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AN ENERGY-DEPENDENT FINITE-ORBIT TREATMENT
FOR PLASMA BUILDUP IN MIRROR FUSION DEVICES

by

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THESIS

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in the Graduate College of the
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AN ENERGY-DEPENDENT FINITE-ORBIT TREATMENT
FOR PLASMA BUILDUP IN MIRROR FUSION DEVICES

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A computer simulation of hot plasma buildup in mirror fusion devices and results from this model are presented here. In a small, hot magnetically confined plasma, the ion orbit radius (ρ_i) can be comparable to the plasma radius (R_p). In a mirror-confined plasma where $\rho_i/R_p > 1/25$ (such as 2XII-B), a "point kinetic" treatment of ion interactions becomes inaccurate and a finite gyro-radius' (FGR) treatment must be used to adequately describe plasma buildup processes. This is particularly true for describing losses due to cold-gas charge exchange (c-x) near the plasma surface, since a particle lost near the vacuum interface may have contributed to the density as far as $2\rho_i$ radially inward from the c-x point. A similar FGR effect applies to beam-deposited ions whose large orbits influence the density up to $2\rho_i$ from the trapping point.

These and other important FGR effects are treated by a dynamic model called FOREMD (for Finite Orbit, Radial, Energy and Magnetic-field Dependent) which was developed for the present studies. Unlike earlier finite-orbit models, FOREMD is fully energy dependent (important because $\rho_i \sim \sqrt{E}$) and includes a quasi-linear treatment for the Drift Cyclotron Loss Cone (DCLC) instability. Also included are effects of the local magnetic-field on ion orbits.

The FOREMD model has been used to study plasma buildup (particularly early stages) in presently operating and proposed mirror devices (2XII-B, TMX, FERF) and in the Twin Beam Mirror (TBM), a concept proposed by

G. H. Miley at UIUC. Results of these simulations compared to experiments or design expectations are presented here.

General factors in plasma buildup and their relative importance are also examined with recommendations on how to achieve plasma buildup under c-x erosion conditions with minimal injected beam power. A detailed description of the model is also included.

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Chapter 1: INTRODUCTORY COMMENTS AND PROBLEM EXAMINATION

1.1 Purpose

The magnetic mirror fusion device and off-shoots such as the tandem mirror are objects of considerable research effort in controlled nuclear fusion. Consequently, accurate modeling of the plasma in these devices is important to investigators trying to design future generations of experiments. In this research a model is developed that treats plasma buildup in a small mirror device. This model considers effects due to large ion orbits, plasma erosion by cold-gas charge exchange, beam ion deposition, magnetic-field effects and other important buildup parameters using an energy dependent, dynamic treatment of greater accuracy and completeness than any previous treatment.

This chapter examines fundamental concepts involved in describing processes and parameters for a small mirror-confined plasma. A brief history of earlier attempts to model such plasmas and a critical review of limitations of these simulations are also included. Finally, an introduction to the "state-of-the-art" buildup code developed in the current research is presented.

1.2 Mirror Reactor Design History

The magnetic-mirror has been considered a candidate for fusion plasma confinement almost from the start of controlled fusion research. The simplest early models (such as Toy Top¹) consisted of two Helmholtz coils and a solenoidal coil (around the vacuum vessel) to provide a magnetic field (B) that increased from the center-plane to the coil

throats. Plasma was injected and trapped in this pulsed device. Although magnetically open-ended, plasma particles orbiting field lines are reflected back toward the low-field region in such a device provided that particles satisfy two restrictions on their motion; otherwise they escape. These restrictions are first, that the ratio of parallel to perpendicular velocities ($v_{||}/v_{\perp}$) (with respect to $\vec{B}$) be less than $\sqrt{R-1}$, where R is the "mirror ratio" = $B_{\text{maximum}}/B_{\text{minimum}}$ and second that the particle not see a significantly large change in B over one orbit $\frac{\delta B}{\delta t} < \frac{B}{2\pi\omega_c}$ so that the "adiabatic invariant" $\mu = \frac{W_{\perp}}{B}$, where $W_{\perp}$ is the perpendicular energy (to $\vec{B}$), is a constant of the motion². Appendix A contains a more detailed discussion.

Early experiments, however, soon showed that simple mirrors were Rayleigh-Taylor (magnetic-interchange) unstable because the magnetic pressure decreases as the perturbed plasma moves radially outward so that the magnetic field does not provide a sufficient restoring force to push the plasma back into equilibrium³. This difficulty was solved by Ioffe who applied an octupole magnetic field to twist the lines so that B increased slightly with radius out from the center⁴. Eventually a way was found to incorporate Ioffe's external conductors into the main windings, yielding a one-coil device that looks like the seam of a baseball. The experiments Baseball I and II (BBI, BBII) were built with these coil configurations⁵. Further improvement was made when two more-open coils were used and placed together rotated 90° relative to each other to form a "Yin-Yang" configuration. This name was adopted because of the similarity between this classical Taoist

figure and the appearance of the coils fitted together (Fig. 1). By adjusting the coil spacing, the minimum-B region can be of varied size and shape in this design⁶.

According to classical estimates, the confinement time for particles in either of these minimum-B wells is proportional to the time it takes particles to scatter in velocity space until the ratio of $v_{\parallel}/v_{\perp}$ exceeds $\sqrt{R-1}$. In early global treatments, the scattering time used was the perpendicular (90°) scattering time for the momentum vector; this time is proportional to $W^{3/2}$ and the mass of the particle. The actual loss rate is proportional to a term called the "loss-cone probability" (P_{LC}) which is related to the ratio $v_{\parallel}/v_{\perp}$ through the associated loss-cone angle:

$$\theta_c = \tan^{-1} \left(\frac{v_{\perp}}{v_{\parallel}} \right) = \tan^{-1} \left(\frac{1}{\sqrt{R-1}} \right). \quad (1-1)$$

Alternately, the corresponding fraction "f" of solid angle in velocity space into which a particle is scattered and lost is:

$$f = \frac{1}{4\pi} \int_0^{\theta_c} 2\pi \sin\theta d\theta. \quad (1-2)$$

Because lower mass particles Coulomb-scatter faster than those of larger mass (at the same energy), the electrons initially scatter into the loss region (loss cone) faster than the ions. This unequal loss rate creates a charge difference in the plasma that results in a "plasma potential" (ϕ_p) which is positive and retards the electron

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Figure 1. Shows the "Yin-Yang" coil geometry which generates a MHD stable, minimum B field configuration to contain the plasma.

losses and enhances ion losses until the rates are equal. At this point ϕ_p exceeds the electron temperature (T_e) and the electrons become electrostatically confined. The presence of ϕ_p depletes the ion distribution at low energies because ions are accelerated by the potential as they approach the mirror region and, for ions with $w_{\perp} \leq |e\phi_p|$, $v_{\perp}/v_{\parallel}$ exceeds the maximum value for reflection, leading to an ion distribution which is not only anisotropic in velocity space (because of the loss cone) but is non-Maxwellian in energy (or speed)⁷.

These anisotropies make the plasma susceptible to various micro-instabilities, the worst observed so far being the Drift Cyclotron Loss Cone (DCLC) instability^{8,9}. This instability causes anomalous losses by driving clusters of higher energy ions into the vacated phase/energy-space region of the distribution, which are then subsequently lost through the mirrors. Energy losses from this instability can be very high, particularly in regions where the density gradient is large (near the boundary).

Various techniques have been proposed for controlling DCLC losses: two that have been tried successfully on 2XII-B are injecting a cool plasma or some cold gas to fill part of the low energy region of the ion distribution. Introducing this plasma, however, with its cold electrons reduces ion confinement through energy loss to the electrons. The amount of cold plasma required to stabilize the device is examined in Appendix B. A very large mirror plasma is not DCLC unstable and does not require a streaming plasma. The criterion for stability is that $\rho_i/R_p \leq \frac{1}{25}$, where ρ_i is the ion gyro-radius and R_p is the plasma radius¹⁰. For present experiments, such as 2XII-B, ρ_i/R_p is about 1/3.

1.3 Previous Models

Modeling of DCLC-stable or "classical" mirror plasmas has traditionally been done with Fokker-Planck or Monte Carlo simulations. Early Fokker-Planck treatments (such as those of Kuo-Petravic, Kileen, and Marx¹¹) have progressed from global (0-D), single species to multidimensional, multispecies codes (such as Hybrid-II¹²), but all have been point-kinetic models which treat ions as point particles ($\rho_i = 0$). However such a model does not account for the spatial (radial) coupling between regions in a plasma where the ion gyro-radius is comparable to the plasma radius. Related effects from beam trapping and charge-exchange (c-x) losses are also described inadequately. The author's model is among the first Fokker-Planck treatments which takes these effects into account.

Monte Carlo treatments (e.g., R. Miller¹³) have been used to study particle transport in the mirror geometry. However due to their statistical nature and, therefore, large computation time such Monte Carlo codes are not generally considered suitable for dynamic operation requiring many iterations and the treatment of many different physical processes occurring nonlinearly on different time scales. To the author's knowledge, no Monte Carlo simulation of plasma build-up including finite orbit, collisional and beam trapping effects has ever been attempted.

In 1975 a model designed to treat spatial (radial) finite-orbit effects in a mirror plasma was developed by B. Stallard at Lawrence Livermore Laboratory¹⁴. This code included a finite-orbit, radial density treatment (based on a treatment proposed by A. Futch¹⁵) in

a mono-energetic target, supported and built up by a neutral beam injected at another specific energy. No energy relaxation was considered and all important plasma parameters were treated as input variables. This model, however, did include cold-gas charge-exchange effects and represented a good initial attempt to describe these important spatial effects. The author was able to use this code to study buildup in a hot (70-keV) plasma and some important conclusions drawn from that work are presented in Chapter 5. This model (now called BUILDUP¹⁶) was later improved by S. DeVoto to include two additional beam injection energies, but otherwise remains essentially the same as the original in its limitations (lack of an energy treatment, lack of overall self-consistency). A more sophisticated model is currently under development at Lawrence Livermore Laboratory by A. Futch, although no results have yet been published¹⁷. This model is discussed in Chapter 5.

1.4 Finite Orbit Considerations

The importance of treating finite ion orbit effects in a small plasma has already been mentioned. In a small mirror device, where R_p/ρ_i is not large, an ion will contribute to the plasma density over a region bounded by its orbit. In such a case, a point particle model for the ions is clearly unsatisfactory. The local-region density is actually a time-averaged contribution from orbits passing through that region. This becomes particularly important when charge exchange ($D^0 + D^+ \rightarrow D^+ + D^0$) between plasma ions and cold atoms impinging on the plasma surface is considered. This reaction neutralizes a warm ion, allowing it to escape across the field lines

to the wall, and deposits in its place a cold ion which rapidly leaves through the loss cone, accelerated by the plasma potential. This mechanism can represent a serious plasma energy and particle loss, particularly if most of the c-x losses are from a small region near the plasma surface. Compounding the severity of this effect is the fact that the lost hot ion may have contributed to the density over a large volume of plasma, thereby affecting the interior by its loss near the surface (Fig. 2).

Stallard has shown, using his mono-energetic code, that surface charge-exchange effects can steepen the density profile near the boundary^{14,16} (recall that this steepening will also affect DCLC modes). If the boundary losses are not adequately replaced, the plasma will erode away to extinction. The steep density profile predicted by this model has been observed in 2XII-B¹⁸. Therefore, a proper treatment of finite ion orbit effects is necessary in modeling a small mirror plasma ($\rho_i/R_p \geq 1/25$).

Two factors that affect the ion's orbit, and therefore the ion's time-averaged density contribution to plasma regions traversed by its orbit are the ion energy and the average magnetic field across an orbit ($\rho_i = \frac{mv_i}{qB}$ where m = mass, q = charge, v_i and B are as before). The ions in a plasma have a wide distribution of energies, so clearly an energy dependent treatment is required to accurately model the density profile. The magnetic field inside the plasma varies with density and temperature because the plasma behaves diamagnetically and lowers the internal field with increasing plasma pressure. These local variations can also affect the ion orbit radius and average density contributions.

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Figure 2. Shows how a large-orbit ion (center at P) contributes to the density over a large region of the plasma (hashed area).

1.5 Introduction to the Present Model

The model developed in the present research takes into account these important, and heretofore neglected effects of the finite orbit treatment and uses this model to simulate various small mirror devices either presently in operation or on the drawing board. This model incorporates the most necessary features of the Fokker-Planck treatment using energy and spatial grids, and includes effects of particle energy and local magnetic fields on the finite gyro-radius (FGR) treatment of radial plasma density and temperature profiles. Additionally, the model treats charge-exchange losses, mirror losses and beam trapping in each radial plasma region (or shell) and calculates the various parameters of interest self-consistently at each time step. A detailed description of this model and the results of plasma simulations using it are given in the following chapters.

Chapter 2: OVERVIEW OF PHYSICS AND METHODOLOGY

2.1 Introduction

In the previous section some of the important effects of the finite ion gyro-radius (FGR) in a small plasma were examined briefly. As mentioned, to the author's knowledge all previous Fokker-Planck and Monte Carlo treatments have used point-kinetic models and consequently have neglected detailed orbit considerations. While these effects might, in principle, be included in such models, they have not been previous to the present energy-dependent model^{19,20}. This chapter discusses the important physical considerations in developing such FGR models.

2.2 Finite Orbit Effects on the Density Distribution.

The author's model includes significant finite orbit effects in a Finite-Orbit Radial, Energy and Magnetic-Field Dependent (FOREMD) treatment. The ion distribution is tracked both as a density of orbit centers (DOC) and as a time-averaged density (TAD), which is found by calculating the sum of the density contributions to each radial plasma region (or shell) from all particles whose orbits pass through that region. This is done for each energy group making up the ion distribution. Each contribution is proportional to the fraction of the particle orbit contained within the shell. Figure 3 illustrates this method. The enclosed arc-length is found by straight forward geometric and trigonometric techniques. Each arc contribution must be weighted by the number of particles whose centers lie in a contributing thin annular subdivision ($2\pi R\Delta R$) of the

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Figure 3. Shows that the density contribution to a radial region is proportional to the arc length passing through that region, given by $\rho\theta$.

plasma¹⁵. This finer mesh is set to $1/20$ the shell spacing. By assuming the DOC is constant across the shell and then integrating (summing) the weighted orbit contributions ($DOC \times 2\pi R \Delta R$) over a shell width, the mean time-averaged density contribution from a given shell to each interacting shell in the plasma is found at each energy (a more detailed description will be given in Section 3.2).

For the case of a nonuniform radial magnetic field ($\nabla B \neq 0$) the ion orbit can be noncircular in high gradient regions, such as near the plasma boundary. The FOREMD model treats this effect to first order by splitting the orbit into radially inward and outward semi-circular portions, each having different radii of curvature determined by the local field and radial gradient (see App. C and Fig. 1C). Both portions of the orbit are then treated as above (the ion speed remains constant so the contribution is still proportional to the circumscribed arc length). These contributions are calculated and stored in a "lookup" table as a function of p_i and the shell number in the FOREMD code.

Once the TAD has been found, it is used in place of the point-particle ion distribution formerly used in build-up simulations. The electron density may still be treated with a point model because the electron orbit radius (ρ_e) is much smaller than the ion radius (ρ_i). Ion distribution relaxation, neutral-beam trapping, loss-cone out-scattering, and cold-neutral charge-exchange processes are all dependent on the local TAD .

2.3 Collisional Energy Transfer and Scattering Losses

The ion distribution relaxation occurs because the loss-cone

losses, neutral-beam injection, and cold electron temperature (a result of high-energy electrons which escape through the plasma-potential barrier) create a non-Maxwellian distribution in which the ions constantly strive to achieve a thermal equilibrium. Ion-ion energy transfer is treated with a simple 1-D Fokker-Planck model which takes into account velocity-space spreading as well as slowing-down due to small-angle collisions. Ion-electron transfer is done with a Sivukhin-type slowing-down model adapted to treat non-Maxwellian distributions, which is discussed in Chapter 3. This is permissible because momentum transfer to the ions and therefore velocity-space scattering for ion-electron collisions is negligible compared to the scattering caused by ion-ion interactions.

An ion whose orbit passes through more than one radial shell experiences collisions and energy transfer according to the average conditions in each shell. The FOREMD model sums these over an orbit to obtain a net energy gain or loss at each particle energy. This information is then used to find the change in the DOC distribution.

Electron equilibration is much simpler to handle because the collision time between electrons is very short, their orbits are small, and they are confined in a potential well so their distribution is essentially Maxwellian (truncated at energies above ϕ_p). Therefore, the electron distribution can be treated as a Maxwellian on time scales as long as those of ion interactions) with energy transfer being averaged over a Maxwellian, as given by Sivukhin²¹.

Because of the open-field configuration in mirrors, the primary energy and particle loss mechanism for both ions and electrons is

loss-cone outscattering. The classical loss rate depends on the ion collision rate, the vacuum mirror ratio, and the plasma pressure and potential. The loss rate also changes with neutral-beam injection angle which determines the initial value of $v_{\perp}/v_{\parallel}$, but studies by the author and others have shown that nearly perpendicular injection is optimal for most cases of interest, thus FOREMD implicitly assumes perpendicular injection. The corresponding loss rate is given by:

$$\frac{dn}{dt} = \frac{n^2}{n\tau} \quad (2-1)$$

where $n\tau$ has been determined through Fokker-Planck studies to have the functional form:

$$n\tau = W_1^{3/2} \log_{10} R_{\text{eff}}, \quad \text{for } R_{\text{eff}} > 1. \quad (2-2)$$

R_{eff} is the effective mirror ratio given as

$$R_{\text{eff}} = \frac{R_{\text{vac}}}{f(\beta)} \left(1 + \frac{|e\phi_p|}{W_1} \right), \quad (2-3)$$

$$\text{where } f(\beta) = \sqrt{1-\beta} \quad [\text{long, thin plasma}] \quad (2-3a)$$

$$= 1 - .5\beta \quad [\text{short, fat plasma}] \quad (2-3b)$$

β equals the plasma pressure over the magnetic pressure (p/p_M), R_{vac} is the vacuum mirror ratio, and ϕ_p is the plasma potential^{22,23}.

Further description of this ion loss formulation is given at the end of Appendix A.

As discussed in Ch. 1, the plasma potential arises from the initial difference between electron and ion loss rates and grows until these rates become equal (i.e., the losses set up an ambipolar potential in about an electron-electron collision time). Ions having energies below $|q\phi_p|$ are accelerated by the plasma potential until $v_{\parallel}/v_{\perp}$ exceeds the critical confinement value and these ions are lost in a transit time

across the length of the mirror. Loss of these low energy particles due to the ambipolar potential creates an "ambipolar hole" in the energy distribution (Fig. 4). Ion (D^+) slowing by electron drag into this hole leads to losses on the same order as those due to velocity-space scattering.

A factor affecting the electron loss rate and, therefore, ϕ_p is the electron temperature (T_e) which rapidly drops with a potential present because escaping electrons carry out an energy of order $|e\phi_p| + T_e$ in climbing this electrostatic barrier. The electron temperature falls to $\leq 1/10$ the ion energy before ion-electron energy transfer makes up this loss^{11,23}.

