LA-UR -78-2341

TITLE: MEASUREMENT OF PLASMA BETA IN HIGH- β FUSION EXPERIMENTS

MASTER MASTER

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SUBMITTED TO: International School of Plasma Physics
"Course on Diagnostics for Fusion Experiments,"
Varenna, Como (Italy) September 4-16, 1978

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ENERGY RESEARCH AND
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CONTRACT W-7405-ENG. 36

UNITED STATES

MEASUREMENT OF PLASMA BETA IN HIGH- β FUSION EXPERIMENTS

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SUMMARY

The plasma beta on axis of a high- β plasma column can be determined by combining measurements of the magnetic flux excluded by the plasma column and the plasma luminosity profile. The excluded flux is measured by a balanced loop-probe arrangement consisting of a small magnetic probe outside the discharge tube, which measures the external magnetic field, and a single-turn loop around the discharge tube which senses $d\phi/dt$ inside the tube and is sensitive to the magnetic flux excluded by the plasma. With the signals from the loop and probe nulled without plasma, the integrated difference signal normalized to the integrated probe signal gives the net magnetic flux excluded by the plasma column or the effective area from which flux is completely excluded.

Self-luminosity is used to determine a relative plasma density profile. The plasma continuum radiation from as many as 20 parallel chords through the plasma is imaged on optical fiber bundles with each bundle connecting to a separate photomultiplier. Data from the multichannel luminosity apparatus is unfolded to give a radial light intensity profile which is proportional to the square of the density profile provided the plasma temperature is not a strong function of radius. The relative density profile is combined with the excluded flux data to give the plasma beta on the axis of the column. The luminosity data also provides plasma position and radius, and when combined with a single-chord density measurement gives an absolute density profile. A combination of the plasma beta, density, and external magnetic field measurements gives the total plasma temperature using pressure balance. An independent measurement of the plasma electron temperature allows the determination of the ion temperature.

The measurement of the magnetic flux excluded by the plasma column with the balanced loop-probe arrangement can be used to determine the plasma energy per unit length. For plasmas of moderate to high beta, the energy line density can be obtained from two directly measurable electromagnetic quantities, the excluded flux radius and the external magnetic field, without requiring a separate determination of the plasma radial density profile.

1. INTRODUCTION

The plasma beta, β , the ratio of plasma pressure to external magnetic field pressure, is an important parameter in magnetic fusion experiments. Ideally the plasma beta profile would be determined from direct local measurements of plasma density, temperature and magnetic field. Although these complete detailed measurements do not exist, one can infer the β profile by combining measurements of the magnetic flux excluded by the plasma column and the radial plasma luminosity distribution. This technique has been extensively used in linear magnetic fusion experiments such as the θ pinch. The excluded flux is measured by a balanced loop-probe arrangement and a relative density profile is obtained from a multichannel luminosity profile apparatus. These measurements give the plasma beta, radius, position, and energy line density, and when combined with a single-chord density measurement, the absolute density profile and total plasma temperature.

2. MEASUREMENT OF PLASMA LUMINOSITY PROFILE

The optical intensity profile of a plasma column can be determined by measuring the plasma continuum radiation from multiple chords through the plasma. This is done by imaging the plasma radiation with a lens on a stack of optical fiber bundles. Each fiber bundle connects with a separate photomultiplier tube. This technique has been used by Eberhagen and Keilhacker[1], Sawyer et al.[2], and Ellis et al.[3] in various high- β experiments.

The data from the multiple channels are unfolded by an Abel inversion to give the light intensity as a function of radius $I(r)$. The square root of relative light intensity is proportional to plasma density provided the plasma temperature is not a strong function of radius. Thus, a relative density profile can be derived from the luminosity profile. In θ -pinch experiments the assumption that the density distribution is Gaussian has usually been made. This assumption is supported by the fact that the density distribution is found interferometrically to be very nearly Gaussian and that the experimental luminosity profiles can be fitted quite satisfactorily by the Gaussian shape. Since the Abel inversion of a Gaussian is also a Gaussian with the same half-width, this assumption eliminates the need for doing numerical Abel inversions to obtain the plasma radius.

