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MASTER
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THROUGH A PLASMA

BY

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Propagation of Cosmic Rays Through a Plasma

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ABSTRACT

The theory of the coupling of energetic particles to a plasma by Alfvén wave instabilities is described. The theory is applied to the problem of propagation of cosmic rays through the interstellar plasma and to the problem of trapping of cosmic rays in an expanding bubble of plasma produced by a supernova explosion.

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PROPAGATION OF COSMIC RAYS THROUGH A PLASMA

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Whenever one has a situation where both energetic particles and background magnetized plasma are present, one is faced with the possibility of unstable waves that strongly couple them together. In particular, if the random speed of the energetic particles is much larger than the Alfvén speed then the most important waves are hydromagnetic waves with frequencies much below the ion-cyclotron frequency. One can give an approximate growth rate for waves of wave number k and whose wave number component along B is $k_{||}$:

$$\gamma = \frac{\pi C \Omega_0}{4} \frac{n_E(p > p_0)}{n^*} \frac{c}{v_A} (\delta - \delta_c) \quad (1)$$

where $\Omega_0 = eB/mc$ is the cyclotron frequency, $n_E(p > p_0)$ is the number density of energetic particles whose momentum exceeds $p_0 \equiv eB/c k_{||}$, n^* is the background ionized plasma density, v_A the Alfvén speed associated with this plasma, δ the anisotropy of the energetic particle distribution whose definition will be given below, and δ_c a critical value for this anisotropy usually of order v_A/c (Kulsrud and Pearce 1969). C is a number of order unity that depends on the momentum distribution of the energetic particles and is independent of p_0 for a power law distribution.

The mechanism of the instability is simple to understand. The energetic particles moving rapidly along the magnetic field see the low frequency wave at a much higher frequency. If this Doppler shifted frequency is comparable to the relativistic gyrofrequency of the particle Ω , there will be a resonant exchange of energy. This condition is

$$\omega - k_{||} v_{||} = \pm \Omega \quad (2)$$

where ω is the wave frequency, and $v_{||} = v \cos \theta$, is the component of the velocity parallel to the magnetic field. Since normally $\omega \ll \Omega$, we have resonance if $k_{||} = \Omega/(v \cos \theta)$, or if the wavelength of the wave is comparable to the gyroradius of the particle.

The net exchange of energy between the wave and all resonance particles depends on the distribution of particles. If they are isotropic (in the plasma frame), the waves are damped. However even a small anisotropy can lead to a

growth of the waves.

Let us picture the interaction in momentum space. Consider a wave propagating in the direction of B . The resonant surfaces of equation (2) are the $(\pm)$ planes indicated in projection in Figure 1. (Waves propagating parallel to

Figure 1

B are circularly polarized and interact only with particles on a single one of these planes. Waves propagating at a finite angle are linearly polarized and interact with particles on both planes.) A wave-particle interaction leads to a small displacement along the line W' , a part of a circle centered at O' , a point a distance $\gamma m v_A$ from the origin. This line should be compared with the line W a part of a circle centered at the origin. A displacement to the right means a gain in energy of the particle, that to the left a loss. The sense of the displacement depends on the relative phase of the interacting particle and wave, either direction being equally likely. However, if the momentum distribution f of the particles increases to the right along W' it is intuitively evident that the preponderance of the interactions will lead to energy flowing to the wave, and thus to wave growth, and vice versa. Thus, for an isotropic distribution f decreasing with increasing momentum, the contrary case of wave damping applies since W are contour curves for f . If the particles have a streaming velocity $v_D > v_A$, the constant f contours are W'' , parts of circles centered at O'' , and f increases to the right along W' so the net energy flow is from the particles to the waves. Symmetrical results apply on the arcs V , V' , and V'' . These results hold for both linear and circularly polarized waves. Finally, if the anisotropy is a symmetric " $p_{\perp} - p_{\parallel}$ ", one, with more energy in the parallel direction, then the constant f contours are like V on the $+$ plane and W'' on the $-$ plane. For this case, only right circularly polarized waves interacting on the $-$ plane are unstable. Left-propagating, left-circularly polarized waves are also unstable.

