_S which decreased with increasing number of loads (see Table 3-3). This behavior results from the fact that with increasing number of loads the probability of all loads having unusually large values

is smaller. This effect is entirely absent in the existing procedures, which use the same load factors regardless of the number of loads combined. It is believed that this problem would especially be critical in design of nuclear power plants, where many independent loads are combined to design for the safety of containment structures. Based on the framework formulated in this report, an appropriate procedure may be developed to improve the existing methods on this item.

The second item above, concerning the stochastic variabilities of loads and the resulting effects on their combinations, is a very complex problem. In the formulation developed in this report, these effects were included through the second moment descriptor functions p and q . These include the most significant parameters that may define stochastic loads, i.e., the occurrence rate, the duration, and the form of the instantaneous distribution.

It was observed in the example of Chapter 5 that the stochastic variabilities of loads have highly significant effects on the load factors in various combinations. In particular, it was observed that when two transient loads were included in a combination, the load factors for the same set of (stochastic) loads were substantially smaller than in combinations involving only one or no transient loads. Meanwhile, the dead load factor remained unaffected (except for a small reduction due to the increase in number of loads). It is obvious that these reductions in load factors are related to the above mentioned parameters pertaining to the stochastic loads in the combination and, therefore, would generally vary from one situation to another.

It can perhaps be stated that the current procedures are most inadequate when it comes to the combination of stochastic loads. The treatment is limited to

- a. Exclusion of certain combinations which may have large combined effects but are thought to be very unlikely, i.e., the

probability of occurrence is small. A good example is the combination of earthquake and wind loads which is almost never considered.

- b. Standard reduction of load factors (or increase of allowable stresses, in WSD) for combinations involving one or more transient loads. An example is the combination of live loads with earthquake or wind loads, wherein a reduction of 25% is specified.

The current practice, as described above, has unfortunately been established on the basis of experience and engineering judgment only. No theoretical studies have been conducted to show, for example, that the combination of earthquake and wind loads is insignificant. Although this may intuitively be obvious for certain groups of structures, for others which demand very high levels of reliability (e.g., nuclear power plants), it may not be obvious. A proper investigation is, therefore, required before such procedures are justified.

The presently specified reduction in load factors is also subjective. While it is expected that the proper reductions depend on the various parameters of the individual loads in a combination, a standard reduction is currently specified with no consideration of the respective load occurrence rates or the corresponding durations. The adequacy of this procedure should also be investigated, especially with regard to design of critical structures such as nuclear power plants.

The theoretical development in this report should form the framework for the above investigations. It may be used to justify the current specifications for a certain class of structures or load combinations; or, it may be used to develop improved procedures as necessary. It is obvious that thorough studies would be required before such vital problems are resolved.

An improvement on the load factor reduction procedure that can be recommended at this time, is as follows. It was observed in Equation 5-23 or 5-25 that the dead load factor is independent of the parameters p and q .^{*} This means that it is independent of combination, except for the *number* of loads involved. In fact, a study of the dead load factor in Tables 5-3 and 5-4 reveals that this factor remains insensitive to the type of loads involved in each combination. This demonstrates, then, that the reduction in load factors should *not* be applied to the dead load factor.

Current procedures, on the other hand, specify a blanket reduction of load factors, including that of the dead load. It is felt that this inconsistency can be avoided by specifying that load factor reductions do not apply to dead loads. Instead of this, a somewhat smaller adjustment of the dead load factor based on the number of loads in a combination may be appropriately specified.

6.2 DESIGN FACTORS FOR UNCONVENTIONAL STRUCTURES

As indicated before, the current procedures and specifications of design have evolved through the years primarily on the basis of engineering experience and intuitive judgment. Because of the relatively long period of experience with design, construction, and inspection of ordinary structures (e.g., residential or office buildings), one may conclude that in fact through a trial and error process we have converged to an optimum set of specifications for such structures (i.e., optimum in the sense that the current procedures produce results which are acceptable from the safety as well as the economic points of view). There is always need, however, to design structures which are markedly different from the ordinary ones, i.e., unconventional structures. These may include structures which require unprecedented levels of reliability (e.g., nuclear power plants), or structures which should resist

^{*}Note that the dead load factor is actually related to q through the parameter α_{S1} . This relationship, however, is invariably insignificant.

unusual environmental conditions (e.g., certain military structures). Since the experience of engineers with these types of structures has been very limited, the engineering intuitive judgment is more prone to errors. Scientific methods are, therefore, needed to guide and direct such judgments.

The theoretical formulation of safety and load factors developed in this report, could particularly be useful in improving or developing the design criteria for unconventional structures. It may form a framework based on which the experience gained for ordinary structures can be extended and applied for unconventional structures.