The electron loss rate as a function of T_e and ϕ_p was originally formulated by Stix²⁴. This was modified by Dolan²⁵ to give the loss rate in the form:

$$\frac{dn_e}{dt} = 3.9 \times 10^{-15} n_e^2 \phi_p^{-3/2} \left[2 \left(\frac{x}{\pi} \right)^{1/2} \exp(-x) + \operatorname{erfc}(x^{1/2}) \right] \quad (2-4)$$

where erfc is the complementary error function and $x = e\phi_p/T_e$; units are m^{-3} for n_e and kV for ϕ_p . This result is essentially identical to that obtained by Pastukhov²⁶. The above formulation allows the self-consistent plasma potential to be calculated in each shell (assuming that local neutrality or, equivalently, equal loss rates for ions and electrons are required in each shell).

2.4 Cold Neutral Gas Effects

Much of the low energy loss-cone particle loss comes from the cold gas (D_2^0 , D^0) incident on the plasma from the vacuum region. This

Figure 4. Shows the confining and loss regions in velocity space for the ions. Particles with velocity vectors such that $\frac{v_{\perp}}{v_{\parallel}} < \frac{1}{\sqrt{R-1}}$, where R is the mirror ratio, escape. Those with energies lower than $q\phi_p$ are accelerated into the "ambipolar hole" region by the plasma potential and escape.

gas primarily comes from recycled charge-exchange (c-x) neutrals which have hit the first-wall and diffused back into the vacuum chamber (however, a large number of cold neutrals can also enter due to gas leakage from the neutral-beam injectors). With good pumping, a neutral density of $\sim 10^{10} - D_2^0/\text{cm}^3$ can be achieved in the plasma chamber prior to start-up²⁷. Once a hot plasma is established in the chamber, the cold gas is "pumped" out near the plasma surface through electron ionization or charge-exchange followed by acceleration of the resulting ion out the loss cone by the plasma potential.

A certain fraction of the D_2^+ ions formed will undergo Franck-Condon dissociation ($D_2^+ + e^- \rightarrow D^+ + D^0 + e^-$) before being lost¹⁴. This process has a very high cross section at moderate T_e (< 500 eV)²⁸. The D^0 atoms resulting from this reaction are created with an average energy of ~ 3 eV and rapidly undergo electron ionization or resonant charge exchange with a plasma ion ($D^0 + D^{+'} \rightarrow D^+ + D^{0'}$). The cold D^+ ion from the D_2^+ dissociation and the one resulting from the resonant c-x event are then accelerated out of the loss cone.

The newly formed c-x neutral escapes to the first wall, is retrapped or charge exchanges again inside the plasma, its direction-of-flight probability being isotropically distributed at this low incident energy. If a secondary hot c-x neutral results, it may escape to the wall. Similarly, beam c-x and passed neutrals can find their way to the wall. After a short time the wall becomes saturated with D_2^0 and incident neutrals are then recycled back into the vacuum region, primarily as cold D_2^0 . An equilibrium is eventually

reached when the neutral reflux balances the plasma "pumping" losses of D^+ and D_2^+ .

The cold c-x region of the plasma extends about two mean-free-paths inward from the surface and most of the c-x particle losses are from this region. If the beam and c-x neutral retrapping in shells that contribute to the TAD of this outer region are insufficient to replace these losses, the plasma surface will erode away until a balance is reached or the plasma disappears. This phenomenon was predicted in earlier work by Stallard⁶ (see Fig. 5) and is discussed further in Chapter 4.

2.5 Neutral Beam Trapping

Lost particles are replaced ("refueled") by neutral beam trapping and retrapping of beam c-x neutrals. The finite size of ion orbits affects this refueling in two ways. First, the hot ions lost near the plasma surface had orbit centers primarily located further toward the center of the plasma (due to the density gradient, ∇n). The principal replacement of these losses comes from trapping in the plasma interior. Second, the beam position and direction relative to the magnetic field and plasma center affect particle deposition (e.g., see Fig. 6). An ionized particle has an orbit tangent to the beam path. The orbit center is displaced perpendicular to the beam path upward or downward by a gyro-radius, determined by $\vec{v} \times \vec{B}$. This new particle in turn, will contribute to densities up to $2\rho_1$ away from its birthpoint. (See Section 6.3 and Figs. 23, 25 for a detailed explanation.) Therefore,

Figure 5. Early FGR results from B. Stallard's code show the importance of c-x erosion effects on plasma buildup. Each graph shows buildup for a plasma at times $t = 0, 5, 20 \text{ ns}$ for 275 A injection at 14 keV and various ratios of thermal gas/beam currents. The boundary profile steepens at high gas currents.

Figure 6. Also shows the displacement of the gyro-center of a trapped beam ion due to the $v \times B$ force. The orbit is tangent to the beam at the point of formation "x". Assuming uniform trapping and a constant ρ_1 over a small plasma region which intercepts the beam, the deposition of orbit centers is found by displacing the trapping region by ρ_1 and superimposing it back onto the original radial shell grid. Depending on the location of the beam and the direction of $v \times B$, the orbit centers may be deposited either inward or outward radially.

internal trapping and beam positioning are important factors in calculating the high-energy particle balance, particularly near the plasma surface.

The FOREMD model calculates beam trapping in each radial shell by splitting the beam into sections tangent to the shells as shown in Fig. 6. For a given section, each segment of beam shown lying between two shell boundaries represents a region where trapping is assumed to be uniform. Then deposition of centers occurs at some average orbital radius upward or downward from this region. A superposition of the trapping region, displaced a gyro-radius, back onto the shell structure gives the "deposition of centers" contribution to each underlying shell in proportion to the enclosed area. This process is illustrated in Fig. 6, also.

The processes responsible for neutral beam trapping (charge exchange and ionization) deposit hot ions in the manner just discussed. However, charge-exchange events (either beam-ion or cold gas-ion) create a new neutral particle which can be retrapped in the plasma. As mentioned earlier, a low-energy neutral that undergoes charge exchange in the plasma will create a neutral whose direction-of-flight probability is isotropic. At higher incident velocities the energy dependence of the c-x cross section (σ_{cx}) makes it more probable for the neutral to exchange with an ion moving in the same general direction (the lower relative velocity giving a higher σ_{cx}) producing a c-x neutral in that direction. This anisotropy is treated approximately by dividing the velocity-averaged cross section $\langle\sigma\rangle$ into an isotropic part and a forward part (Fig. 7). This method was

chosen instead of using forward/backward fractions as in earlier models in an attempt to improve retrapping estimates for the large fraction of c-x neutrals which travel in a direction roughly perpendicular to the beam path (Fig. 7). Eventually, to make the retrapping tractable, the isotropic part is treated as a uniformly distributed shell source of neutrals which is then divided into a forward and backward component. These components are then treated as beams. The forward fraction of this source and the forward-directed beam c-x neutrals generated in the shell are added to the beam that passes through each shell. Retrapping then occurs as a part of the beam trapping. That this method models c-x retrapping paths fairly accurately (at least through the first generation) is illustrated in Fig. 8, which shows in part "A" that for an isotropic point source the fraction of the trajectory solid-angle intercepted by an inner shell is approximately equal to the fraction of "source" shell area used in FOREMD source directed at that same inner shell. Part "B" of Fig. 8 shows a different viewpoint by dividing an isotropic point source into sections of width (arc length) which are nearly equal to the shell spacing (ΔR) at the "source distance" (R) from the origin. These sections then correspond to the beam paths used in the model. The figure also shows that as the intercepting (inner) shell approaches the source shell this model becomes less correct, approaching the accuracy of the forward/backward type approximation.

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Figure 7. Shows possible paths through the plasma for c-x neutrals. In addition to the traditional forward/backward directed components, many other paths are possible, some of which may lead to a high escape probability. Instead of approximating these paths by using the forward/backward directed c-x "beam" fractions (lower left) a more accurate forward/isotropic breakdown is used (lower right).

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Figure 8A. Part "a" shows how a portion of the isotropic neutral source is nearly equivalent to a beam (parallel lines) covering the same solid angle (θ). The intensity of the isotropic source is proportional to θ and the beam source calculated in FOREMD is proportional to the arc-length "e-f" which also subtends an angle θ . Therefore, the functional form of the "beam" approximation is the same as that of an actual isotropic source.

b

Figure 8B. Part "b" shows the isotropic c-x neutral paths for a constant distance of plasma penetration (bounded by the arc of radius R). In the "beam" approximation these paths are approximated by tangents to each shell (rather than secants) because each beam is tangent to a shell. The error from this approximation increases for shells close to the source of c-x neutrals and approaches the forward/backward treatment in degree of error.

Chapter 3: A FURTHER DESCRIPTION OF THE MODEL

3.1 Introduction

The previous section has touched briefly on the method of treating beam deposition and density used in the energy-dependent, finite-orbit FOREMD model. This chapter is devoted to a more detailed examination of the actual techniques used. As noted earlier, the major feature that distinguishes this model from earlier finite gyro-orbit models is that it treats the ion energies as a distribution and then calculates time-averaged densities from the orbits generated by this distribution. A discussion of this energy treatment and its spatial extension, and other aspects of plasma treatment follow.

3.2 Ion Density/Energy Distribution Treatment

In order to speed computation the FOREMD code uses a nonuniformly spaced energy grid of about 40 mesh points. These points are more closely spaced at low and intermediate energies to provide adequate resolution of the distribution. The energy range, is typically - 50 eV to 700 keV. A multiplicative factor allows this grid to be scaled to an energy range appropriate for the problem under consideration. Typically, if the highest energy group is set at about 3 times the injected beam energy the resulting mirror distribution will be adequately resolved. Problems involving plasma development from a cold target to a hot plasma can be done in stages, using the output of the start up run to initialize a hot run using an extended

energy grid (and perhaps greater spatial shell width). In addition to this grid, a low-energy group (-3 to 5 eV) has been added to handle cold charge-exchange ions. Provision has been made to treat this lowest group and other low-energy groups apart from the others (i.e., not in the Fokker-Planck relaxation) because of the short time scale for particle interactions at low energy.

This energy scaling is set up in the initializing routine FOREMSET and is then used to calculate cross sections for collisions between the various energy groups averaged over all possible collision angles between incident velocity vectors. This yields values of $\langle \sigma v \rangle$ that are placed in a "lookup" table to be weighted by the appropriate densities of each group to yield reaction rates (i.e., $RR_{jk} = n_j n_k \langle \sigma v \rangle_{jk}$). Calculations for Coulomb scattering, charge-exchange, ion-ionization and fusion rates are done in this manner and stored in tables, eliminating computations which would be redone redundantly in the time dependent code.

This routine also generates a "lookup" table for transforming from the density of orbit centers to a time-averaged density in each shell. The radial spatial grid is input (typically 20-50 shells of 0.5-1.0 cm thickness) as well as the range of orbit radii (- 0.5 cm to - 8 cm) and the number of different orbit sizes treated (typically 15, evenly spaced). An ion contributes to the density in a shell through which its orbit passes in proportion to the contained arc-length (the orbit speed is assumed constant). If the density of centers is considered to be uniformly spread across a shell, a volume-weighting proportional to $2\pi R L_m dR$ (L_m is the half-length

between mirrors) must be integrated over the contributing shell, as mentioned previously, to generate the net contribution to adjacent regions. These weighted figures are converted to a fraction of the total possible contribution to all shells (i.e., a fraction of 1) and then to a two digit percentage. These two digit numbers are encoded, along with the radial extrema of the orbit, into three floating-point variables (words) for each possible gyro-radius and shell. Encoding is done to generate smaller (and faster) "lookup" arrays. Because on the computer (CRAY-I) three-word storage is limited to only 16 to 17 two-digit numbers, there is a restriction in the present model on the maximum orbit radius that it not span more than 17 shell-thicknesses. In most cases of interest, this restriction is not important. The reason that this limit is placed on the orbit radius rather than its diameter is that the radially inward and outward halves of an orbit are calculated and coded separately. This permits looking up half-orbits of different radii to approximate slightly noncircular orbits which arise from the magnetic field gradient, particularly in regions of large density gradient near the edge as discussed in section 2.2 (Fig. 9, also see App. C, Fig. C1).

A final step in determining the local time-averaged density requires finding the average values for n and n^2 along the axial length of the plasma. These values are used to calculate reaction rates and losses for the entire shell volume using the central plasma parameters. The axial (z) averages are obtained by integrating an analytic fit to the density distribution (as a function of B) over the length (volume) of a flux tube. The analytic density distribution

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Figure 9. Shows the noncircular orbit (drift orbit) resulting from the presence of a magnetic field gradient ∇B (left) and the FOREMD "bi-radial" approximation to this orbit. Two gyro-radii (ρ_1 , ρ_2) with the same center of curvature (denoted by line C.O.C.) are used to define the inward and outward radial extent of the orbit.

along the field used by the author is given by L. S. Hall²⁹ (based on earlier work by Cordey and Taylor³⁰):

$$\frac{n(B)}{n_0} = (2R_0 - 3)(\tanh^{-1}(x) - x) + \frac{1}{3} B^{-1} x^3, \quad (3-1)$$

where $x = (1-B)^{1/2}$ and $B = \frac{B}{B_0}$. B_0 is $|\vec{B}|$ at the mirror throat and R_0 is the centerplane effective mirror ratio, with n_0 the central density (at $B = B_{\min}$).

The integration over a flux tube was first done by Moir and Taylor for a simple mirror³¹. Using flux conservation ($\phi_M = \vec{B} \cdot \vec{A}$), the cross sectional area of the flux tube varies with B as:

$$BA = B_0 A_0, \quad (3-2)$$

where the subscript "0" refers to the centerplane value. The volume integral then becomes:

$$V = \int A dl = \int \frac{B_0}{B} \frac{dl}{L_m}, \quad (3-3)$$

and the density integrals become

$$\bar{n} = n_c \int n(B) \frac{B_0}{B} \frac{dl}{L_m} \quad \text{and} \quad (3-4)$$

$$\overline{n^2} = n_c^2 \int n^2(B) \frac{B_0}{B} \frac{dl}{L_m}, \quad (3-5)$$

where L_m is the half-length between mirrors.

The local magnetic field B is given as a function of the axial parameter l using a cubic fit and observing the mirror-field boundary conditions listed below, which determine the values of the coefficients:

$$B = Al^3 + Cl^2 + Dl + E, \quad \text{and at} \quad (3-6)$$

$$l = 0, \quad B = B_{\min},$$

$$\ell = L_m, B = B_{\max},$$

$$\ell = 0, \frac{dB}{d} = 0,$$

$$\ell = L_m, \frac{dB}{d} = 0.$$

Substituting $S = \frac{\ell}{L_m}$ into the equation for B as a dimensionless parameter and solving for A, C, D and E gives:

$$\frac{B}{B_{\min}} = -2R_0 S^3 + 3R_0 S^2 + 1 \quad (3-7)$$

This expression for B is then used to evaluate the above integrals.

The average n and n^2 thus obtained can be used to find neutral trapping rates ($\propto n$) and reaction rates for fusion and Coulomb scattering ($\propto n^2$).

3.3 Beam Deposition

The incident neutral beam currents as a function of energy and position are also generated in FOREMSET. This is done by assuming that each injected beam is spatially and energetically Gaussian with mean position (XB), energy (WIB) and e^{-1} widths (DXB, DWB) given. The beams are then broken down into sections a shell width across (Fig.10) and into the plasma energy-group distribution. Because trapped ion deposition depends on the direction of $\vec{v} \times \vec{B}$ at the creation point, each beam section must also be treated for left or right incidence.

Advantages to this method of treatment are in the great accuracy of beam deposition (spatially and energetically) and that any number of beams with various radial positions, widths and energies can be treated as easily as a single beam. This model represents a vast

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Figure 10. Shows the decomposition of a beam with a Gaussian spatial distribution of half-max-width HMW and centered a distance X_B above the plasma center. The current under the Gaussian (e.g., shaded regions) is assumed to be distributed across the beam sections (parallel lines) of width ΔR (equal to the radial shell spacing). The " I_5 " component is the beam fraction which misses the plasma.

improvement over the one-or two-energy, square-current profile beams of earlier codes and is especially useful in considering low energy components of injected beams created by acceleration of the parasitic D_2^+ and D_3^+ ions formed inside the injector³².

Deposition of the gyro-centers of trapped beam ions has been discussed in some detail already. The fractional beam-trapping region is effectively displaced vertically by one gyro-radius, as shown in Fig. 6, for each energy and is mapped back onto the radial spatial grid, as already discussed. The portion of trapped ions (at a given energy) whose centers are deposited in an affected shell is proportional to the fraction of the displaced area overlaying that shell. The deposition of centers is functionally dependent only on beam position, event (trapping) shell position (and thickness) and gyro-radius (the same as used in the density conversion) so that a "lookup" table can be generated for use in the time dependent code, saving redundant computation. The initialization routine DEPOSE does this encoding of deposition fractions and radial extrema into a single floating-point word, similar to the density conversion lookup method described earlier. This routine is separate from FOREMSET because its output is a function only of the radial and gyro-radius meshes which often do not change from case to case.

These lookups are then used in the time dependent FOREMD model to calculate changes in the density-of-centers and time-averaged density. A beam deposition routine generates the change in density-of-centers for newly trapped beam ions using the decoded weighting factors just described. Losses due to charge exchange, slowing

down, loss cone outscattering, etc., require converting from the local time-averaged loss to an actual loss of ions contributing to that region (i.e., the change in ion gyro-centers). This is done by assuming that the local losses occur at random and do not depend on the precise location of the original source ion contributing to the local density. Then the loss of ions from each donor region is proportional to its original contribution to the reactive (event) region. Thus, converting local loss to a density-of-centers loss is done by working backwards from the initial density contributions to each loss region. This technique is more complicated in the case of ion slowing, where the original contributions to the adjacent (higher or lower) energy group must be considered to find the density change at a given energy. For such a case, a sum of the energy gain/loss rates for an ion contributing to affected shells is made, weighted by the original density contributions as before, and a net energy gain or loss is calculated from the initial to the appropriate adjacent energy group (this method is accurate because the ion orbital period is less than the time step used in FOREMD).

3.4 Energetics Using Time-Averaged Density

The previous sections have described how the time-averaged density is calculated in each shell from the density of orbit centers at each energy. Once this local density distribution has been found, it can be used to find energy transfer rates between ions and electrons, between very low energy and hot ions, and also Coulomb 90° scattering times for ion-ion collisions. These

processes are treated using a formulation derived by Sivukhin²¹ which has been modified to permit the use of an arbitrary energy distribution constructed as a sum of (Kroneker) delta functions in energy (i.e., $f(E) = \sum_j a_j \delta(E-E_j)$). By substituting this form of the distribution into Sivukhin's integrals one finds that the ion collision rate for the j^{th} energy group onto the k^{th} group is:

$$\tau_{jk} = C_{jk} \times \begin{cases} v_j^3/v_k^2, & v_k \leq v_j \\ v_j^2/v_k, & v_k > v_j \end{cases}; \quad (3-8a)$$

$$(3-8b)$$

$$\text{where } C_{jk} = \frac{m_j^2 v_{I_k}^2}{8\pi e^4 Z_j^2 Z_k^2 \Lambda} \quad (3-8c)$$

and C_{jk} contains factors of ion mass (m), charge (C), Coulomb logarithm (Λ) and the R.M.S. velocity of the target plasma (V_I). To find the overall scattering time for a particle in the j^{th} energy group, a sum is done over all collision times giving a net 90° scattering time:

$$\tau_{ij} = \left[\sum_{k=1}^n (\tau_{ijk} / (1 + \delta(j-k)))^{-1} \right]^{-1} \quad (3-9)$$

Ion-ion energy transfer is calculated in a similar fashion.

