LA--9505-T

DE83 002347

LA-9505-T
Thesis

UC-34c
Issued: September 1982

Measurement of the Spin-Dependent Parameter D , R , A , and P , and Analyzing Power for $\bar{p}$ - d Elastic Scattering at 500 and 800 MeV

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ABSTRACT

A specific set of spin dependent parameters for elastic scattering of polarized protons from an unpolarized deuterium target have been measured over the angular range of 15° and 65° in the Laboratory system. The experiment was performed at the Los Alamos Meson Physics Facility (LAMPF) using a polarized proton beam of 0.5 and 0.8 GeV incident kinetic energy. A carbon analyzer was used to measure the scattered proton polarization. This comprises the first set of measurements in the intermediate energy range.

Of particular interest, the analyzing power, (A_y), has been measured for the pd elastic scattering reaction at both energies. A test of Time-Reversal Invariance (TRI) has been made for this reaction. This was accomplished by comparing the polarization (P) with the analyzing power A_y together with the depolarization parameter, (D), which was measured in this experiment. No evidence of time-reversal violation was found for this reaction in the region of non-zero spin flip probability.

The measurements also furnished very useful and selective information on the p-d collision matrix, as well as the double-spin-flip nucleon-nucleon amplitudes.

MEASUREMENT OF THE SPIN DEPENDENT PARAMETER D , R , A , AND P , AND
ANALYZING POWER FOR $\vec{p}$ - d ELASTIC SCATTERING AT 500 AND 800 MeV

by

Ali Akbar Rahbar

Introduction

In recent years attention has been drawn to the interaction of nucleons with deuterons with the hope that the problem of two- and three-nucleon systems may be better understood. The deuteron is the simplest nuclear system and its interaction with nucleons can be treated by several models. For example, the multiple scattering theory of Glauber¹⁾ has proven to be very successful in the interpretation of high energy (small angle) hadron-nuclear interactions,^{2,3)} and is also a remarkably good first approximation at intermediate energies (~ 1 GeV, where the transferred momentum is small and interaction should be dominated by the impulse approximation.

Since the difference between the sum of the magnetic moments of the proton and neutron, $\mu_p + \mu_n$, and of the deuteron, μ_d differs from zero and the electric quadrupole moment of the deuteron is zero, it is clear that the deuteron is not in a purely spherically symmetric S-state. In addition, early experiments with unpolarized beam and target proved that p - d elastic scattering at intermediate energies is very sensitive to the D-state of deuteron, particularly in the minimum region of the differential cross section. Thus one can conclude, the nuclear force is spin-dependent at intermediate energies and so it is difficult to predict such a force directly (e.g., by a few simple measurements). Rather difficult experiments, such as those with polarized beam and/or with polarized targets, are required.

In early measurements of spin correlation or polarization transfer parameters, three scatterings were necessary (triple scattering experiments): the first prepared the polarized beam, in the second, the nucleon-nucleon (or nucleon-nucleus) scattering takes place, and in the third, the polarization of the outgoing particle is determined.

Later many polarization experiments were possible, since polarized beams in 3 orthogonal directions of polarization could be prepared and/or increased possibilities to polarize the target became available to vastly improve the precision of these measurements.

Generally polarization experiments can be divided as follows:

- i) $B(a, \vec{c})D$ polarization experiments
- ii) $B(\vec{a}, c)D$ analyzing power experiments
- iii) $B(\vec{a}, \vec{c})D$ polarization transfer experiments
- iv) $\vec{B}(\vec{a}, c)D$ spin correlation experiments (initial channel)
- v) $B(a, \vec{c})\vec{D}$ spin correlation experiments (final channel)

where the arrows indicate the particle whose spin polarization is known or measured. The availability of polarized ion sources and polarized targets has considerably simplified polarization experiments and made it possible to make quantitatively precise measurements.

The experiment described in this work has the following goals:

- 1) To measure as many as possible of the spin-rotation and depolarization observables in the elastic scattering of polarized protons from the deuteron target. These may provide selective

information on the p-d collision matrix, and in particular, information on the double-spin-flip NN amplitudes. Bleszynski⁹⁾ has suggested that the measurement of all the Wolfenstein parameters D , R , R' , A , and A' can be used to determine the depolarization parameters D_0 , D_x , D_y and D_z . These parameters are of considerable interest with respect to their dependence of the double spin flip component of NN amplitude.

- 2) To test the equality of $P(\theta)$ and $A_y(\theta)$.
- 3) To provide data to test several theoretical questions.
 - a) Are the empirically determined NN amplitudes from NN experiments sufficient to predict the p-d scattering amplitudes at intermediate energies?
 - b) Since the deuteron is the simplest bound state, the effect of non-eikonal corrections to the Glauber Model and also the effect of the double scattering can be checked.

We report here on the measurements of the following Wolfenstein parameters D , R , A and P , and the analyzing power for p-d elastic scattering at 500 MeV and 800 MeV, between 28° and 52° , and between 17° and 62° laboratory angles, respectively. This is the first measurement of these parameters in this angular region at these energies. Our measurement will be complemented in the future with R' and A' . This will also allow us to determine the parameters of D_0 , D_x , D_y and D_z .

The experiment was performed using the External Proton Beam (EPB) at the Clinton P. Anderson Meson Physics Facility (LAMPF) of the Los Alamos National Laboratory.

In Chapter 1, a review of nucleon-nucleon and nucleon-deuteron formalism is given. The experimental conditions and setup are discussed in Chapter 2. The method of analyses along with experimental results and discussion are presented in Chapter 3.

CHAPTER 1

The Scattering Theory

The principal aims of nuclear physics are to 1) ascertain many nuclear properties , 2) answer questions about nuclear structure and binding energies. It is important to understand the nature of the forces existing between nucleons in the nucleus. In order to obtain such information, a fruitful way has been to investigate the scattering phenomena. The most fundamental systems which we have to study are, the nucleon-nucleon and nucleon-deuteron systems. Here the theory becomes simpler. This may hopefully lead to a better understanding of the problem in complex nuclei.

1.1 Structure of Nucleon-Nucleon Scattering

The scattering of the two-spin 1/2 particles (two nucleons) is described by the scattering amplitude considered as a matrix in spin space.¹⁾ The scattering wave function for nucleon-nucleon scattering may be written as²⁾

$$\psi = \chi e^{i\vec{k} \cdot \vec{r}} + M\chi \frac{e^{ikr}}{r} . \quad (1-1)$$

where M (spin-scattering matrix for the two-nucleon system) is a 4 x 4 matrix operating on the initial spin state. It depends on the Pauli

spin operators $\vec{\sigma}_1$ and $\vec{\sigma}_2$ for the two particles, and on $\vec{k}_i$ and the $\vec{k}_f$ the initial and the final momenta of one of the particles (in the center-of-mass frame), i.e.

$$M(\vec{\sigma}_1, \vec{\sigma}_2, \vec{k}_i, \vec{k}_f) \quad (1-2)$$

The most general function of the form (1-2) invariant under rotation, reflection and time reversal is¹⁾

$$\begin{aligned} M = & \alpha + \beta(\vec{\sigma}_1 \cdot \hat{n})(\vec{\sigma}_2 \cdot \hat{n}) + \gamma(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{n} + \delta(\vec{\sigma}_1 \cdot \hat{\ell})(\vec{\sigma}_2 \cdot \hat{\ell}) \\ & + \epsilon(\vec{\sigma}_1 \cdot \hat{m})(\vec{\sigma}_2 \cdot \hat{m}) \end{aligned} \quad (1-3)$$

where the coefficients are functions of the energy and the c.m. scattering angle.

The unit vectors $\hat{X}$, $\hat{Y}$, and $\hat{Z}$ form a right-handed coordinate system (in the center-of-mass frame) and are defined by:

$$\begin{aligned} \hat{X} &= \frac{\vec{k}_i + \vec{k}_f}{|\vec{k}_i + \vec{k}_f|}, \\ \hat{Y} &= \frac{\vec{k}_f - \vec{k}_i}{|\vec{k}_f - \vec{k}_i|}, \text{ and} \end{aligned} \quad (1-4)$$

$$\hat{Z} = \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_i \times \vec{k}_f|} .$$

Because quantum mechanics is a linear theory, the spinor χ_f that describes the outgoing spin 1/2 particles is related linearly to the spinor χ_i that describes the incoming spin 1/2 particles, i.e.

$$\chi_f = M \chi_i . \quad (1-5)$$

A polarization state for the system is, in general, described as a mixture of many separate pure spin states in which the system may be found at any particular time. The most general scattering problem is that of finding the final polarization state and intensity as a function of the scattering angle, if the initial polarization state of the particle is known. This problem can be formulated in terms of the density matrix, ρ , for a mixture of states:

$$\rho_i = \sum_n a_n \chi_i^n \chi_i^{n\dagger} \quad (1-6)$$

where a_n is the relative probability of finding the system in the state χ^n . And the corresponding density matrix for the final state can be obtained from (1-6) by replacing χ_i by $M\chi_i$

$$\rho_f = \sum_n a_n M \chi_i^n \chi_i^{n\dagger} M^\dagger = M \rho_i M^\dagger \quad (1-7)$$

where for a beam of particles, the sum may be considered as a sum over

the states of the individual particles in the beam. For a given density matrix ρ , the expectation value of any Hermitian operator Ω in the spin space can be written as

$$\langle \Omega \rangle = \frac{\sum_n a_n \chi^{n\dagger} \Omega \chi^n}{\sum_n a_n \chi^{n\dagger} \chi^n} = \frac{\sum_n \sum_{\alpha\beta} a_n \chi^{n\dagger} |\alpha\rangle \langle \alpha| \Omega |\beta\rangle \langle \beta| \chi^n}{\sum_n \sum_{\alpha} a_n \chi^{n\dagger} |\alpha\rangle \langle \alpha| \chi^n} \quad (1-8)$$

$$= \frac{\sum_{\alpha} \langle \alpha | \rho \Omega | \alpha \rangle}{\sum_{\alpha} \langle \alpha | \rho | \alpha \rangle}$$

or

$$\langle \Omega \rangle = \frac{\text{Tr}(\rho \Omega)}{\text{Tr} \rho} \quad (1-9)$$

If ρ_i is normalized to unity ($\text{Tr} \rho_i = 1$), the differential cross section for a polarized beam is given by

$$I(\theta, \phi) = \frac{\text{Tr} \rho_f}{\text{Tr} \rho_i} = \frac{1}{2} \text{Tr}(M \rho_i M^\dagger) \quad (1-10)$$

For unpolarized beam, one has

$$\rho_i = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (1-11)$$

If also the target is unpolarized, Eq. (1-10) reduces to

$$I_o(\theta) = \frac{1}{4} \text{Tr}(MM^\dagger). \quad (1-12)$$

Consider a matrix

$$B = b_0 I + \mathbf{b} \cdot \vec{\sigma} = \begin{pmatrix} b_0 + b_z & b_x - ib_y \\ b_x + ib_y & b_0 - b_z \end{pmatrix}$$

It is clear that an arbitrary 2×2 matrix can be written in this fashion. Therefore the Pauli matrices, together with the two-dimensional unit matrix, form a complete set of 2×2 matrices.

Then for a polarized beam of spin $\frac{1}{2}$ particles, the density matrix can be described in terms of the set of matrices I , σ_x , σ_y , and σ_z , namely

$$\rho_i = \frac{1}{2} (I + P_x \sigma_x + P_y \sigma_y + P_z \sigma_z) \equiv \frac{1}{2} (I + \sum_{j=1}^3 P_j \sigma_j) \quad (1-13)$$

where I is a unitary matrix, and σ_j are the Pauli matrices, and P_j are the components of the beam polarizations,. Explicitly, inserting the form of ρ_i into Eq. (1-10), one finds

$$I(\theta, \phi) = \frac{1}{4} \text{Tr}(MM^\dagger) + \frac{1}{4} \sum_{j=1}^3 P_j M \sigma_j M^\dagger \quad (1-14)$$

$$= \frac{1}{4} \text{Tr}(MM^\dagger) + \frac{1}{4} \langle \vec{\sigma}_1 \rangle_i \cdot \text{Tr}(M \vec{\sigma}_1 M^\dagger)$$

where $\vec{P}_b = |\langle \sigma_1 \rangle_i|$ is the polarization of the incoming particles (beam).

On the basis of invariance under time reversal, one can obtain the following relation (Wolfenstein⁴) and also see App. E)

$$\text{Tr}(M \sigma_1 M^\dagger) = \text{Tr}(M M^\dagger \sigma_1) \quad , \quad (1-15)$$

(note that this is not a mathematical identity, since the $\vec{\sigma}$ matrices do not commute with M). Therefore Eq. (1-14) becomes

$$I(\theta, \phi) = I_o + I_{pol} = I_o (1 + \vec{P}_b \cdot \hat{P}_n) \quad (1-16)$$

where P is the induced polarization of the incoming particle which would be produced in the scattering of unpolarized beam from unpolarized target. Note that, the first term on the right-hand side of Eq. (1-16), I_o , is the differential cross section for an unpolarized beam and an unpolarized target. Because of rotational invariance, it is azimuthally symmetric (i.e., independent of ϕ). The second term is the contribution to the cross section due to initial polarization, the asymmetry comes entirely from this term. Equation

(1-16) is irrelevant to the spin of the target particles and hence holds true for any target particle. For a polarized incident beam with known p_b (transverse component of beam polarization, let us assume to be normal to the scattering plane), one can see from Eq. (1-16), the scattering of particles to the left ($\phi = 0$) is different from the one to the right ($\phi = \pi$), thus

$$I_L \equiv I(\theta, 0) = I_0(1 + p_b P) \quad (\text{for } \phi = 0) \quad (1-17)$$

and

$$I_R \equiv I(\theta, \pi) = I_0(1 - p_b P) \quad (\text{for } \phi = \pi) \quad (1-18)$$

Hence, for a given polarization P^* , one can define the azimuthal asymmetry, ϵ , i.e.

$$\epsilon = p_b A(\theta) \equiv \frac{I_L - I_R}{I_L + I_R} = p_b P(\theta) \quad , \quad (1-19)$$

where $A(\theta)$ is the analyzing power of the reaction.

The outgoing polarization vector, $\vec{P}'$, (in Laboratory coordinate system) can be expressed in terms of the Wolfenstein parameters $P(\theta)$.

* The notation of $P(\theta)$ will be used instead of P in the rest of the text.

D ($= D_{nn'}$), A ($= D_{\ell s'}$), R ($= D_{ss'}$), A' ($= D_{\ell \ell'}$), R' ($= D_{s\ell'}$) and the incident polarization vector,^{2,3} (see also App. E) i.e.

$$IP' = I_0 [(Rp_s + Ap_\ell)\hat{s}' + (P(\theta) + Dp_n)\hat{n}' + (R'p_s + A'p_\ell)\hat{\ell}'] \quad (1-20)$$

where the prime is referred to the outgoing reactant helicity frame, and the unit vectors $(\hat{s}, \hat{n}, \hat{\ell})$ and $(\hat{s}', \hat{n}', \hat{\ell}')$ in the laboratory coordinate system are defined by:

$$\begin{aligned} \hat{\ell} &= \frac{\vec{k}_i}{|\vec{k}_i|} \quad (\text{longitudinal}), \\ \hat{n} &= \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_i \times \vec{k}_f|} \quad (\text{normal}), \\ \hat{s} &= \hat{n} \times \hat{\ell} \quad (\text{sideway}) \end{aligned} \quad (1-21)$$

for incident particles, and

$$\hat{\ell}' = \frac{\vec{k}_f}{|\vec{k}_f|}, \quad \hat{n}' \parallel \hat{n}, \quad \hat{s}' = \hat{n}' \times \hat{\ell}' \quad (1-22)$$

for outgoing particles.

The D is called depolarization parameter and the R, A, R', and A', are called spin rotation parameters.

Using Eq. (1-16), one can obtain the polarization components of scattered particle, i.e.

$$p_s' = \frac{p_s R + p_\ell A}{1 + p_n A(\theta)} , \quad (1-23)$$

$$p_n' = \frac{P(\theta) + p_n D}{1 + p_n A(\theta)} , \quad (1-24)$$

$$p_\ell' = \frac{p_s R' + p_\ell A'}{1 + p_n A(\theta)} . \quad (1-25)$$

In practice, one can utilize each of the possible incident beam polarization directions in the experiment and measure the final polarization component of outgoing particles. Then, from Eqs. (1-23) to (1-25), one can obtain the parameters $P(\theta)$, D , A , R , A' , and R' . The D parameter, however, relies on the measurement of the analyzing power, $A(\theta)$. $A(\theta)$ can be obtain from the polarizaton, $P(\theta)$, in the elastic scattering case. Otherwise it is necessary to have a measurement of $A(\theta)$.

1.2 Spin Structure of the p-d Collision Matrix and the Nucleon- Nucleon Amplitudes

The most general form of the p-d collision matrix F can be constructed, using symmetry principles (similar to NN scattering in the previous section).