It is now clear why only particles with momentum greater than p_0 interact

with waves with projected wave numbers $k_{||}$. Further, because the number of energetic particles falls off with increasing momentum p the principle interaction is with particles of momentum about p_0 . Conversely, particles on a sphere of constant p interact with waves for which $k_{||} > eB/pc$ but again the preponderance of interactions are with waves with $k_{||} \approx eB/pc$, or for waves for which $p_0 \approx p$. Thus, there is a rough one-to-one correspondence between particles of a given momentum and waves with a given projected wave number. The exchange of energy and momentum leads to a growth or damping of the wave and a diffusion of the particle in momentum space. If $v_A \ll v$, the velocity of the resonant particle, the diffusion is primarily in pitch angle.

Equation (1) represents the effect of the particles on the waves. The effect of the waves on the particles is a pitch angle diffusion which scatters the particles reversing their direction and leading to a mean free path of order

$$\ell = r_L \frac{B^2}{\langle \delta B^2 \rangle_{k_{||}}} \quad (2)$$

where $\langle \delta B^2 \rangle_{k_{||}}$ is the fluctuation "energy" in waves with $k_{||}$ within a factor two of $k_{||}$, and $r_L = v/\Omega$ is the relativistic gyroradius of the particle. Because r_L is normally very small, equation (3) represents a very strong coupling between the energetic particles and the plasma even at small wave amplitudes. In any given unstable situation the waves will grow according to equation (1) until $\langle \delta B^2 \rangle$ is large enough that the mean free path ℓ according to equation (3) is short enough to reduce the anisotropy δ below δ_c . A steady-state situation is thus rapidly reached with $\langle \delta B^2 \rangle_k$ determined by this value of ℓ and the anisotropy δ of the particles equal δ_c .

Let us consider two such examples:

Propagation of Cosmic Rays Through the Interstellar Medium.

From a study of the observed composition it may be concluded that the cosmic rays spend an average time of order three million years in the galactic disk. Thus, they must be continually produced and leave the disk with a streaming velocity v_D along the galactic magnetic field consistent with this time. The actual mode of escape from the disk is uncertain, however, because the topology of the magnetic field is not known. In any event the streaming velocity of the cosmic rays through the background interstellar plasma leads to a build up of Alfvén waves with wave scattering of the cosmic rays. In evaluating the growth rate by equation (1) one must replace δ by v_D/c , so the relevant formula for γ is

$$\gamma = \frac{\pi C \Omega_0}{4} \frac{n_E (p > p_0)}{n^*} \frac{v_D - v_A}{v_A} \quad (4)$$

For a value for the interstellar magnetic field of 3×10^{-6} gauss, the wavelength of the resonantly unstable waves is, for protons, $k_{||}^{-1} = 10^{12} \epsilon$ cm where ϵ is the energy in BeV. For such a short wavelength one must take into account the partially ionized nature of the interstellar medium. The force of the perturbed magnetic field is exerted only on the ionized component of the medium. If the wave period is short compared to the ion-neutral collision frequency, the ionized component oscillates essentially independently of the neutral component. The occasional ion-neutral charge-exchange collisions act to damp the wave. The propagation speed of the waves is thus $v_A = B/(4\pi n^*)^{1/2}$ where n^* is the density of the charged component, the damping by ion-neutral collisions is (Kulsrud and Cesarsky 1971)

$$-\gamma^* = \frac{n_0}{2} \langle \sigma v \rangle = 3 \times 10^{-9} n_0 s^{-1} \quad (5)$$

where n_0 is the neutral density, and n must be replaced by n^* in v_A .

Since Alfvén waves are amplified by the cosmic rays according to equation (4) and damped according to equation (5) we get the relative streaming velocity for equilibrium by equating these two expressions. We see that propagation in clouds is rapid since γ^* is very large while γ is small. In the normal partially ionized intercloud medium we assume $n_0 = .1 \text{ cm}^{-3}$, $n^* = .03 \text{ cm}^{-3}$ to obtain an Alfvén speed of 40 km/s and

$$\frac{v_D}{v_A} = 1 + 2 \epsilon^{1.5} \quad (6)$$

indicating a very strong dependence of v_D on ϵ in the intercloud medium. (We may ignore the clouds since they occupy only 10% by volume of the interstellar region.) This strong dependence of v_D on particle energy arises because the damping rate of waves is independent of wavelength while the growth depends on the number of resonant particles, and there are very few high energy particles. The observed dependence of age on energy (and thus of v_D on energy) is much weaker $v_D \propto \epsilon^{.2}$ instead of $\epsilon^{1.5}$.