The electron-ion energy exchange rate ($\frac{dE}{dt}$) has already been evaluated by Sivukhin for a Maxwellian electron distribution and is given by the formula (for the j^{th} ion energy group):

$$\frac{dE}{dt_j} = \frac{-4\pi Z_i^2 e^4}{v_{ij}} n_e \Lambda \left[\frac{\phi(\frac{v_{ij}}{v_e})}{m_e} - \frac{2(m_i+m_e)v_{ij}}{\sqrt{\pi} m_i m_e v_e} \exp(-\frac{v_{ij}^2}{v_e^2}) \right] \quad (3-10)$$

where the i subscript refers to ions and the e subscript refers to electrons. The function ϕ here is just the Gaussian error function (erf) defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-s^2) ds. \quad (3-11)$$

The relaxation of the ion distribution is obtained from a Fokker-Planck treatment (1-D [energy], 1-species [ions], developed by MacDonald and Rosenbluth^{33,34}) which solves the separated, time-dependent energy equation to obtain the speed distribution. This formulation is modified slightly to allow treatment of a non-uniformly spaced energy grid. This requires evaluating the velocity derivatives over unequally spaced intervals and uses a net spacing which spans across the group of interest to adjacent groups on either side.

The deconversion from local density changes to that of the density of orbit centers requires a careful "bookkeeping" of particle transfers in each group. This information is not readily available from the Fokker-Planck equations used in the present model. Fortunately, only a knowledge of the net change to and from each group is required to find the gross particle transfers to and from each grid point. By assuming that small-angle collisions dominate the scattering, gross particle shifts can be found by comparing the initial and final distributions after each time step. Working from the lower edge of the distribution (low energy) where the flow of energy (and thus particles) to or from the single adjacent (i.e., higher) group is obvious, the source and magnitude of changes for each succeeding group are calculated using a running sum of the number of particles shifting energy levels and the apparent change in the distribution over a time step. The particle transfers are then transformed into a change in the orbit-center distribution as previously discussed.

3.5 Inclusion of Drift-Cyclotron Instability

In a small, hot mirror ($\rho_i \sim R_p$) ion energy transport can be strongly influenced by velocity diffusion due to the Drift Cyclotron Loss Cone (DCLC) instability³⁵. Therefore, the addition of a DCLC diffusion term to the Fokker-Planck treatment is required in order to correctly calculate ion losses. This term can be obtained for a simple model using the diffusion equation:

$$\begin{aligned} \frac{\partial F}{\partial t_D} &= \nabla_v \cdot D_{\perp} \cdot \nabla_v F \quad \text{or alternately,} \\ \frac{\partial F}{\partial t_D} &= \frac{1}{v^2} \frac{\partial}{\partial v} D_{\perp} (v^2 \sin^2 \theta \frac{\partial F}{\partial v} + v \sin \theta \cos \theta \frac{\partial F}{\partial \theta}) + \\ &\quad \frac{1}{v^2 \sin \theta} \frac{\partial}{\partial \theta} D_{\perp} (v \sin^2 \theta \cos \theta \frac{\partial F}{\partial v} + \sin \theta \cos^2 \theta \frac{\partial F}{\partial \theta}) \end{aligned} \quad (3-12)$$

in spherical velocity space, where $D_{\perp}$ is the perpendicular DCLC velocity diffusion coefficient given by Rognlien³⁶:

$$D_{\perp} = \frac{J_m^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_{ci}} \right)}{v_{\perp}^2} \cdot \left(\frac{q \phi_m}{m_i} \right)^2 \frac{m^2 \omega_{ci}}{2 \omega_m} \quad (3-13)$$

Here m is the mode number (usually taken to be 1), J is the common Bessel's function, $v_{\perp}$ is the velocity component perpendicular to the magnetic field ($v \sin \theta$), $k_{\perp}$ is the wave number, ω_{ci} is the ion cyclotron frequency, ϕ_m is the radio-frequency potential (m th mode), m_i is the ion mass and $\Delta \omega_m$ is the correlation frequency.

Averaging Eq. 3-12 over the angular part of the velocity distribution gives:

$$\begin{aligned}
\langle \nabla_v \cdot D_{\perp} \cdot \nabla_v F \rangle_{\theta} = & \frac{1}{\int_{\theta_c}^{\pi-\theta_c} \sin \theta d\theta} \left[\frac{1}{v^2} \left(\frac{\partial}{\partial v} \left\{ D_{\perp} v^2 \int_{\theta_c}^{\pi-\theta_c} \sin^3 \theta F d\theta + \right. \right. \right. \\
& D_{\perp} v \int_{\theta_c}^{\pi-\theta_c} \sin \theta \cos \theta \frac{\partial F}{\partial \theta} d\theta \left. \left. \right\} + \right. \\
& \frac{1}{v^2} \int_{\theta_c}^{\pi-\theta_c} d\theta \frac{\partial}{\partial \theta} \left\{ D_{\perp} v \sin^2 \theta \cos \theta \right\} \frac{\partial F}{\partial v} + \\
& \left. \left. \frac{1}{v^2} \int_{\theta_c}^{\pi-\theta_c} d\theta \frac{\partial}{\partial \theta} \left\{ D_{\perp} \sin \theta \cos^2 \theta \right\} \frac{\partial F}{\partial \theta} \right] , \right.
\end{aligned}$$

(3-14)

where θ_c is the loss cone angle.

If we now assume that the distribution function F is separable (to first order) into $f(v)g(\theta)$ and that $g(\theta)$ can be approximated as $g(\theta) \sim \delta(\theta - \pi/2)$ for small mirror ratios, then Eq. 3-14 becomes:

$$\langle \nabla_v \cdot D_{\perp} \cdot \nabla_v F \rangle_{\theta=\pi/2} = \frac{1}{v^2} \frac{\partial}{\partial v} \left[D_{\perp} v^2 \frac{\partial f(v)}{\partial v} + D_{\perp}' v f(v) \right] \quad (3-15)$$

Expanding Eq. 3-15 gives:

$$\begin{aligned}
\langle \nabla_v \cdot D_{\perp} \cdot \nabla_v F \rangle_{\theta=\pi/2} = & D_{\perp} \left(\frac{\partial^2 f}{\partial v^2} + 3v^{-1} \frac{\partial f}{\partial v} + v^{-2} f \right) + \\
& \frac{\partial D_{\perp}}{\partial v} \left(v^{-1} f + \frac{\partial f}{\partial v} \right) . \quad (3-16)
\end{aligned}$$

The derivative $\frac{\partial D_{\perp}}{\partial v}$ can be shown to reduce (for $\theta = \pi/2$) to:

$$\frac{\partial D_{\perp}}{\partial v} = \left(\frac{e\phi_1}{m_i} \right)^2 \frac{\omega_{ci}^2}{\Delta\omega_i} \left(\frac{\omega_{ci}}{k_{\perp}} \right) \cdot \left[\frac{2J_1(x) J_0(x)}{x^2} \right] - \frac{4}{x} D_{\perp} \quad (3-17)$$

where $x = \frac{k_{\perp} v}{\omega_{ci}}$ and $m = 1$ has been assumed.

In 2XII-B, typical values for the parameters in $D_{\perp}$ are (from experiments); $\Delta\omega_i \geq 0.1 \omega_{ci}$, $\phi_1 = 10 - 40$ v and $k_{\perp} = 1 \text{ cm}^{-1}$, where the 10 Volt figure for ϕ_1 is for a cold-stream stabilized plasma.³⁶

3.6 Time Step Considerations

In order to ensure numerical stability the time step is reduced until the change in density due to charge-exchange, slowing-down, and beam trapping is less than 1% (usually, much less) in any energy group above the plasma potential energy $|e\phi_p|$. Below this energy, ions are lost so quickly due to acceleration by the plasma potential that this criterion cannot be used and still obtain a reasonable time step. Therefore, these low-energy groups maintain only very small numbers of ions on the average (proportional to the ratio of the streaming time (τ_s) to the time step (Δt)). The time step must be small to ensure the stability of the Fokker-Planck treatment. Step sizes range typically between about 0.005 and .1 ms.

Other details of this model, such as the plasma potential calculation, have been considered in sufficient detail in Chapter 2 and will not be treated again in this section. In summary, the FOREMD code solves a set of coupled, nonlinear, first-order differential equations. These equations express the particle and energy balances in the plasma as a function of time and radial position. The actual equations are set up in matrices, but the basic balances are as follows (for the local time-averaged density without a stream plasma):

PARTICLE BALANCE:

IONS:

$$i^{th} \text{ group: } \frac{dn_i(E_i, R_\ell)}{dt} = \sum_k^{\max} \frac{I_B(E_i, R_k)}{v(R_\ell)} (1 - \exp(-\Delta R / \lambda_{tB_{ik\ell}})) -$$

$$\begin{aligned}
& \sum_k^{k_{\max}} \sum_j \frac{I_B(E_j, R_k)}{V(R_k)} (1 - \exp(-\Delta R / \Lambda_{tB_{jk\ell}})) \frac{\sigma_{cxB_j}}{\sigma_{tB_j}} + \\
& - \frac{I_c(R_k)}{V(R_k)} (1 - \exp(-\Delta R / \Lambda_{tc\ell})) \frac{\sigma_{cxc\ell}}{\sigma_{tc\ell}} - \\
& n_i(R_k) \left[\sum_j \frac{n_j}{\tau_{ij}} P_{Lc_{ij\ell}} - \sum_j n_j \langle \sigma v \rangle_{F_{ij\ell}} \right]
\end{aligned}$$

ELECTRONS:

$$\text{fluid: A) } \frac{dn_e(R_k)}{dt_{LC}} = 3.9 \times 10^{-15} n_e^2(R_k) \phi_e(R_k)^{-3/2}$$

$$\left[2 \left(\frac{x}{\pi} \right)^{1/2} \exp(-x) + \operatorname{erfc}(x^{1/2}) \right],$$

$$\text{where } x = \frac{q\phi_e(R_k)}{\frac{1}{2} m v_e^2}$$

$$\text{B) } \frac{dn_e(R_k)}{dt} = \frac{-dn_e(R_k)}{dt_{LC}} + \sum_j I_B(E_j, R_k) (1 - \exp(-\Delta R / \Lambda_{t\ell})).$$

$$\frac{\sigma_{ion_j}}{\sigma_{t_j}} + \frac{I_c(R_k)}{V(R_k)} (1 - \exp(-\Delta R / \Lambda_{tc\ell})).$$

$$\frac{\sigma_{ion_{c\ell}}}{\sigma_{tc\ell}}$$

ENERGY BALANCE:

IONS:

$$i^{\text{th}} \text{ group: } \left. \frac{dn_i E_i}{dt} \right|_{R_L} = E_i \left. \frac{dn_i}{dt} \right|_{R_L} - (E_i - E_e) \left. \frac{n_i n_e}{\tau_{i-e}} \right|_{R_L} +$$

S.R.E.

ELECTRONS:

$$\text{fluid: } \left. \frac{dn_e E_e}{dt} \right|_{R_L} = - \left(\frac{2}{3} E_e + q \phi_p(R_L) \right) \left. \frac{dn_e}{dt_{LC}} \right|_{R_L} +$$

$$\sum_j (E_j - E_e) \left. \frac{n_i n_e}{\tau_{i-e}} \right|_{R_L} +$$

$$\frac{m_e}{m_i} \sum_j I_B(E_j, R_L) E_j (1 - \exp(-\Delta R / \lambda_{TjL}))$$

$$\frac{\sigma_{ionj}}{\sigma_{tBj}}$$

$$- P_{rad}$$

Symbols

n	density
E	energy
V	volume of radial shell
ΔR	thickness of radial shell
R	radius
I	current
σ	cross section
ϕ_p	plasma potential
P_{LC}	loss-cone probability
P_{rad}	cyclotron and bremsstrahlung radiation
Λ	mean free path
τ	collision time/(particles/cm ³)
m	mass
q	electron charge
S.R.	secondary retrapping (gains-losses)
S.R.E.	secondary retrapping energy (gains-losses)

Subscripts

i, j	i^{th} or j^{th} energy group
k, ℓ	k^{th} or ℓ^{th} radial grid point (shell)
k_{max}	maximum contributing shell number
B	beam
t	total
c	cold gas
cx	charge exchange
ion	ionization or ion
F	fusion
LC	loss cone
e	electron
i-e	ion-electron

The ion particle balance equates the rate of change of the local density (n_i) at each radial position (R) for each energy group (i) to the various sources and losses shown on the right-hand-side (RHS). The RHS terms in order of appearance are the neutral beam trapping rate, beam c-x loss, secondary (c-x generated) beam retrapping, cold gas c-x loss, loss-cone loss (includes D.C.L.C., if considered), and fusion loss.

The electrons, because of their small gyro-radii are rapid self-equilibration time and are treated as a point-particle Maxwellian distribution. The electron density buildup rate is balanced by the loss-cone loss (dn_e/dt_{LC}) (from T. Dolan)²⁵ and the beam and cold gas ionization contributions, respectively, in the expression.

The rate-of-change of nE , the density-energy product, is determined by the particle loss-rate for each energy group times the energy of that group minus the energy loss due to electron drag. The electron energy loss-rate is a balance between the loss-cone loss, which removes an energy $|q\phi_p| + T_e$ in crossing the plasma potential (ϕ_p) barrier, and the energy gained from the ions, with the third term, the ionized-beam contribution being very small.

The particle and energy balances outlined above are solved self-consistently by FOREMD at each time step. A flow chart of calculations done by FOREMD is given in Fig. 11. The FOREMD model is a complex and sophisticated treatment of mirror plasma buildup. The following chapters will show that this model functions correctly to generate useful information on plasma buildup in proposed and operating mirror devices.

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Figure 11. Shows a flow chart of processes in the time-dependent FOREMD model.

Chapter 4: SOME EARLY CASE STUDIES

4.1 Radial Shell Spacing Criteria

Prior to beginning work on the present FOREMD model, the author was able to use the Livermore FGR buildup code (by B. Stallard) to examine charge exchange erosion in FERF (Fusion Engineering Research Facility, now the Mirror Fusion Test Facility, MFTF)³⁷. In the course of these studies, it was found that for correct treatment of finite orbit effects on plasma buildup it is important to have:

$$\Delta R \leq \rho_i \quad , \quad (4-1)$$

where ΔR is the radial shell grid spacing and ρ_i is the (mean) ion orbit radius. This is illustrated in Fig. 12 which shows the equilibrium density profile resulting for cases run using various spatial-grid spacings. Rapid convergence occurs for spacing smaller than the gyro-radius.

Another criterion, which can be important for mirror plasmas, is to require that:

$$\Delta R \leq \Lambda_{cx} \quad , \quad (4-2)$$

where Λ_{cx} is the mean-free-path for cold-neutral charge exchange (D^0 or D_2^0) in the first shell where a significant fraction of the incident neutrals (20-40%) undergo charge exchange.

The first criterion simply implies that if finite orbit contributions are to be considered, the spatial mesh must be fine enough to resolve them. A too-coarse grid results in the vast majority of the ions being treated as point particles (fully contained

Figure 12. Shows the rapid convergence for a c-x boundary erosion case in a MFTF-type plasma. For radial shell spacing $\Delta R > \rho_i$, finite orbit effects are unresolved and the model approaches a point particle treatment. For $\Delta R < \rho_i$, rapid convergence to the steep FGR-type density profile is observed.

in a shell). As discussed earlier, point particle models are inadequate for treating plasmas where the ion gyro-radius is of the same order as the plasma radius because such models ignore coupling between plasma regions spanned by these large orbits.

The second statement requires that the cold neutrals experience less than an e-folding of charge-exchange attenuation in passing through any one shell, at least until cumulative degradation has reduced the cold flux considerably (by $\geq 60\%$). This prevents overly optimistic estimates of beam trapping versus loss in a shell (if most of the c-x losses actually occur in a thin outer layer but, because of the inadequate resolution, are treated as being lost from the entire shell volume). Replacement of these losses would be overestimated to be from trapped beam ions contributing to the entire shell rather than just those contributing to the local region. This constraint is not as severe for D_2^0 as D^0 because D_2^0 charge exchange is non-resonant and, therefore, the cross section $\langle\sigma v\rangle$ is generally smaller and Λ_{cx} larger.

For D^0 at thermal energies ($\sim .03$ eV) the c-x mean-free-path (in cm.) in a plasma of ion energy $W_1 < 15$ keV is about

$$\Lambda_{cx} \sim \frac{10^{12}}{n}, \quad (4-3)$$

where n is the ion density cm^{-3} , which can be a serious constraint, depending on the density profile. For a typical mirror plasma (e.g., density 10^{13} – 10^{14}cm^{-3}) a realistic computational accuracy in the boundary profile does not extend much beyond an order of magnitude, possibly two, below the peak density. Below this point the initial

target choice may still strongly affect the density.

Fortunately, thermal reflux from the first wall is expected to be primarily D_2^0 boiloff, with the D^0 from hot c-x neutral bombardment refluxing with a minimum energy at least comparable to the recombination energy ($2D^0 \rightarrow D_2^0$ or $> 1-2$ eV). Franck-Condon neutrals (D^0) from electron dissociation of D_2^+ have energies of order ~ 1 eV, as well. Finally, it is reasonable to expect that most neutrals leaking through the injection ports will also be either D_2^0 or D^0 with an accelerated/neutralized spectrum of energies above $\sim 1-2$ eV. At energies of a few electron-volts the c-x mean-free-path (MFP) for D^0 becomes, less restrictively for the model being used, an order of magnitude larger than the thermal case, which has been found acceptable for most 2X-type target and early buildup-stage plasmas of interest with shell sizes on the order of 0.5 to 1 cm. At higher plasma energies Λ_{cx} is a rapidly decreasing function of plasma ion temperature and the restriction from this effect on shell width becomes negligible.

In the multi-energy treatment criteria similar to the above are expected. Eq. 4-1 is satisfied for the smallest value of ρ_i treated as a finite orbit (at an energy well below the injection energy) in the FOREMD model. Eq. 4-2 is also satisfied for the 0.5 to 1 cm shell widths typically used.

4.2 Plasma Erosion

Another effect was observed in early test cases where isotropically incident cold gas caused severe charge-exchange erosion. In such

cases the plasma either eventually disappeared altogether $[n(D^+) = n(D_2^0)]$ or collapsed in radius to within about a distance $\sim 2\rho_i$ of the beam edge (for a square beam), as evident in Fig. 12, for example.