Following Ref. 8, let us express the operator F in terms of the Pauli spin operators $\sigma_o, \sigma_x, \sigma_y, \sigma_z$, and the deuteron spin operators $1, J_x, J_y, J_z$, and the quadrupole operators

$$Q_{ik} = 1/2(J_i J_k + J_k J_i) - 2/3 \delta_{ik}$$

with the following properties

- a) F must be rotationally invariant
- b) F must be symmetric with respect to the parity (P) and time reversal (T) operations. In this context, the expression for F in the standard reference frame has the following form.

$$F = F^O \sigma_o + F^X \sigma_x + F^Y \sigma_y + F^Z \sigma_z, \quad (1-26)$$

with

$$F^O = F_o^O + F_y^O J_y + F_{xx}^O Q_{xx} + F_{yy}^O Q_{yy} \quad (1-27a)$$

$$F^X = F_{xx}^X J_x + F_{xy}^X Q_{xy} \quad (1-27b)$$

$$F^y = F_o^y + F_y^y J_y + F_{xx}^y Q_{xx} + F_{yy}^y Q_{yy} \quad (1-27c)$$

$$F^z = F_z^z J_z + F_{yz}^z Q_{yz} \quad (1-27d)$$

where

$$x || \vec{k}_i - \vec{k}_f ,$$

$$y || \vec{k}_i \times \vec{k}_f ,$$

$$z || \vec{k}_i + \vec{k}_f ,$$

and where $\vec{k}_i$ and $\vec{k}_f$ denote the initial and final projectile momenta. This form follows from the properties of the operators σ_i , J_i , and Q_{ik} which are given in Table (1-1). As an example, let us write the most general form for the F^o term in the Eq. (1-26) without imposing T-reversal and P-conservation

$$F^o = F_o^o 1 + F_x^o J_x + F_y^o J_y + F_{xx}^o Q_{xx} + F_{yy}^o Q_{yy} \\ + F_{yz}^o Q_{yz} + F_{xy}^o Q_{xy} + F_{zx}^o Q_{zx}$$

There is no F_{zz}^o term because $Q_{zz} = -(Q_{yy} + Q_{xx})$. By using Table (1-1) and because F must be invariant under time reversal, this equation becomes

$$F^O = F_O^O \cdot 1 + F_y^O J_y + F_z^O J_z + F_{xx}^O Q_{xx} + F_{yy}^O Q_{yy} + F_{yz}^O Q_{yz} .$$

Under parity conservation it will reduce to

$$F^O = F_O^O + F_y^O J_y + F_{xx}^O Q_{xx} + F_{yy}^O Q_{yy}$$

which is the equation (1-27a). Thus, for pd elastic collisions, the scattering operator contains one spin-independent amplitude and eleven spin-dependent amplitudes. In the scattering experiments where the spin states of the proton can be analyzed in the initial and final states, therefore the following quantities can be determined.

$$1) \quad D_{ss'} = \text{Tr}\{F\sigma_s F^\dagger \sigma_{s'}\} / S_O$$

$$2) \quad D_{\ell\ell'} = \text{Tr}\{F\sigma_\ell F^\dagger \sigma_{\ell'}\} / S_O$$

$$3) \quad D_{nn'} = \text{Tr}\{F\sigma_n F^\dagger \sigma_{n'}\} / S_O$$

$$4) \quad D_{\ell s'} = \text{Tr}\{F\sigma_\ell F^\dagger \sigma_{s'}\} / S_O \quad (1-28)$$

$$5) \quad D_{s\ell'} = \text{Tr}\{F\sigma_s F^\dagger \sigma_{\ell'}\} / S_O$$

$$6) \quad P(\theta) = \text{Tr}\{FF^\dagger \sigma_n\} / S_O$$

$$7) \quad A_y = \text{Tr}\{F\sigma_n F^\dagger\} / S_O$$

$$8) \quad S_0 = \text{Tr}\{FF^\dagger\}$$

where s , n , and l denote the particle polarization directions (see Eqs. 1-21,22), and $S_0 = \frac{1}{6} I_0$.

Table (1.1) Properties of Parity and Time-Reversal Invariance

<u>Parity Conservation</u>	<u>Time-Reversal Invariance</u>
$\sigma_x \rightarrow -\sigma_x$	$\sigma_x \rightarrow -\sigma_x$
$\sigma_y \rightarrow \sigma_y$	$\sigma_y \rightarrow \sigma_y$
$\sigma_z \rightarrow -\sigma_z$	$\sigma_z \rightarrow \sigma_z$
$J_x \rightarrow -J_x$	$J_x \rightarrow -J_x$
$J_y \rightarrow J_y$	$J_y \rightarrow J_y$
$J_z \rightarrow -J_z$	$J_z \rightarrow J_z$
$Q_{xx} \rightarrow Q_{xx}$	$Q_{xx} \rightarrow Q_{xx}$
$Q_{yy} \rightarrow Q_{yy}$	$Q_{yy} \rightarrow Q_{yy}$
$Q_{zz} \rightarrow Q_{zz}$	$Q_{zz} \rightarrow Q_{zz}$
$Q_{xy} \rightarrow -Q_{xy}$	$Q_{xy} \rightarrow -Q_{xy}$
$Q_{xz} \rightarrow Q_{xz}$	$Q_{xz} \rightarrow -Q_{xz}$
$Q_{yz} \rightarrow -Q_{yz}$	$Q_{yz} \rightarrow Q_{yz}$

For simplicity of the discussion, let us assume that the direction of s , n , ℓ , and s' , n' , ℓ' coincide with x , y , z . However, in the actual experimental conditions one must account for this correction by performing the rotation as in Ref. 8.

Let us rewrite the quantity $D_{\alpha\beta}$ in the following form

$$S_0 D_{\alpha\beta} = \text{Tr}(F \sigma_\alpha F^\dagger \sigma_\beta)$$

where α and β are one of s , n , ℓ , s' , n' , and ℓ' , respectively.

By considering $s = s' = x$, $n = n' = y$, and $\ell = \ell' = z$, one can get, e.g.

$$S_0 D_{ss'} = \text{Tr}(F^0 F^{0\dagger}) + \text{Tr}(F^x F^{x\dagger}) - \text{Tr}(F^y F^{y\dagger}) - \text{Tr}(F^z F^{z\dagger}) .$$

From Eq. (1-33)

$$S_0 D_0 = \text{Tr}(F^0 F^{0\dagger})$$

$$S_0 D_x = \text{Tr}(F^x F^{x\dagger})$$

$$S_0 D_y = \text{Tr}(F^y F^{y\dagger})$$

$$S_0 D_z = \text{Tr}(F^z F^{z\dagger})$$

hence

$$D_{ss'} = D_0 + D_x - D_y - D_z \quad (1-29a).$$

Similarly

$$D_{nn'} = D_0 - D_x + D_y - D_z \quad (1-29b)$$

and

$$D_{\ell\ell'} = D_0 - D_x - D_y + D_z, \quad (1-30)$$

$$D_{\ell s'} = -D_{s\ell'} = Q, \quad (1-31)$$

$$P(\theta) = A_y(\theta), \quad (1-32)$$

where $D_j (j = 0, x, y, z)$ are the parameters introduced by Bleszynski,⁹⁾ and Eqs. (1-31,32) followed from the symmetries with respect to the T and P - transformation.

The quantities D_j are related to the traces of the amplitudes F^j as follows:

$$S_0 D_j = \text{Tr}(F^j F^{j\dagger}) \quad , \quad j = 0, x, y, z \quad (1-33)$$

and the polarization and spin rotation functions are

$$\begin{aligned} P(\theta) &= \text{Tr}\{FF^\dagger\sigma_y\} / S_0 \\ &= \text{Tr}\{(F^0 + F^x\sigma_x + F^y\sigma_y + F^z\sigma_z)(F^{0\dagger} + F^{x\dagger}\sigma_x \\ &\quad + F^{y\dagger}\sigma_y + F^{z\dagger}\sigma_z)\sigma_y\} / S_0 \end{aligned}$$

or

$$S_o P(\theta) = 2 \operatorname{Re} \operatorname{Tr}\{F^{o\dagger} F^y\}$$

and the spin rotation function

$$S_o Q = 2 \operatorname{Im} \operatorname{Tr}\{F^{o\dagger} F^y\}$$

Thus

$$S_o (P(\theta) + iQ) = 2 \operatorname{Re} \operatorname{Tr}\{F^{o\dagger} F^y\} + i2 \operatorname{Im} \operatorname{Tr}\{F^{o\dagger} F^y\}$$

or

$$S_o (P(\theta) + iQ) = 2 \operatorname{Tr}\{F^{o\dagger} F^y\} \quad (1-34)$$

with

$$S_o = \sum_{j=o,x,y,z} \operatorname{tr}\{F^{j\dagger} F^j\} . \quad (1-35)$$

Eq. (1-33) can be written as

$$S_o \sum_j D_j = \sum_j \operatorname{Tr}\{F^j F^{j\dagger}\} = S_o \quad (1-35b)$$

From eq. (1-35b) one can see the following relation

$$\sum_j D_j = D_o + D_x + D_y + D_z = 1 \quad (1-35c)$$

Using Eq. (1-35c) the parameters D_{ss}' , D_{nn}' , and $D_{\ell\ell}'$ can be expressed in terms of the quantities D_j , in the following form, i.e.

$$D_{ss}' = 1 - 2(D_y + D_z) \quad (1-36)$$

$$D_{nn}' = 1 - 2(D_x + D_z) \quad (1-37)$$

$$D_{\ell\ell}' = 1 - 2(D_x + D_y) \quad (1-38)$$

Thus, it is clear that the measurement of the Wolfenstein parameters (D_{ss}' , D_{nn}' , $D_{\ell\ell}'$, $D_{\ell s}'$, $D_{s\ell}'$) provides us with the information on various components of the F operator.

For example, D_{ss}' is sensitive to the F^y and F^z components because

$$D_y = \text{Tr} \{ F^y \dagger F^y \}$$

$$D_z = \text{Tr} \{ F^z \dagger F^z \}$$

and D_{nn}' is sensitive to the F^x , F^z components, whereas $D_{\ell\ell}'$ is sensitive to the F^x and F^y components.

The deviation of D_{nn}' , $D_{\ell\ell}'$, and D_{ss}' from 1 gives us the information on the strength of the corresponding double-spin flip and single spin flip component of the NN amplitude.⁹⁾

To be more specific, let us rewrite the NN amplitude (Eq. 1-3) in the following form.

$$F_{NN} = \alpha(q) + \beta(q)\sigma_y^1\sigma_y^2 + \gamma(q)(\sigma_y^1 + \sigma_y^2) + \delta(q)\sigma_x^1\sigma_x^2 + \epsilon(q)\sigma_z^1\sigma_z^2 \quad (1-39)$$

where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the Pauli spin operator of two protons. The calculation of D_x , D_y , and D_z quantities in the framework of the Glauber model^{9,10)} shows the single scattering contributions to D_x and D_z depend only on the δ and ϵ (i.e., the double spin flip parts of NN amplitudes). Therefore, in the small momentum transfer region when the single scattering term dominates in pd elastic scattering, the measurement of D_x and D_z provides selective information on double-spin flip part of the pp and pn amplitudes.

The double scattering contributions to D_x and D_z are also sensitive to δ and ϵ . Because one of the scatterings in the double scattering amplitude involves double-spin flip transition described by δ and ϵ amplitudes, respectively. Figure 1-1 shows the sensitivity of D_x and D_z to the slope of the double-spin flip amplitudes. The solid curves represent the observables D_x and D_z calculated with NN amplitudes parameterized as⁹⁾

$$\alpha(q) = (-0.042 + i 5.06) \exp(-5q^2/2) ,$$

$$\gamma(q) = (-0.38 + i 3.8)q \exp(-10q^2/2) ,$$

$$\delta(q) = (-1.52 - i 2.5) \exp(-3q^2/2) ,$$

$$\epsilon(q) = (1.05 - i 1.5) \exp(-3q^2) ,$$

in GeV. The dashed curves correspond to D_x and D_z evaluated with the same parameters, except for the slopes of $\delta(q)$ and $\epsilon(q)$ which were taken to be 10 (GeV/c)^2 .

Figure 1-1: The behaviour of the observables D_x and D_z as functions of q^2 , calculated with two different values of the slope of δ and ε .⁹⁾

In summary, the experimental study of double p-d scattering provide us with the selective information on the p-d collision matrix.

CHAPTER 2

2. The Experiment

In order to carry out this experiment, one should perform the so-called "triple scattering" experiment of the following form

TRIPLE SCATTERING

Such experiments are originally designed to determine how the second scatterer changes the direction and/or magnitude of the polarization of the incoming beam; thus, the first scatterer serves simply as a polarizer and the third one as an analyzer.

The availability of a polarized ion source reduces the required number of scatterings for this type of experiment to two, that is to double scattering experiment. We performed a double scattering experiment, using a polarized proton beam emerging from the ion source of the linear accelerator (linac) at the Clinton P. Anderson Meson Physics Facility (LAMPF), with polarization oriented either in the $\hat{s}$, $\hat{n}$, or $\hat{t}$ direction.

POLARIZED PROTON BEAM

The following sections give a detailed description of the experiment and apparatus.

2.1 The Accelerator and Polarized Proton Beam

The LAMPF linear accelerator is designed to accelerate simultaneously two beams, the one protons (H^+), and the other one negative hydrogen ions [polarized (p^-) or unpolarized (H^-)]. The polarized p^- beam is produced by a Lamb-Shift ion source (see Appendix A). This machine uses a Cockcroft Walton injector at 750 keV in making the initial acceleration from the source. The next stage of acceleration is an Alvarez drift-tube linac¹⁾ which consists of four copper-lined tanks. This section is about 202.5 ft. or 61.75 m long and operates at a frequency of 201.25 MHz. It accelerates the injected particles from 750 keV to 100 MeV.

After leaving the drift-tube linac, the beam enters the final stage of the accelerator which is 731 m long and consists of a 44-side coupled cavity guide¹⁾ operating at 805 MHz. This stage increases the beam energy continuously from 100 MeV to up to 800 MeV. The final beam energy is stepwise variable from 100 MeV to 800 MeV. Since particles can only be accelerated during a limited phase interval of the RF, the beam from this kind of accelerator has a time structure. A very sharp pulse (called a micro pulse) has a microstructure consisting of a 0.25 nsec (FWHM $\approx$ 0.25 ns) bursts every 5 nsec. The time sequence of the accelerator consists of 120 pulses (macropulses) per second, each pulse about 500 μ sec long. This means that the particles are accelerated only for a small fraction of the time. The macro duty of the accelerator is thus about 6% (the duty factor).

At the end of the accelerator, where the beam has achieved its final energy, the H^+ and P^- (or H^-) beam are magnetically separated from each other and then steered into separate experimental areas. Fig. 2-1 shows a schematic layout of the switchyard and the experimental areas. From the switchyard the H^+ beam travels to Area A while the P^- (or H^-) goes down to Line X where it passes through four anti-strippers which can be used to improve beam emittance by phase space trimming. A fifth stripper at the end of Line X is then used to provide a polarized proton beam for Line C and the remaining P^- beam goes to Line B. In line B the P^- beam is split for use in three different experimental areas. Figure 2-2 shows these areas along with some of the beam instrumentation and beam control magnets.

Figure 2-1: The experimental areas and switchyard layout of LAMPF.

Figure 2-2: Schematic layout of Areas B and EPB.

The orientation of beam polarization ($\hat{s}$, $\hat{n}$, or $\hat{l}$ -type) depends on the needs of the different experimental areas, and can be set to any desired direction at the source. However, there is no such freedom to give each area a desired polarization orientation independent of the others only by adjusting the orientation at the source.

The LAMPF External Proton Beam (EPB), where the experiment described in this thesis was done, has the advantage of having the spin orientation almost independent of the others. This is accomplished by use of a spin precessor. The spin precessor consists of a solenoid and two bending magnets. The field of the superconducting solenoid (LB-05-01) is parallel to the beam momentum, thus precessing the spin from, e.g. $\hat{n}$ -type to $\hat{s}$ -type without changing the proton's direction. Precession from $\hat{s}$ -type to $\hat{l}$ -type can be done by the two bending magnets (LB-BM-04 and LB-BM-05). Since both spin and momentum are in a plane perpendicular to the magnetic fields, both change the direction. For 800 MeV proton they change in the ratio $(\theta_p - \theta_b)/\theta_b = 3.32$. The total bend by LB-BM-04 and LB-BM-05 is 32° (16° each), giving a net spin precession of 106° , this is 16° more than desired, but since $\cos(16^\circ) = 0.96$ this is not a serious problem. Thus a $\hat{l}$ type beam is produced from a $\hat{n}$ or $\hat{s}$ type with nearly the same polarization.