A possible resolution of this problem presented itself with the discovery that large regions of the interstellar medium are very hot and of low density, so that neutrals are absent entirely (Jenkins 1978). Therefore, the only remaining damping is nonlinear and the cosmic rays cannot drift faster than

perhaps 1.5 times the Alfvén speed (Wentzel, 1969). In these regions the magnetic field strength is weaker by a factor ϵ so that the drift velocity must be less than or equal to $1.5 x v_A$. Along a tube of force the cosmic ray flux divided by the magnetic flux $n_E v_D / B$ must be constant. We assume $n^* = .001$ in the hot regions and find that the mean streaming velocity, in the cold regions, is

$$v_D = \min \left[60 \left(\frac{.1}{.001} \right)^{1/2}, 40 + 80 \epsilon^{1.5} \right] \quad (7)$$

(If the second number in the brackets is smaller, the cosmic rays are slowed down by wave scattering in the cold regions. This reduces the net streaming velocity in the hot regions below the critical value and there is no scattering in the hot regions.)

At 3 BeV, $v_D = 450$ km/sec, while at 100 BeV, $v_D = 600$ km/sec, values consistent with an energy dependence ϵ^{-1} of the lifetime. The lifetime is given by AL/v_D where L is the half length of the line of force in the disk and A is a number of order 0.1 which takes into account the details of the diffusion process in the presence of wave scattering. Taking a lifetime of 3×10^6 yrs. at 3 BeV, we find $L = 4.5$ kpc. The mean free path in the cool regions ℓ is 30 pc. and the corresponding amplitude of the waves is given by $B/B_0 = (r_L/\ell)^{1/2} = 10^{-3.5}$. The smallness of the wave amplitude indicates the efficiency of the wave scattering process. These numbers are for illustrative purposes only; the actual state of the interstellar medium is too uncertain to take them more seriously.

Cosmic Rays from Supernova

Supernova have been seriously considered to be the site for the origin of cosmic rays (Ginzburg and Syrovatskii 1964). The supernova produces a hot shock-heated rapidly expanding bubble of interstellar plasma around itself and cosmic rays are presumably produced and injected into the bubble when it is still quite small. Since the interstellar plasma is magnetized, the plasma instabilities quickly set in and limit the streaming of the cosmic rays along the magnetic field. In addition the cosmic rays are locked to the plasma transverse to the field by the small gyration radius. The high cosmic ray pressure will do work on the expanding bubble of plasma giving up energy to it. Thus, each cosmic ray will lose energy adiabatically by an amount equal to the ratio of the radius at which it finally escapes, to the radius at which it first starts to be adiabatically decompressed. The initial energy can be as large as thirty times that observed finally at the earth. A 5 GeV cosmic ray had to be accelerated to 150 GeV. In addition an initial amount of energy 30

times larger than earlier thought must be supplied by the supernova to accelerating cosmic rays. This means nearly the entire energy in the supernova is used to accelerate cosmic rays. The trapping of cosmic rays around the supernova thus represents a very severe difficulty for the supernova origin of cosmic rays. (A similar problem arises in connection with injection of particles into extra-galactic radio sources.)

Let us apply equation (4) to obtain the growth rate of waves in the case where the streaming velocity is equal to the expansional velocity, u , since this is the velocity necessary for the cosmic rays to run ahead of the bubble and escape. We take $u = 3 \times 10^9 R^{3/2}$ cm/s from the Sedov solution with R the radius of the bubble in parsecs. The accepted number of cosmic rays per supernova is 10^{52} . If we take the shock heated plasma to have $n = .1 \text{ cm}^{-3}$ and $B = 3 \times 10^{-6}$ gauss, we get $\gamma = 4 \times 10^{-2} / R^{4.5} \text{ s}^{-1}$ where we assume the cosmic rays uniformly distributed in the bubble. The growth time $25R^{4.5}$ seconds is extremely short compared to the dynamical time, so that the waves grow very rapidly and lock the cosmic rays to the expanding plasma (Kulsrud and Zweibel 1975).

The exact amount of adiabatic decompression depends on the fraction of the bubble the cosmic rays occupy and this depends on the mechanism for injecting them onto the interstellar field lines in the first place. Since, if they occupy only a fraction of the bubble they will decompress faster, as $R^{5/4}$ (Chevalier 1974), we make the simplifying assumption that they occupy the entire bubble.