For example, consider the development of design specifications for nuclear power plants. Assuming that current design specifications of ordinary structures are adequate, the formulations developed in this report can be used to estimate needed increases in the safety or load factors in order to meet the higher reliability that is required for these structures.

6.3 DEVELOPMENT OF PROBABILISTIC DESIGN PROCEDURES

In the above sections, the significance of the present study for critical examinations and further developments of the current deterministic design procedures were discussed. While such developments are urgently needed at this time, the real value of this and similar studies will in the long run be in their contribution towards the eventual formulation of a probabilistic design procedure involving rational criteria of safety and serviceability.

Towards that end, the major contributions of the present study are believed to be in the method developed for the evaluation of extreme combined loads, and in the formulations of safety and load factors for the case of multiple load combinations.

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APPENDIX A

DERIVATION OF p AND q FUNCTIONS FOR GAUSSIAN PROCESS

Let S be a stationary Gaussian process with mean $\bar{s}$ and standard deviation σ_S . Let $\dot{S}$ denote the time derivative process of S , i.e., $\dot{S} = (d/dt) S$. It can be shown (Lin, 1967) that the mean upcrossing rate of S from a level s , i.e., the mean number of times per unit time that S crosses the level s from below, is

$$v_s = \frac{\sigma_{\dot{S}}}{2\pi\sigma_S} \exp\left[-\frac{1}{2} \frac{(s - \bar{s})^2}{\sigma_S^2}\right] \quad (\text{A-1})$$

where $\sigma_{\dot{S}}$ is the standard deviation of $\dot{S}$. In particular, the mean upcrossing rate from the mean level $\bar{s}$, denoted by v , is

$$v = \frac{\sigma_{\dot{S}}}{2\pi\sigma_S} \quad (\text{A-2})$$

Equation A-1 can therefore be written as

$$v_s = v \exp\left[-\frac{1}{2} \frac{(s - \bar{s})^2}{\sigma_S^2}\right] \quad (\text{A-3})$$

The value of v can be obtained either experimentally using actual realizations of S , or analytically using the power spectral density of the process (see Lin, 1967).

If the upcrossings from a level s are modeled as a Poisson process, the probability of having n upcrossings during time T becomes

$$P(n \text{ upcrossings}) = (v_s T)^n \frac{e^{-v_s T}}{n!} \quad (\text{A-4})$$

Now, note that in order for the extreme value of S during T , i.e., S_T , to be smaller than s , it is necessary that S be smaller than s at time zero and that there be no upcrossings from level s during the time T . Thus, the cumulative distribution of S_T is obtained as,

$$\begin{aligned}
 F_{S_T}(s) &= P(S_T < s) \\
 &= P(S < s)_{t=0} P(\text{no upcrossings from level } s)_{0 \rightarrow T} \\
 &= F_S(s) \exp(-v_s T)
 \end{aligned} \tag{A-5}$$

where

$$F_S(s) = \int_{-\infty}^s \frac{1}{\sqrt{2\pi} \sigma_S} \exp\left[-\frac{1}{2} \frac{(s - \bar{s})^2}{\sigma_S^2}\right] ds$$

Using Equation A-3, this becomes

$$F_{S_T}(s) = F_S(s) \exp\left\{-vT \exp\left[-\frac{1}{2} \frac{(s - \bar{s})^2}{\sigma_S^2}\right]\right\} \tag{A-6}$$

Consider the normalized variate $U = (S_T - \bar{s})/\sigma_S$.

The cumulative distribution function is

$$\begin{aligned}
 F_U(u) &= F_{S_T}(\bar{s} + u\sigma_S) \\
 &= \left[\int_{-\infty}^u \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du \right] \exp\left[-vT \exp\left(-\frac{u^2}{2}\right)\right]
 \end{aligned} \tag{A-7}$$

The corresponding density function is

$$\begin{aligned}
 f_U(u) &= \left[\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) + \nu T u \exp\left(-\frac{u^2}{2}\right) \int_{-\infty}^u \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du \right] \exp\left[-\nu T \exp\left(-\frac{u^2}{2}\right)\right] \\
 &= \left[\frac{1}{\sqrt{2\pi}} + \nu T u \Phi(u) \right] \exp\left[-\frac{u^2}{2} - \nu T \exp\left(-\frac{u^2}{2}\right)\right] \quad (A-8)
 \end{aligned}$$

where

$$\Phi(u) = \int_{-\infty}^u \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du$$

is the standard normal probability.