During the experiment the beam polarization is measured concurrently with polarimeter calibration by cycling the source between polarized (55 sec) and quenched (5 sec). Typical beam polarizations are $P_b = 0.85$, with spin direction reversals every minute such that the magnitude of the polarization is unchanged but the direction is reversed 180° .

It is possible to manipulate beam intensity and pulse sequence at the ion source from the Central Control Room (CCR) to match the experimental requirements. The proton beam current was typically 10 nA. A triplet of quadrupole magnets allowed the beam in EPB to be focused at any desired location on a target, with a beam spot of a few mm diameter.

The actual energy of the 800 MeV beam was determined to be 796 ± 2 MeV instead of 800 MeV. This figure was obtained from a measurement with the LAMPF High Resolution Spectrometer (HRS). The energy obtained from the HRS was found to agree to better than 1 MeV with a simultaneous determination made by laser dissociation of H^- ions.²⁾

2.2 Targets

The targets used in this experiment consisted of a liquid deuterium (LD_2) target and a liquid hydrogen (LH_2) target. The LH_2 target was used during the initial setup and testing of the polarimeter (JANUS) and data acquisition systems. The cryogenic target system was used in order to liquify the hydrogen or deuterium which were supplied in gaseous form. This system consists of two parts.

- a) A LAMPF CTI compressor refrigerator system with a nominal cooling capacity of 10 W at 20° K, using helium as a coolant.
- b) A target flask and vacuum chamber. The target flask has a 1.5-inch diameter cylindrical shape and hemispherical end caps. Figure 2-3 shows the schematic layout of the system.

In the normal operation of the target, the target flask is filled with

Figure 2-3: A schematic diagram of the cryogenic target plumbing system.

liquid deuterium (hydrogen) and kept at a pressure of 15 psi within the vessel. Some additional amount of $LD_2(LH_2)$ is kept in the reservoir. The target flask has two carbon resistor level sensors at the top and the bottom, in order to verify that the target is full or empty. The operation of $LD_2(LH_2)$ target can be found in Ref. 3.

2.3 Beam Line Polarimeter and Monitors

Since this experiment strongly depends on the proton beam polarization we need some sort of devices to measure and monitor the polarization of the beam. For these purposes we used the beam line polarimeters⁴⁾ (EPB and Line B polarimeter). They are used as secondary standards for the beam polarization and calibrated primarily by the ion source quench ratio technique⁵⁾ (see Appendix A).

Before describing the beam line polarimeter, let me point out some facts about scattering. If a polarized beam, $P_b > 0$ (polarization in $\hat{n}$ -direction), scatters from a target, then the probability to scatter to the left (L) is different from the one to the right (R), giving an asymmetry

$$\epsilon = A_y P_b = \frac{L - R}{L + R} \quad (2-1)$$

where A_y is the analyzing power. The subscript y is added to indicate that the polarization direction is perpendicular to the scattering plane; (note that $A_x = A_z = 0$ by parity conservation).

A beam polarized UP ($\uparrow$) and scattered right ($R\uparrow$) is equivalent to a beam polarized down ($\downarrow$) and scattered left ($L\downarrow$), so that the yields of two detectors (Left and Right) may be replaced by the yields in one detector for beam spin up and down. By scattering to both left and right simultaneously and flipping the beam spin (up and down) periodically (every one minute in this experiment), we can make the geometric mean of the yields, namely:

$$L = [(L\uparrow)(R\downarrow)]^{1/2} \quad \text{and} \quad R = [(R\uparrow)(L\downarrow)]^{1/2} \quad (2-2)$$

It has been shown that using values for L, R, and ϵ obtained this way eliminate almost all experimental uncertainties and instrumental asymmetries to a high order.⁵⁾

The beam line polarimeters consist of a CH_2 analyzer (0.25 inch for EPB and 0.0625 inch for Line B) and four pairs of scintillation detectors placed to detect both primary and conjugate protons in coincidence elastically scattered from the hydrogen in CH_2 , near 40° c.m., where the analyzing power has a broad maximum.

The schematic layout of the polarimeter is shown in Fig. 2-4 (only for two pairs, left and right).

Protons from scattering of $pp \rightarrow pp$ reaction are detected near 17° and 67° lab (these are the conjugate angles for pp elastic scattering near 800 MeV) in each of two directions, left and right (or up and down). A left-right scattering is defined by a coincidence between detector 3 (detector 1, both mounted at 17°_{lab} relative to the beam) which detects the scattered beam particle and detector 4 (detector 2) which detects the recoil proton at 67°_{lab} from the hydrogen in CH_2

Figure 2-4: A schematic view of beam line polarimeter (only for two pairs).

target. The left-right detectors measure the $\hat{n}$ component of the beam polarization while the up-down detectors monitor the $\hat{s}$ component. Since there is no asymmetry corresponding to the longitudinal ($\hat{l}$) component, it is impossible to measure this component of beam polarization directly. The left-right (or up-down) experimentally observe asymmetry, ϵ , is directly related to the beam polarization, P_b , via Eq. (2-1). Then if the analyzing power A of the target is known at the beam energy, the beam polarization is given by

$$P_b = \frac{\epsilon}{A_y} .$$

Because of the ^{12}C contained in the CH_2 target, approximately 5% of the total events are from $^{12}\text{C}(p,2p)$ quasi-free scattering. These events have about half the asymmetry ($A_c = 0.275 \pm 0.015$ near 800 MeV) which leads to a calibrated total analyzing power $A = 0.48 \pm 0.02$ for each polarimeter at 796 MeV⁴⁾. (Note that this is a few percent lower than $^1\text{H}(p,p)^1\text{H}$ reaction.)

Another concern is a possibility of accidental (random) events. The random coincidences were determined by adding a delay (40 ns) to one arm of a detector. By subtracting the random coincidences from the measured rate, one obtained the true rate of coincidences that was used in the calculation of the beam polarization. In order to calculate the random rate, let the single rate in each detector be about 20 times the coincidence rate and the resolving

time be about 10 ns. Hence, the random rate for 4% duty factor can be obtained from⁴⁾

$$\text{Random Rate} \equiv R = \frac{(20 \times C)^2}{0.04} \cdot 10^{-8}$$

where C is the coincidence rate, e.g., if $C = 10^3$ Hz, then $R = 100$ Hz.

The LAMPF Line B and EPB polarimeters are shown in Figs. 2-5 and 2-6. The only essential differences between the beam line polarimeter are in geometry. However, their setups have been made to minimize the need for geometrical changes to accommodate energy changes.

In addition to knowing the beam polarization, it is necessary to know the beam intensity. The intensity of the incident beam was monitored by the EPB beam polarimeter. This is accomplished by adding the properly corrected left and right yields for the given beam polarization.

2.4 The Recoil Polarimeter (JANUS) and Construction

The measurement of spin rotation (triple scattering) parameters for nucleon-nucleus or nucleon-nucleon interaction (P, D, R, R', A, and A' as defined by Wolfenstein⁶⁾ requires a special device, namely a polarimeter. The recoil polarimeter is designed to measure the polarization of scattered particles in double or triple scattering experiments. (The word "recoil" is used to distinguish from beam-line polarimeters).

Figure 2-5: Line B polarimeter setup

Figure 2-6: EPB polarimeter setup

Figure 2-7 shows the schematic layout of the experimental setup. The first scatter is from the liquid deuterium target. Both primary and conjugate particles are detected in multiwire drift chambers (MWDC), (section 2.4.1), selecting elastic scattering by the angular correlations, which is characteristic of the two-body final state.

The particle identification (PID) was obtained using time-of-flight (TOF) and pulse height (PH) information derived from the event-defining scintillators. A typical two-dimensional dot plot of TOF versus X7 (x-position of chamber 7) is shown in Fig. 2-8. These figures show the recoil deuteron group, distinguished from other kinds of reactions. The polarization components of the elastically scattered protons is then measured by observing the azimuthal distribution after the second scattering (Section 3.2) from the carbon analyzer (the carbon polarimeter). In the intermediate energy range, carbon is a suitable polarization analyzer. It is easy to handle, has large analyzing power (≈ 0.3) and scattering cross section, and introduces relatively little multiple scattering.

The polarimeter (JANUS), which was used in this experiment, (similar to polarimeters at SIN⁷) and TRIUMF⁸) consists essentially of a carbon scatterer (density $\approx 1.8 \text{ g/cm}^2$ and purity better than 99.5%) $24" \times 24"$ with thickness ranging from 1.25" to 10" depending on the incident proton energy and scattering angle.

An elevation view of the polarimeter along with some critical dimensions is shown in Fig. 2-9. SF and SB are two identical scintillation counters; each consists of two $12" \times 24"$ scintillators, which overlap slightly. SF and SB where each gives an active area of

Figure 2-7: A schematic layout of the experimental setup.

Figure 2-8: a) Typical two-dimensional particle identification dot plot.
b) With a linear polynomial correction.

Figure 2-9: A schematic side view of the polarimeter (JANUS).

24" x 24", with a total of four LAMPF standard photomultipliers (EMI9813). Figure 2-10 shows the schematic layout of the back scintillator (SB) which is similar to the front one (SF).

The particle trajectories before and after scattering from a selected thickness of carbon were defined by six identical drift chambers, C1-C3 at the front and C4-C6 at the rear of the analyzer. Each chamber has one x and y drift plane. The planes have an active area $60 \times 60 \text{ cm}^2$ (Fig. 2-11). All scintillators and chambers are mounted on a movable frame. The frame is designed to allow the separation of the scintillators, chambers, and carbon to be changed. The carbon thickness can be changed without disturbing the chambers.

2.4.1 Multiwire Drift Chamber (MWDC)

Each multiwire drift chamber used in this experiment is two dimensional with the anode and cathode wires of one coordinate in the same direction and with the X- and Y-planes parallel to each other. The chamber consists of two cover planes and three ground planes separated by two signal planes in the sequence cover-ground-signal-ground-signal-ground-cover, respectively. All planes are assembled on an aluminum frame.

Each ground plane consists of 0.5 mil aluminum foil. Each wire (signal) plane consists of anode wires with 0.8 mil goldplated tungsten sense wires and separation of 0.8128 cm, alternating with 3 mil goldplated aluminum field shaping wires (cathode wires) placed halfway between the anode wires as shown in Fig. 2-12. The two planes

Figure 2-10: Back view of polarimeter, showing the back scintillator configuration.

Figure 2-11: A schematic layout of the polrrimeter.

Figure 2-12: Drift chamber construction.

are separated by 0.952 cm such that the X-plane is in front of Y-plane (Fig. 2-11). Each anode wire is connected to a 200-ns long delay line, with a 2.5 ns per wire separation. The delay lines are microstrip⁹⁾ delay lines printed on copper teflon-glass laminate. The properties of a 30 cm length of delay line are summarized in Table 2-1. Both ends of the delay line are capacitively coupled to a discriminator (Fig. 2-13). The signals from these discriminators are delayed about 200 ns, then used to stop TDCs (Fig. 2-14). The delay lines are held at a high voltage (HV), which are dependent on the gas mixture. The chambers are operated with a typical gas mixture of 66% argon, 33% isobutane and 0.03% freon. For this gas, mixture positive 2250 volts are applied to anode wires while the cathode wires are grounded. The gas pressure is kept slightly above atmospheric pressure by submerging the end of the exit line of the chamber about 3 mm into low-vapor pressure oil. Figure 2-15 shows the gas handling system. The flow rate is such that the gas in a chamber is replaced every few hours.

The gas mixture serves as a medium for ionization. When a charged particle passes through the chambers, it ionizes the gas; the ionization is detected by the wires nearest the trajectory, and the pulse generated travels to each end of the delay line. These signals are used to start and stop time to amplitude converters (TAC).

Figure 2-13: A schematic layout of delay line in MWDC.

Figure 2-14: Multiwire drift chamber electronics (only for x-plane)

Figure 2-15: Wire chambers gas handling system.

Table 2-1: Delay Line Properties

Length	30.0 cm
Width	7.6 cm
Thickness	0.8 mm
Conductor Width	0.5 mm
Conductor Length	20.0 m
Inductance	9.0 μ H
Capacitance	680.0 PF
Attenuation (100 MHz)	3.0 db
Delay	2.5 ns/cm
Delay/Risetime	19:1
Impedance	110 Ω
Resistance (D.C.)	40 Ω
Resistance (100 MHz)	80 Ω

In the drift chambers, the shortest distance from the anode wire to the track of an ionizing particle is determined with high accuracy.¹⁰⁾ However, a unique determination of the track location in a multiwire structure is prohibited by the left-right ambiguity, i.e., two tracks located at the same distance but on either side of the anode wire cannot be distinguished.

Among the solutions adopted to lift the ambiguity problem, such as using the "double wire" system,^{11,12)} the "double chamber" system,^{13,14)} and the latest method by Walenta¹⁵⁾ and Breskin,¹⁶⁾ we selected the method of Walenta and of Breskin.

To get unambiguous information on the position, one must pay attention to two things:

- 1) the precise boundary of the drift region belonging to one wire;
- 2) the side of the wire on which the particle has passed.

The simple method suggested by Walenta and Breskin requires no mechanical change in the chamber structure. The events that originate from opposite sides of a given wire are distinguished by picking up the image of the avalanche on the adjacent field shaping wires. The wire closest to the avalanche has the larger pulse. Alternate cathode wires are bussed together to give two pulses known by convention as "odd" and "even" (Fig. 2-12). The pulses from the "odd" and "even" busses are added ($O+E$) and subtracted ($O-E$). Pulses from the $O-E$ output are fed into a charge sensitive analog to digital converter (ADC) and integrated over the first 100 ns of the pulse. The resulting spectrum consists of two peaks which correspond to events occurring on the opposite side of the anode wire (Fig. 2-16). The summed ($O+E$) pulse output is fed into a time to digital converter (TDC) and used for determination of the drift time (see Appendix C).

The reason for using six chambers (three in front and three after the carbon analyzer) is not only to determine the trace of incident and outgoing protons, but also to perform a consistency check on each trace. These are further discussed in Section 2.4.4 .

Figure 2-16: Odd-even pulse height spectrum.

2.4.2 Electronic Circuitry

The electronics used in this experiment can be divided into four parts, namely, event trigger, fast reject, spin logic, and data acquisition system. These will now be discussed.

The MASTER TRIGGER is defined as a coincidence of SF, SB, and SC, all in anti-coincidence with the MASTER INHIBIT signal. The coincidence between SF and SB insures that the system will not be triggered by unwanted particles, such as low-energy charged particles (these particles do not pass through the entire system), or by noise in the scintillators. Each scintillator (SF and SB) is an "OR" of the top and the bottom paddles shown in Fig. 2-10 (Fig. 2-17). The schematic layout of MASTER TRIGGER along with MASTER INHABIT is shown in Fig. 2-18. The MASTER INHIBIT is an "OR" of the GROSS INHIBIT (Fig. 2-19), the LAMPF Trigger Module (LTM) busy, and master and event trigger busies. The GROSS INHIBIT prevents MASTER triggers except during a pulse by the BEAM GATE (macropulse) and when spin signal is in an allowed state. The LAMPF Trigger Module¹⁷⁾ is used to generate a BUSY signal from the time an event is registered until the system is read, cleared, and another event can be accepted. The master trigger BUSY is triggered by a master trigger and inhibits further master triggers until a decision is made about accepting the event (the entire process takes about 400 μ sec).

The EVENT TRIGGER has two modes. The so-called ACCEPT mode is defined such that the EVENT TRIGGER is the same as the MASTER TRIGGER.

Figure 2-17: Scintillator, SF (or SB), electronics.

Figure 2-18: Master trigger and master inhibit logics.

In the second mode a hardware (FAST REJECT) system is being used, and the coincidence of MASTER TRIGGER and FAST REJECT forms the EVENT TRIGGER. This is called the REJECT mode (about 80% of data were taken in this mode). A schematic drawing of the EVENT TRIGGER is shown in Fig. 2-20; the dashed line indicates how these two modes are different. The FAST-REJECT system is used in order to eliminate the small angle scatters through the hardware, and therefore to enhance the percentage of events scattering at an angle $>3^\circ$ in carbon. A large percentage of events would scatter at small angles in the carbon, where the analyzing power of the carbon is small. At 800 MeV, with a 10"-thick carbon slab, use of this system increased the percentage of scatters $>3^\circ$ written to tape from 20% to 33%. The system is adjusted such that no events scattering $>4^\circ$ are rejected.