The initial radius at which the adiabatic decompression starts cannot be smaller than a few tenths of a parsec since a sphere of a smaller radius contains initially less mass than 10^{52} cosmic rays. If the cosmic rays were injected earlier, this mass would be accelerated to the speed of light without removing much energy from the cosmic rays. It is further unlikely that any serious adiabatic expansion occurs before the Sedov phase of the explosion sets in, since up to this time the magnetic field is not changing its field strength and no transverse energy is being lost. Thus, one could estimate the initial radius as one or two parsecs.

The final radius is that at which the bubble stop expanding or is determined by the condition that the damping of the waves overbalances the growth rate quoted above evaluated at a radius for which the cosmic ray streaming velocity is comparable to the expansional velocity.

Before discussing the various damping processes we must reexamine the properties of hydromagnetic waves under the extreme condition in the plasma bubble. The plasma is very hot so the mean free path is effectively infinite.

This means collisionless damping processes are very strong and no waves will be excited propagating in directions making a finite angle with the magnetic field. Further, the Alfvén speed is orders of magnitude smaller than the sound speed, c_s , so that most plasma particles are travelling at speeds faster than the wave. The dispersion relation for parallel-propagating waves in this limit is

$$(\omega^2 - k^2 v_A^2)^2 = \frac{1}{2} \omega^2 k^4 v_i^4 / \Omega_o^2 \quad (8)$$

where $v_i^2 = T/M$ (Foote and Kulsrud 1977). The solutions of this equation are plotted in Figure 2 and are quite different from the cold-plasma relation for Alfvén waves. For frequencies in excess of $(v_A^2/v_i^2) \Omega_o$ the right circular polarized wave propagates much faster than the Alfvén speed, but still slower than c_s . The left circularly polarized wave propagates considerably slower. Waves in this region are resonant when $v_i^2 - u^2 > v_A^2$, or during the first few parsecs of expansion. Below $(v_A^2/v_i^2) \Omega_o$ the speed is nearly v_A , but the dispersion of the waves is still substantial.

Figure 2

The damping of waves in the bubble is different from that in the interstellar medium since clearly no neutrals are present. In fact, waves propagating parallel to $\underline{B}$ suffer no linearized damping at all.

There are two possible nonlinear damping processes that apply to parallel propagating Alfvén waves. One consists of a "decay" of Alfvén waves into sound waves and lower frequency Alfvén waves (Sagdeev and Galeev 1969). If the sound speed exceeds the Alfvén speed, this process cannot operate.

The second possible nonlinear damping process is the analogue of nonlinear Landau damping of plasma oscillations (Sagdeev and Galeev 1969). Two Alfvén waves together produce a nonresonant beat wave with a variation in the transverse field strength, and thus a second order variation in the magnitude of the field. This variation produces a force on the thermal ions travelling at the

speed of the beat wave increasing their energy on the average. As a result photons of the higher frequency wave are converted to those of the lower frequency wave. (The beat wave is nonresonant because of the dispersion just discussed.) The exact calculation of this induced scattering process has not yet been done. One can estimate its rate as follows (Ott and Dum 1971, Johnston and Kulsrud 1977). The two circular polarized waves together produce a variation of the magnetic field strength of the form

$$\delta|B| = -\frac{1}{B} \frac{\delta B_1 \delta B_2}{2} \cos [(k_1 - k_2)z - (\omega_1 - \omega_2)t] \quad (9)$$

(assuming one may linearly superimpose these waves). The beat wave exerts a force $(mv_{\perp}^2/2B) (\partial|B|/\partial z)$ on an ion. By analogy with the case of plasma oscillation one finds that the rate of work done on the plasma ions resonant with the beat wave is

$$\dot{E} = \left(\frac{\pi}{2}\right) \frac{1/2 \langle v_{\perp}^4 \rangle}{8v_i^3} (k_1 - k_2) \frac{(\delta B_1)^2 (\delta B_2)^2}{4\pi B^2} \quad (10)$$

and $\langle v_{\perp}^4 \rangle$ is an average over $v_{\perp}$ for the resonant particles. This must be equated to $-E_1 (1 - k_2/k_1)$ to get the loss of energy from wave 1, $\dot{E}_1$. If we ignore the gain from shorter waves, we get the nonlinear damping rate due to wave 1.

$$-\gamma_{NL} = \left(\frac{\pi}{2}\right) \frac{1/2 \langle v_{\perp}^4 \rangle}{16v_i^4} \frac{v_i}{c} \frac{(\delta B_2)^2}{B^2} \Omega \quad (11)$$

for waves resonating with cosmic rays of cyclotron frequency Ω , where $\langle v_{\perp}^4 \rangle / v_i^4 = 8$ for a thermal distribution.