The plasma luminosity along as many as 20 parallel chords is measured as a function of time using the optical arrangement of Freese and Siemon[4] shown in Fig. 1. A variable aperture stop one focal length behind the objective lens serves the dual function of collimating the viewing direction and providing adjustment for plasma light intensity. By using such an aperture, the light for each channel is collected along nearly parallel chords through the discharge tube. The use of a zoom lens to relay the plasma image formed by the collimating optics onto the detector array allows the size of the plasma image to be varied. This feature, which provides a continually adjustable plasma image on the detector provides a high degree of flexibility to the luminosity profile measurement. It can be used for detailed profile measurements when the plasma position is relatively constant or to follow a wide range of plasma motion at the expense of highly resolved profile information. The light detector uses an array of photomultiplier tubes (EMI 9798B) which have a high gain and broad spectral response (S-20). The photomultiplier signals are amplified by individual line drivers which have a risetime of 5 ns for signal levels up to 10 V. The luminosity apparatus also includes a calibration light pulse which enters the optical system via a beam splitter in front of the objective lens so that the calibration light passes through the same optics as the plasma

light. A tion is automatically taken just before each discharge. It has been found in 0 pinches within shot-to-shot variations, the profiles obtained are independent of whether one a narrow band-pass filter, selected to view only continuum, or the entire S-4 or S-20 spec response. This results since the integrated line intensity is small compared to the integrated continuum. The broader spectral band provides a larger intensity which permits highly collimated viewing and thus better spatial resolution.

The luminosity signals on a given discharge are sampled at a 10 MHz rate, digitized and stored by data acquisition channels. After the discharge, the data is read into a computer for analysis. The computer normalizes the signals to the respective calibration pulses and collates this data into a series of luminosity profiles $I(x)$ at successive instants of time.

The raw data $I(x)$ must be Abel inverted to yield the light emission and obtain the relative radial density distribution $n_e(r)$ at a particular instant of time. If the plasma luminosity arises solely from bremsstrahlung, then from a small volume d^3x in the plasma,

$$dI = \text{constant} \cdot n_e^2(r) T_e(r)^{1/2} d^3x. \quad (1)$$

The weak radial dependence on the electron temperature allows the assumption, $T_e(r) = T_e = \text{constant}$ in high- β experiments. Abel inversion is sensitive to the tail of the $I(x)$ profile, where the signal to noise ratio is always poor. One approach, which has been used in high- β , θ -pinch experiments, to analyzing the data is to assume a profile shape, suggested by the appearance of the data, and dispense with the inversion procedure and its associated errors. Generally the data can be fitted to a Gaussian profile with good accuracy. Assuming $T_e(r) = T_e = \text{constant}$ and a Gaussian profile,

$$n_e(r) = n_0 e^{-(r/a)^2} \quad (2)$$

where a is the inflection point of the Gaussian and the plasma radius which accounts for the line density. The observed intensity distribution $I(x)$ is therefore also a Gaussian,

$$I(x) = I_p \exp[-2(x-x_0)^2/a^2] + C, \quad (3)$$

such that the profile half-width of $I(x)$ is the 0.707 width of $n_e(r)$ and 1/4 width of $I(x)$ is the half-width of $n_e(r)$.

The luminosity data is analyzed by assuming a Gaussian profile, which the $I(x)$ closely resembles, and fitting to the data, by the method of least squares, a profile of the form given by eq. (1). Thus four experimental parameters are obtained in fitting the profile: a , the radius of the density profile; x_0 , the plasma column position; I_p , the peak intensity of the light emission; and C , the constant which measures the background light level.