The second-moment descriptor functions p and q can now be obtained as

$$\begin{aligned}
 p &= E[U] \\
 &= \int_{-\infty}^{\infty} u f_U(u) du \quad (A-9)
 \end{aligned}$$

and

$$\begin{aligned}
 q &= \left\{ \text{VAR}[U] \right\}^{1/2} \\
 &= \left[\int_{-\infty}^{\infty} u^2 f_U(u) du - p^2 \right]^{1/2} \quad (A-10)
 \end{aligned}$$

APPENDIX B

EXTREME VALUE DISTRIBUTION FOR FILTERED POISSON PROCESS

Consider a filtered Poisson process with mean number of occurrences ν . Let $F_S(s)$ denote the distribution of intensity at each occurrence. For a general treatment, assume that negative as well as positive values of S are possible.

Let n denote the random number of occurrences during time T . On the basis of the total probability theory, the cumulative distribution of the extreme of S , denoted as S_T , is obtained as

$$\begin{aligned} F_{S_T}(s) &= P(S_T < s) \\ &= \sum_{n=0}^{\infty} P(n \text{ occurrences}) P(S < s \text{ in } n \text{ occurrences}) \end{aligned} \quad (\text{B-1})$$

where

$$P(n \text{ occurrence}) = \frac{(\nu T)^n \exp(-\nu T)}{n!} \quad (\text{B-2})$$

and

$$\begin{aligned} P(S < s \text{ in } n \text{ occurrence}) &= [F_S(s)]^n; \text{ for } n = 1, 2, 3 \dots \\ &= \begin{cases} 0.0 & s < 0 \\ 1.0 & s > 0 \end{cases}; \text{ for } n = 0 \end{aligned} \quad (\text{B-3})$$

The above equation neglects the overlapping of occurrences. Using Equations B-2 and B-3, Equation B-1 can be written as

$$\begin{aligned}
 F_{S_T}(s) &= \exp(-vT) H(s) + \sum_{n=1}^{\infty} \frac{(vT)^n \exp(-vT)}{n!} [F_S(s)]^n \\
 &= \exp(-vT) H(s) + \exp(-vT) \sum_{n=1}^{\infty} \frac{[vT F_S(s)]^n}{n!} \\
 &= \exp(-vT) H(s) + \exp(-vT) \exp[vT F_S(s)] - \exp(-vT) \\
 &= \exp\{-vT[1 - F_S(s)]\} + [H(s) - 1] \exp(-vT)
 \end{aligned} \tag{B-4}$$

where $H(s)$ is the unit step function, i.e.,

$$H(s) = \begin{cases} 1.0 & \text{for } s > 0 \\ 0.0 & \text{for } s < 0 \end{cases} \tag{B-5}$$

APPENDIX C

DERIVATION OF EQUATION 4-32

Let $S_1, S_2, \dots, S_n$ be stationary independent Gaussian processes. It is well known (e.g., Lin, 1967) that their sum

$$S = S_1 + S_2 + \dots + S_n \quad (C-1)$$

is also a stationary Gaussian process. The instantaneous mean and standard deviation of S are obtained as

$$\bar{s} = \bar{s}_1 + \bar{s}_2 + \dots + \bar{s}_n \quad (C-2)$$

and

$$\sigma_S = \sqrt{\sigma_{S_1}^2 + \sigma_{S_2}^2 + \dots + \sigma_{S_n}^2} \quad (C-3)$$

where $\bar{s}_i$ and σ_{S_i} are the mean and standard deviation of S_i .

The derivative process $\dot{S}$ is obtained as

$$\dot{S} = \frac{d}{dt} S \quad (C-4)$$

$$= \dot{S}_1 + \dot{S}_2 + \dots + \dot{S}_n \quad (C-4)$$

Therefore,

$$\sigma_{\dot{S}} = \sqrt{\sigma_{\dot{S}_1}^2 + \sigma_{\dot{S}_2}^2 + \dots + \sigma_{\dot{S}_n}^2} \quad (C-5)$$

where $\sigma_{\dot{S}_i}$ can be expressed in terms of the mean upcrossing rate from level $\bar{s}_i$, denoted by ν_i , as

$$\sigma_{\dot{S}_i} = 2\pi \nu_i \sigma_{S_i} \quad (C-6)$$

Thus

$$\sigma_{\dot{S}} = 2\pi \sqrt{v_1^2 \sigma_{S_1}^2 + \dots + v_n^2 \sigma_{S_n}^2} \quad (C-7)$$

Using Equation A-3 of Appendix A the mean upcrossing rate of S becomes

$$v = \sqrt{\frac{\sum_i v_i^2 \sigma_{S_i}^2}{\sum_i \sigma_{S_i}^2}} \quad (C-8)$$