This effect can be explained by observing the ratio of trapped beam fraction to shell volume near the boundary as illustrated in Fig. 13. Assuming $\Delta R \ll \lambda_t$ (the trapping MFP), the rate of density increase due to beam trapping is approximately

$$\frac{dn_i}{dt}_B = I_B (\Delta R / \lambda_t) / V_{\text{shell}}, \quad (4-4)$$

where I_B is the beam current, and V_{shell} is the shell volume given for large R by

$$V_{\text{shell}} = 2\pi R \Delta R L_s, \quad (4-5)$$

where L_s is the length of the cylindrical shell. This shows that $\frac{dn_i}{dt}$ varies inversely with R . Actually, this calculation for V_{shell} (4-5) is local and does not give a true picture of the distributed volume occupied by a large-orbit ion. Assuming that for an accurate calculation, as previously discussed, we have chosen $\Delta R < \rho_i$, then the volume throughout which trapped ions are distributed is (ignoring any radial displacement of the orbit center)

$$V = 2\pi R (2\rho_i) L_s, \quad (4-6)$$

assuming $R > \rho_i$. This then gives:

$$\frac{dn_i}{dt}_B = \frac{I_B (\Delta R \lambda_t)}{4\pi R \rho_i L_s}. \quad (4-7)$$

The charge exchange loss rate, again assuming $\Delta R < \lambda_{cx}$ (which is likely only approximately true) is

$$\frac{dn_i}{dt}_{cx} = -K n_o a(\Delta R \lambda_{cx}), \quad K = \text{const. and} \quad (4-8)$$

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Figure 13. Illustrates the fractional plasma volume over which beam trapping takes place (hashed). The trapped beam ions must replace losses in the entire plasma volume, which is larger (added white region) when the plasma radius exceeds the beam width. At low density, trapping is proportional to R but plasma volume is proportional to R^2 , making a larger plasma more difficult to sustain against c-x losses.

where the surface area A is given by

$$A = 2\pi R L_s. \quad (4-9)$$

If we take the ratio of this loss rate to the trapping gain (magnitude only) the functional relationship becomes clearer;

$$\text{LDR} = \frac{dn_i}{dt_{cx}} / \frac{dn_i}{dt_B} \propto K' R^2, K' = \text{const.} \quad (4-10)$$

The ΔR dependence cancels and the "loss deficit ratio" (LDR) increases as R^2 . (i.e., $\text{LDR} < 1 \rightarrow$ buildup, $\text{LDR} > 1 \rightarrow$ decay).

If the plasma radius falls much inside the beam width, two factors come into play that usually lead to a "runaway collapse" situation. First, any beam missing the plasma will probably contribute to an increased cold gas density because of reflux from the wall (or beam trap). Second, and more immediately important, the beam intercepted and trapped by the plasma is reduced by a factor of about R/W_b , where W_b is the beam width, giving:

$$\text{LDR} \propto CR, C = \text{const.} \quad (4-11)$$

Looking at this in the converse sense, for $R > W_b$ reducing R will rapidly decrease the $c-x$ loss deficit ($\propto R^2$) allowing the possibility that a stable equilibrium may be reached at some smaller value of R . But if $R < W_b$, the advantage gained from reducing R is proportionately less ($\propto R$) and an equilibrium becomes much harder to achieve, any excess of gas then requiring a proportionate reduction in plasma radius. With the gas increase from a large passed beam fraction, a runaway collapse is very likely and is frequently observed in various test cases as mentioned previously.

This decay to within $\sim 2\rho_1$ of the beam boundary has also been

observed in FOREMD simulations, but is not as sharply defined because the "edge" is much more diffuse than for a square-beam case. This effect is demonstrated for the present model in Chapter 6.

Chapter 5: VERIFYING THE MODEL

5.1 Convergence

Numerous tests have been performed to determine the correctness, accuracy and self-consistency of results from the FOREMD code. Particle and energy conservation are satisfied to the expected accuracy of the numerical calculations. The case of a collisionally decaying, unsupported plasma agrees with theoretical predictions for classical decay and local nT values are found to be proportional to $W_1^{3/2}$, as expected.

Convergence of the energetic solution was tested by comparing the energy distribution in equilibrium to that of other Fokker-Planck results for global treatments with satisfactory agreement (e.g., see Fig. 14).

Convergence of the spatial solution, critical to proper FGR treatment, is illustrated in Fig. 15A which shows radial density profiles for the same 2XII-B-like case run with 20 and 40 shells of 1 cm and 0.5 cm spacing, respectively. The corresponding temperature profile is shown in Fig. 15B. The density profile agreement is very good, even at the boundary, where the slopes are nearly the same and the low density edge has contracted about 1 cm in the smaller shell case, the limit of resolution in the former case. The central region shows a small inaccuracy in the density calculation for the innermost two shells. This error is non-cumulative and results from an inaccuracy in treating this innermost region. As will be shown, this inaccuracy is not very important. To minimize this error a small shell width was

Figure 14. Shows a comparison between the energy distribution generated by the FOREMD Fokker-Planck package (in an earlier, single-shell version) versus Riviere's fit to Kuo-Petravic's Fokker-Planck results (65-keV injection) at equilibrium ($t > 1 \text{ s}$). Excellent agreement is observed.

Figure 15A. Shows convergence of finite orbit/c-x erosion effects for $\Delta R < \rho_i$ as in Fig. 13 for B. Stallard's model. The slope near the edge remains fairly constant and the edge moves "inward" about 1 cm at the e^{-1} point in going from the $\Delta R \sim 1$ -cm case to the $\Delta R \sim .5$ -cm case, distance equal to the 1-cm spacing accuracy of the large-spacing case.

Figure 15B. Shows a similar convergence (at the boundary) for the ion temperature profile. The boundary change is somewhat more marked than for the density.

used in all calculations. This problem was later corrected.

5.2 Comparison to Other Models

In order to show that the FOREMD model generates results which are correct within the limitations of the physics used in the model, comparisons with experimental data or results from other, similar models were necessary. Early versions of the FOREMD code did not include DCLC effects on the ion-electron loss rates and therefore could not accurately simulate 2XII-B experimental results. While the present model was being developed a similar model was under development at Lawrence Livermore Laboratory by A. Futch.¹⁷ Futch's code includes an energy-dependent FGR treatment and DCLC effects but does not include any local magnetic field effects. Also, treatment of cold gas processes and beam trapping are much more simplified in this Livermore model. It was necessary to alter both codes slightly to make them more compatible so that test simulations could be run for a small, classical (no DCLC) plasma similar to 2XII-B.

The plasma parameters for this comparison are given in Table I. Fig. 16A shows the density profiles generated by the two models, at time $t = 0.9$ ns, early into buildup but on the order of the target displacement time. The solid line represents the Livermore result for a parabolic initial radial density profile. The dashed lines show the FOREMD results at the same time starting from somewhat square-profile targets of 8 cm and 10 cm radius. The results agree remarkably well, both in the interior and at the boundary, considering that the two models contain considerably different treatments of the

Table I: Test Case Parameters

PLASMA TARGET PARAMETERS

$$W_I = 2 \text{ keV}$$

$$W_e = 0.2 \text{ keV}$$

$$N_I = 2.5 \times 10^{13} \text{ cm}^{-3}$$

$$R = 8 \text{ cm}$$

$$B = 1.0 \text{ T}$$

$$N_D^0 = 1 \times 10^{10} \text{ cm}^{-3}$$

$$L_M = L_S = 30 \text{ cm}$$

INJECTION PARAMETERS

$$E_{INJ} = 18 \text{ keV}$$

$$I_{INJ} = 200 \text{ A}$$

$$WIDTH = 8 \text{ cm}$$

$$CENTER: R = 0 \text{ cm}$$

CODE SPECIFICATIONS

ENERGY RANGE 5 eV - 100 keV

NO. ENERGY GROUPS 38

NO. RADIAL SHELLS 40

SHELL THICKNESS 0.5 cm

NO DCLC LOSS

Figure 16A. Shows a comparison between results from A. Futch's code (A.F.) and the FOREMD model (M.C.) using two different targets. The Futch results are for a parabolic target density profile (8-cm half-max) and this is compared to FOREMD cases using 8-cm and 10-cm step-function (square) radial profiles. Excellent agreement is observed at time $t=1 \text{ ns}$. Later tests show that the agreement remains very good for times of at least 10 ns.

cold gas, beam trapping and other factors. Temperature profiles are compared in Fig. 16B, where both parabolic and square target results from FOREMD are compared to Futch's data (the parabolic density profile being very close to the square profiles already shown in Fig. 16A). At this early time in plasma evolution the temperature profile depends greatly on the initial target conditions. A steep rise in temperature occurs over a region near the boundary where recent trapping of hot beam ions, whose large orbits extend out into the low density region, constitute most of the local edge density. Where this transition occurs depends greatly on the initial density profile and hence, the location of near-edge trapping. This region moves radially outward as the target profile becomes more diffuse. The lower minimum temperature generated by FOREMD is probably due to slight differences in the targets and in methods of cold gas treatment. At a later time ($t = 1.9\text{ms}$), Fig. 16C shows that the temperature profiles have converged as expected if the earlier difference were due to initial target conditions (of a transitory nature). The flatness of this profile can be understood with the realization that all cold target ions, with their small orbits have been replaced by hot beam ions with large orbits ($\rho_1 \approx 3\text{ cm}$) by $t = 1.9\text{ ms}$. These large orbits tend to smear the "local" temperature over a region $2\rho_1$ wide.

The comparisons with the Livermore results show such excellent agreement (even to times of several milliseconds) that there is no doubt that the FOREMD code functions properly and adequately treats

Figure 16B. Shows the radial ion energy profile comparison between Futch (A.F.) and FOREMD (M.C.) results. The 8-cm and 10-cm radius FOREMD cases are similar and only the 8-cm case is shown. A parabolic-target FOREMD case was run and is also shown. At early times ($t \sim 1$ ms) the initial target profile is still important in determining the temperature profile, particularly at the edge where large, beam-ion orbits extend beyond the target into the vacuum region, accounting for the rapid temperature rise at the edge.

Figure 16C. Shows the same case (either 8- or 10-cm target) as 16A + B but at $t = 1.9$ ms. The ion temperature profile has flattened and converged with the Futch parabolic-target result.

the finite orbit effects. The next section presents various buildup results obtained using the FOREMD model.

6.1 Introduction

The FOREMD model can provide considerable insight into factors important to the achievement of plasma buildup in a wide variety of mirror devices. This section includes a study of the importance of some basic buildup parameters (e.g., beam position and energy, nT , c-x erosion rate) followed by a survey study of plasma evolution predictions for various proposed or on-line mirror devices. These devices include 2XII-B, TMX end cell, FERF (MFTF) and TEM. For reference, the major parameters for these devices are listed in Table II.

6.2 An Analytic Model

As discussed in earlier sections and particularly in Chapter 4, plasma buildup from a cool, low density target will occur only if losses due to cold gas charge-exchange can be overcome. This loss is confined to a boundary region (ΔR_{CX}) near the plasma surface (see Fig. 17) due to the small c-x mean-free-path (Λ_{CX}), and if beam trapping is not sufficient to replace this loss the boundary will move radially inward until such an equilibrium is reached or the plasma is destroyed. Because the orbits of hot, beam ions are comparable in size to the plasma radius, c-x losses in the region ΔR_{CX} can be replaced by beam particles trapped at points as far away as $2\rho_i$ from the c-x region. This concept is illustrated in Fig. 17, where the "x"s at R_1 , R_2 and R_3 are various locations of particle trapping from the beam which lead to orbits contributing to the TAD in the c-x loss region. Replacement trapping for these losses, therefore, occurs over a larger volume than just the cold c-x region itself (as would be predicted by a point particle model).

Table II: Design Parameters for Various Mirror Devices*

Parameter/Device	2XII-B ^{17,44}	40 keV ⁴⁴ TMX	FERF/ ^{45,46} MFTF	TBM ⁴⁷
E _{START} (keV)	(Gun)	18	20	70**
I _{START} (A)	(Gun)	100-500	~1000	600**
E _{inj} (keV)	~18	40	65-80	300
I _{inj} (A)	300-600	100-200	600-1000	60
W _i (keV)	~13	~36	~50-70	~140
T _e (keV)	~0.15	≥0.3	~1-5	~7
n _i (10 ¹⁴ cm ⁻³)	0.3-1.0	0.3-0.7	1-3	1-2
n _T (10 ¹⁰ cm ⁻³ -s)	7	~30	100-600	>10 ³
β	~0.5-1.0	0.4-0.8	0.5-0.7	~0.9
STREAM	Yes	(Yes)	Little	—
B _{VAC} (T)	0.4-1.0	1.0	2.0-3.7	~3.7
R _M (B _M /B _C)	2	2	2	2
L _M (cm) (vacuum)	~50	~50	170-210	~150
V _p (1)	~4.5	~5-10	~500	>100
R _p (cm)	~8-10	~10	~30	~25

* A range of design parameters is given in most cases. All values are plasma-averaged.

**Both 70 and 300 keV beams are injected simultaneously into a 2X-like target (W_i ~ 20 keV, n_i ~ 10¹⁴).

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Figure 17. Shows three beam trapping events (Xs) which generate orbits contributing to replacement of losses in the cold-gas c-x region of e^{-1} gas attenuation thickness ΔR_{C-X} . The ion trapped at R_1 contributes to the c-x region at a distance $2\rho_1$ from the trapping, at R_3 this distance is about ρ_1 and at R_2 the trapping event has occurred inside the c-x region.

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Figure 18. Illustrates the time evolution of two possible, theoretical plasma buildup schemes. In "a" the beams are directed so that the trapped beam ions are deposited toward the plasma center. The plasma boundary c-x decays at first (e^{-1} densities at r_1, r_2, r_3) but buildup in the center proceeds until central trapping is so large that the boundary also builds up from centrally trapped ions whose orbits extend into the c-x region. Part "b" shows the alternative method of directing beam deposition outward so that edge losses are immediately replaced and the boundary builds with the plasma radius increasing or nearly constant at all times.

A first impression of how to solve the cold c-x problem might be to somehow increase beam trapping or increase the fraction of trapped ions with orbits extending into the c-x loss region. The trapping can be increased by raising the beam current or by varying plasma parameters that affect beam trapping, such as the target density, electron and ion energies, and plasma radius or by varying the factors that affect the density or density profile such as nT or B (which changes the profile through ρ_i). Orbit contributions to the c-x region can be increased via radial beam positioning so that the $\vec{v} \times \vec{B}$ force on the deposited beam ion directs the orbit to pass through the cold c-x region, as will also be discussed later.

A different plasma buildup method from that described above is to consider the problem as a race between plasma decay and collapse due to erosion in the surface c-x region and buildup from beam trapping in the interior. The density can fall at the plasma edge and build at the center (largely shielded from cold c-x losses) until eventually beam trapping in the interior becomes sufficiently large to replace the boundary losses occurring about $2\rho_i$ further out (radially). In order to promote a rapid buildup in the interior, a deposition of beam ions that orbit through the plasma center rather than the c-x edge region as suggested before, is desired. It will be shown later that beam positioning (directing the beams at particular plasma regions) can give the desired, preferential central buildup. Hypothetical buildup cases showing the radial density profile at various stages for a "loss replacement" and an "edge-decay/center-buildup" scenario are illustrated in Fig. 18. [Profile shapes are approximations to FOREMD results which will be discussed later.]

A restriction on the "decay/buildup" method is that the plasma edge must neither fall far inside the beam "edge" (e^{-1} point), so that

a large fraction of passed beam hits the wall, nor remain slightly (~10%) inside the beam for a long period of time (>~2ms) since gas reflux and wall heating can become serious problems under either condition (wall heating can also be a restriction on devices with "gettered" walls and beam dump, which usually saturate on a time scale of milliseconds). The reader is referred back to Chapter 4 on this topic.

Regardless of which build up concept is chosen, the net result desired is to build the plasma density to a point where beam trapping is sufficient to replace losses in the cold c-x region. For a small (2X-like), classical (no DCLC) plasma the analysis of this problem can be simplified to obtain an analytic expression which estimates the minimum density required to obtain this equilibrium.

The particle balance is given by the local beam trapping rate less the local c-x loss rate given approximately as:

$$\frac{dN}{dt} = I_B (1 - \exp(-\Delta R_B / \Lambda_B)) F_{FGR} - \quad (6-1)$$

$$I_{cx} (1 - \exp(-\Delta R_{cx} / \Lambda_{cx})) F_{cx}$$

where I is the incident current, ΔR is the appropriate "trapping" length for beam-replacement or cold-c-x-loss, Λ is the trapping mean-free-path and each F is a correction factor (explained below) with subscripts B and CX referring to the injected-beam and cold-gas components, respectively.

The factor F_{cx} corrects the trapped-gas fraction to include only the component that charge exchanges and is equal to

$$F_{cx} = \frac{\langle \sigma v \rangle_{cx}}{\langle \sigma v \rangle_{cx} + \langle \sigma v \rangle_i} \quad (6-2)$$

where σ is the cross section and cx and i refer to charge exchange and ionization, as before.* The factor F_{FGR} corrects beam trapping to include only the ionized fraction of the beam ($c-x$ events do not, in general, increase the density) and also corrects the beam contribution for finite-orbit effects (the orbiting ion spends only a fraction of its time locally).

For a typical 2X-type case we use the typical parameters given in Table III. The F_{FGR} is partially determined by the average fraction of the density contributed by an ion to the $c-x$ region (versus) the interior). If the ion is assumed to spend an equal amount of time in each radial shell containing part of its orbit, this factor is estimated to be typically $\geq \frac{\Delta R_{cx}}{2\rho_i} \sim 0.25$. This is then multiplied by the ionized fraction, $\frac{\langle \sigma v \rangle_i}{\langle \sigma v \rangle_{cx} + \langle \sigma v \rangle_i} \sim 0.5$ at 18 keV, to give $F_{FGR} \sim 0.13$ for the 2X-like case. The value of R_g is determined to be the maximum radial extent over which ions trapped from the beam will contribute to the $c-x$ region (see Fig. 17).

* For crosssection values used see Ref. 40 (and Fig. 31).

Table III: Analytic Buildup Terms

$$I_B \sim 450 \text{ A}$$

$$I_{cx} \sim 50 \text{ A}$$

$$\Delta R_B \sim 2\rho_i + \Delta R_{cx} \approx 7.5 \text{ cm (maximum)}$$

$$\Delta R_{cx} \sim 1.5 \text{ cm (from FOREMD result)}$$

$$\rho_i \sim 3 \text{ cm (18-keV D}^+, 10 \text{ kG)}$$

$$F_{cx} \sim 0.5$$

$$\Lambda_{cx} \sim 1.5 \text{ cm}$$

$$F_{FGR} \sim 0.13$$

Setting dN/dt in Eq. 6-1 equal to zero at equilibrium and solving for Λ_B gives:

$$\Lambda_B = \frac{7.5 \text{ cm}}{-\ln(0.65)} = 17 \text{ cm.} \quad (6-3)$$

We can now use Eq. 6-3 to solve for the density required for equilibrium by noting that:

$$\Lambda_B = \frac{v_B}{n \langle \sigma v \rangle_B}, \quad (6-4)$$

where the beam velocity (v_B) is about 1.0×10^8 cm/s for D^+ at 18 keV and $\langle \sigma v \rangle_B$ is of order 1.4×10^{-7} so that solving for n gives

$$n \sim 4 \times 10^{13} \text{ cm}^{-3}, \quad (6-5)$$

or the interior density must be $\geq 4 \times 10^{13} \text{ cm}^{-3}$ to guarantee plasma buildup. Cases where the cold gas current approaches or exceeds the total injected beam current are, of course, unstable and will decay regardless of the initial target density.