One further correction need be made. If the beat wave is too strong, particles with large $v_{\perp}^4$ will be trapped and oscillate before passing through a wave packet. From the results for Alfvén waves in a hot medium equation (8), the condition for trapping is roughly $\omega_{\text{Trap}} = (kF/m)^{1/2} < k(v_p - v_g) = kv_A (v_i^2/2v_A c)$ where v_p and v_g are the phase and group velocities. This gives the condition

$$\frac{v_{\perp}^2}{v_i^2} \frac{\delta B_1 \delta B_2}{B^2} < \frac{v_i^2}{c^2} \quad (12)$$

that the particles not be trapped. Thus, in computing the average $\langle v_{\perp}^4 \rangle$, we

should take into account this constraint. Now let us estimate v_D by cl/R with l given by equation (3), set γ from equation (4) equal to $-\gamma_{NL}$ from equation (11), and determine the radius R at which $v_D = v_1 = u = 3 \times 10^9/R^{3/2}$. Ignoring restriction (12), at first, we find $R = 90 \epsilon^{-3/7}$ pc. For $\epsilon = 1$, $u = v_1 = 3 \times 10^6$ cm/s, $\delta B/B = 5 \times 10^{-3}$. Applying equation (12) we see that thermal particles with $v_1 > .02 v_1$ are trapped. If we attempt to estimate the effect of trapped particles by replacing $\langle v_1^4 \rangle$ in equation $2v_1^2 v_1^2$ with v_1^4 given by equation (12), we find $-\gamma_{NL}$ is always less γ and the cosmic rays are never released during the Sedov phase.

In any event the release radius must be greater than or equal to $90 \epsilon^{-3/7}$ pc and the trapping of cosmic rays in the bubble presents a very serious problem to any theory for the origin of cosmic rays in the early phases of a supernova explosion. It is thus gratifying that a new theory of cosmic ray production by the strong shock propagating away from the supernova has arisen. This theory due to Bell (1978) and Blandford and Ostriker (1978) also depends on the same coupling between cosmic rays and plasmas by Alfvén wave instabilities. Its advantage is that the cosmic rays can be produced during the late stages of the supernova bubble evolution and do not suffer adiabatic expansion.

In conclusion we have described two problems in which plasma instabilities play an important role. These examples illustrate the physics which can arise in connection with these instabilities. The work on the supernova expansion was done in collaboration with Ellen Zweibel; that on Alfvén waves in a hot medium with Elizabeth Foote.

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References

- Bell, A. R. 1978, M.N.R.A.S., 182, and 147 and 182, 443.
 Blandford, R. D. and Ostriker, J. P. 1978, Ap. J., 221, L29.
 Chevalier, R. A. 1974, Ap. J., 188, 501.
 Foote, E. A. and Kulsrud, R. M. 1977, Bulletin of the American Physical Society, 22, 1072.
 Galeev, A. A. and Sagdeev, R. Z. 1969, "Nonlinear Plasma Theory," (Benjamin).
 Ginzburg, V. L. and Syrovatskii, S. I. 1964, "The Origin of Cosmic Rays," (MacMillan and Co.).
 Jenkins, E. B. 1978, Ap. J., 220, 107.
 Johnston, S. and Kulsrud, R. M. 1977, Physics of Fluids, 20, 829.
 Kulsrud, R. M. and Cesarsky, C. J. 1971, Ap. Letters, 8, 189.
 Kulsrud, R. M. and Pearce, W. A. 1969, Ap. J., 156, 445.
 Kulsrud, R. M. and Zweibel, E. G. 1975, 14th International Conference on Cosmic Rays, Munich Paper, O-9-1-12.
 Ott, C. and Dum, C. T. 1971, Physics of Fluids, 14, 959.
 Wentzel, D. G. 1969, Ap. J., 156, 303.