Typical luminosity data as recorded on an oscilloscope from a 10-channel apparatus is shown in Fig. 2(A). A baseline and calibration pulse are recorded automatically $\sim 100 \mu s$ before the actual plasma discharge. The initial intensity peak is due to the implosion phase of the θ -pinch discharge and is followed by variations in intensity due to motion of the

plasma column perpendicular to the viewing direction. A typical computer-generated least-squares-fitted Gaussian profile is shown in Fig. 2(B) with experimental points shown as circles. The accuracy of the detector system is about 15% due to calibration difficulties and nonlinearities of the photomultiplier. Simultaneous measurements of the relative density profile by the luminosity apparatus and the absolute density profile by side-on holographic interferometry has shown agreement of the profiles to better than 10%.

3. EXCLUDED-FLUX MEASUREMENTS

The magnetic flux excluded by the plasma can be measured by a differential probe technique first suggested by Green[5] and further improved and used by Goldman et al.[6], Harris and McClintock[7], Sawyer et al.[2], and Ellis et al.[3]. This technique is particularly applicable to high- β plasmas and can be easily applied in simple plasma-magnetic field configurations such as in linear systems with axial magnetic fields. The excluded flux is defined as

$$\Delta\phi = \int_0^R [B_{z0} - B_z(r)] 2\pi r dr, \quad (4)$$

where B_{z0} is the magnetic field far from the plasma and R is large enough for the integrand to vanish.

The balanced loop-probe arrangement for measuring the excluded flux is shown schematically in Fig. 3. The small magnetic probe positioned outside the discharge tube produces a voltage

$$V_p = K_1 N A_p dB_{z0}/dt, \quad (5)$$

where K_1 is a constant adjusted by a potentiometer in the differencing circuit, $N A_p$ is the probe area, and B_{z0} is the magnetic field well outside the plasma column and is approximately constant independent of radius. The larger single-turn loop of area A_L encircles the discharge tube and is therefore sensitive to flux displacements caused by the plasma. This loop generates a voltage

$$V_L = \frac{K_2}{C_1} \int_{A_L} \dot{B}_z(r) 2\pi r dr, \quad (6)$$

where C_1 is the step down ratio of the voltage divider in series with the loop, typically ~ 100 .

In vacuum without plasma K_1 is adjusted to obtain $V_p = V_L$, the balance condition, which from eqs. (4) and (5) gives $K_1 = (K_2/C_1)(A_L/N A_p)$ in terms of which eq. (4) becomes

$$V_p = \frac{K_2}{C_1} A_L \dot{B}_{zo} \quad (7)$$

When plasma is present, the difference signal is given by

$$\Delta V = V_p - V_L = \frac{K_2}{C_1} \int_{A_L} [\dot{B}_{zo} - \dot{B}_z(r)] 2\pi r dr \quad (8)$$

If a perfect balance is not obtained, as is usually the case in practice, then any imbalance from a vacuum shot must be subtracted from ΔV . After integration by RC circuits, having time constants $(RC)_1$ for V_p and $(RC)_2$ for V_L , we obtain the dimensionless ratio

$$\frac{\Delta\phi}{\phi_p} = \frac{\int \Delta V dt}{\int V_p dt} = \frac{(RC)_p}{(RC)_L} \frac{1}{A_L B_{zo}} \int_{A_p} [B_{zo} - B_{z1}(r)] 2\pi r dr \quad (9)$$

where A_p is the plasma cross-sectional area, $B_{z1}(r)$ is the magnetic field distribution inside the plasma column, B_{zo} is the outside field, $\Delta\phi$ and ϕ_p are experimental measurables given by the probes.