As will be shown in following sections the results from a self-consistent FOREMD calculation show that Eq. 6-5 overestimates the critical density for plasma buildup by a factor of ~ 2 . This overestimate is likely due to the fact that we are solving for a term which

lies in an exponential and is therefore very sensitive to errors in initial assumptions and cross section values (estimated vs. distribution-averaged).

6.3 Charge-Exchange Erosion

In order to show that gas c-x erosion is still an important factor in the energy-dependent model, several buildup cases were run with a flux of cold D^0 (~50 A) incident on a 2X-like plasma (a few times the estimate for 2X, simulating a high injector leakage of D^0 gas, assumed to be fully "on" at $t=0$). Figure 19 shows the rapid c-x decay of a classical plasma (2X, but no DCLC) supported by injection of 375 A at 18 keV (beam e^{-1} width at ± 4 cm, centered on plasma). Although the plasma begins to build up in the central region ($t=1$ ms), c-x erosion proceeds inward and eventually destroys the plasma. Figure 20 shows the same case for 450-A injection where the edge begins to decay but buildup in the interior is fast enough to eventually overcome the c-x losses and the plasma proceeds to buildup classically. The 375-A beam current used in the first run ordinarily would be sufficient to support a dense plasma ($n_i \sim 3 \times 10^{13}$ with DCLC, $n_i \sim 4 \times 10^{14} \text{ cm}^{-3}$ without DCLC) if charge exchange were absent. The reason that such a high injection current is required to overcome c-x is that these losses all take place in a region about ρ_i thick as explained in Chapter 5. In order to trap the ~50 A of beam required to replace the c-x losses at the plasma boundary the density must reach some critical value. At this critical density the beam trapping mean-free-path is short enough that trapping replaces all losses in the c-x region (see section 6.2). The boundary moves inward until such an equilibrium is reached, otherwise the plasma decays to

Figure 19. Shows the c-x erosion (due to ~50-A D⁰) and decay of a 2X-like plasma supported by 375 A of 18-keV beam. Plasma parameters other than the injection current are the same as in Fig. 20, listed in Table IV.

Figure 20. Shows plasma buildup for the same case as in Fig. 19 (see Table IV) but for 450-A injection. In this case the boundary decays and the interior density increases ($t = 1$ ms) until the edge c-x losses are replaced by beam ions trapped in the interior ($R \sim 5-6$ cm) whose orbits ($\rho_i \sim 3$ cm) extend into the boundary ($R \sim 9-10$ cm). This case is used as a "base" or reference case for various following comparisons.

extinction. At high gas levels the equilibrium radius (if any) or e^{-1} density point is about $2\rho_1$ beyond the e^{-1} width of the beam (see Ch. 4).

6.4 Effects of Plasma Parameters on Buildup

Having established that charge-exchange erosion can cause serious plasma losses, we next consider what plasma parameters are most important in governing the severity of this loss and what can be done to assure plasma buildup. Section 6.2 mentioned some of the various parameters that can be varied and briefly discussed some possible techniques for achieving plasma buildup. In this section we examine results from FOREND for cases where these parameters are varied independently in order to ascertain their relative importance to plasma build up. For this study a simple, 2X-like plasma was chosen with a base 2X-like case (Table IV) of 450-A injection at 18 keV (same centered, 8 cm beam width as in Fig. 19 case) onto a target of density $\sim 8 \times 10^{12} \text{ cm}^{-3}$ and energies $W_i \sim 2.4 \text{ keV}$ and $W_e \sim 250 \text{ eV}$ (the same as the 450-A case in Fig. 20). No DCLC losses or stabilizing stream were considered in these cases. A list of the parameters which will be examined include:

- 1) Beam current
- 2) Beam position
- 3) Beam energy spectrum
- 4) Magnetic Field Strength
- 5) Target Radius
- 6) Target Density
- 7) Other target parameters (Temperature, n_T)

Table IV: Basic 2X-like Case

PLASMA TARGET PARAMETERS

$$W_I \sim 2.4 \text{ keV}$$

$$W_e \sim 200 \text{ eV}$$

$$N_I \sim 8.2 \times 10^{12}$$

$$R_T \sim 8 \text{ cm}$$

$$B \sim 1.0 \text{ T}$$

$$L_M \sim 30 \text{ cm (effective)}$$

INJECTION PARAMETERS

$$W_{INJ} \sim 18 \text{ keV}$$

$$I_{INJ} \sim 450 \text{ A (various)}$$

$$\text{Beam } e^{-1} \text{ pt. } \sim \pm 4 \text{ cm (centered)}$$

CODE SPECIFICATIONS

$$\text{ENERGY RANGE} \quad 5 \text{ eV} - 100 \text{ keV}$$

$$\text{NO. ENERGY GROUPS} \quad 38$$

$$\text{NO. RADIAL SHELLS} \quad 40$$

$$\text{SHELL THICKNESS} \quad 0.5 \text{ cm}$$

NO DCLC OR STREAM

These effects will be treated in the order of their importance. In order to facilitate these studies, some were done using a simplified, mono-energetic version of the FOREMD code which allowed the variation of a number of parameters (e.g., T_e , nT) independently that would normally have to be calculated self-consistently in the multigroup model. The primary justification for using a mono-energetic treatment for studying buildup in a classical plasma (no DCLC) is that buildup in a cool target (~ 2 keV) is dominated by electron ionization and c-x displacement during early stages. Beam c-x displacement of the target occurs typically in less than 1 ms in 2X. This time is much shorter ($\sim 1/50$) than either the ion or electron collision times (τ_{i-i} or τ_{i-e}), so there is little spreading of the injected beam energy distribution over this early buildup time period (illustrated in Fig. 21). Both multi-energetic and mono-energetic simulation show that the time required for cold gas (D^0) c-x erosion of the plasma (if destructive) is also ~ 1 ms under typical conditions (radius $R_p \sim 8$ cm, D^0 current ~ 50 A). Therefore, a mono-energetic ion treatment is satisfactory for studying early buildup ($t \sim 1$ ms) in a classical 2X-like plasma.

That the mono-energetic model can give good agreement with a multi-group treatment during early buildup stages is illustrated for a classical 2X-type plasma in Fig. 22. Further considerations in using a mono-energetic treatment, such as the choice of an ion-electron energy transfer rate coefficient and plasma nT coefficient will be discussed later.

Figure 21. Shows the energy distribution of a 2X-like plasma (Fig. 20) where 450 A at 18 keV is injected into a target plasma (see Table IV). By time $t = 1 \text{ ms}$ the distribution is dominated by the injection peak at 18 keV with the target remnant (at 2-3 keV), originally Gaussian, rapidly decaying as the plasma potential increases and drives out low-energy ions. Charge exchange with the beam leads to the nearly mono-energetic distribution.

Figure 22. Shows a comparison between the multi-energy base case (Table IV) and a mono-energetic test case (described in the text), both with 450-A injection at 18 keV and 50-A D^0 incident. The profile agreement is very good, verifying the use of a mono-energetic treatment for early buildup ($t \leq 2 \text{ ms}$) in a classical (no DCLC) plasma.

Injected beam positioning has a dramatic effect on plasma buildup. As discussed in Chapters 2 and 3, the trapped beam ion is deposited so that its orbit center is displaced a gyro-radius up or down depending on the direction of $\vec{v} \times \vec{B}$ (Fig. 23). In a small plasma (2X, TMX) where restrictions on the beam width cause it to be about equal to R_p , there is little room for positioning the beam without missing the target. However, the direction of the beam and, hence, the direction of $(\vec{v} \times \vec{B})$ deposition can be readily changed and will accomplish much the same effect as changing beam position would in a larger plasma.

Studies were done for a classical, 2X-type plasma with left and right incident beams (giving inward and outward depositions, respectively) directed at the upper half of the target. The ratio of the two beam currents was varied while keeping the total injected current constant (450 A at 18 keV). Results for these cases with their respective ratios of inward to outward depositing beam currents are given in Fig. 24 for time $t \approx 1$ ms. Somewhat surprisingly, the cases which build up and overcome c-x erosion are those with more than 50% of the beam depositing ions inward toward the plasma center rather than outward toward the c-x loss region. This is due to the requirement that the central density must rise above some critical value before cold gas penetrates to the interior, as discussed earlier. This critical value can be estimated by comparing the peak central ($R \lesssim 4$ cm) density achieved for the borderline-decay case of Fig. 19 and the central density of the borderline-buildup case of Fig. 20 at about the time of the density peak in Fig. 19 ($t \sim 1$ ms, as nearly as possible). The empirical value of this density is about $1-2 \times 10^{13} \text{ cm}^{-3}$ for

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Figure 23. Illustrates the direction of deposition of trapped-ion orbit centers. Right and left directed beams deposit orbit centers a gyro-radius up or down depending on the direction of $\vec{v} \times \vec{B}$. For the $\vec{B}$ direction shown and a beam displacement "h" above the plasma center the right-incident neutral becomes an ion with center located at R_1 , radially further out than the point of ionization and the left incident ion center is deposited at R_2 located radially inward from the right-directed case. These cases deposit ions which contribute primarily to the edge (c-x) and central plasma regions, respectively.

Figure 24. Illustrates the effect of changing the inward/outward beam deposition ratio for a fixed total injected current. This is done by varying the ratio of left/right (out/in) incident current while holding all other parameters at the base-case values (no DCLC). The two cases shown where beam deposition is directed primarily inward build up (.15/1 and .53/1 cases) do not build to high enough densities to overcome c-x erosion and eventually decay (explanation in text). The base case uses .53/1 out/in directed beams.

injection of 450 A at 18 keV with 50 A of D^0 for a classical 2X case. At densities low enough that beam trapping is roughly $\propto n_i$, this "critical density" increases roughly linearly with the incident D^0 current and decreases linearly (very roughly) with higher beam current. The reason why inward deposition reaches the critical density more readily is due to the smaller volume of an inner flux surface (Fig. 25), which leads to a higher density for a given number of trapped ions (as indicated in previously discussed cases). For example, an ion of orbit radius 2 cm created at $R = 2$ cm will be contained in a cylindrical flux tube of volume $V \approx \pi \cdot (2)^2 \cdot L$ if deposited inwardly and $V \approx \pi \cdot (4^2 - 2^2) \cdot L$ if outwardly directed, giving a respective density ratio of 3:1. [Note that previously discussed cases have all had an out/in ratio of 0.53/1].

Various cases were run to check the relationships between beam current, c-x losses and the critical density. Figure 24 shows a centered, 450-A injection case which is marginally unstable to c-x erosion and eventually decays. Figure 26 shows the base case but with 600-A, centered injection with 50-A D^0 gas incident compared to the same 600-A case with 65-A D^0 ($t=0.5$ ms). This latter case should also be destroyed by c-x erosion according to the beam current/c-x loss relationships discussed in Section 6.2 (i.e., buildup is proportional to beam current and inversely proportional to the cold gas current). The increase in beam current is 33% while the increase in the D^0 flux is about 30%. Therefore, the plasma buildup behavior is consistent with predictions in Section 6.2 based on "critical density" arguments, the critical density at $t=5$ ms for 600-A injection being $>8.5 \times 10^{12}$ but $<1.1 \times 10^{13} \text{ cm}^{-3}$ for centered injection and the theoretical value is found to be between 7×10^{12} to $1.4 \times 10^{13} \text{ cm}^{-3}$ (2/3 rds of the 450-A case).

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Figure 25. Inward or outward deposition of trapped beam ion depends on the direction of $\mathbf{v} \times \mathbf{B}$ and the beam position. Inward deposition places the ion orbit in a smaller volume of plasma (V_2) than in the outward deposition case where the flux surface occupied by the ion has a larger volume (V_1) and therefore the ion contributes to a smaller change in the density of this region (V_1) than if deposited in the inner volume (V_2).

Figure 26. Shows a case with 600-A, 18-keV injection for D^0 currents of 50 and 65 A at $t=0.5 \text{ ms}$. The 65-A D^0 case eventually decays away, verifying predictions based on the analysis in Section 6.2.

A rapid density buildup in the center is more critical in preventing c-x destruction than immediately replacing boundary losses, as already discussed. A limitation on this method in an actual device, though, is that if the plasma shrinks due to decay to an extent where a large fraction of the beam misses the plasma, the resulting cold reflux from the beam dump may cause a runaway erosion as described in Chapter 4. In 2X and other small devices "gettering" is used to delay first-wall reflux for about 1 ms which allows sufficient time for the critical density to be reached well before c-x erosion becomes severe. In somewhat larger devices this technique may not be practical.

Additionally, the gyro-orbit size can affect buildup. This factor enters through two parameters, the magnetic field strength (B) and the energy spectrum of the injected beam. The ion gyro-radius ($\rho_1 = \frac{mv_1 c}{qB}$) is inversely proportional to the magnetic field and proportional to $\sqrt{E}$, where E is the injection energy.

Figure 27A shows a comparison between a standard, single-beam case with $B = 1.0$ T and multi-energetic injection and low B cases (all run with the multi-energy version for a classical, 2X-like plasma). In the multi-energy injection case the beam is divided into a main and two "subharmonic" energy components 18/9/6 keV in the ratios 1/0.8/0.2 respectively (occurring from acceleration of D_2^+ and D_3^+ to 18 keV before eventual neutralization) with the total current remaining 450 A, as in the base case. A higher central density results from the overall improved beam trapping (Λ_t is shorter for the low-energy components and more beam is trapped). The boundary profile, however, is much steeper than in the 18 keV case. The ion temperature profile in Fig. 27B offers an explanation for this steeper density gradient. The lower boundary ion temperature (energy) leads to a larger density gradient in

Figure 27A. Shows a comparison between the base-case result and results for two separate methods of altering the mean ion orbit radius, the first being multiple-energy injection (18/9/6 keV at 225/180/45 A, respectively, to simulate various beam components) and the second changing the vacuum magnetic field. In the former, buildup is enhanced by improved beam trapping due to the relatively shorter mean-free-path ($\Lambda_T \sim (\sqrt{W_i})^{-1}$) for the low energy components. The low $|B|$ case shows a lower density because the larger ρ_i causes ions to occupy a larger plasma volume (see text, Fig. 25) thus reducing their average density contribution. The larger ρ_i also leads to a less-steep density gradient at the edge, which falls an order of magnitude in a distance of about ρ_i .

Figure 27B. Compares the ion temperature profiles for the single- and multiple-energy injection schemes. The 18/9/6-keV injection case produces a lower mean ion energy, as expected.

two ways. First, the gas c-x cross section is somewhat higher for the low-energy beam ions present in this case, enhancing losses. Second, the plasma density tends to fall off exponentially beyond the beam edge, as discussed in Chapter 5. An order of magnitude drop in density occurs over a distance on the order of ~ 1 gyro-radius so that the smaller average orbit radius at the boundary in this multi-energy injection case implies a steeper density gradient, which is observed. Quantitatively, this effect accounts for the edge profile being "pulled in" by about 0.5 cm, additional steepness being due to enhanced losses as discussed.

Returning to Fig. 26, we see that a 20% drop in the magnitude of $\vec{B}$ causes the internal density to drop by 20-30% and causes a less-steep edge profile than in the base case. These effects are primarily due to each orbit extending over a larger volume of plasma. The increase in ρ_1 is $\sim 25\%$ (ρ_1 at 18 keV is ~ 3 cm in the base case for deuterium) and the plasma volume occupied by the ion (given approximately as $V \sim \pi R(2\rho_1)L_p)$ is increased proportionately causing a drop of about 25% in the peak density but generating a flatter profile with a more gradual fall-off at the edge (again a ten-folding in about ρ_1 distance). Therefore, this effect accounts for the entire difference between the base and reduced B profiles. Two other factors that affect plasma buildup in the presence of c-x erosion are the target radius and density. An increase in target size of 50% results in a $\sim 50\%$ increase in central target density (compare Fig. 28 to Fig. 20). Increasing the target density by 30% over the base case gives the result shown in Fig. 29. The total beam trapping is very sensitive (exponentially) to these parameters and large, dense targets are desirable but, as mentioned, a given device may be restricted as to maximum target density and radius

Figure 28. Shows plasma buildup at two stages ($t = .5 \text{ ms}$, $t = 1.0 \text{ ms}$) for injection (with 50-A D^0) onto a 12 cm target (vs. 8 cm for the base case). All parameters other than the target radius are held at base-case values. The density builds more rapidly due to improved beam trapping and shielding of the central region ($R < 9 \text{ cm}$) from c-x erosion effects and reaches a peak value nearly twice that of the 8-cm target case by $t = 1 \text{ ms}$.

Figure 29. Shows that increasing the target density (by $\sim 30\%$) improves the target buildup rate. Again all other parameters are those of the base case. By time $t = 1$ ms the density peak for the dense-target case is twice that of the base case.

by the experimental arrangement used (e.g., continuous stream or pulsed guns).

As mentioned in Section 6.3, the choice of ion-electron energy transfer and coefficients must be considered in designing the mono-energetic model. For the cases considered here, the electron energy (W_e) was calculated using a simple rate equation:

$$\frac{dW_e}{dt} = \frac{W_i - W_e}{\tau_{ie}} - \frac{P_{LC}}{n_i} \left(\frac{dn_i}{dt} \right)_{LC} \cdot (|e\phi_p| + \frac{2}{3} W_e) ,$$

where W_i is the ion energy, τ_{ie} the ion-electron equilibration time, P_{LC} the loss-cone probability, n_i the ion density, $\frac{dn_i}{dt}_{LC}$ the loss-cone loss rate, and ϕ_p the plasma potential as before. The quantity in brackets at the far right is the average energy lost by an electron climbing the potential barrier.²⁴

A question arises as to what form should be used for the electron-ion energy transfer rate. The τ_{i-e} calculated by Sivukhin²¹ is an instantaneous loss rate and represents an e-folding time for the energy loss. The ions would thermalize in about 2-3 e-folding times if allowed to equilibrate with the electrons. For the early buildup cases being considered here, the energy transfer rate is recalculated at each time step. Since the time step is much shorter than τ_{i-e} (several milliseconds) the only inaccuracy in calculating energy transfer arises from using an average ion energy (W_i) instead of a discrete distribution. This error is small, particularly since the total elapsed time for cases considered here is still less than τ_{i-e} , as mentioned previously.