The electronic circuits of the system which used to reject the events from the small-angle scattering region is shown in Fig. 2-21. Signals from the delay line ends of the chamber are used to start and stop a time to amplitude converter (TAC). For a linear delay line, the TAC output will be linearly related to the wire number. By using a pulse summer, one can calculate the relative slopes of two (incident and outgoing) rays. Once the sum is formed, the output pulse of the summer is fed into a linear gate, which allows only the middle portion of input pulse to be output. This output is then fed into a single-channel analyzer (SCA), which outputs a pulse only if the input pulse is within some adjustable range. Therefore the single-channel analyzer (SCA) triggers when the scattering angle projected on the X-plane is small, and one other monitors the Y-projection. A coincidence between the two is formed from an event with small

Figure 2-19: Gross inhibit logic

Figure 2-20: Event trigger logic

Figure 2-21: A block diagram of fast-reject logic

scattering angles in both the X- and Y-direction (identical setups are used for both the X and Y-directions). In addition, a coincidence of the valid stop outputs from the TACs of the MWDC's is formed, in order to eliminate any event in which one delay line end did not fire. The coincidence of these two signals provide the final FAST-REJECT signal.

Another part of the electronic logic is called the spin logic (Fig. 2-22). Two-spin signals (NORMAL and REVERSE) are supplied from the ion source control room. Normally one signal will be 5 volts and the other zero volts, while during the spin flip both drop to zero volts. When the signals go from a one up and one down state to both down, a signal pulse is generated which triggers a read of the scalers. If both signals are up or both down, data taking is inhibited. There is a one-second delay on the circuit for the following reasons:

1. This delay lets any existing noise in the circuit die down and prevents multiple triggering,
2. It prevents an unwanted scaler read trigger, since one spin signal occasionally drop then immediately return to its proper state.

The spin logic has two purposes: 1) to trigger a scaler read at the time of the spin change and, 2) to inhibit data acquisition during spin changes or at the time of faulty spin signals.

Once an event is triggered by the hardware, it is read by the PDP 11/45 computer and written to magnetic tape. This transfer is controlled by the data acquisition program "Q", which will be discussed in the next section. Each event written to tape consisted of about 80 data words. These contain three TDC outputs and one ADC for each

Figure 2-22: A schematic diagram of scaler control and spin logic.

polarimeter plane, two coincidence buffers, the wire information from the MWDC's, the time of flight between SC and SB, the time of the event relative to the RF signal and the SC pulse height.

In addition a total of 100 scalars are also read in for each spin reversal. These contain the O+E signals, the MWDC signals, the signals from each end of the delay lines, all from scintillators information, and beam line polarimeter telescopes (EPB and Line B), and three real time scalars. Each real-time scaler is hardware gated differently, such that one counts during all data taking, one during beam normal times, and the third one during beam reverse time. This allows scalars to be split into normal and reverse sets without duplicating all modules and hardware gating them.

2.4.3 Polarimeter Calibration

A polarimeter such as the one used here measures asymmetry resulting from proton-carbon scattering. This asymmetry (ϵ) is related to the beam polarization (P_b) via Eq. 2-1. Let us re-write this equation

$$\epsilon = P_b \cdot A_c(\theta) \quad .$$

Since the polarimeter does not distinguish between elastic and inelastic scattering, and since the analyzing power, $A_c(\theta)$, is a function of both energy and scattering angle, the polarimeters must be calibrated in proton beams of known energy and polarization.

The actual calibration of the polarimeter has been done several times previously at different energies. This has been carried out by using different and independent techniques, such as, 1) double scattering calibration, 2) quench ratio calibration, and 3) polarized target calibration. Details of these calibrations can be found in Refs. 4, 21, and 22. We repeated some of these calibrations. They will now be discussed.

2.4.4 Chamber Alignment, Calibration and Checks

The first step in the calibration procedure is the alignment of the chambers. The chambers were physically aligned such that the first wire of each chamber was within a few mm of being on a straight line drawn through the chambers. However, this alignment is not crucial, since the anode position calibration (see Appendix C), including the phase correction, effectively shifts the wire positions. Therefore, an alignment correction of the following form was made to final X (or Y) positions

$$X(Y) = X(Y) + R$$

where the Rs are constants which can be determined with no carbon analyzer. These constants correct the positions X(Y) so that a straight track through chambers XY1 to XY6 appears as a straight line to the software. This "alignment" is affected by the physical alignment of chamber frames, the wires within the frames, the anode delay lines, the delay cables and the TDC calibrations. In order to

determine this constant, one needs to calculate a quantity called "residual."

The so-called residuals are defined by the difference between the best fit line to the three points of one set of planes such as X1,X2 and X3 and the actual position in the middle plane X2 (similarly for Y1-Y3, X4-X6 or Y4-Y6 planes). The histogram of a residual should show a peak centered on zero. This is achieved by adjusting the R-constants. Once this is done and the R-constants determined, one adjusts the position of the rear chambers with respect to the front chambers. It is a consistency check of the above procedure, that the residual distribution is a sharp peak and centered on zero, otherwise we should repeat the procedure.

During data taking the C2 and C5 residuals (front and back residuals) were used for consistency checks on incident and outgoing rays. Figures 2-23 and 2-24 show X2 and Y2, residuals, with sharp peaks at zero with a FWHM of about 0.3 mm. Events with bad residuals may have scattered from the material of a chamber or a frame and are normally rejected.

Once the chambers are aligned, one can calculate the scattering angles. Before doing this, however, some checks are made to determine internal consistency of the polarimeter, known as checks for good events. These are the checking of the left-right determination, and the monitoring of the quantity called the "checksum."

In other words, this is a check of whether the two pulses from delay line ends agree on the wire number (n), and the pulses from anode and cathode consent on the drift time (DTIM).

Figure 2-23: x-plane residual spectrum for chamber 2, software cut is made for ± 1 mm

Figure 2-24: y-plane residual spectrum for chamber 2, software cut is made for ± 1 mm

The checksum is derived for each chamber plane as follows. The drift time may be derived either from the average time of the two ends of the anode delay line or from the time of the O+E pulse (TOE). The difference between these two estimates of the drift time is the checksum, that is

$$CKSM = T_+ + T_- - 2TOE$$

where T_+ and T_- are the times obtained from the anode delay line ends and TOE is the O+E pulse time.

It can be shown for a good event, this checksum is constant. In order to see this, let N be the number of wires in each plane, and D delay per wire, thus

$$T_+ = nD + DTIM + R1 ,$$

$$T_- = (N-n)D + DTIM + R2 ,$$

$$TOE = DTIM + R3 ,$$

where R_s are constant offsets.

A good event requires the identical values for $DTIM$ and n measured by the different TDCs, hence

$$CKSM = ND + R = \text{Const} , \quad (\text{ where } R = R1 + R2 - 2R3).$$

Figure 2-25 shows a checksum spectrum. This spectrum consists of two peaks; the main peak corresponding to good events followed by a long tail, the second one corresponding to time out of all TDCs. This peak

Figure 2-25: Checksum spectrum.

is separated from the main peak and excluded by cuts in ALLTST. The broad background corresponds to bad events including time out of one or two TDCs and events in which more than one track passed through the chamber. Events which lie outside of the checksum peak (main peak) were rejected.

The other check is made in order to identify the incorrect odd-even assignment. The residual spectrum (Fig. 2-23,24) should show a sharp peak, however, the result of incorrect left-right identification causes a region with a width of about 1.2 cm containing about 5% of the events. We are taking advantage of having three planes in order to determine the ray left-right assignment. After this correction, a cut of ± 1 mm is made on the residual.

The determination of a residual requires three good corresponding checksums (because a good checksum indicates a good check on internal consistency of each plane). We found that about 3% of the events which passed good checksum do not pass good residual.

Once the above checks have been completed, the scattering angles (polar angle, θ_c , and azimuthal angle, ϕ_c) in the carbon along with other quantities such as X-, Y-, Z-, and D-close are calculated. A review of the calculations is given in Appendix D.

2.5 Data Acquisition Program and Software

The primary goal of the data acquisition system was to develop a system that can support the generation of valid experimental data as fast and efficiently as possible. This goal implies the ability to acquire, record, analyze, and display multiparameter experimental data.

A general data acquisition system called "Q"¹⁸) was written by the LAMPF computer support group (MP-1). This system provides a simple "Q" language called QAL, use of this language allowed the experimenters to write all the required data acquisition programs in Fortran. The original Q system has been modified by LAMPF MP-10 group, and this modified versions have been used in the experiment.

The software used during the experiment can be divided into three main groups: the ANALYZER, the ALLTEST package, and the DISPLAY package. The ANALYZER, which is written in QAL language, contains a list of the devices used in data acquisition, their location in CAMAC and a short identifier. After all devices have been specified, the EVENTS desired are specified. An EVENT is a listing of devices to be read whenever a hardware or software trigger for that event is received. In addition, all Fortran subroutines which are needed for the analysis are called. Once the QAL program is written and compiled, the ANALYZER is ready for data acquisition. The ANALYZER is responsible for the disposition of data associated with each event TRIGGER. Two types of events (Event 6 and Event 7) were used during the experiment. Event 6 was the data event, while the Event 7 was the scaler event.

There are three modes of running with regard to the on-line analysis: "must process", "may process", and "no process". On every polarization reversal Event 7 was triggered to read, clear the scalers and add them to appropriate buffer locations according to whether the spin had been normal or reverse. A "must process" required that all events be processed. On-line data acquisition was performed in a "may process" mode whereby the raw data words associated with each TRIGGER pulse are written to tape and if time permits, the data are analyzed on line. Finally, a "no process" means the data is only written to tape and not analyzed. Normally "may process" mode will be used for Event 6 and "must process" mode for Event 7.

The ALLTEST package¹⁹⁾ is used to define conditions which raw or calculated data must satisfy in processing an event. The test file consists of two parts: the first part contains a series of tests and gates on data words called "microtests;" the second part contains a series of logic tests on combinations of previously defined tests known as "macrotests". The test file used in analyzing the experimental data along with histogram file, are given in Appendix B.

Along with the other packages, a DISPLAY²⁰⁾ software is also available. This software enables the experimenter to define as many as 40 histograms from raw and calculated data, view the histograms as data accumulates on-line (or off-line), and store them on disk when the run is completed. (Note that the "Q"-system histogram package only histograms positive numbers. Therefore the histograms are offset by 10,000, i.e., true zero is displayed as channel 10,000). Another capability of this package is that to create and display a live, two-dimensional dot plots of the data. Histograms are maintained in

central memory and they can be displayed on a Tektronix 4000 series graphic terminal.

CHAPTER 3

Off-Line Analysis and Results

Since all the experimental data are acquired in the "may process" mode in which the raw data are written directly to magnetic tape, and if time permits, analyzed on-line (typically about 20 to 30% of the data can be analyzed on-line). It is therefore necessary to analyze all the experimental data off-line.

In addition, the off-line analysis allows the experimenter to revise the software to provide additional calculations or information not determined on-line. In this chapter the software and methods used in the off-line analysis of data will be described and the results of the analysis will be presented.

3.1 Software

In this section all the modifications made to the software for the data replay along with some description will be given.

The software packages used in replay are similar to those used for on-line data acquisition except the ANALYZER operates in a "must process" mode, thus ensuring that all taped data are analyzed. In addition, some modifications were made. The ANALYZER were modified to define and calculate additional data words that were used in determining good events such as correcting SC timing, Figs. 2-8a,b, as well as to calculate the true scattering angles (θ_s and ϕ_s) at the target.

The ALLTEST file was also modified for the off-line analysis (different test files were used for the above mentioned analysis). Additional histograms and several associated tests were defined. Certain tests were of particular importance in defining acceptable events to be histogrammed. Several tests were combined to create a GOOD-EVENT test. Some tests were made to get the events associated with each bin of scattering angle (NORMAL and REVERSE). Among these tests, there was a test with all good events and necessary cuts. This test was used to make a condensed tape of only useful events that can be used for any further replay.

During the replay of the data tapes the completed necessary sums (see Section 3.2.3) were accumulated in the ASY section of ALLCOM. At the end of replay of each data tape, the ASY data file containing all accumulated sums was created. This file was used for the actual calculations of the asymmetries (spin rotation parameters). Note that the extraction of deuterium analyzing power required different information, which will be discussed later.

3.2 Principle of Data Analysis

The analysis consists of two parts: 1) extraction of the analyzing power of deuterium, and 2) measurement of the spin rotation parameters. Each part of the analysis will be discussed separately.

3.2.1 Coordinate System

In order to describe the result of polarization transfer experiments (triple scattering experiments), one must choose the coordinate systems associated with the scattering. These reference frames are defined as follows:

The coordinate system of incident beam $(\hat{x}_0, \hat{y}_0, \hat{z}_0)$, are defined in the laboratory to be with $\hat{z}_0$ along $\vec{k}_{in}$, $\hat{y}_0$ is the up-down laboratory direction, and $\hat{x}_0$ is defined consistent with a right-handed coordinate system.

The scattering reaction can be described in the plane defined by the incident momentum vector, $\vec{k}_{in}$, and the scattered momentum vector, $\vec{k}_{out}$. For the incident particle coordinate system, $(\hat{x}, \hat{y}, \hat{z})$. $\hat{z}$ is defined to be along $\vec{k}_{in}$ with $\hat{y}$ being along $\vec{k}_{in} \times \vec{k}_{out}$ and $\hat{x}$ defined in a right-handed coordinate system. The scattered particle coordinate system, $(\hat{x}', \hat{y}', \hat{z}')$, has $\hat{z}'$ along $\vec{k}_{out}$ and $\hat{y}'$ along $\vec{k}_{in} \times \vec{k}_{out}$ (i.e. $\hat{y}$ and $\hat{y}'$ are parallel). $\hat{x}'$ is again defined by a right-handed coordinate system. These coordinate systems are shown in Fig. 3-1. In this figure, θ_s is the true scattering angle, and ϕ is the azimuthal scattering angle.

The description of particle polarizations are given in the reference system $(\hat{s}, \hat{n}, \hat{l})$, where $\hat{l}$ (longitudinal) is along the particle momentum, $\vec{k}$, $\hat{n}$ (normal) is perpendicular to the scattering plane (i.e. parallel to $\hat{y}$ and $\hat{y}'$), and $\hat{s}$ (sideways) is perpendicular to these in a right-handed coordinate system.

Figure 3-1: Coordinate systems associated with the scattering.

3.2.2 Definition or Physical Meaning of Wolfenstein Parameters

As mentioned in Section 1.1, the five triple scattering observables corresponding to the five Wolfenstein parameters, $D_{nn'}$ (= D), $D_{\ell\ell'}$ (= A'), $D_{s\ell'}$ (= R'), $D_{\ell s'}$ (= A), and $D_{ss'}$ (= R) together with the induced polarization, $P(\theta)$, and analyzing power, $A(\theta)$, completely describe the elastic scattering in a general parity-conserving and time-reversal invariant form (Eq. 1-20):

$$\vec{P}' = \begin{pmatrix} R & 0 & A \\ 0 & D & 0 \\ R' & 0 & A' \end{pmatrix} \cdot \vec{P} + \begin{pmatrix} 0 \\ P(\theta) \\ 0 \end{pmatrix} \frac{1}{1 + P_n \cdot A(\theta)} \quad (3-1)$$

where P_n is the incident beam polarization in $\hat{n}$ -direction.

The parameter D is called depolarization, and the R, A, R', A', rotation parameters. The physical meaning of each parameter can simply be described by considering each of the possible incident beam polarization orientations and derive these parameters from Eq. (3-1).

Let the beam be completely polarized in $\hat{n}$ -direction, thus

$$\vec{P} = \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix} P_n \quad .$$

By substituting this into Eq. (3-1), one shows the outgoing particles are polarized in $\hat{n}'$ -direction, and Eq. (3-1) becomes

$$\vec{P}' = \pm \left(\frac{D \pm P(\theta)}{1 \pm A(\theta)} \right) \hat{n}' \quad (3-2)$$

(Note that for elastic scattering the polarization $P(\theta)$ is equal to analyzing power $A(\theta)$, by time reversal¹⁾ which was checked during the experiment and, detailed in Section 3.6).

For $D = 1$, one obtains $P'_n = \pm 1$, that is, if the incident beam is completely polarized, the scattered beam remains completely polarized. If $D < 1$, one obtains $P'_n < 1$, thus the initial beam is depolarized by scattering. From the condition that $|\vec{P}'| < 1$, the value of D is limited by

$$-1 + 2|P(\theta)| < D < 1 \quad .$$

Figure (3-2) shows the physical meaning of D and $P(\theta)$ parameter.