Typical data from the experiment are shown in Fig. 4. Any departures from a null vacuum balance, as shown in the bottom trace of Fig. 4 must be subtracted from the integrated difference signal before calculating $\Delta\phi$. Any losses in the differencing circuit must also be taken into account. Small additional corrections to the value of the excluded flux must be made if radial components of the magnetic field are present. The diamagnetic loop-probe systems can be calibrated in situ by discharging the primary energy storage bank with a copper tube inserted into the discharge tube to simulate the plasma. The L/R decay time of the currents induced in the copper tube are usually much greater than the experimental observation time and thus the tube acts as a perfectly diamagnetic body. Generally, the effective radius calculated from the excluded flux measurement with the copper tube in place is within 10% of the measured tube radius. However such an in situ calibration provides calibration factors which can be introduced into the excluded flux analysis to improve the accuracy of the measurement.

4. DETERMINATION OF PLASMA β

The relative density profile derived from the luminosity profile, Sec. 2, is combined with the magnetic probe data to determine the plasma beta. From the pressure-balance relation

$$B_{zo}^2/8\pi = B_{z1}^2(r)/8\pi + n(r)k(T_e + T_i) \quad (10)$$

we solve for $B_{z1}(r)$

$$B_{z1}(r) = \sqrt{1-\beta(r)} B_{z0}, \quad \text{where,} \quad \beta(r) = 8\pi n(r) k(T_e + T_i) / B_{z0}^2. \quad (11)$$

From the definition of excluded flux, eq. (4) and eq. (11) we have

$$\Delta\phi = 2\pi B_{z0} \int_0^a [1 - \sqrt{1-\beta(r)}] r dr. \quad (12)$$

First we derive β for a square, sharp-boundary profile and later for a diffuse Gaussian profile. In the square profile case B_{z1} is independent of radius and substituting for B_{z1} from eq. (11) in eq. (9) and integrating yields

$$\frac{\Delta\phi}{\phi_p} = \frac{(RC)_p}{(RC)_L} \frac{a^2}{r_L^2} [1 - \sqrt{1-\beta}] \quad (13)$$

$$\text{or} \quad a = r_L \left[\frac{\Delta\phi}{\phi_p} \frac{(RC)_L}{(RC)_p} \right]^{1/2} [1 - \sqrt{1-\beta}]^{-1/2} \quad (14)$$

where a and r_L are the respective plasma and loop radii. It is convenient to introduce an "effective plasma radius," or excluded flux radius, r_ϕ , defined by

$$r_\phi = r_L \left[\frac{\Delta\phi}{\phi_p} \frac{(RC)_L}{(RC)_p} \right]^{1/2}, \quad (15)$$

where $a = r_\phi$ when $\beta = 1$. From eqs. (14) and (15) we obtain the plasma β for a square profile,

$$\beta = 1 - [1 - (r_\phi/a)^2]^2. \quad (16)$$

The effective plasma radius is determined from the excluded flux measurables $\Delta\phi$ and ϕ_p . This is combined with the plasma radius, a , determined from the relative density profile to give the plasma beta.

In the following we derive the plasma beta for a diffuse Gaussian profile in terms of the experimentally measurable quantities. When the plasma profile is diffuse, B_z and β are functions of radius as indicated in eq. (11). In particular we take $\beta(r)$ proportioned to $n(r)$ assuming that $(T_e + T_i)$ is independent of radius as given in eq. (11). As discussed in Section 2 on the measurement of the relative density profile, we assume the density profile is Gaussian, giving

$$\beta(r) = \beta_A e^{-(r/a)^2}, \quad (17)$$

where β_A is the β on axis.

Substituting eqs. (11) and (19) into eq. (9) gives

$$\frac{\Delta\phi}{\phi_p} = \frac{(RC)_P}{(RC)_L} \frac{1}{A_L} \int_0^a [1 - \sqrt{1 - \beta_A} e^{-(r/a)^2}] 2\pi r dr, \quad (18)$$

relating $\Delta\phi/\phi_p$ to the Gaussian profile. Integrating eq. (18) yields

$$\frac{\Delta\phi}{\phi_p} = \frac{(RC)_P}{(RC)_L} \frac{a^2}{r_L^2} \left\{ 2(1 - \sqrt{1 - \beta_A}) + \ln \left[\left(\frac{\beta_A}{4} \right) \left(\frac{1 + \sqrt{1 - \beta_A}}{1 - \sqrt{1 - \beta_A}} \right) \right] \right\}, \quad (19)$$