Figure 30 shows a comparison between various initial assumptions about the electron temperature on plasma buildup. A 30% decrease in the initial target electron (and ion) temperature has little effect on the density profile at $t = 1$ ms. An increase in the electron-ion transfer rate by a factor of three results in a significantly lower density since most of the 50 A of incident D^0 is not ionized by the hotter (700-eV vs. 350-eV) electrons. Consequently, the D^0 penetrates the target further and causes c-x losses. [For $100 \text{ eV} < W_e < 450 \text{ eV}$ this effect is not severe because $\langle \sigma v \rangle_i$ is fairly constant.] Figure 31 shows velocity-averaged (Maxwellian) cross-section values for H^0 which can be used to estimate the cold gas attenuation for various T_e and T_i .³⁸ The effect of changing the ion collisional $n\tau$ is illustrated in Fig. 32 where a reduction of more than an order of magnitude creates only a 10% drop in density at $t = 1$ ms. The reason this effect is small is that during early buildup in a low-density target, collisional losses are negligible (about 3% of the c-x loss because n^2 is small) and do not greatly affect plasma development at this stage. Therefore, the early evolution and c-x resistance of the plasma is fairly insensitive to what $n\tau$ value is chosen in the monoenergetic treatment. A case run using the $n\tau$ inputted in B. Stallard's model, given by $n\tau \sim 0.45 \times 10^{10} \cdot \log_{10} R \cdot W_i^{3/2}$ which is a factor of ~ 6 smaller than the low $n\tau$ in the figure, again shows only a slightly lower density. In order to obtain the good agreement shown in Fig. 22 for classical 2X cases, however, the latter $n\tau$ was used. A wide range of choices for $n\tau$ is possible because ion losses are due primarily to

Figure 30. Shows target temperature effects on plasma buildup. A 50% target temperature difference (ions and electrons, $W_i = 1.5, 2.1 \text{ keV}$; $W_e = 100, 150 \text{ eV}$) shows no significant change in density profiles by $t = 1 \text{ ms}$. Increasing W_e (to $\sim 600 \text{ eV}$) by decreasing the electron-ion collision time (τ_{ie}), or any other parameter that increases energy transfer decreases the electron-ionization rate for beam and cold neutrals resulting in less beam trapping and greater c-x losses (only $\sim 15\%$, however).

Figure 31. Shows c-x (C-X H°), electron (E-I) and ion (I-I) ionizations reaction rate $\langle \sigma v \rangle$ for H° and c-x (C-X H_2°) $\langle \sigma v \rangle$ for H_2° .⁴⁰ Because these cross sections are velocity dependent values for I-I and C-X reactions are shifted upward in energy by $\sqrt{2}$ for D° , D_2° , and $\sqrt{3}$ for T° , T_2° . For most reasonable values of the electron energy the E-I curve is independent of the neutral velocity (as assumed here) and is about the same for any hydrogen atomic isotope. Also the molecular D-I cross section curve is accurately represented by the atomic curve at energies normally encountered in a plasma.

Figure 32. Shows the effect of changing plasma confinement ($n\tau$) on buildup (often $n\tau$ is not known to better than a factor of 5-10). At early buildup stages ($t < 2 \text{ ms}$) the dominant loss mechanism for cold-target ions (target density $\approx 3 \times 10^{13}$) is beam charge exchange. An increase in the ion collision rate (τ_{i-1}) of $> 10^1 \times \tau_{i-1}$ causes only a slight ($\sim 10\%$) change in density at $t \sim 1 \text{ ms}$ (less at the boundary) showing that buildup is only weakly dependent on $n\tau$. In DCLC-loss cases, where $n\tau$ is a factor of ~ 25 -100 lower, the c-x loss is still somewhat ($\sim 2\times$) larger.

electron slowing into the ambipolar hole (Chapter 2) and DCLC losses which, combined, lead to $n\tau$'s that can range over two orders of magnitude, depending on the loss model used. Fortunately, as we have demonstrated, the exact value used is not critical during early buildup. The value chosen by the author of $n\tau \sim \frac{1}{13} n\tau$ was obtained by trial and error to match the mono- and multi- energetic profiles and lies well within the permissible range of values ($n\tau \sim 10^{11}-10^{13} \text{ cm}^{-3}\text{s}$).^{23,35}

6.5 Buildup With DCLC Losses in 2X

In 2XII-B (and the TMX end cell) the Drift Cyclotron Loss Cone (DCLC) instability has been observed experimentally to dominate plasma losses. The presence of this micro-instability can reduce plasma n_T by more than two orders of magnitude, but stabilization of this instability with a stream plasma reduces the ion loss rate to levels where build up to densities of order 10^{13} to 10^{14} cm^{-3} is possible using less than 600 A of injected beam. Experiments in 2X without stream stabilization yield typical densities on the order of 10^{11} cm^{-3} compared to densities $> 10^{13}$ cm^{-3} with a streaming plasma present. This implies an n_T without stabilization at least 100 times lower than for the stabilized case and > 1000 less than classical Fokker-Planck n_T 's.⁴¹

With a streaming plasma present to fill in the low-energy hole in the ion distribution, DCLC losses are greatly reduced, but ion energy loss to cold stream electrons greatly decreases the ion confinement time. Consequently, even with the stream, n_T remains about 1/10-1/30 of classical.⁴¹ Because stream electron drag of ions into the ambipolar hole dominates, the ion loss rate is proportional to $T_e^{3/2}$ and to n^2 ($n_i = n_e$).

Figure 33A shows a FOREMD simulation of a 2X-like plasma for injection of 600 A at 14 keV (to simulate a multi-energy component beam) into a ~2-keV target of density 5×10^{13} cm^{-3} and a total, penetrating stream current of about 150 A. The density decays until a near-equilibrium is reached at $t \sim 0.6$ ms and a mean density of

Figure 33A. Shows plasma "decay" to near-equilibrium ($t \sim .4 \text{ ms}$) for a stream stabilized Z-X-like case with 600-A injection at 14 keV and a total stream current of 150 A into a dense target ($W_{i,T} = 2.4 \text{ keV}$, $n_T = 5 \times 10^{13} \text{ cm}^{-3}$). The central density "dip" is a result of beam attenuation and the problem discussed in Ch. 5.

about $3.5 \times 10^{13} \text{ cm}^{-3}$. The central "dip", due to beam attenuation exaggerated by a slight under estimate of the central density in the code (later corrected, its lack of importance is discussed in Chapter 5) leads to a lower T_e and n_r in this region. The ion temperature evolution is shown in Fig. 33B and equilibrium is reached in about a c-x displacement time. The temperature rise near the plasma edge during buildup is a result of trapped, hot beam ions whose orbits extend out into the target/vacuum interface region. Near equilibrium, $W_i(\sim T_i)$ approaches an average 12.5 keV and W_e approaches $\sim 260 \text{ eV}$ ($T_e \sim 170 \text{ eV}$). This result agrees fairly well with that obtained by Berk for a similar 2X case using a global DCLC treatment (within 30% in both density and temperature without taking into account any differences between models).³⁵

The ion energy distribution for this 2X-like case is shown in Fig. 34 at time $t=0.4 \text{ ms}$ and radius $R=2.5 \text{ cm}$. The low-energy peak occurs at the stream energy with a "tail" due partly to electron and ion heating by the hot plasma. The distribution falls rapidly from the stream energy and begins to rise for particle energies above the plasma potential energy ($q\phi_p$). Above energy ($q\phi_p$) the amplitude rises rapidly and levels off as the target energy (W_T) is approached. The flat region around this point is primarily the target remnant which is slowly decaying as the cooler ions are c-x displaced by beam ions. A second peak occurs at the injection energy (E_{inj}) as expected. Superimposed on this peak is Riviere's⁴² numerical fit of the classical Fokker-Planck distribution of Kuo-Petravic¹¹ (K-P) for 14 keV injection. The K-P distribution would be generated for the classical (no DCLC or stream) case. In the

Figure 33B. Shows the ion temperature evolution for the case of 33A. The high temperature boundary at $t = 0.1 \text{ ms}$ is due to hot ion orbits extending into the near-vacuum region (as already discussed in Fig. 16B). Later c-x replacement of target ions brings the internal ion temperature up to that at the boundary.

Figure 34. Shows the ion energy distribution, $f(E)$, for the case of Fig. 33A,B at $t = 0.4$ ms and at $R = 2.5$ cm. The distribution is peaked at the stream energy (E_{STR}) and at the injection energy (E_{inj}) with a remnant of the original target observable at the target energy (W_T). The steep "valley" occurs near the plasma potential energy below which loss-cone losses become very large. For comparison a Riviere⁴² fit to the Fokker-Planck generated, classical 14-keV distribution (squares) is superimposed over the 14-keV peak.

DCLC/stream case, electron drag rapidly pulls hot ions down in energy until the critical loss energy ($q\phi_p$) is reached and the ions escape. The high energy "tail" of this distribution requires about an ion-ion collision time (~3 milliseconds) to fill out completely.

Previously, the effect of cold-gas c-x erosion was examined for a classical 2X-type plasma and was shown to place a severe limitation on the minimum injection requirement. In the stream stabilized 2X case, however, it is possible to partially shield the plasma if the stream is radially extended out into the vacuum region. This "shielding" effect, observed by the author using Stallard's code, is due to cold-electron ionization of D^0 and D_2^0 traveling through this sheath region.³⁷ A mean stream density of 10^{11} to 10^{12} cm^{-3} will produce an e-folding drop in the cold neutral flux over a distance of several centimeters and needs to be maintained during buildup only until the plasma reaches a sufficiently high density to overcome c-x erosion.

If shielding with the stream should prove impractical because of excessive heat conduction or energy expenditure then c-x losses can be significant. Figure 35 shows a comparison of plasma boundary density profiles for cases with and without incident cold gas (the interior profile is unaffected at such high plasma densities with 14-keV injection and is not shown). This case is again for 600-A injection and differs from that of Fig. 33A only in having a slightly different stream profile (reduced 10-20% in the interior with a steep drop off beyond the plasma edge). Boundary erosion has steepened the density profile in the case with 50 A of D^0 incident. The stream current

Figure 35. Shows edge profiles for a 2X-like plasma with parameters essentially the same as that in Fig. 33A,B (slightly different (~15%) stream profile) for cases with (50-A) cold gas and without. The boundary profile steepens somewhat while the interior ($R < 7-9 \text{ cm}$) is unaffected.

(which falls off rapidly beyond $R \sim 9$ cm, so that "shielding" is very small) as well as the high plasma density inhibit any deep penetration by the D^0 .

That the cold charge exchange is affected by the presence of the stream is evident in examining the energy spectrum of neutrals escaping to the first wall (Fig. 36). A significant number of secondary c-x neutrals (est. $\sim 10\%$, below E_{inj}) come from c-x interactions between D^0 and the 50-eV stream ions as seen by comparing the difference in areas under the 50-A D^0 /no-gas curves for the energy ranges E_{STR} to $q\phi_p$ vs. $q\phi_p$ to E_{inj} [a much larger fraction of the D^0 is electron ionized]. The peak for escaping neutrals comes at the injection energy and is primarily due to the passed fraction and beam charge exchange. In contrast, the cold D^0 charge exchange occurs at lower energies where $\langle \sigma v \rangle_{cx}$ is higher. Omitting the passed beam fraction decreases the peak at E_{inj} shown by the "squares" in this graph. (This distribution of neutrals hitting the first wall is also useful in estimating wall sputtering rates and gas reflux fractions which depend on incident energy.)^{39,43}

6.6 Buildup in TMX End Cell

Buildup in the TMX end cell (one of two conventional mirrors used to electrostatically stopper the ends of a long solenoidal region whose effects are presently ignored) for 14 to 20 keV injection yields a plasma essentially identical to 2X (particularly for 2X operated at high $\vec{B}$). Therefore, simulations were done for cases with 40-keV injection (soon to come on line) into a target built up by the 20-keV injectors ($W_i \sim 15$ keV, $W_e \sim 350$ eV, $n \sim 5 \times 10^{13}$). Since the proposed injection current for these new beams will be 100-200 A, a near-median value of 160 A was assumed.⁴⁴ Reflux of cold ions from the central cell region of this device was simulated by a streaming plasma similar

Figure 36. Shows the energy distributions of neutrals escaping the plasma to the first wall for the cases with and without cold gas in Fig. 35. The cold gas contributes to $e-x$ losses from the stream and low energy part of the ion distribution. Most of the high energy loss comes from beam charge exchange. The high energy peak is decreased if the "passed beam fraction" (P.B.F.) contribution is omitted (squares).

to that used in the 2X-DCLC cases already discussed.

As in the 2X case, charge-exchange erosion can be an important plasma loss mechanism. Although the c-x cross section is lower at 40 keV than at 20 keV (see Fig. 31), the TMX 40-keV beam current is lower than in the 2X 20-keV case. Also the trapping mean-free-path (Λ_t) is longer and electron ionization of D^0 is less at this higher energy. These latter effects reduce the beam trapping per unit length and, therefore, the number of beam ions trapped per unit volume to replace c-x losses. Competing against the increase in Λ_t is a larger ρ_1 at 40 keV which increases the size of the trapping region that contributes to replacing c-x losses in a given shell ($\sim 2\rho_1 + \Delta R_{cx}$, see Section 6.1). Since ρ_1 increases only as $\sqrt{W_1}$, this positive effect is not sufficient to completely compensate for the Λ_t effect.

The plasma density evolution was calculated for a 50-A incident D^0 case. This neutral flux represents a moderately high injector leakage and is approximately equivalent to a cold neutral density in the vacuum region of order 4 to $5 \times 10^{10} \text{ cm}^{-3}$, which is close to the ~ 2 to 4×10^{10} estimated for the actual device. To be conservative, the cold gas flux is taken to be fully "on" at time $t=0$. The radial ion density profile for this case is shown in Fig. 37A. The density falls gradually (to about $3.7 \times 10^{13} \text{ cm}^{-3}$) in the interior region but remains very flat, indicating a fairly uniform trapping of the 40-keV beam. The boundary expands outward at the far plasma edge due to hot-ion orbits extending out beyond the target edge, but the density of the near-edge drops somewhat due to c-x erosion. Figure 38 shows a comparison between this case and a "no gas" case at $t = 0.4 \text{ ms}$, the slight steepening of the boundary profile due to c-x losses being evident.

Figure 37A. Shows plasma "decay" in the 40-keV/160-A - injection TMX case. The target was assumed to be a 2X-like plasma ($W_{iT} = 15 \text{ keV}$, $n_T = 5 \times 10^{13} \text{ cm}^{-3}$) with 50 A of D^0 present. The equilibration time is longer for this case than for 2X because collisional cross sections (except for ion ionization) are lower at the higher plasma energy and is not quite at equilibrium by $t = 1.2 \text{ ms}$.

Figure 37B. Shows the ion temperature profile evolution. The profile shape is very similar to the 2X case (Fig. 33B), but equilibration occurs after a longer time due to the slower reaction rates at $W_i \sim 30\text{--}40 \text{ keV}$.

Figure 38. Shows a comparison between the with vs. without cold gas (50-A D^0) buildup cases. As in the 2X case the edge gradient is steeper with the cold c-x present, but the effect is spread out over a larger region near the boundary ($R \geq 6 \text{ cm}$) because of the larger mean-free-path of cold D^0 in this 40-keV injection case.

Evolution of the ion temperature (energy) profile is given in Fig. 37B for the corresponding times. An overall, gradual increase is apparent indicating a longer c-x displacement time in TMX than in 2X. The temperature increase is fastest in the central region where the higher density improves beam trapping. Also, any low-energy beam components and cold gas have a shorter mean-free-path and are trapped closer to the plasma edge than hotter components, reducing the ion temperature in the outer region. Including the subharmonic beam-energy component raises the density only slightly but reduces the mean ion energy to ~25 keV (see Table V).

For comparison, a TMX case with 100-A injection (all other parameters the same) was also run. The density profile evolution is shown in Fig. 39. For this low injection current the initial density of $5 \times 10^{13} \text{ cm}^{-3}$ is too low for a c-x equilibrium to occur and the plasma (eventually) decays away (beam trapping is already <30 A by $t = 1.4 \text{ ms}$). This run verifies the importance of c-x losses in 40-keV TMX buildup.

The electron energy and plasma potential radial profiles for the case of Fig. 37A,B are shown in Fig. 40. Both fall off gradually from the central region to the plasma edge. This is an expected result since a self-consistent solution for ϕ_p should give $\phi_p \sim 4T_e$ for typical streaming cases, and T_e increases with density and ion temperature, $(\frac{dE}{dr})_{i-e} \propto n^2 (W_i - W_e)$, which are largest in the central region. The plateau at $r = 8 \text{ cm}$ occurs at the plasma/beam edge where the ion temperature begins to increase again. The plasma potential profile determines the radial confinement (density) profile in the central plug region. While not a self-consistent solution for the full TMX device, this result implies a density peak in the central solenoid region where ions tied to these flux surfaces see the highest potential barrier.

Figure 39. Shows a TMX end cell case for 100-A injection at 40 keV into a target with the same parameters as in Fig. 37A. The initial density of $5 \times 10^{13} \text{ cm}^{-3}$ is not sufficient to trap enough beam to replace the c-x losses and the plasma eventually decays away.

Figure 40. Shows the electron temperature and plasma potential profiles for the case of Fig. 37A,B. The plateaus occur at the edge where W_i is increasing although n is decreasing. Beyond this point W_e and ϕ_p fall rapidly to about the stream energy ($\sim 50 \text{ eV}$).

Evolution of the ion distribution (at $R \sim 2.5\text{cm}$) is shown in Fig. 41. The target distribution (approximately Maxwellian) is predominant at $t = 0.1\text{ ms}$ with only a small peak at the injection energy (40 keV) evident. At later times the target decays (probably primarily through cold-gas charge exchange) and is nearly gone by $t = 1.2\text{ ms}$. As in the 2X case, this distribution will tend to spread out more on times of the order of τ_{1-1} ($>10\text{ ms}$). Ions also slow down rapidly on the cold electrons until an energy close to $|\phi_p|$ is reached and the ion is lost. The low-energy peak is due to the stream ions (as in 2X) and as ϕ_p increases with time a widening "gap" opens up between this peak and the main plasma distribution. Also as in the 2X case, electron slowing has spread the distribution to an extent that using a mono-energetic treatment would prove unsatisfactory.

6.7 FERF Buildup

Simulations were also done of FERF (Fusion Engineering Research Facility), now redesigned and called at first MX and now MFTF (Mirror Fusion Test Facility) plasma buildup.^{45,46} The MFTF device is large enough ($\rho_i/R_p < 1/15$) that DCLC losses and, therefore, stream requirements will be much smaller than in earlier devices. Also, the plasma is much hotter than in 2X or the THX end cells ($W_i \sim 70\text{ keV}$) so that cold-gas c-x erosion is very small ($\langle\sigma v\rangle_{cx}$ for D^0 and D_2^0 is low and ion-impact ionization $\langle\sigma v\rangle_1$ is large).

Cases were for 600-A, 70-keV injection into a 2X-like target ($n_i \sim 10^{14}\text{ cm}^{-3}$) with and without the effect of β on R_{eff} (see Chapter 2) and Fig. 42 shows this comparison for buildup to $\beta \sim 0.4$. Earlier

Figure 41. Shows the TMX end cell ion energy distribution at $R = 2.5$ cm and various times. As in the 2X case the stream is responsible for the low-energy peak which remains nearly constant as the target peak (at W_T , $t = 0.1$ ms) rapidly decays and the injection peak (at E_{inj}) rapidly grows. The increasing plasma potential causes a widening "valley" in the profile just below ~ 1 keV.