If the beam is polarized in $\hat{s}$ -direction, i.e.

$$\vec{P} = \begin{pmatrix} P_s \\ 0 \\ 0 \end{pmatrix} \quad .$$

Then Eq. (3-1) reduces to

$$\vec{P}' = \begin{pmatrix} R p_s \\ P(\theta) \\ R' p_s \end{pmatrix} \quad \text{or} \quad \begin{aligned} P'_s &= R p_s \\ P'_n &= P(\theta) \\ P'_l &= R' p_s \end{aligned} \quad (3-3)$$

Therefore, for a given P_s and measuring the final state polarization

Figure 3-2: Wolfenstein parameters which can be measured from $\hat{n}$ -type beam polarization.

components, one gives R , R' and $P(\theta)$. Figure 3-3 shows these parameters. Finally, for $\hat{\ell}$ -type beam polarization, i.e.

$$\vec{P}' = \begin{pmatrix} 0 \\ 0 \\ p_{\ell} \end{pmatrix},$$

one can obtain from Eq. (3-1)

$$\vec{P}' = \begin{pmatrix} Ap_{\ell} \\ P(\theta) \\ A'p_{\ell} \end{pmatrix}, \quad (3-4)$$

or $p'_S = Ap_{\ell}$, $p'_N = P(\theta)$ and $p'_L = A'p_{\ell}$. Therefore, in the same way the parameters A , A' and $P(\theta)$ can be measured. These parameters are shown in Fig. 3-4. Since there is no asymmetry corresponding to longitudinal ($\hat{\ell}$) component of polarization, therefore it is not possible to measure this component directly. The measurement of this component would require a special experimental setup. But this experiment was limited to measurements of those parameters which were related to the transverse components of the scattered proton polarization, namely, D , R , A , and $P(\theta)$.

3.2.3 Asymmetry Calculation

Let us consider an incident proton beam moving along the $\hat{z}$ direction with a polarization

$$\vec{P} = p_S \hat{s} + p_N \hat{n} + p_L \hat{\ell}. \quad (3-5)$$

Figure 3-3: Wolfenstein parameters which can be measured from $\hat{s}$ -type beam polarization.

Figure 3-4: Wolfenstein parameters which can be measured from l -type beam polarization.

The transverse polarization components along $\hat{n}'$ and $\hat{s}'$ of the scattered proton can be measured by a second scattering on carbon. This measurement is accomplished by measuring the corresponding asymmetry. There are a number of methods for extracting the asymmetries. Here the methods from which the data was analyzed²⁾ will be discussed.

The nucleon-nucleon interaction obeys certain symmetry principles. These imply that the observed angular distribution after the second scattering on carbon is given by

$$I(\theta_c, \phi_c) = I_0(\theta_c) [1 + \epsilon_n(\theta_c) \cos \phi_c + \epsilon_s(\theta_c) \sin \phi_c] A(\theta_c, \phi_c) \quad (3-6)$$

where

θ_c and ϕ_c are the polar and azimuthal scattering angles in carbon, Fig. 2-11,

$I_0(\theta_c)$ is the unpolarized intensity,

$\epsilon_n = p_n A_c(\theta_c)$, the asymmetry on the $\hat{n}$ direction,

$\epsilon_s = -p_s A_c(\theta_c)$, the asymmetry on the $\hat{s}$ direction, (see Ref. 3 for sign convention)

$A_c(\theta)$ is the carbon analyzing power,

$A(\theta_c, \phi_c)$ is the acceptance function for the polarimeter as defined by the back scintillator (SB). Note that we will omit the index c for θ and ϕ in the rest of the text.

The asymmetries $\epsilon_n(\theta)$ and $\epsilon_s(\theta)$ are the quantities of interest and need to be determined. Since Eq. (3-6) contains only a cosine and sine term, the immediately obvious method of extracting their coefficients is to do a Fourier analysis. Therefore by taking the Fourier analysis of Eq. (3-6) under varying assumptions on $A(\theta, \phi)$, one gives:

a) Cone Test

This test requires the rejection of events if after rotating the outgoing track about the incoming track to form a cone, part of the cone fell outside the acceptance of the polarimeter. In this case, one has

$$A(\theta, \phi) = A(\theta) \quad .$$

Under this assumption, Eq. (3-6) becomes

$$I(\phi) \sim (1 + \epsilon_n \cos \phi + \epsilon_s \sin \phi) A(\phi) \quad ,$$

where the new acceptance function $A(\phi_c)$ is either one or zero for accepted or rejected events, respectively. Hence, with

$$\frac{1}{2\pi} \int_0^{2\pi} I(\theta, \phi) d\phi = N(\theta) \quad (3-7a)$$

one gets

$$\frac{1}{2\pi} \int_0^{2\pi} I(\theta, \phi) \cos \phi d\phi = \frac{1}{2} N(\theta) \epsilon_n(\theta) = \frac{1}{2} N(\theta) A_c(\theta) p_n \quad (3-7b)$$

$$\frac{1}{2\pi} \int_0^{2\pi} I(\theta, \phi) \sin \phi d\phi = \frac{1}{2} N(\theta) \epsilon_s(\theta) = -\frac{1}{2} N(\theta) A_c(\theta) p_s \quad . \quad (3-7c)$$

In the actual analysis the integrals are replaced by sums (i.e. $\int_0^{2\pi}$ $\frac{d\phi}{2\pi} \rightarrow \sum_{\phi}$). Hence for a given θ , e.g. $\theta = \theta_i$, the Eqs. (3-7a,b,c) can be written as

$$\sum_{\phi} I(\theta_i, \phi) = N(\theta_i) \quad (3-8a)$$

$$\epsilon_n(\theta_i) = \frac{2 \sum_{\phi} I(\theta_i, \phi) \cos \phi}{\sum_{\phi} I(\theta_i, \phi)} = A_c(\theta_i) P_n \quad (3-8b)$$

$$\epsilon_s(\theta_i) = \frac{2 \sum_{\phi} I(\theta_i, \phi) \sin \phi}{\sum_{\phi} I(\theta_i, \phi)} = -A_c(\theta_i) P_s \quad (3-8c)$$

The error for ϵ in Eqs. (3-8b,c) can be obtained by considering $I(\theta_i, \phi)$ as a measurable quantity, thus

$$\delta_{\epsilon_n}^2 = \left\{ \frac{\partial}{\partial I(\theta_i, \phi)} \left[\frac{2 \sum_{\phi} I(\theta_i, \phi) \cos \phi}{\sum_{\phi} I(\theta_i, \phi)} \right] \right\}^2 \delta_I^2 \quad (3-9)$$

where

$$\delta_I^2 = I(\theta_i, \phi) \quad (3-10)$$

CONE-TEST

ACCEPTED

REJECTED

Figure 3-5: Examples of accepted and rejected events for cone-test.

By considering Eq. (3-8a) and (3-10), then

$$\begin{aligned}\delta_{\epsilon_n}^2 &= 4 \sum_{\phi} \left[\frac{N(\theta_i) \cos \phi - \Sigma I(\theta_i, \phi) \cos \phi \cdot 1}{N^2(\theta_i)} \right]^2 I(\theta_i, \phi) \\ &= 4 \sum_{\phi} \left\{ \left[N^2(\theta_i) I(\theta_i, \phi) \cos^2 \phi + [\Sigma I(\theta_i, \phi) \cos \phi]^2 I(\theta_i, \phi) - \right. \right. \\ &\quad \left. \left. 2 I(\theta_i, \phi) \cos \phi \Sigma I(\theta_i, \phi) \cos \phi \cdot N(\theta_i) \right] / N^4(\theta_i) \right\}\end{aligned}$$

Using Eq. (3-8b) and noticing that

$$\Sigma I(\theta_i, \phi) \cos^2 \phi = \frac{1}{2} \Sigma I(\theta_i, \phi) = \frac{1}{2} N(\theta_i),$$

one obtains

$$\begin{aligned}\delta_{\epsilon_n}^2 &= 4 \left(\frac{N^2(\theta_i) \cdot \frac{1}{2} N(\theta_i) + \frac{1}{4} N^3(\theta_i) \epsilon_n^2 - \frac{1}{2} N^3(\theta_i) \epsilon_n^2}{N^4(\theta_i)} \right) \\ &= \frac{2}{N(\theta_i)} - \frac{\epsilon_n^2}{N(\theta_i)}\end{aligned}$$

or

$$\delta_{\epsilon_n}^2 = \frac{2}{N(\theta_i)} \left(1 - \frac{\epsilon_n^2}{2}\right) \quad (3-11)$$

since $\epsilon_n^2 \ll 1$, Eq. (3-11) becomes

$$\delta_{\epsilon}^2 = \frac{2}{N(\theta_i)} \quad (3-12)$$

Figure (3-5) shows examples of accepted and rejected events.

Note that Eqs. (3-8a,b,c) and (3-12) still have explicit θ dependence, since they are only summed over ϕ . In order to combine the information from different θ bins, there are two options; the so-called "unweighted" and the "weighted" methods.

a.1) Unweighted Cone Test

This technique does not require prior knowledge of $A_c(\theta)$, and can be accomplished by averaging the asymmetries over all θ bins and dividing by an average carbon analyzing power. Thus Eq. (3-8b) becomes

$$p_n \sum_{\theta} A_c(\theta) N(\theta) = \sum_{\theta} \left[2 \sum_{\phi} I(\theta, \phi) \cos \phi \right] = 2 \sum_{\theta, \phi} I(\theta, \phi) \cos \phi \quad (3-13)$$

dividing each side by $\sum_{\theta} N(\theta)$, one obtains:

$$p_n \frac{\sum_{\theta} A_c(\theta) N(\theta)}{\sum_{\theta} N(\theta)} = p_n \bar{A}_c = \frac{2 \sum_{\theta, \phi} I(\theta, \phi) \cos \phi}{N}$$

where $N = \sum_{\theta} N(\theta)$ is the total number of events and finally

$$p_n = \frac{2 \sum I(\theta, \phi) \cos \phi}{N \bar{A}_c} \quad (3-14)$$

similarly

$$p_s = \frac{2 \sum I(\theta, \phi) \sin \phi}{N \bar{A}_c} \quad (3-15)$$

From Eqs. (3-9, -10), the statistical error for p becomes

$$\delta_{p_n}^2 = \delta_{p_s}^2 \cong \frac{2}{N(\bar{A}_c)^2} \quad (3-16)$$

Therefore, for the unweighted cone test, the following sums must be accumulated

$$\sum I(\theta, \phi) \cos \phi, \sum I(\theta, \phi) \sin \phi, \text{ and } \sum I(\theta, \phi).$$

In addition, a histogram of $N(\theta)$ is needed for the $\bar{A}_c$ calculation.

a.2) Weighted Cone Test

In principle, the different θ bins give independent measurements for P_n and P_s . In the weighted method one constructs an average where the different θ bins are weighted with their statistical accuracies. That is

$$P_n = \frac{\sum ((p_n)_i / (\delta_{p_n}^2)_i)}{\sum (1 / (\delta_{p_n}^2)_i)} \quad (3-17)$$

where the index i refers to different θ bins.

Let us rewrite Eqs. (3-8b) and (3-12) for p_n at a given θ

$$(p_n)_i = \frac{2 \sum_{\phi} I(\theta, \phi) \cos \phi}{N(\theta) A_c(\theta)}$$

$$(\delta_{p_n}^2)_i = \frac{2}{N(\theta) A_c^2(\theta)}$$

Introducing these two equations in Eq. (3-17) gives

$$P_n = 2 \frac{\sum_{\theta, \phi} I(\theta, \phi) \cos \phi A_c(\theta)}{\sum_{\theta, \phi} I(\theta, \phi) A_c^2(\theta)} \quad (3-18)$$

Similarly,

$$P_s = 2 \frac{\sum I(\theta, \phi) \sin \phi A_c(\theta)}{\sum I(\theta, \phi) A_c^2(\theta)} \quad (3-19)$$

where the Σ implies summing over all events. The statistical error can be calculated as before, by means of the equation

$$\begin{aligned}\delta_{p_n}^2 &= \left\{ \frac{\partial}{\partial I} \left[\frac{2 \sum_{\theta, \phi} I \cos \phi A_c(\theta)}{\sum I A_c^2(\theta)} \right] \right\}^2 \delta_I^2 \\ &= 4 \Sigma \left[\frac{A_c(\theta) \cos \phi \sum I A_c^2(\theta) - \sum I A_c(\theta) \cos \phi \cdot A_c^2(\theta)}{[\sum I A_c^2(\theta)]^2} \right]^2 I\end{aligned}$$

By using Eq. (3-18) and noticing that

$$\sum_{\theta, \phi} I A_c^2(\theta) \cos^2 \theta = \frac{1}{2} \sum_{\theta, \phi} I A_c^2(\theta)$$

one can write

$$\begin{aligned}\delta_{p_n}^2 &= 4 \Sigma \left[\frac{I A_c^2(\theta) [\sum I A_c^2(\theta)] - \frac{1}{4} \epsilon_n^2 \cdot I A_c^2(\theta) [\sum I A_c^2(\theta)]^2}{[\sum I A_c^2(\theta)]^4} \right] \\ &= \frac{2}{\sum I A_c^2(\theta)} \left[1 - \frac{\epsilon_n^2}{2} \right]\end{aligned}$$

or

$$\delta_{P_n}^2 = \delta_{P_s}^2 \cong \frac{2}{\sum I(\theta, \phi) A_c^2(\theta)} \quad (3-20)$$

Thus, for the weighted cone test, the following sums must be accumulated:

$$\sum I(\theta, \phi) A(\theta) \cos \phi, \sum I(\theta, \phi) A_c(\theta) \sin \phi, \text{ and } \sum I(\theta, \phi) A_c^2(\theta).$$

Comparing with the unweighted case, one sees the equal expression for the errors if A_c is not a function of θ . In general, the weighted analysis yields better accuracy. In our case, a Monte Carlo calculations show that the same results (within 1%) for the two methods, since A_c varies little with θ for $\theta > 4^\circ$.

b) $\phi + \pi$ Test

This method of analysis is similar to the one which was developed at SIN.¹⁰⁾ This method requires only that the $\phi + \pi$ scatter be allowed, (see Fig. 3-6) which is much less restrictive than the cone test. In other words, this method involves an acceptance function $[A(\theta, \phi)]$ that has periodicity of π . That is, for an event at coordinate (θ, ϕ) , one must be able to detect an event at $(\theta, \phi + \pi)$ as well, i.e.

$\phi + \pi^-$ TEST

Figure 3-6: Examples of accepted and rejected events for $\phi + \pi^-$ test.

$$A(\theta, \phi) = A(\theta, \phi + \pi) .$$

If this is true, Eq. (3-6) becomes

$$I(\theta, \phi) \sim [1 + \epsilon_n \cos \phi + \epsilon_s \sin \phi] A(\theta, \phi) .$$

Thus, the Fourier integrals can be expressed in the following form

$$\begin{aligned} \int_0^{2\pi} I(\theta, \phi) \cos \phi d\phi &= \int_0^{\pi} A(\theta, \phi) (1 + \epsilon_n \cos \phi + \epsilon_s \sin \phi) \cos \phi d\phi \\ &+ \int_0^{\pi} A(\theta, \phi + \pi) [1 + \epsilon_n \cos(\phi + \pi) + \epsilon_s \sin(\phi + \pi)] \cos(\phi + \pi) d\phi \\ &= \text{1st term} - \int_0^{\pi} A(\theta, \phi) [1 - \epsilon_n \cos \phi - \epsilon_s \sin \phi] \cos \phi d\phi \\ &= \int_0^{2\pi} A(\theta, \phi) [\epsilon_n \cos^2 \phi + \epsilon_s \sin \phi \cos \phi] d\phi . \end{aligned}$$

But

$$\begin{aligned} \int_0^{2\pi} I(\theta, \phi) [\epsilon_n \cos^2 \phi + \epsilon_s \sin \phi \cos \phi] d\phi &= \\ \int_0^{\pi} A(\theta, \phi) [1 + \epsilon_n \cos \phi + \epsilon_s \sin \phi] [\epsilon_n \cos^2 \phi + \epsilon_s \sin \phi \cos \phi] d\phi \\ &+ \int_0^{\pi} A(\theta, \phi + \pi) [1 + \epsilon_n \cos(\phi + \pi) + \epsilon_s \sin(\phi + \pi)] \\ &\quad [\epsilon_n \cos^2(\phi + \pi) + \epsilon_s \sin(\phi + \pi) \cos(\phi + \pi)] d\phi \\ &= \text{1st term} + \int_0^{\pi} A(\theta, \phi) [1 - \epsilon_n \cos \phi - \epsilon_s \sin \phi] [\epsilon_n \cos^2 \phi + \epsilon_s \sin \phi \cos \phi] d\phi \end{aligned}$$

$$\begin{aligned}
&= \int_0^\pi A(\theta, \phi) [\epsilon_n \cos^2 \phi + \epsilon_s \cos \phi \sin \phi] d\phi \\
&+ \int_0^\pi A(\theta, \phi) [\epsilon_n \cos^2 \phi + \epsilon_s \cos \phi \sin \phi] d\phi \\
&= \int_0^{2\pi} A(\theta, \phi) [\epsilon_n \cos^2 \phi + \epsilon_s \cos \phi \sin \phi] d\phi .
\end{aligned}$$

Therefore,

$$\int_0^{2\pi} I(\theta, \phi) \cos \phi d\phi = \epsilon_n \int_0^{2\pi} I(\theta, \phi) \cos^2 \phi d\phi + \epsilon_s \int_0^{2\pi} I(\theta, \phi) \cos \phi \sin \phi d\phi . \quad (3-21)$$

Similarly

$$\int_0^{2\pi} I(\theta, \phi) \sin \phi d\phi = \epsilon_n \int_0^{2\pi} I(\theta, \phi) \cos \phi \sin \phi d\phi + \epsilon_s \int_0^{2\pi} I(\theta, \phi) \sin^2 \phi d\phi , \quad (3-22)$$

and

$$\int_0^{2\pi} I(\theta, \phi) d\phi = \int_0^{2\pi} A(\theta, \phi) d\phi = N(\theta) . \quad (3-23)$$

The integrals can be approximated by sums and expressed as two linear equations

$$\sum_{\phi} I(\theta, \phi) \cos \phi = \epsilon_n \sum_{\phi} I(\theta, \phi) \cos^2 \phi + \epsilon_s \sum_{\phi} I(\theta, \phi) \cos \phi \sin \phi \quad (3-24)$$

$$\sum_{\phi} I(\theta, \phi) \sin \phi = \epsilon_n \sum_{\phi} I(\theta, \phi) \cos \phi \sin \phi + \epsilon_s \sum_{\phi} I(\theta, \phi) \sin^2 \phi . \quad (3-25)$$

Figure 3-6 shows examples of accepted and rejected events.