which is analogous to eq. (13) in the sharp-boundary case. By noting that

$$\ln \left[\left(\frac{\beta_A}{4} \right) \left(\frac{1 + \sqrt{1 - \beta_A}}{1 - \sqrt{1 - \beta_A}} \right) \right] = 2 \ln \left(\frac{1 + \sqrt{1 - \beta_A}}{2} \right), \quad (20)$$

and using the definition of r_ϕ in eq. (15), we obtain the result

$$\frac{a}{r_\phi} = \left\{ 2 \left[1 - \sqrt{1 - \beta_A} + \ln \left(\frac{1 + \sqrt{1 - \beta_A}}{2} \right) \right] \right\}^{-1/2}. \quad (21)$$

This transcendental equation relates β on axis for a Gaussian profile to the experimentally determined quantities a and r_ϕ . A computer code computes β_A from eq. (21) by an iterative process.

Figure 5 compares curves for β_A versus a/r_ϕ for the Gaussian profile, eq. (21), the sharp-boundary square profile, eq. (16), and for a rigid rotor profile. The difference between the Gaussian and rigid rotor profiles is quite small. The difference between these and the square profile approach a maximum value of 30% at $\beta_A = 1$. Experimentally, a is determined by curve-fitting the luminosity data, and r_ϕ is found from the excluded-flux quantities $\Delta\phi$ and ϕ_p from eq. (15). The choice of which profile to fit with the data is made empirically. Usually a Gaussian profile is adequate.

Figure 6 shows the plasma beta, radius, position and excluded flux as obtained from the luminosity and excluded-flux data from the Scylla IV-P, 5-m linear θ pinch. These results can be displayed by an on-line computer within a few minutes after the plasma discharge. The scatter in the plasma radius at early times is transferred to the plasma beta since in the β calculation there is a strong functional dependence on the plasma radius.

5. PLASMA ENERGY LINE DENSITY FROM EXCLUDED FLUX MEASUREMENTS

The measurement of the magnetic flux excluded by the plasma column with the balanced loop-probe arrangement can be used to determine the plasma energy per unit length. For plasmas of moderate to high β , the energy line density can be obtained from two directly measurable electromagnetic quantities, the excluded flux radius, r_ϕ , and the external magnetic field, B_{z0} , without requiring a separate determination of the plasma radial density profile. This approximate method of measuring the energy line density has been applied to the linear θ pinch to determine the energy density as a function of axial position.

The plasma internal energy (energy per unit volume) can be written as

$$E = (F/2) P \quad , \quad (22)$$

where $P = nk(T_e + T_i)$ is the plasma pressure and F is the number of degrees of freedom. From pressure balance

$$P = [B_{z0} + B_z(r)] [B_{z0} - B_z(r)] / 8\pi \quad . \quad (18)$$

For low β , $[B_{z0} + B_z(r)] \approx 2 B_{z0}$. Using this approximation, substituting eq. (23) in eq. (22) and integrating the result over the coil bore gives the energy line density,

$$E_L = \int_0^b E \, 2\pi r \, dr = \frac{F}{2} \frac{1}{4\pi} B_{z0} \int_0^b [B_{z0} - B_z(r)] 2\pi r \, dr = \frac{F}{2} \frac{B_{z0}}{4\pi} \Delta\phi \quad , \quad (24)$$

where b is the coil radius and $\Delta\phi$ is the excluded flux measured by the diamagnetic loop-probe arrangement as discussed in Section 3. The excluded flux can be expressed as $\Delta\phi = \pi r_\phi^2 B_{z0}$, so the energy line density becomes

$$E_L = \frac{F}{2} \left(\frac{r_\phi B_{z0}}{2} \right)^2 \quad , \quad (25)$$

which is the desired result, i.e., the energy line density expressed as a function of two directly measurable electromagnetic quantities, r_ϕ and B_{z0} . The excluded flux radius is given in terms of measurable quantities in eq. (15) of Section 4 and B_{z0} is obtained by integrating the signal from a magnetic probe external to the discharge tube.