Figure 42. Shows a comparison between radial density profiles for FERF (MFTF)-type cases with and without the effect of plasma β on the effective mirror ratio (R_{eff}) with $E_{\text{inj}} = 70 \text{ keV}$ and $I_{\text{inj}} = 600 \text{ A}$, showing a "bump" developing at the edge by time $t = 18 \text{ ms}$. This bump later grows due to beam trapping in this high-density edge region. By repositioning the injectors off-axis to inject into the lower-density flanges, a more uniform trapping can be obtained.

stages show no difference since β is small, but by $t = 18$ ms the improved confinement in the $R_{\text{eff}}(\beta)$ causes increased plasma buildup near the edge where the beam intensity is greatest. The edge density for this case increases with time (as β increases) to about $n_1 \sim 1.5 \times 10^{14} \text{ cm}^{-3}$ and beam attenuation creates a somewhat hollow profile (just beginning to appear by $t = 18$ ms) which may be unstable to inward fluting. To correct this problem, a fraction of the beam would have to be injected near the axial plasma edge (flared region) where the density is lower, as examined by DeVoto.¹⁶

Another problem is that with a centered beam the plasma does not build out to the expected 30-cm radius starting from a small target (~ 16 cm). Using a larger target or positioning the beams so as to deposit ions outward will enhance the hollow-profile buildup problem. Careful positioning of the various injectors will be required to obtain a large diameter, uniformly dense plasma in this device.

A comparison between FOREND results and experimental or theoretical results for 2X, TMX and FERF (MFTF) are given in Table V.

6.8 Twin Beam Mirror

The Twin-Beam Mirror (TBM) is a conceptual design for an enhanced Q mirror with smaller injection requirements than a conventional mirror.⁴⁷ The TBM adds a small amount of injected current at a very high energy (300 keV) in addition to conventional injection (40-90 keV). Although the hot-beam current is only on the order of 20-80 A the power input from this beam can be 30-50% of the total injected

Table V: Comparison of FOREMD to experimental and design results.

	2XII-B Expt.	FOREMD Simulation	FERF ⁴⁶ (MFTF)	FOREMD Simulation	TMX ⁴⁴ End Cell	FOREMD Simulation	
						40 keV	40/20 keV
n_i	$\sim 3 \times 10^{13} \text{ cm}^{-3}$	$\sim 3.6 \times 10^{13} \text{ cm}^{-3}$	$\sim 3 \times 10^{14} \text{ cm}^{-3}$	$\sim 2 \times 10^{14} \text{ cm}^{-3}$	$5 \times 10^{13} \text{ cm}^{-3}$	$3.8 \times 10^{13} \text{ cm}^{-3}$	$3.9 \times 10^{13} \text{ cm}^{-3}$
W_i	$\sim 13 \text{ keV}$	$\sim 13 \text{ keV}$	$\sim 70 \text{ keV}$	$\sim 70 \text{ keV}$	$\sim 26 \text{ keV}$	$\sim 35 \text{ keV}$	$\sim 26 \text{ keV}$
W_e	$\sim 210 \text{ eV}$	$\sim 260 \text{ eV}$	$\sim 7 \text{ keV}$	$\sim 6 \text{ keV}$	$\geq 300 \text{ eV}$	$\sim 750 \text{ eV}$	$\sim 280 \text{ eV}$
R_p	$\sim 8 \text{ cm}$	$\sim 8 \text{ cm}$	$\geq 30-35 \text{ cm}$	$\sim 23 \text{ cm}$	$\sim 9 \text{ cm}$	$\sim 9 \text{ cm}$	$\sim 9 \text{ cm}$
E_{inj}	$\sim 14 \text{ keV}$	$\sim 14 \text{ keV}$	$\sim 80 \text{ keV}$	$\sim 70 \text{ keV}$	*	$\sim 40 \text{ keV}$	40/20 keV
I_{inj}	$\sim 300-400 \text{ A}$	$\sim 600 \text{ A}$	$\sim 500 \text{ A}$	$\sim 600 \text{ A}$	*	$\sim 160 \text{ A}$	80/80 A

* Injected power = 9 MW

power, greatly reducing current requirements for the low energy beam. Additionally, TBM's Q-value is higher because of beam-background fusions occurring as the hot ions thermalize. Applications of this device extend to use in hybrid fusion/fission reactors and a MFTC-type facility.

Previously, the author investigated the effect of hot-beam injection angle on optimizing Q using a 2-D ($v_{\parallel}$, $v_{\perp}$), global Fokker-Planck code supplied by A. Mirin at LLL.^{48,49} Results of these studies show that nearly-perpendicular injection is optimal for TBM studies. With this conclusion, the author felt that the slightly greater accuracy in calculating the ion energy distribution by using a 2-D treatment was outweighed by the necessary increase in computation time for such a model. Therefore, a 1-D Fokker-Planck treatment (energy) has been used in the FOREND model.

A hot injection energy of 300 keV was picked for two reasons. First, this energy is high enough to give a net injection advantage and beam-ion fusions, but is low enough that it represents a reasonable extrapolation from present injector technology. Second, 300 keV is about the highest energy possible for efficient beam trapping (>80%) in a small (25 to 30-cm radius) plasma at typical densities ($2 \times 10^{14} \text{ cm}^{-3}$).

For a typical case the ratio of hot to warm beam currents was chosen to be about 0.1. This gives an equilibrium ratio of hot to background ions (Γ_H) of about 0.2, the hot ions forming an extended tail on the mirror distribution (Fig. 43 shows this for an earlier global simulation).

The radial ion density and temperature profiles obtained by

Figure 43. Shows a comparison between a 60-keV Riviere⁴² distribution (analytic fit to Fokker-Planck data) and a Twin Beam Mirror distribution with 60- and 300-keV injection (at a current ratio of ~10/1, respectively). Both are normalized to a peak density of 1. This result is from an earlier global model developed to test the presently used Fokker-Planck model.

building up from a 2X-type target ($W_i \sim 20$ keV, $n_i \sim 10^{14}$ cm $^{-3}$) using neutral beam injection of 600/60 A at 70/300 keV, respectively, is shown at two different times in Fig. 44A. Earlier, ($t=22$ ms) the density and temperature profiles are very flat but at later times ($t=38$ ms) this flatness disappears as the overall plasma density and radius approach the maximum supportable by 660-A injection (i.e., losses equal injection). Figure 44B again shows the $t=38$ ms density and temperature profiles along with the β profile for this same case. This linear graph shows that at this later time, the ion temperature decreases and the density increases steadily from the plasma center to a radius of about 21 cm, however β (roughly proportional to the density and temperature product) remains nearly constant ($\pm 5\%$) out to ~ 20 cm. These profiles are the result of plasma trapping characteristics for the two beams: the low-energy beam has the shorter mean-free-path and is trapped radially further out while the hot beam is trapped more uniformly across the plasma. Attenuation of the low-energy beam causes a lower central density and a resulting higher ratio of hot to warm trapped ions which leads to the higher central temperature.

The Q-value (fusion power/injected power) for this case is ~ 0.54 . A conventional mirror with the same ion temperature and average radial density uniformly distributed over the plasma volume has a calculated Q about 10% lower than the TBM case. Although these Q values are very close, a twofold advantage in neutron flux still occurs due to the peaked off axis density of the TBM. First, the peak density is higher (over a large region of the plasma) than in a smaller plasma with

Figure 44A. Shows plasma buildup in a Twin Beam Mirror for injection of 600/60 A at 70/300 keV, respectively, into a 2X-like target. At fairly early times ($t \sim 22 \text{ ms}$) the density and ion temperature profiles are flat. At later times ($t \geq 38 \text{ ms}$) the density rises and the temperature falls off near the edge. This effect is due to the difference in trapping mean-free-path for the two energy components which results in uniform trapping of the 300-keV beam but rapid attenuation of the 70-keV component near the high-density edge.

Figure 44B. Shows on a linear scale how the density rise and temperature drop of Fig. 44A at $t=38 \text{ ms}$ compensate each other to allow β to remain nearly constant ($\pm 5\%$) over most of the plasma. A uniform β prevents the development of pressure-driven MHD instabilities.

uniform trapping causing the peak reaction rate to be greater in TBM. Secondly, neutron production ($\propto n_1^2$) peaks at a point nearer the first wall. Both of these effects combine to increase the first-wall neutron flux greatly (by an estimated factor of ~ 3) over the equivalent, conventional (uniform density) case. This improvement could be very significant in a hybrid or MFTF reactor.⁵⁰

It is also important to note that since β (or the pressure) is fairly uniform there is little chance for a pressure-driven instability to develop in the plasma interior. Also, the only added cost for TBM is in developing the high-energy beam technology, which may be necessary anyway to power advanced-fuel systems.

6.9 Summary of Conclusions for Plasma Buildup (2X)

- 1) Charge-exchange erosion can place a severe restriction on the minimum beam current requirement for buildup (a factor of $\sim 5-6$ times larger than the no-c-x case).
- 2) A critical density of about $2 \times 10^{13} \text{ cm}^{-3}$ must be achieved with 50 A of D^+ (I_{CX}) incident and an injection current of 450 A (I_B) at 18 keV. If beam trapping scales linearly with density to first order then the critical density scales approximately as $n \sim 2 \times 10^{14} \text{ cm}^{-3} \left(\frac{I_{CX}}{I_B} \right)$.
- 3) Beam positioning is a critical factor in determining plasma buildup. By directing the beam so that trapped-ion deposition is primarily to the central region a significant reduction in the injected current required to achieve plasma buildup can be obtained (about a factor of $\sim 50\%$ in 2X).

- 4) A large fraction (~50%) of low-energy beam component (at half the peak energy) can increase the rate of plasma buildup at early stages ($t < 1$ ms) because of the shorter trapping mean-free-path for this component. Densities in 2X cases were observed to be twice the "hot-only" density by $t=1$ ms, but this effect was much smaller in the 40-keV, $n_T=5 \times 10^{13} \text{ cm}^{-3}$ TMX with stream present.

In addition to these observations, some general conclusions were also reached:

- 1) The target radius and vacuum magnetic field values also affect buildup (~50% in this study) although not as dramatically as the above parameters. The maximum values of these two factors are generally fixed by other design considerations for a given device (although $|\vec{B}|$ can usually be lowered).
- 2) Plasma nT and target temperature have much smaller effects (~10-20%) on early buildup than any other parameters studied.

To maximize the probability for successful buildup in any mirror plasma where charge exchange is thought to be important as a loss mechanism ($W_i \lesssim 80$ keV), certain startup procedures should be followed for optimum results.

- 1) Use a target where the density-radius product is as large as possible ($n_i R_p > 10^{13} \text{ cm}^{-2}$).
- 2) Position a beam of sufficiently low energy to have a large trapping fraction (>70%) across the target chosen in (1) so that trapped beam ions are deposited toward the plasma center (care being taken that the plasma doesn't decay so much that a large fraction of the beam misses the plasma before buildup occurs).

3. When the plasma has built up to the critical density (approximately $> 2 \times 10^{14} \left(\frac{I_{cx}}{I_B} \right) \left(\frac{\Lambda_B}{\Lambda_{cx}} \right)$ ignoring collisional and stream effects) the low energy beam is switched off (if desired) and the high energy beam (if any) turned on. (Note that Λ_B in the trapping correction term $\frac{\Lambda_B}{\Lambda_{cx}}$ is that for a hot beam neutral). This scenario achieves rapid startup with minimum injected power.

If I_{cx} increases due to reflux at a rate faster than the beam trapping rate, the beam current may need to be larger than the estimates given here. Under such conditions a generalized analytic approach fails and modeling by FOREMD must be done to obtain the data for estimating the necessary beam current.

In summary, studies on a 2X-like plasma using the FOREMD model indicate that neutral beam positioning, a procedure that adds no extra cost to a designed mirror system, can reduce injection requirements to achieve plasma buildup by up to 50%. Other parameters also affect plasma buildup significantly (B , target density, target radius) but, by contrast, require a system design change (bigger magnets, more target plasma guns, a larger containment vessel) that increases the cost of the device.

The FOREMD model has also demonstrated an ability to handle problems that a mono-energetic code can only roughly approximate. While the mono-energetic model is fairly accurate for classical build-up on a short time scale, in stream stabilized DCLC cases, where the velocity distribution spreads much more rapidly, the mono-energetic treatment is not applicable. The FOREMD model has been used to verify

the importance of cold-gas c-x erosion in DCLC dominated plasmas. [TMX requires >100-A, 40-keV injection for buildup.] Multi-energy beam components have a significant effect on buildup (particularly in TBM) and also require a multi-group treatment. For these reasons, the FOREMD model represents a great advance over all previous single-energy models and will be an important tool for accurately modeling mirror plasmas in future design studies.

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APPENDIX A

BASIC MIRROR EFFECTS

In a magnetized plasma, ions and electrons are generally constrained to move along magnetic field lines due to the $q\vec{v} \times \vec{B}$ force which acts on a charged particle (mass "m," charge "q") moving within a $\vec{B}$ field. This force is balanced by the centrifugal pseudo-force:

$$\frac{q(\vec{v} \times \vec{B})}{c} = \frac{mv_{\perp}^2}{\rho} \quad (\text{A-1})$$

which, rearranging, gives the orbit (gyro) radius (ρ):

$$\rho = \frac{mv_{\perp}c}{qB}, \quad (\text{A-2})$$

where CGS units are assumed. The orbital period is given by

$$\tau_c = \frac{2\pi\rho}{v_{\perp}} = \frac{2\pi mc}{qB} \quad (\text{A-3})$$

and the cyclotron frequency is

$$\omega_c = \frac{2\pi}{\tau} = \frac{qB}{mc}. \quad (\text{A-4})$$

In a $\vec{B}$ field with a radial gradient the gyro-radius changes with particle position creating non-circular orbits that do not, in general, close on themselves which leads to a $\vec{v} \times \vec{B}$ drift as (see Appendix C and Fig. 1C). The presence of this drift causes the plasma to be distributed uniformly within a ring of constant flux on a time scale of the order of a few bounce times. The drift velocity is found from the velocity-averaged fluid equations to be:

$$v_d = \frac{\mu \nabla B \times B}{B^2 c}, \quad (\text{A-5})$$

where μ is the "adiabatic invariant" ($\mu = \frac{W_{\perp}}{B}$).

To show that μ is a constant of the particle motion with the constraint of small magnetic field gradients (i.e., the change in B encountered by the particle is small over one orbit), we examine the change in the flux contained within an orbit. From Maxwell's equations the induced electric field changes the perpendicular energy according to:

$$\Delta W_{\perp} = \int q \mathbf{E} \cdot d\mathbf{l} = \int q (\nabla \times \mathbf{E}) \cdot d\mathbf{s} = -\frac{q}{c} \int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}. \quad (\text{A-6})$$

Over an orbit, $\frac{\partial \mathbf{B}}{\partial t}$ becomes, on the average:

$$\frac{\partial \mathbf{B}}{\partial t} \sim \frac{\Delta \mathbf{B}}{\tau_c}, \quad (\text{A-7})$$

where τ_c is the cyclotron period. Assuming that, to first order, the area of the orbit is the unperturbed area ($\nabla B = 0$) and substituting the expression for $\frac{\partial \mathbf{B}}{\partial t}$ into Eq. A-6, we obtain to first order:

$$\Delta W_{\perp} = -\frac{q}{c} \frac{\Delta B}{\tau_c} \int d\mathbf{s} = -\frac{\pi \rho^2 q \Delta B}{c \tau_c}; \quad (\text{A-8})$$

$$\text{but } \rho^2 = \frac{m^2 v_{\perp}^2}{q^2 B^2} c^2 = \frac{2 W_{\perp} m}{q^2 B^2} c^2$$

and

$$\tau_c = \frac{2\pi mc}{qB},$$

so that:

$$\Delta W_{\perp} = q \frac{\pi \Delta B}{c} \frac{2 W_{\perp} m c^2}{q^2 B^2} \left(\frac{qB}{2 mc} \right) = \frac{\Delta B}{B} W_{\perp}. \quad (\text{A-9})$$

Therefore, we obtain to first order that

$$\frac{\Delta W_{\perp}}{W_{\perp}} = \frac{\Delta B}{B}, \quad (\text{A-10a})$$

or integrating both sides,

$$\mu \equiv \frac{W_{\perp}}{B} = \text{constant}. \quad (\text{A-10b})$$

Because $\frac{\partial E}{\partial t} = 0$ is assumed inside the mirror plasma, the total energy (W) must remain constant. Thus any change in $W_{\perp}$ must be accompanied by a corresponding reduction in $W_{\parallel}$, until $v_{\parallel}$ falls to zero at which point the particle is reflected ($v_{\parallel}$ changes sign) and proceeds back toward the plasma center. This "mirroring" effect confines particles on the open magnetic field lines provided the particle encounters a sufficiently large change in $|\vec{B}|$ as it moves along the field. This reflection will take place if $v_{\parallel}/v_{\perp}$ is not too large and $B_{\max}/B_{\min}$ is not too small. The following describes this:

$$W = W_{\perp} + W_{\parallel} \text{ and} \quad (\text{A-11a})$$

$$\mu = \frac{W_{\perp}}{B} = \text{const.} \quad (\text{A-11b})$$

At reflection $v_{\parallel} \rightarrow 0$, $B \rightarrow B_{\max}$ and at some point in the motion $B \rightarrow B_{\min}$ so that:

$$\frac{W_{\perp \max}}{B_{\max}} = \frac{W}{B_{\max}} = \frac{W_{\perp \min}}{B_{\min}}, \quad (\text{A-12})$$

$$\text{or } W = \frac{B_{\max}}{B_{\min}} W_{\perp 0} \quad (\text{A-13})$$

and, using this,

$$\frac{B_{\max}}{B_{\min}} W_{\perp \min} = W_{\perp 0} + W_{\parallel 0} \quad (\text{A-14})$$

and

$$\frac{W_{\parallel 0}}{W_{\perp 0}} = \frac{B_{\max}}{B_{\min}} - 1 \equiv R-1,$$

where R is the mirror ratio ($R = \frac{B_{\max}}{B_{\min}}$) evaluated for the absolute maximum and minimum fields seen by the particle and "o" refers to the $B_{\min}$ -point value. The restriction on the velocity for reflection to take place becomes:

$$\frac{W_{\perp 10}}{W_{\perp 0}} \leq R-1, \quad (\text{A-15a})$$

or

$$\frac{v_{\perp 10}}{v_{\perp 0}} \leq \sqrt{R-1}. \quad (\text{A-15b})$$

In velocity space, this ratio describes a cone of angle

$$\sin \theta_c = \frac{v_{\perp 0}}{v_{\perp 10}} = \frac{1}{\sqrt{R-1}} \quad (\text{A-16})$$

where θ_c is known as the "loss-cone critical angle" and particles with $\theta < \theta_c$ are not reflected (are lost from the system) because $v_{\perp}$ never goes to zero. This cone subtends a solid angle in velocity space given by

$$\text{S.A.} = \pi \int_0^{\theta_c} d(\cos \theta) = 4\pi \sin^2 \theta_c. \quad (\text{A-17})$$

The probability that a particle randomly scattered in velocity space by collisions (through 90°) ends up with its velocity vector inside the loss cone is given by the fraction of the total solid angle making up the loss cone:

$$\frac{4\pi \sin^2 \theta_c}{4\pi} = \sin^2 \theta_c \equiv P_{\text{Lc}} \quad (\text{A-18})$$

In reality, particles scatter through various angles to enter the loss region. The effective loss cone probability is frequently described by an empirical fit to Fokker-Planck data:⁵¹

$$P_{\text{Lc}} = \frac{1}{C(\log_{10} R)}, \quad C = 3-5. \quad (\text{A-19})$$

This form of the loss probability due to velocity scattering is used by the author. The loss rate for scattering into the loss cone is therefore:

$$\frac{dn}{dt} = \frac{n}{\tau_{i-i_1}} P_{Lc}, \quad (A-19)$$

where n is the ion density and τ_{i-i_1} is the perpendicular (90°) scattering time for ion-ion collisions (proportional to $W_i^{3/2}$).