As in the cone test, in order to average over all θ bins, one can apply both the weighted and the unweighted methods.

b.1) Unweighted $\phi + \pi$ Test

As mentioned in Section a.1, Eqs. (3-24) and (3-25) can be summed without prior knowledge of carbon analyzing power $[A_c(\theta)]$. Thus by performing the sums of the Eqs. (3-24,25) over all θ bins, one obtains:

$$\begin{pmatrix} \sum_{\theta, \phi} I(\theta, \phi) \cos \phi \\ \sum_{\theta, \phi} I(\theta, \phi) \sin \phi \end{pmatrix} = \begin{pmatrix} \sum_{\theta, \phi} I(\theta, \phi) \cos^2 \phi & \sum_{\theta, \phi} I(\theta, \phi) \sin \phi \cos \phi \\ \sum_{\theta, \phi} I(\theta, \phi) \sin \phi \cos \phi & \sum_{\theta, \phi} I(\theta, \phi) \sin^2 \phi \end{pmatrix} \begin{pmatrix} \epsilon_n \\ \epsilon_s \end{pmatrix} \quad (3-26)$$

which may be inverted to solve for ϵ_n and ϵ_s . That is, considering the Eq. (3-26) in matrix form

$$\underline{B} = \underline{F} \underline{\hat{\epsilon}} \quad , \quad (3-27)$$

then

$$\underline{\hat{\epsilon}} = \underline{F}^{-1} \underline{B}$$

where the superscript " $\hat{}$ " indicates an estimator.

With

$$\underline{B} = \begin{pmatrix} B_n \\ B_s \end{pmatrix}$$

the co-variance matrix of vector B is defined by

$$\underline{V(B)} \equiv \begin{pmatrix} \delta_{B_n}^2 & \delta_{(B_n B_s)} \\ \delta_{(B_s B_n)} & \delta_{B_s}^2 \end{pmatrix} \quad (3-28)$$

where

$$\delta_{(B_n B_s)} = \Sigma \frac{\partial B_n}{\partial I(\theta, \phi)} \frac{\partial B_s}{\partial I(\theta, \phi)} \quad I^2 \quad (3-29)$$

Using the same method as before, one gets for the elements of $\underline{V(B)}$:

$$\delta_{B_n}^2 = \Sigma I(\theta, \phi) \cos^2 \phi \quad , \quad (3-30a)$$

$$\delta_{B_s}^2 = \Sigma I(\theta, \phi) \sin^2 \phi \quad (3-30b).$$

and

$$\delta_{(B_n B_s)} = \Sigma I(\theta, \phi) \cos \phi \sin \phi \quad . \quad (3-30c)$$

Therefore

$$\underline{\tilde{V}}(\underline{\tilde{B}}) = \begin{pmatrix} \Sigma I(\theta, \phi) \cos^2 \phi & \Sigma I(\theta, \phi) \cos \phi \sin \phi \\ \Sigma I(\theta, \phi) \sin \phi \cos \phi & \Sigma I(\theta, \phi) \sin^2 \phi \end{pmatrix} \quad (3-31)$$

Hence

$$\underline{\tilde{V}}(\underline{\tilde{B}}) = \underline{\tilde{F}} \quad (3-32)$$

For this method, the following sums must be accumulated

$$\begin{aligned} & \Sigma I(\theta, \phi) \cos \phi, \quad \Sigma I(\theta, \phi) \sin \phi, \\ & \Sigma I(\theta, \phi) \cos^2 \phi, \quad \Sigma I(\theta, \phi) \sin^2 \phi, \text{ and} \\ & \Sigma I(\theta, \phi) \sin \phi \cos \phi. \end{aligned}$$

and a histogram of $N(\theta)$ to calculate $\bar{A}_c$.

b.2) Weighted $\phi + \pi$ Test

Once again, let us use the same technique as we used in the cone test, that is, rewriting Eq. (3-27) for a given θ , (e.g. $\theta = \theta_i$)

$$\underline{\tilde{B}}(\theta_i) = \underline{\tilde{F}}(\theta_i)(\underline{\tilde{\epsilon}}(\theta_i)) = \underline{\tilde{F}}(\theta_i)A_c(\theta_i)(\underline{\tilde{P}})_i \quad (3-33)$$

or

$$(\underline{\tilde{P}})_i = [\underline{\tilde{F}}(\theta_i)A_c(\theta_i)]^{-1} \underline{\tilde{B}}(\theta_i) \quad (3-34)$$

with the co-variance matrix

$$\underline{V}(\underline{\hat{P}}_i) = [\underline{F}(\theta_i)A_c(\theta_i)]^{-1}\underline{V}(\underline{B})[\underline{F}(\theta_i)A_c(\theta_i)]^{-1} .$$

From Eq. (3-33), one can see,

$$\underline{V}(\underline{\hat{P}}_i) = [\underline{F}(\theta_i)]^{-1}/A_c^2(\theta_i) . \quad (3-35)$$

Once again here, we construct an average where different θ bins are weighted with their statistical accuracies (see Eq. 3-17), i.e.

$$\underline{\hat{P}} = \frac{1}{\sum_i [\underline{V}(\underline{\hat{P}}_i)]^{-1}} \{ \sum_i [\underline{V}(\underline{\hat{P}}_i)]^{-1} \underline{\hat{P}}_i \} .$$

Introducing Eqs. (3-34,35) in this equation gives

$$\underline{\hat{P}} = \frac{1}{\sum_i \underline{F}(\theta_i) A_c^2(\theta_i)} \{ \sum_i [\underline{F}(\theta_i) A_c^2(\theta_i)] [\underline{F}(\theta_i) A_c(\theta_i)]^{-1} \} \underline{B}(\theta_i)$$

or

$$\underline{\hat{P}} = \frac{1}{\sum_i A_c^2(\theta_i) \underline{F}(\theta_i)} \left[\sum_i A_c(\theta_i) \underline{B}(\theta_i) \right] \quad (3-36a)$$

and

$$\underline{\tilde{v}}(\underline{\hat{P}}) = \frac{1}{\sum_1 [\underline{\tilde{v}}(\underline{\hat{P}}_1)]^{-1}} \cong \frac{1}{\sum_1 A_c^2(\theta_1) \underline{\tilde{F}}(\theta_1)} \quad (3-36b)$$

Using the definition of $\underline{B}(\theta_1)$ and $\underline{\tilde{F}}(\theta_1)$, Eqs. (3-36a,b) becomes

$$\begin{pmatrix} P_n \\ -P_s \end{pmatrix} = \begin{pmatrix} \sum I(\theta, \phi) A_c^2(\theta) \cos^2 \phi & \sum I(\theta, \phi) A_c^2(\theta) \cos \phi \sin \phi \\ \sum I(\theta, \phi) A_c^2(\theta) \cos \phi \sin \phi & \sum I(\theta, \phi) A_c^2(\theta) \sin^2 \phi \end{pmatrix}^{-1} \begin{pmatrix} \sum I(\theta, \phi) A_c(\theta) \cos \phi \\ \sum I(\theta, \phi) A_c(\theta) \sin \phi \end{pmatrix} \quad (3-37)$$

and

$$\underline{\tilde{v}}(\underline{\hat{P}}) = \begin{pmatrix} \sum I(\theta, \phi) A_c^2(\theta) \cos^2 \phi & \sum I(\theta, \phi) A_c^2(\theta) \cos \phi \sin \phi \\ \sum I(\theta, \phi) A_c^2(\theta) \cos \phi \sin \phi & \sum I(\theta, \phi) A_c^2(\theta) \sin^2 \phi \end{pmatrix}^{-1} \begin{pmatrix} \sum I(\theta, \phi) A_c(\theta) \cos \phi \\ \sum I(\theta, \phi) A_c(\theta) \sin \phi \end{pmatrix} \quad (3-38)$$

This method requires to accumulate the following five sums:

$$\begin{aligned} & \Sigma I(\theta, \phi) A_c(\theta) \cos \phi \quad , \quad \Sigma I(\theta, \phi) A_c(\theta) \sin \phi \quad , \\ & \Sigma I(\theta, \phi) A_c^2(\theta) \cos^2 \phi \quad , \quad \Sigma I(\theta, \phi) A_c^2(\theta) \sin^2 \phi \quad \text{and} \\ & \Sigma I(\theta, \phi) A_c^2(\theta) \sin \phi \cos \phi \quad . \end{aligned}$$

The advantage of the $\phi + \pi$ test is that the cone test becomes useless near the edge of the polarimeter, while the $\phi + \pi$ test will still give information on asymmetries. In order to satisfy the required acceptance function for the $\phi + \pi$ test, the x and y coordinate at the back scintillators (SB) were calculated for the actual ray (θ, ϕ) , (XSB, YSB) , and for the alternate ray $(\theta, \phi + \pi)$, $(XSBA, YSBA)$ in the subroutine called "PROC06" of the ANALYZER (see Appendix D).

In order to avoid instrumental asymmetries, the acceptance of the apparatus must be determined by these software cuts. This was done by putting the cuts within the acceptance of the apparatus as viewed on the XSB and YSB histograms. Figures (3-7a) and (3-7b) show these histograms and the places where the proper cuts were made. Note that the cuts for XSB and YSB were identical to those for XSBA and YSBA, respectively.

This method still rejects a fairly large percentage of events scattering near the edge of the analyzer. A third method, which imposes almost no symmetry on $A(\theta_c, \phi_c)$ and allows all events to be detected. That is, events are accepted everywhere that has $A(\theta, \phi) \neq 0$. This has the appeal of not throwing out any information.

Figure 3-7a: X-coordinate of back scintillator (SB) spectrum with proper cut at the edge of SB.

Figure 3-7b: Y-coordinate of back scintillator (SB) spectrum with proper cut at the edge of SB.

The price to be paid is a more complicated analysis. It was not essential to use this method in this experiment. For more details of this method, one may refer to Ref. 2.

3.3 Determination of Spin Rotation Parameters

Once the proper sums (mentioned in Section b.1) were accumulated and stored in the ASY files, the observables could be calculated. The calculations were done with a program called WPRCAL, which was based on the weighted $\phi + \pi$ test. Before discussing WPRCAL, the sources employed to obtain the carbon analyzing power will be presented. The carbon analyzing power has been measured at several energies. These measurements can be used to develop an energy-dependent parametrization of the analyzing power. Such a parametrization was done by Waters⁴⁾ for the data taken at TRIUMF. Since then, analyzing powers have been measured at SIN⁵⁾ and LAMPF,⁶⁾ the major part of LAMPF data come from experiment by McNaughton et al. They extracted the data from an experiment designed to measure the pp spin transfer parameters at 800 MeV. This was accomplished by a polarized proton beam scattered from an unpolarized target. The situation is identical to an unpolarized beam scattering from an unpolarized target, if the two beam polarizations are averaged. Using this technique and knowing pp analyzing power,^{7,8)} and the scattered proton polarization, the carbon analyzing power can be determined. This method allows the analyzing power to be measured over a wide range of energies without requiring a variable energy beam.

One other measurement was from the LAMPF High Resolution Spectrometer (HRS).⁹⁾

In order to parameterize the results Waters⁴⁾ has found an empirical fit of the following form

$$A_c(p, \theta) = \frac{a(p)P_T}{1 + b(p)P_T^2 + c(p)P_T^4} \quad (3-39)$$

where $P_T = pR \sin\theta$, p is the momentum of the proton at the center of carbon in GeV/c and $R = 13.922$ GeV/c characterizes the radius of the carbon nucleus. The energy dependence of the coefficients a , b , and c was given by a polynomial fit of the following form

$$\begin{aligned} a(p) &= a_0 + a_1p + a_2p^2 + a_3p^3 + a_4p^4 \\ b(p) &= b_0 + b_1p + b_2p^2 + b_3p^3 \\ c(p) &= c_0 + c_1p + c_2p^2 + c_3p^3 + c_4p^4 \end{aligned}$$

The SIN group suggested a similar formula in order to parametrize the angular dependence only, and no attempt was made to find an energy dependent form to fit their data. They found the need of an extra term in Water's formula (Eq. 3-28), thus,

$$A_c(p, \theta) = \alpha \left(\frac{\sin\theta}{1 + \beta\sin^2\theta + \gamma\sin^4\theta} + \delta \right) \quad (3-40)$$

was used for their lower energy experimental data. Here α , β , γ , and δ are free parameters.

Recently, Ransome²⁰⁾ attempted to find and parametrize an energy dependent fit to the carbon scattering data. Among a number of functions used, he found a function similar to that used by SIN, except the additional term (δ) discussed above in the SIN expression was replaced by another one.

Therefore, he suggested the following formula

$$A_c(p, \theta) = \frac{\alpha(\text{psin}\theta)}{1 + \beta(\text{psin}\theta)^2 + \gamma(\text{psin}\theta)^4} + \delta(\text{psin}5\theta) \quad (3-41)$$

The SIN data not only can be fit by this equation but gives a better result at smaller angles, since the analyzing power goes to zero at 0° , which must be zero. The parameters α , β , γ , and δ are energy dependent and different for lower and higher energies.

Two fits have been made, one for energies between 100-500 MeV and one for energies between 400-800 MeV. These have been accomplished by using the following functions:

$$\alpha = \alpha_0 + \alpha_1 R + \alpha_2 R^2 + \alpha_3 R^3$$

$$\beta = \beta_0 + \beta_1 R + \beta_2 R^2 + \beta_3 R^3$$

$$\gamma = \gamma_0 + \gamma_1 R + \gamma_2 R^2 + \gamma_3 R^3$$

$$\delta = \delta_0 + \delta_1 R + \delta_2 R^2 + \delta_3 R^3 \quad .$$

For the higher energy fit (400-800 MeV) $R = (p - 1.2)$ in GeV/c, and for lower energy fit (100-500 MeV) $R = (p - 0.7)$ in GeV/c and $\delta = 0$. (p is particle momentum at the center of carbon). Figure (3-8) shows

Figure 3-8: The carbon analyzing power as a function of energy; the solid lines are the error corridor of an energy dependent fit by Ransome.²⁰⁾

the Ransome fit to the three different data which were obtained at LAMPF, SIN, and TRIUMF.

As mentioned earlier, the actual calculation of the D, R, A, and $P(\theta)$ parameters and associated errors was performed using the WPRCAL program. These calculations as mentioned above, are based on the $\phi + \pi$ technique (Section 3.2.3). The new weighted sums were accumulated for each 12 bins of θ_c . This was done by reading the ASY files containing the unweighted sums ($\Sigma I \sin\phi$, $\Sigma I \cos\phi$, $\Sigma I \sin^2\phi$, $\Sigma I \cos^2\phi$, and $\Sigma I \sin\phi\cos\phi$). Then, the accumulated sum along with the proper carbon analyzing power of each θ_c bin were added up to make a weighted sum. Note that the unweighted sums have been accumulated for 72 bins, i.e., 3 bins of θ_{scatt} , 12 bins of θ_c for every θ_{scatt} and 2 bins for normal and reverse ($3 * 12 * 2 = 72$ bins), and for each bin five sums need to be accumulated. Therefore, a total number of $72 * 5 = 360$ different sums were calculated and stored in the ASY file.

Once these weighted sums were determined, the scattered proton polarization components can be calculated (see Section 3.2.3-b), and then the corresponding parameters (depend on the orientation of the beam polarization) can be extracted (see Section 3.2.2). For $\hat{n}$ -type beam, Eq. (3-2) can be written as

$$P'_+ = \frac{P(\theta) + p_b D}{1 + p_b A_y(\theta)} \quad (3-42a)$$

$$P'_- = \frac{P(\theta) - p_b D}{1 - p_b A_y(\theta)} \quad (3-42b)$$

where signs + and - refer to beam normal and reverse.