Using the relation $P = \beta B_{z0}^2 / 8\pi$, an exact expression for the energy line density can be written in terms of the plasma beta as

$$E_L = \frac{F}{2} \left(\beta \frac{B_{z0}^2}{8\pi} \right) \pi a^2 \quad , \quad (26)$$

where a is the plasma radius. Equating eqs. (25) and (26) yields an expression for β as a function of r_ϕ and a in the low β approximation given by,

$$\frac{a}{r_\phi} = \sqrt{2/\beta} \quad . \quad (27)$$

For a Gaussian radial density profile the exact solution for a/r_ϕ as a function of β on axis, β_A , is given by eq. (21) of Section 4. At $a/r_\phi = 1.48$ the exact analysis, eq. (21), gives $\beta_A = 0.8$ compared with the approximate analysis, eq. (27), which gives $\beta_A = 0.91$ an overestimate of 12%. For higher a/r_ϕ ratios eq. (27) more closely approximates the exact solution; at $a/r_\phi = 2$ the approximate analysis only overestimates β_A by 6%.

From the plasma parameters measured at the midplane of the Scylla IV-P 5-m linear θ pinch, the energy line density has been calculated using both the approximate expression given by eq. (25) and the more exact form given by eq. (26). The results are shown in Fig. 7, which represents the data averaged over 15 plasma discharges. The dotted line, obtained from eq. (25), is calculated from the diamagnetic loop-probe data while the solid curve, obtained using eq. (21) is generated using β_A from the loop-probe data and a determined from the luminosity profile data. The validity of the approximate form of E_z given by eq. (25) for the β range obtained in Scylla IV-P is clearly demonstrated in Fig. 7. This approximate method of measuring the energy line density as a function of axial position is easily incorporated into an on-line data analysis system.

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FIGURE CAPTIONS

- Fig. 1. Schematic diagram of luminosity profile apparatus. (Courtesy of Dr. K.B. Freese, Los Alamos Scientific Laboratory, Ref. [4].)
- Fig. 2. Typical luminosity data. (A) shows photomultiplier traces from two adjacent luminosity channels. (B) shows a typical computer-fitted luminosity profile at a single instant of time, determined by simultaneous data from 10 output channels. (Reference [3].)
- Fig. 3. Schematic arrangement of the excluded-flux measuring apparatus. (Reference [3].)
- Fig. 4. Typical differential loop-probe signals with and without plasma in the Scyllac θ pinch. The upper traces in each oscillogram show the integrated probe signals giving the magnetic field B_{z0} outside the plasma column. The integrated difference signal with plasma is proportional to the excluded flux $\Delta\phi$. The vacuum shot shows the imbalance signal in the loop-probe arrangement.
- Fig. 5. Graphs of the plasma beta on axis versus the ratio of the plasma radius, a , to the excluded-flux radius $r_{eff} = r_\phi$ for the indicated plasma profiles.
- Fig. 6. Results obtained from the luminosity and excluded flux data. (Courtesy of Dr. K.B. Freese, Los Alamos Scientific Laboratory.)
- Fig. 7. Temporal history of the energy line density, E_ℓ , in kJ/m at the center of the 5-m Scylla IV-P θ pinch, averaged over 15 plasma discharges. Dotted line: E_ℓ determined from the excluded flux measurements and the low β approximation. Solid line: E_ℓ determined from luminosity and excluded-flux data with the assumption of a Gaussian profile. (Courtesy of Dr. C.A. Ekdahl, Los Alamos Scientific Laboratory.)

(A)

(B)

WITH
PLASMA

WITHOUT
PLASMA

$1\ \mu\text{s}$