In the presence of a positive plasma potential (ϕ_p , created because the electron scattering time is much shorter than that of the ions, generally) the "loss cone" becomes a "loss parabola" with low energy ions gaining sufficient $v_\perp$ to be lost through acceleration by the potential. This loss occurs for all particles with energies below $|q\phi_p|$ (the "ambipolar hole"). The potential improves electron confinement so that only those with energies greater than the potential barrier can escape. Ions with energies $W_i > |q\phi_p|$ exhibit an enhanced loss rate. To describe this effect the loss cone probability must be altered to:

$$P_{Lc} = \frac{1}{C \log_{10} R_{eff}}, \quad C \sim 3-5 \quad (A-20)$$

and

$$R_{eff} = R_{vac} / (1 + q\phi_p / W_i), \quad (A-21)$$

where R_{vac} is the vacuum mirror ratio.

High beta ($\beta \propto \frac{nkT}{B^2}$) plasmas have an additional factor due to the diamagnetic effect of the orbital magnetic moments. For large β , this can significantly lower the internal magnetic field by a factor of order $(1-\beta)^{1/2}$ so that the mirror ratio becomes:

$$R_{eff} = \frac{R_{vac}}{\sqrt{1-\beta}} \left/ \left(1 + \frac{q\phi_p}{W} \right) \right. \quad (A-22)$$

This is the effective mirror ratio used in the FOREMD model.

APPENDIX B

DCLC INSTABILITY STABILIZATION

The drift cyclotron loss cone (DCLC) instability is an important particle and energy loss mechanism in small mirror devices such as Baseball II and 2X-IIB, as well as in devices outside the U.S.⁴¹ This section is devoted to an empirical (rather than theoretical) treatment based on experimental scaling laws observed in 2X-IIB.^{41, 52} Such an approach was deemed preferable for use in the present code.

The DCLC instability is a micro-instability which occurs in plasmas where the ion gyro-radius (ρ_i) is comparable to the plasma radius and a mirror-type energy distribution is present with a large density gradient at the plasma vacuum interface. The instability arises because of coupling between the ion-cyclotron motion and the tangential drift motion due to $\nabla B \neq 0$. At the edge ∇B is large and because $\rho_e \ll \rho_i$, ion orbits extend out beyond the "edge" of the electron plasma. The absence of electrons in this region inhibits shielding of the electrostatic fluctuations permitting strong wave-particle coupling. Energy from the non-Maxwellian, anisotropic velocity distribution is channeled by these interactions into scattering clusters of ions into the loss cone in an attempt to fill the low-energy hole caused by the plasma potential.⁵³ These losses can reduce plasma nT by more than an order of magnitude from classical expectations.

Possible solutions for alleviating this problem are to considerably reduce $\frac{\nabla n}{n}$ (create a larger plasma or a density inversion by some

means) or shield the boundary region with cold plasma so that the electrostatic coupling between ions is cut off. This last approach has received the most interest and has been used successfully to stabilize 2X-IIB and Baseball II, with an order of magnitude increase in n_T being obtained in the former.⁵⁴

This cold plasma can be introduced via plasma guns which inject along the field lines or by ionization of cold gas molecules from the plasma chamber. The ratio of the cold to hot ion density ($\frac{n}{n_H} \equiv \Delta$) required to suppress this instability depends on ρ/R_p , and the ratio of the stream-ion velocity to the mean plasma-ion velocity $\frac{v_n}{v_H}$. Fig. 1B shows these dependences for $v_n/v_H = 10$.⁵⁵ The value for Δ can be obtained empirically from 2X-IIB experimental results as follows:

$$\Delta \sim 1.5 \left(\frac{v_n}{v_H}\right)^2 \left(\frac{\rho}{R_p}\right)^{4/3} \quad \text{for } \frac{\rho}{R_p} > \frac{v_n}{v_H} \quad (\text{B-1a})$$

$$\Delta \sim .36 \left(\frac{v_n}{v_H}\right)^{1/2} \left(\frac{\rho}{R_p}\right)^2 \quad \text{for } \frac{\rho}{R_p} < \frac{v_n}{v_H} \quad (\text{B-1b})$$

From B-1, Δ is typically on the order of $\sim 10^{-2}$ to 10^{-3} for 2X-IIB and about $\leq 10^{-3}$ to 10^{-4} for an MFTF type reactor.

The DCLC loss begins to disappear when $R_p/\rho \geq 25$ (the density gradient becomes too small to excite it) and will not be as important in large devices. This is fortunate because the cold plasma electrons cause unacceptable energy loss by cooling the hot ions and transporting this energy out the loss cone. Also, secondary cold electrons attracted by the plasma potential, return

Figure 1-B. Shows the stream density fraction (Δ) required for DCLC stabilization for various values of β and R_p/ρ_i . Values for Δ as a function of β and the local value of R_p/ρ_i for $E_{\text{hot}} = 50$ keV and $E_{\text{hot}}/E_{\text{stream}} = 10^{5.5}$.

through the mirror throat and enhance this problem.³⁶ These electron energy losses make stream stabilization of a large, power producing reactor highly impractical.

Incorporation of this instability into the Fokker-Planck treatment has already been considered in an earlier section. The formula given in B-1 is used to find the amount of streaming plasma required for DCLC stabilization in the present model.

APPENDIX C

FUTURE WORK

Various improvements in the present FGR orbit treatment are possible for a very high β plasma (~ 1) where the semicircular orbit approximation becomes too inaccurate. One such method is outlined below and may be included in a future version of the FOREMD model.

Let:

$$B_z = \nabla_r \times A_\phi = \frac{1}{r} \frac{\partial}{\partial r} r A_\phi \quad (C-1)$$

in cylindrical coordinates; r is defined in Fig. 1C which shows a typical noncircular orbit and the radial maxima and minima are denoted by r_1 and r_2 respectively.

A Taylor's expansion of Eq. C-1, about $r = r_0 + \Delta r$ gives:

$$r A_\phi = (r A_\phi)|_{r_0} + \left[\frac{\partial}{\partial r} (r A_\phi) \right] |_{r_0} \Delta r + \left[\frac{\partial^2 (r A_\phi)}{\partial r^2} \right] |_{r_0} \frac{\Delta r^2}{2} + \dots \quad (C-2)$$

But Eq. C-1 also can be rewritten as

$$r B_z = \frac{\partial}{\partial r} (r A_\phi) \quad (C-3)$$

and substituting Eq. C-3 into Eq. C-2 and rearranging gives to third order

$$\begin{aligned} r A_\phi - (r A_\phi)|_{r_0} &= (r B_z)|_{r_0} \Delta r + (B_z + r \nabla_r B_z)|_{r_0} \frac{\Delta r^2}{2} + \\ &+ (2 \nabla_r B_z + r \nabla_r^2 B_z)|_{r_0} \frac{\Delta r^3}{6} \end{aligned} \quad (C-4)$$

If we assume that $\nabla_r B_z$ is about B_z/R_s and the second derivative is similarly of order $\frac{B_z}{R_s^2}$ (where R_s is some scale length) and additionally

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Figure 1C. Shows a noncircular orbit due to the $\nabla B \times B$ drift, r_2 and r_1 are radial extrema of the orbit (tracked from the central point "O").

that $R_s > \Delta r$ then we can drop the $\nabla^2 B$ term in Eq. C-4 to obtain:

$$\begin{aligned} rA_\phi - (rA_\phi)|_{r_0} &= (rB_z)|_{r_0} \Delta r + \\ & (B_z + r\nabla_r B_z)|_{r_0} \frac{\Delta r^2}{2} + (\nabla_r B_z)|_{r_0} \frac{\Delta r^3}{3} \end{aligned} \quad (C-5)$$

Near the plasma boundary, the region of primary interest, where $r \gg \Delta r$ the term in Δr^3 can be dropped also. The perpendicular energy ($E_\perp$) of the particle is given by

$$E_\perp = \frac{1}{2} m v_r^2 + \frac{(P_\phi - q r A_\phi)^2}{2 m r^2} \quad (C-6a)$$

At the maximum and minimum of the orbit Eq. C-6 can be evaluated to find rA_ϕ :

$$rA_\phi|_{r=r_1, r_2} = \begin{cases} \frac{1}{q} (P_\phi + r \sqrt{2E/m}), & r = r_1 \\ \frac{1}{q} (P_\phi - r \sqrt{2E/m}), & r = r_2 \end{cases} \quad (C-6b)$$

$$(C-6c)$$

Substituting this into Eq. C-5 (without the Δr^3 term) and letting $r = r_0 \pm \rho_i^\pm$ where $\rho_i^\pm$ is the classical gyro-radius for the upper (+) or lower (-) half-orbit we obtain (dropping the superscripts (+) on ρ_i but placing them on ∇r as a reminder):

$$\begin{aligned} \frac{1}{q} (P_\phi + r \sqrt{2E/m}) - (rA_\phi)|_{r_0} &= (rB_z)|_{r_0} \rho_i + \\ (B_z + r\nabla_r B_z)|_{r_0} \frac{\rho_i^2}{2} \end{aligned} \quad (C-7a)$$

for the upper half-orbit and

$$\frac{1}{q} (P_\phi - r \sqrt{2E/m}) - (rA_\phi)|_{r_0} = -(rB_z)|_{r_0} \rho_i + (B_z + r \nabla_r B_z)|_{r_0} \frac{\rho_i^2}{2} \quad (C-7b)$$

for the lower half-orbit. It is clear that once r_0 and P_ϕ are defined the upper and lower gyro-radii can be found. If we let r_0 be the point in the orbit where particle motion is purely radial, we obtain from Eq. C-6 that $E_\perp = 1/2 m v_r^2$ and $P_\phi = q r A_\phi|_{r_0}$.

This simplifies equations C-7a, b considerably to become:

$$\pm \frac{1}{q} r \sqrt{2E/m} = \pm (rB_z)|_{r_0} \rho_i + (B_z + r \nabla_r B_z)|_{r_0} \frac{\rho_i^2}{2}, \quad (C-8)$$

where $r = r_0 \pm \rho_i$ and ρ_i and Δr are evaluated in upper and lower half-orbits as before. If the "density of tangents" defined at r_0 is known for each radial shell the time-averaged density in each shell can be calculated in a straightforward manner. The time a particle spends in a radial shell bounded by R_1 and R_2 is given by the integral:

$$\tau = \int_{R_1}^{R_2} \frac{dr}{v_r}, \quad (C-9)$$

where v_r can be found from 6e to be

$$v_r = \frac{\sqrt{2mEr^2 - (P_\phi - qrA)^2}}{mr} \quad (C-10)$$

Using Eq. C-3 we find that

$$r'A_\phi = \int_0^{r'} rB_z dr = \int_0^{R_1} rB_z dr + \int_{R_1}^{r'} rB_z dr, \quad (C-11)$$

where r' is R_2 for the case in Eq. C-9. If we assume that B_z is some average value in the shell defined as:

$$B_z = B_z|_{R_1} + \Delta B_z|_{R_1} \frac{(R_2 - R_1)}{2}, \quad (C-12)$$

then a good estimate of $qr'A_\phi$ in the shell is

$$qr'A_\phi = q \left\{ \int_0^{R_1} rB_z dr + \left[B_z + \nabla B_z \frac{(R_2 - R_1)}{2} \right] \Big|_{R_1} (r'^2 - R_1^2) \right\}, \quad (C-13a)$$

which can be written $r'A_\phi = Cr'^2 - D$, where

$$C = \left[B_z + \nabla B_z \frac{(R_2 - R_1)}{2} \right] \quad \text{and} \quad D = CR_1^2 - \int_0^{R_1} rB_z dr. \quad (C-13b)$$

This form can be used with Eq. C-9 in Eq. C-10 to obtain:

$$t = \int_{R_1}^{R_2} \frac{2\pi r dr}{\sqrt{2\pi E r^2 - (P_\phi + D - Cr^2)^2}}, \quad (C-14)$$

where the prime on r has been dropped. The expression under the radical can be expanded and, using a substitution of variables to complete the square, yields the following form for Eq. C-14:

$$t = \int_{\mu_1}^{\mu_2} \frac{m d\mu}{C \sqrt{F^2 - \mu^2}}, \quad (C-15)$$

where

$$F^2 = (2\pi E - (P_\phi + D)C)^2 / 4C^2 \quad (C-15a)$$

and

$$\mu = Cr^2 - (2mE - (P_\phi + D)C)/2C. \quad (C-16)$$

and where μ_2 and μ_1 are μ of Eq. C-16 evaluated at $r = R_1$ and $r = R_2$, respectively.

The integral in Eq. C-15 is evaluated easily, giving:

$$t = \frac{m}{CF} \arcsine \left(\frac{\mu}{F} \right). \quad (C-17)$$

Thus for these approximations, the time spent in each shell (and therefore the density contribution to that shell) by a particle with a noncircular orbit reduces to evaluating an arcsine as in the previous semi-circular orbit model where the orbit arclength in a shell is proportional to the orbital angle subtended by the shell boundaries (See Fig. 3).

The remaining problem, then, is to set up the "density of tangents" conversion routine required to use this treatment. As in the present FOREMD model, the conversions back and forth can be done using a "lookup" table of weighting functions, now dependent on shell number, energy and orbit eccentricity.

For beam deposition, the mean displacement of the "tangent" can be calculated in a straightforward manner from equations C-7a, b (the ion enters the plasma at either a maximum or minimum in its orbit) with the values of v_r and P_ϕ known at the entry point and an estimate used for B_z . This is illustrated in Fig. 2C which shows that v_r and P_ϕ are initially just:

$$v_r = v_b \cos\phi \text{ and } P_\phi = mv_b r \sin\phi, \quad (C-18)$$

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Figure 2C. Shows the resolution of the beam velocity (v_b) into radial and ϕ components (v_r and v_ϕ) at the trapping point x in an outer shell of width ΔR .

where ϕ is the angle between the radius and velocity vectors. The guess for $\nabla B_z|_{R_1}$ can be $\nabla B_z|_{R_c}$ where R_c is equal to $r_{1,2} \pm \rho_1$ (depending on the $\vec{v} \times \vec{B}$ direction) and ρ_1 is the ion gyro-radius calculated at the entry point.

Another approach to finding this displacement is to recall that at the tangent point,

$$qrA_\phi \approx P_\phi, \quad (C-19)$$

and

$$qrA_\phi = q \int_0^r r B_z dr. \quad (C-20)$$

Numerically integrating Eq. C-20 starting from the kernel defined by $\int_0^{R_1} r B_z dz$, where R_1 is the entry radius, until the point where qrA_ϕ is just greater than P_ϕ of the trapped ion will give the shell in which the tangent falls (i.e., just before $qrA_\phi > P_\phi$). Either approach represents an improvement over the simpler method of approximation described in the text, although a larger lookup table is required.

An additional improvement in the model that would allow the time step (Δt) to be increased considerably is to convert the Fokker-Planck treatment to an implicit, rather than the present explicit, time dependent form. In the explicit model, any change in a velocity group is calculated from values obtained during the previous time step. That is, the value of a grid element f_j at time step $n + 1$ is given (for the Fokker-Planck) by:

$$f_j^{n+1} = f_j^n + \Delta t \cdot F(A_{j-1}^n f_{j-1}^n, A_j^n f_j^n, A_{j+1}^n f_{j+1}^n), \quad (C-21)$$

where n refers to the previous time step. Δt is the present time step and F is a function (the Fokker-Planck equation) with coefficients (A_j)

that contain derivatives, etc., which determines the change in the distribution.

In an implicit time description the f_j^{n+1} element is given by:

$$f_j^{n+1} = f_j^n + \Delta t \cdot (1-s) \cdot F(A_{j-1}^{n+1} f_{j-1}^{n+1}, A_j^{n+1} f_j^{n+1}, A_{j+1}^{n+1} f_{j+1}^{n+1}) + \Delta t \cdot s \cdot F(A_{j-1}^n f_{j-1}^n, A_j^n f_j^n, A_{j+1}^n f_{j+1}^n) \quad (C-22)$$

where s is a fraction between 0.0 and 1.0, and other notations are as before. The coefficients A_j^{n+1} are estimated by a linear extrapolation of previous values and the explicit part is also evaluated from the previous values. This results in a set of N coupled linear equations in $A_j^{n+1} f_j^{n+1}$ and the $j-1$ and $j+1$ adjacent groups, using the extrapolated A_j^{n+1} values. These equations can be solved by standard matrix/determinant methods using the coefficients in a tri-diagonal matrix ($j-1, j, j+1$ positions; all other off-diagonal terms are zero).²³

By using an implicit time dependence, an estimated first order correction is included in the time-rate-of-change of the distribution. This method allows a larger time step to be used before obtaining a non-convergent solution, but at a loss of accuracy due to the use of extrapolated (rather than exact) coefficients in the matrix. By mixing implicit and explicit solutions using the weighting factor " s " in Eq. C-22, the best compromise between speed and accuracy for a particular type of problem can be obtained (by trial and error). The code could also be speeded up by calculating neutral beam deposition only every few time steps and using an extrapolation to find intermediate values.

The present explicit model is relatively fast (at least as fast as the A. Futch model) in calculating plasma profiles at each time step but suffers from the explicit time dependence (discussed above) in that more, shorter time steps must be used for convergence. Including the improvements outlined in this appendix will decrease the running time of FOREMD by more than an order of magnitude.

VITA

Mark M. Campbell was born on October 4, 1949 in South Bend, Indiana, where he spent most of his youth and attended J. W. Riley High School. After graduating from high school in 1968 with various honors, he attended Rose Polytechnic (now Rose-Hulman) Institute in Terre Haute, Indiana and graduated *magna cum laude* in 1972 with a B. S. in Physics. He was then accepted at the University of Illinois (UIUC) Department of Physics on a fellowship and received his M. S. degree in Physics in 1973. He then entered the Nuclear Engineering Program at UIUC and obtained his Ph.D. in 1980.

He has been active in doing computer simulations of plasma buildup in magnetic-mirror fusion devices and has several publications on this topic. In 1976 he worked at the Lawrence Livermore Laboratory (LLL) at Livermore, California on studies of plasma buildup in a proposed mirror device and produced an internal publication (UCID-17458, 1977) on the results of this research. He is presently collaborating with Dr. A. Futch at LLL in creating a computer simulation of the Tandem Mirror Experiment (TMX). He can be contacted through his advisor, George H. Miley, Nuclear Engineering Program, UIUC.