By measuring P_+^* and P_-^* and knowing the analyzing power, $A_y(\theta)$, of target (deuterium), one can obtain the parameters D and $P(\theta)$. The procedure of extracting the quantity $A_y(\theta)$ is further discussed in Section 3.5.

Note that Eqs (3-42a,b) can be solved without knowing $A_y(\theta)$, if for elastic scattering the statement of "polarization equals analyzing power" is accepted. We did not make this assumption, that is, we actually wanted to check this truism, implied by the time reversal invariance (see Section 3.6 for more details).

When the WPRCAL program is executed, the user is prompted to specify several options, e.g., incoming proton energy, laboratory scattering angular bins, the type and magnitude of beam polarization, and carbon thickness. The program then asks for the names of the ASY files to be summed. After the file names are specified, the program reads the files, adds them, and calculates the parameters associated with the type of beam polarizations being used and also calculates the associated errors.

3.4 Experimental Measurements of Spin Rotation and Depolarization Observables

The measurement for proton-deuteron elastic scattering at laboratory scattering angles of 30° , 36° , 40° , and 50° for 500 MeV and 20° , 25° , 30° , 40° , 50° , and 60° for 800 MeV are presented in Tables 3-1 through 3-6. Each spectrometer setting was divided in to two or three angular bins. The measured parameters D , R , and A as a function

Table 3-1: Measured $D (=D_{nn'})$ at $T_p = 500$ MeV

$\theta_{lab}(deg)$	$-t(GeV/c)^2$	$D \pm \delta$
28.40	0.265	0.817 ± 0.060
31.80	0.316	0.798 ± 0.073
36.00	0.403	0.753 ± 0.101
38.40	0.449	0.713 ± 0.094
41.30	0.507	0.788 ± 0.100
48.30	0.650	0.737 ± 0.082
51.30	0.709	0.707 ± 0.083

Table 3-2: Measured $R (=D_{ss'})$ at $T_p = 500$ MeV

$\theta_{lab}(deg)$	$-t(GeV/c)^2$	$R \pm \delta R$
28.40	0.265	0.676 ± 0.049
31.30	0.316	0.724 ± 0.068
34.40	0.373	0.740 ± 0.081
37.30	0.428	0.586 ± 0.098
38.40	0.449	0.455 ± 0.082
41.30	0.507	0.391 ± 0.092
48.40	0.650	0.404 ± 0.081
51.30	0.709	0.175 ± 0.085

Table 3-3: Measured A ($=D_{ls}'$) at $T_p = 500$ MeV

$\theta_{lab}(\text{deg})$	$-t(\text{GeV}/c)^2$	$A \pm \delta A$
28.40	0.265	0.666 ± 0.057
31.30	0.316	0.415 ± 0.079
38.40	0.449	0.243 ± 0.093
41.30	0.507	0.346 ± 0.102
48.40	0.650	0.297 ± 0.102
51.30	0.709	0.390 ± 0.116

Table 3-4: Measured D ($=D_{nn}'$) at $T_p = 800$ MeV

$\theta_{lab}(\text{deg})$	$-t(\text{GeV}/c)^2$	$D \pm \delta D$
19.80	0.242	0.979 ± 0.042
24.90	0.370	0.824 ± 0.055
30.00	0.517	0.874 ± 0.043
40.00	0.839	0.959 ± 0.049
49.90	1.172	0.887 ± 0.055
59.90	1.494	0.821 ± 0.023

Table 3-5: Measured $R (=D_{ss'})$ at $T_p = 800$ MeV

$\theta_{lab}(deg)$	$-t(GeV/c)^2$	$R \pm \delta R$
19.80	0.242	0.843 ± 0.039
24.90	0.370	0.725 ± 0.042
30.00	0.517	0.570 ± 0.046
40.00	0.763	0.574 ± 0.055
49.90	1.172	0.479 ± 0.061
59.90	1.494	0.424 ± 0.072

Table 3-6: Measured $A (=D_{ls'})$ at $T_p = 800$ MeV

$\theta_{lab}(deg)$	$t(GeV/c)^2$	$A \pm \delta A$
19.80	0.242	0.400 ± 0.041
24.90	0.370	0.419 ± 0.040
30.00	0.517	0.524 ± 0.052
40.00	0.839	0.513 ± 0.055
49.90	1.172	0.612 ± 0.062
59.90	1.494	0.591 ± 0.048

Figure 3-9: Measured $D_{nn'}$ (=D) versus laboratory scattering angle at $T_p = 500 \text{ MeV}$

Figure 3-10: Measured D_{ls} , (= A) versus laboratory scattering angle at $T_p \approx 500 \text{ MeV}$

Figure 3-11: Measured $D_{ss'}$, ($\approx R$) versus laboratory scattering angle at $T_p = 500 \text{ MeV}$

Figure 3-12: Measured $D_{nn'}$, (=D) versus laboratory scattering angle
at $T_p = 800 \text{ MeV}$

Figure 3-13: Measured $D_{ls'}$ (=A) versus the laboratory scattering angle at $T_p = 800 \text{ MeV}$

Figure 3-14: Measured D_{ss} , (=R) versus laboratory scattering angle at $T_p = 800$ MeV

of θ_{lab} , the laboratory scattering angle, are shown in Figs. 3-9 through 3-14. The polarization, $P(\theta)$, was calculated from the polarization, $P_i(\theta)$, obtained from $\hat{s}$, $\hat{n}$, and $\hat{k}$ type beam by

$$P(\theta) = \{ \sum_i [P_i(\theta) / \delta_{P_i}^2] \} / \{ \sum_i [1 / \delta_{P_i}^2] \}$$

where $i = 1, 2$, and 3 , refer to the type of the beam polarization, and the error is given by

$$\frac{1}{\delta_P^2} = \sum_i \frac{1}{\delta_{P_i}^2}$$

or

$$\delta_P = \left[\frac{\delta_{P_1}^2 \delta_{P_2}^2 \delta_{P_3}^2}{(\delta_{P_1} \delta_{P_2})^2 + (\delta_{P_2} \delta_{P_3})^2 + (\delta_{P_1} \delta_{P_3})^2} \right]^{1/2} .$$

The results obtained for $P(\theta)$ at 500 and 800 MeV are given in Tables 3-7,8, and are shown in Figs. 3-15,16, respectively.

3.5 Extraction of the Deuterium Analyzing Power

The analyzing power had an important role in this experiment, since we wanted to check the time reversal invariance for this reaction. Very little data exists on the analyzing power at 500 and

Table 3-7: Measured $P(\theta)$ at $T_p = 500$ MeV

$\theta_{lab}(\text{deg})$	$-t(\text{GeV}/c)^2$	$P \pm \delta P$
28.40	0.265	-0.040 ± 0.027
31.30	0.316	-0.188 ± 0.033
36.00	0.403	-0.380 ± 0.042
38.40	0.449	-0.421 ± 0.042
41.30	0.507	-0.286 ± 0.046
48.40	0.650	-0.204 ± 0.039
51.30	0.709	-0.221 ± 0.041

Table 3-8: Measured $P(\theta)$ at $T_p = 800$ MeV

$\theta_{lab}(\text{deg})$	$-t(\text{GeV}/c)^2$	$\delta P \pm \delta P$
17.60	0.193	0.372 ± 0.030
19.80	0.242	0.164 ± 0.032
22.10	0.297	0.085 ± 0.063
22.70	0.312	0.204 ± 0.038
24.90	0.370	0.026 ± 0.032
27.20	0.435	-0.015 ± 0.049
27.80	0.452	0.060 ± 0.042
30.00	0.517	0.120 ± 0.032
32.20	0.585	0.182 ± 0.048
37.70	0.763	0.175 ± 0.044
40.00	0.839	0.211 ± 0.036
41.10	0.910	0.262 ± 0.056
47.70	1.099	0.181 ± 0.046
49.90	1.172	0.178 ± 0.038
52.10	1.245	0.102 ± 0.060
57.70	1.426	0.088 ± 0.055
59.90	1.494	-0.075 ± 0.040
62.10	1.560	-0.102 ± 0.067

Figure 3-15: Measured polarization P versus laboratory scattering angle at $T_p = 500$ MeV.

Figure 3-16: Measured polarization P versus laboratory scattering angle at $T_p = 800$ MeV.

800 MeV.¹¹⁾ Hence the accumulation of precise data for the analyzing power appeared to be useful and justified.

The actual experiment was designed to measure the spin rotation and depolarization parameters, and no provisions were made to measure the analyzing power during data taking. Having completed the off-line analysis for the Wolfenstein parameters, the attention was focused on the analyzing power problem, and the following decision was made:

1. Select data accumulated with $\hat{n}$ - type beams only.
2. To obtain front chamber information.
3. Divide the data obtained at each angular setting of the spectrometer into three angular bins. Figure 3-17 shows histogram at $\theta_s = 20^\circ$.
4. Correct the yield (NORMAL and REVERSE) for each angular bin.

Note that this was an independent measurement.

First action was to modify the ALLTST file in order to satisfy our requirements, then to replay the data with the new test file. At the end of each tape, the proper information (such as, computer dead time, chamber efficiency, beam intensity, yield for NORMAL and REVERSE) were stored in the ANP file.

Once the replay was completed, the analyzing power was determined from the corrected yields for both NORMAL and REVERSE and the beam polarization itself. The measured corrected yields for each beam polarization state was obtained from the raw yield after correcting for computer dead time and chamber efficiency. The corrected yield (for either NORMAL or REVERSE) was defined as

Figure 3-17: Laboratory scattering angle spectrum for 20° lab setup.

$$\text{CORRECTED YIELD} = \frac{\text{MEASURED YIELD} * \text{CORRECTION FACTOR}}{\text{IC}}$$

where

$$\text{CORRECTION FACTOR} = \frac{1}{\text{COMP. LIVE TIME} * \text{CHMBR. EFF.}}$$

and IC is the beam flux.

The beam flux, IC, was obtained from the beam line polarimeter. The beam polarization was taken either from quench ratio method or from beam line polarimeter (EPP or Line B). The geometric average of the beam polarizations with spin up and spin down was computed and used for the analyzing power calculations. (See Section 2.3).

Once the corrected yields are determined, the analyzing power can be computed. As mentioned earlier the analyzing power is defined as

$$A_y = \frac{\epsilon}{p_b} \quad (3-43)$$

where ϵ is the asymmetry defined in Eq. (2.1), and p_b is the beam polarization. Since the polarimeter was to the left of the beam, it detects only particles scattered to the left of the beam (see Fig. 2-7). Therefore, the asymmetry (ϵ) may be redefined as:

$$\epsilon = \frac{Y_N - Y_R}{Y_N + Y_R} \quad (3-44)$$

where Y_N is defined as the corrected yield scattered to the left with

beam polarization NORMAL (spin up) while YR is defined as the corrected yield scattered left with beam polarization REVERSE (spin down). By symmetry arguments, it can be shown that the YR equivalent to the yield scattered right with beam polarization NORMAL. Therefore, the analyzing power was obtained from

$$A_y = \frac{1}{p_b} \cdot \frac{Y_N - Y_R}{Y_N + Y_R} \quad (3-45)$$

The error associated to the analyzing power is straight forward, the equation used to compute the error can be given by

$$\delta_{A_y} = \frac{1}{p_b} \left[\frac{2\alpha\beta}{(1 + \beta)^2} \right] \quad (3-46)$$

where

$$\alpha = \left(\frac{1}{Y_N} + \frac{1}{Y_R} \right)^{1/2}$$

$$\beta = \frac{Y_R}{Y_N} \quad .$$

It was attempted to reduce the background as much as possible. It was finally better than 1%. Figure 3-18 shows the primary result along with the final one for the analyzing power. One can see from the first result that some errors exist which arise from false asymmetries. Several actions were taken in order to eliminate this error. Finally, an instrumental asymmetry was found which was due to the angular bins near the edge of the polarimeter.

Figure 3-18: Analyzing power before and after correction for systematic error, the open squares correspond to the final result.

Because of the experimental setup, one should expect these problems, since for instance there was no way to change the event trigger. That is, the scattered proton was rescattered from carbon, and hence, may not be triggered for both beam orientations (NORMAL and REVERSE) particularly near the edges. This problem caused false asymmetries in the analyzing power. The use of the $\phi + \pi$ method significantly removed this type of events and hence eliminated the false asymmetries. The results of the analyzing power after using the $\phi + \pi$ test is shown in Fig. 3-19 along with earlier measurements by Winkelmann.¹¹⁾ The agreement between these two sets of measurements is good. The same technique as used for extracting the analyzing power at 800 MeV, was applied for 500 MeV. The result of the analyzing power at 500 MeV is shown in Fig. 3-20 as a function of laboratory scattering angle. The results obtained for A_y at 500 and 800 MeV are given in Tables 7-9,10.

Table 3-9: Measured A_y at $T_p = 500$ MeV

$\theta_{lab}(deg)$	$-t(GeV/c)^2$	$A_y \pm \delta A_y$
27.60	0.252	0.006 ± 0.009
29.95	0.292	-0.143 ± 0.010
32.20	0.332	-0.279 ± 0.011
33.60	0.358	-0.318 ± 0.017
35.95	0.401	-0.379 ± 0.017
37.60	0.436	-0.394 ± 0.010
38.20	0.445	-0.378 ± 0.020
39.95	0.480	-0.349 ± 0.010
42.20	0.525	-0.296 ± 0.011
47.60	0.634	-0.228 ± 0.010
59.95	0.681	-0.226 ± 0.009
52.20	0.728	-0.206 ± 0.011

Table 3-10: Measured A_y at $T_p = 800$ MeV

$\theta_{lab}(deg)$	$-t(GeV/c)^2$	$A_y \pm \delta A_y$
17.70	0.195	0.345 ± 0.007
19.80	0.242	0.238 ± 0.009
22.20	0.300	0.086 ± 0.014
22.70	0.312	0.065 ± 0.011
24.90	0.370	0.008 ± 0.011
27.20	0.435	0.037 ± 0.013
27.80	0.452	0.055 ± 0.009
29.95	0.515	0.114 ± 0.008
32.20	0.585	0.166 ± 0.010
37.60	0.759	0.205 ± 0.007
39.95	0.837	0.223 ± 0.007
42.20	0.913	0.213 ± 0.009
47.65	1.096	0.166 ± 0.009
49.95	1.174	0.131 ± 0.009
52.20	1.249	0.098 ± 0.011
57.60	1.422	0.045 ± 0.010
59.95	1.492	0.023 ± 0.009
62.20	1.563	-0.016 ± 0.011

Figure 3-19: Measured analyzing power, A_y , versus θ_{lab} at $T_p = 500 \text{ MeV}$

Figure 3-20: Measured analyzing power, A_y , versus θ_{lab} at $T_p = 800$ MeV; the solid circles are this work.

3.6 Comparison of Polarization and Analyzing Power as a Test for T-Invariance Violation in pd Elastic Scattering

Since the discovery of time-reversal-invariance (TRI) violation in the decay (weak decay) of the K_2^0 meson by Christenson,¹²⁾ no further evidence of TRI violation in other processes such as electromagnetic or strong interactions has been found.

It has been shown that TRI is a necessary and sufficient condition for the equality between analyzing power and polarization of the corresponding inverse reaction.¹³⁾ That is, the analyzing power, $A_y(\theta)$, observed in the $B(\vec{a},c)D$ reaction is equal to the polarization, $P(\theta)$, that would be induced in the $D(c,\vec{a})B$ reaction.

Earlier search for TRI violation in nuclear reactions has centered on tests of detailed balance by comparison of $d\sigma/d\Omega$ of inverse reactions and these have given no evidence of TRI violation. However, these TRI tests are not sensitive to spin dependent violations. Another test concerning the validation of TRI, is based on spin rotation measurements in p-p scattering at 430 MeV and checking the relation¹⁴⁾ $\tan\theta_{lab} = (A + R')/(A' - R)$. This may not provide a TRI test, since it is known to be a consequence of rotational invariance.¹⁶⁾

Since elastic scattering is its own inverse process, it has been used in most of the tests of TRI that require the polarization-analyzing power equality. However, it is necessary to scatter target with a non-zero spin, since for a target with zero spin parity conservation alone requires that $P(\theta) = A_y(\theta)$ regardless of invariance under time reversal.

This is because from Ch. 1

$$P(\theta) = \text{Tr}(MM^\dagger \sigma_n) / \text{Tr}(MM^\dagger) \quad (3-47)$$

and

$$A_y(\theta) = \text{Tr}(M \sigma_n M^\dagger) / \text{Tr}(MM^\dagger) \quad (3-48)$$

where $\hat{n} = \vec{k}_i \times \vec{k}_f / |\vec{k}_i \times \vec{k}_f|$ is normal to the scattering plane (y axis), and M is the target scattering operator of the target-projectile system. For spin-zero targets and spin $\frac{1}{2}$ projectiles the operator M at any angle may be expanded in terms of the projectile Pauli spin operator, $\vec{\sigma}$:

$$M = f_1 + g_1 \sigma_x + g_2 \sigma_y + g_3 \sigma_z \quad (3-49)$$

and invariance under the parity operation (Table 1-1) restrict us to the form

$$M = f + g \vec{\sigma} \cdot \hat{n} \quad (3-50)$$

Then one has

$$\begin{aligned} P(\theta) &= \text{Tr}\{(f + \vec{\sigma} \cdot \hat{n} g)(\vec{\sigma} \cdot \hat{n})(f^\dagger + (\vec{\sigma} \cdot \hat{n})g^\dagger)\} / |f|^2 + |g|^2 \\ &= 2 \text{Re}(fg^*) / |f|^2 + |g|^2 \end{aligned}$$

and

$$A_y(\theta) = \text{Tr}\{(f + \vec{\sigma} \cdot \hat{n} g)(f^\dagger + \vec{\sigma} \cdot \hat{n} g^\dagger)\vec{\sigma} \cdot \hat{n}\} / |f|^2 + |g|^2$$

$$= 2\text{Re} (f g^*) / |f|^2 + |g|^2$$

hence

$$P(\theta) = A_y(\theta) \quad (3-51)$$

The most accurate measurements by this method were made in $p + {}^3\text{He}$ and $p + {}^{13}\text{C}$ elastic scattering.¹⁵⁾ They were designed to determine deviations of $P(\theta) - A_y(\theta)$ from zero. However, these experiments were carried out in a region where $D \approx 1$ (this was not known at the time of the measurements), and therefore $P(\theta) - A_y(\theta) \approx 0$ even if TRI is not valid, as proved below.

To illustrate the way in which the magnitude of $P(\theta) - A_y(\theta)$ depends on D , let us follow Ref. 15 where the yield for scattering of a proton with spin up ($\uparrow$) to the right with final spin down ($\downarrow$) is denoted by $R^{\uparrow\downarrow}$. The polarization of a scattered beam (unpolarized) is defined by

$$P(\theta) = \frac{(R^{\uparrow\downarrow} + R^{\downarrow\downarrow}) - (R^{\uparrow\uparrow} + R^{\downarrow\uparrow})}{\Sigma R} \quad (3-52)$$

where $\Sigma R = R^{\uparrow\uparrow} + R^{\uparrow\downarrow} + R^{\downarrow\downarrow} + R^{\downarrow\uparrow}$.

If the beam is completely polarized ($p_b = 1$), then the asymmetry $A_y(\theta)$ is given by

$$A_y(\theta) = \frac{(L^{\uparrow\uparrow} + L^{\downarrow\downarrow}) - (R^{\uparrow\downarrow} + R^{\downarrow\uparrow})}{\Sigma R} \quad (3-53)$$

The positive $\hat{n}$ direction is along $\vec{k}_i \times \vec{k}_f$. By assuming rotational invariance ($L^{\uparrow\downarrow} = R^{\downarrow\uparrow}$ and $L^{\uparrow\uparrow} = R^{\downarrow\downarrow}$), Eq. (3-53) becomes

$$A_y(\theta) = \frac{(R^{\downarrow\downarrow} + R^{\downarrow\uparrow}) - (R^{\uparrow\uparrow} + R^{\uparrow\downarrow})}{\Sigma R} \quad (3-54)$$

From Eqs. (3-52) and (3-54) one can obtain

$$P(\theta) - A_y(\theta) = \frac{2(R^{\uparrow\downarrow} - R^{\downarrow\uparrow})}{\Sigma R} \quad (3-55)$$

One sees from Eq. (3-55) that the polarization-analyzing power equality depends on the equality of the two possible spin-flip yields ($R^{\downarrow\uparrow}$ and $R^{\uparrow\downarrow}$).

The depolarization parameter, D , is given by (see Chapter 1)

$$D = \text{Tr}\{\sigma_y, F \sigma_y F^\dagger\} / \text{Tr}\{FF^\dagger\} \quad (3-56)$$

and

$$F = F^o \sigma_o + F^x \sigma_x + F^y \sigma_y + F^z \sigma_z \quad (3-57)$$

Let us consider the quantization direction to be along the y axis, then

$$\sigma_x = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

and

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Eq. (3-56) then gives

$$\begin{aligned} D &= \frac{|F^0|^2 - |F^x|^2 + |F^y|^2 - |F^z|^2}{|F^0|^2 + |F^x|^2 + |F^y|^2 + |F^z|^2} \\ &= 1 - \frac{2(|F^x|^2 + |F^z|^2)}{|F^0|^2 + |F^x|^2 + |F^y|^2 + |F^z|^2} \end{aligned} \quad (3-58)$$

By considering

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

we find

$$\langle + | F | - \rangle = F^z - iF^x$$

and

$$\langle - | F | + \rangle = F^z + iF^x$$

It follows from the definition of yield that

$$R^{\downarrow\uparrow} = |\langle + | F | - \rangle|^2 = |F^Z|^2 + |F^X|^2 - 2 \operatorname{Re} F^Z F^{X*}$$

and

$$R^{\uparrow\downarrow} = |\langle - | F | + \rangle|^2 = |F^Z|^2 + |F^X|^2 + 2 \operatorname{Re} F^Z F^{X*}.$$

Thus

$$R^{\uparrow\downarrow} + R^{\downarrow\uparrow} = 2(|F^X|^2 + |F^Z|^2). \quad (3-59)$$

Similarly

$$\Sigma R = R^{\uparrow\uparrow} + R^{\uparrow\downarrow} + R^{\downarrow\downarrow} + R^{\downarrow\uparrow} = 2(|F^0|^2 + |F^Z|^2 + |F^X|^2) \quad (3-60)$$

From Eqs. (3-58), (3-59) and (3-60) it follows that

$$D = 1 - \frac{R^{\uparrow\downarrow} + R^{\downarrow\uparrow}}{1/2 \Sigma R} \quad (3-61)$$

or

$$1 - D = \frac{2(R^{\uparrow\downarrow} + R^{\downarrow\uparrow})}{\Sigma R} \quad (3-62)$$

The quantity

$$S = \frac{R^{\uparrow\downarrow} + R^{\downarrow\uparrow}}{\Sigma R} \quad (3-63)$$

can be interpreted as a total spin-flip probability, and it is

related to the depolarization parameter, D , through

$$D = 1 - 2S \quad (3-64)$$

Let us assume breaking of TRI, by means of the equation

$$R^{\downarrow\uparrow} = TR^{\uparrow\downarrow} \quad (3-65)$$

where T is a TRI-violating factor, e.g. $T = 1$ would correspond to no TRI violation. Then from Eqs. (3-55, 62 and 65) one obtains

$$P(\theta) - A_y(\theta) = k(1 - D) \quad (3-66)$$

with

$$k = \frac{1 - T}{1 + T} \quad (\text{no violation of TRI if } k = 0).$$

Therefore, as one can see from this discussion, the relation $P(\theta) \simeq A_y(\theta)$ will hold in the region where the depolarization parameter, D , is near unity even when there is a violation of TRI ($T \neq 1$ or $k \neq 0$). In particular, Conzett¹⁶⁾ has concluded that the elastic scattering experiments may not provide a suitable test of time reversal invariance. Moreover, tests of TRI using the polarization-analyzing power equality should be made through measurements in reactions where the spin-flip probability is expected or known to be large.

Meanwhile, some nuclear physicists have tried to face the problem differently. Aprile et al.¹⁷⁾ have introduced an additional term, λ , (a sixth complex T-noninvariant amplitude) to the general

form of NN scattering amplitudes, thus

$$F_{NN} = \alpha + \beta(\vec{\sigma}_1 \cdot \hat{Y})(\vec{\sigma} \cdot \hat{Y}) + \gamma(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{Y} + \delta(\vec{\sigma}_1 \cdot \hat{Z})(\vec{\sigma}_2 \cdot \hat{Z}) + \\ \epsilon(\vec{\sigma}_1 \cdot \hat{X})(\vec{\sigma}_2 \cdot \hat{X}) + \lambda[(\vec{\sigma}_1 \cdot \hat{Z})(\vec{\sigma}_2 \cdot \hat{X}) + (\vec{\sigma}_1 \cdot \hat{X})(\vec{\sigma}_2 \cdot \hat{Z})] .$$

The experimental reconstruction of this amplitude would provide new information concerning any TRI violation in the nucleon-nucleon interaction, which may not be obtained from any other tests or experiments. Their test was made by measuring a set of sixteen polarization parameters, together with $d\sigma/d\Omega$ and asymmetry $A_y(\theta)$, for pp elastic scattering at 579 MeV. These parameters were sufficient for determination of six complex amplitudes. It was found a value for $|\lambda|^2 \approx 0$. This shows no violation of TRI in pp elastic scattering.

In our experiment, the polarization $P(\theta)$, the analyzing power $A_y(\theta)$, and the depolarization parameter D , were measured. It was found that $P(\theta) - A_y(\theta) \approx 0$ even in the region where D is different from unity. The experimental results are shown in Figs. 3-21 and 3.22.

In order to provide some quantitative arguments, we have calculated the value of k for each θ bin. That is, from Eq. (3-66) one gets

$$k = \frac{P(\theta) - A_y(\theta)}{1 - D} \quad (3-67)$$

and

$$\delta_k^2 = \left(\frac{1}{1-D}\right)^2 [\delta_p^2 + \delta_{A_v}^2 + k^2 \delta_D^2] \quad . \quad (3-68)$$

The results are listed in Tables 3-11 and 3-12 and shown in Fig. 3-23.

If one assumes k is not a function of angle, then the weighted mean, $\bar{k}$, can be obtained as follows:

$$\bar{k} = \frac{\sum_i (k_i / \delta_{k_i}^2)}{\sum_i (1 / \delta_{k_i}^2)}$$

and with error $\bar{\delta}_k$

$$1/\delta_k^2 = \sum_i (1/\delta_{k_i}^2) \quad .$$

This gives an average value for k :

$$\bar{k} \pm \bar{\delta}_k = 0.01 \pm 0.04 \quad (\text{for 500 MeV data})$$

$$\bar{k} \pm \bar{\delta}_k = 0.02 \pm 0.08 \quad (\text{for 800 MeV data}) \quad .$$

This is consistent with the validity of TRI in p-d reaction within the accuracy of this experiment.

Table 3-11: Numerical value for k with error at $T_p = 500$ MeV

θ_{lab} (deg)	$-t$ (GeV/c) ²	$P \pm \delta P$	$A_y \pm \delta A_y$	$D \pm \delta D$	$k \pm \delta k$
28.4	0.265	-0.04 $\pm$ 0.027	-0.045 $\pm$ 0.007	0.817 $\pm$ 0.06	0.027 $\pm$ 0.15
31.3	0.316	-0.188 $\pm$ 0.033	-0.225 $\pm$ 0.008	0.798 $\pm$ 0.073	0.185 $\pm$ 0.183
36.0	0.403	-0.380 $\pm$ 0.042	-0.3800 $\pm$ 0.016	0.753 $\pm$ 0.101	0.0 $\pm$ 0.05
38.4	0.449	-0.421 $\pm$ 0.042	-0.375 $\pm$ 0.009	0.713 $\pm$ 0.094	-0.160 $\pm$ 0.159
41.3	0.507	-0.286 $\pm$ 0.046	0.317 $\pm$ 0.008	0.788 $\pm$ 0.100	0.146 $\pm$ 0.231
48.4	0.650	-0.204 $\pm$ 0.034	-0.228 $\pm$ 0.008	0.737 $\pm$ 0.082	0.091 $\pm$ 0.154
51.3	0.709	-0.221 $\pm$ 0.041	-0.218 $\pm$ 0.008	0.707 $\pm$ 0.083	-0.01 $\pm$ 0.143

$$\bar{k} \pm \delta k = 0.01 \pm 0.04$$

Table 3-12: Numerical value of k with error at $T_p = 800$ MeV

θ_{lab} (deg)	$-t$ (GeV/c) ²	$P \pm \delta P$	$A_y \pm \delta A_y$	$D \pm \delta D$	$k \pm \delta k$
19.8	0.242	0.246 ± 0.02	0.238 ± 0.008	0.979 ± 0.042	0.38 ± 1.28
24.9	0.370	0.026 ± 0.02	0.008 ± 0.011	0.824 ± 0.055	0.01 ± 0.13
30.0	0.517	0.118 ± 0.02	0.115 ± 0.009	0.874 ± 0.043	0.03 ± 0.18
40.0	0.839	0.20 ± 0.02	0.223 ± 0.008	0.951 ± 0.049	-0.25 ± 0.5
49.9	1.172	0.124 ± 0.02	0.132 ± 0.009	0.887 ± 0.055	-0.07 ± 0.20
59.9	1.494	-0.035 ± 0.02	0.023 ± 0.009	0.821 ± 0.068	-0.3 ± 0.17

$$\bar{k} \pm \delta k = -0.04 \pm 0.08$$

Figure 3-21: Comparison of polarization and analyzing power at $T_p = 500 \text{ MeV}$

Figure 3-22: Comparison of polarization and analyzing power at $T_p = 800 \text{ MeV}$

Figure 3-23: The value of k versus θ_{lab} at $T_p = 500$ and 800 MeV

3.7 Theoretical Analysis

In Figs. 3-24 through 3-30 we display our data together with the theoretical predictions derived using the multiple scattering theory of Ref.18 (Alberi, Bleszynski, Jaroszewicz and Santos) based on the Arndt phase shift NN amplitudes.

The resulting parameter free calculation is seen to reproduce the data reasonably well. We also see that the double spin flip components of the NN amplitude play a considerable role and are necessary to obtain better agreement. The discrepancies between the theory and the data observed for A_y are under current investigation. It is interesting to note that the Arndt phase shift amplitudes lead to similar disagreement between the theory and the data found for polarization and spin rotation observables in $p-^{40}\text{Ca}$ scattering at 500 MeV.¹⁹⁾ We believe that the present experiment will be helpful in resolving the origin of these rather disturbing discrepancies.

The interpretation of our experiment will also be facilitated when the planned measurement of the present A' and R' will be considered. This will allow us to determine the parameters D_0, D_x, D_y and D_z introduced by Bleszynski.

Figure 3-24: The theoretical prediction of D at $T_p = 500 \text{ MeV}$.

Figure 3-25: The theoretical prediction of A at $T_p = 500 \text{ MeV}$.

Figure 3-26: The theoretical prediction of R at $T_p = 500 \text{ MeV}$.

Figure 3-27: The theoretical prediction of A_y at $T_p = 500$ MeV.

Figure 3-31: The theoretical prediction of A_y at $T_p = 800$ MeV. The dashed line was obtained by considering no double spin flip terms of NN amplitude in the calculation.

Figure 3-28: The theoretical prediction of D at $T_p = 800$ MeV.

Figure 3-29: The theoretical prediction of A at $T_p=800$ MeV.

Figure 3-30: The theoretical prediction of R at $T_p = 800 \text{ MeV}$.

APPENDIX A

Early polarization experiments have made use of the double-scattering technique, where polarized protons were produced by scattering an initially unpolarized beam (i.e., from a target of helium or carbon). The partially polarized protons were then scattered from the target under investigation. Usually the geometric requirements were such that only particles scattered into a small solid angle ($\sim 10^{-2}$ sr) could be accepted by the second target. Therefore, the available intensity of polarized particles was small, particularly because the desire for particles of well-defined energy limits the permissible thickness of the first target. Additional problems arose from background radiation which the incident beam caused in the collimators and in the beam stop. However, some of these difficulties had been overcome quite successfully (i.e., by collecting them with quadrupole lenses and by piping the beam to scattering chamber several meters from the first scattering chamber).

Considerable advantages can be gained if the ions are polarized before injection into the accelerator. Devices that produce beams of low-energy ions for this purpose are called Polarized Ion Sources.

It is often assumed that the main advantage of the use of polarized ion sources rests in the increased beam intensity at the target.

Besides the promise of higher intensity in the future, polarized ion sources offer other important advantages: a) the beam polarization is independent of beam energy; b) the polarization can be reversed and turned on or off; c) deuteron beams with pure vector or pure tensor polarization can be prepared; d) one obtains beams of high ion optic quality (emittance) and small energy spread; e) the background problems inherent in double scattering experiments are absent.

The following presents a short review of the most important method used to prepare beams of polarized particles.

A.1 Polarized Atom

Sources of polarized protons (or deuterons) involve the preparation of neutral hydrogen atoms in such a way that the spins of the protons in the atoms preferentially point either along or opposite to an externally applied magnetic field. The polarization of the protons is maintained when the atom is ionized, (for instance by electron impact). The first experimental work directed to the preparation of polarized ion beams by an atomic beam method¹⁾ was undertaken by Clausnitzer et al.,²⁾ in 1956. The use of atomic hydrogen allows one to take advantage of the large magnetic moment of the electron. The nuclear spin, in turn, is affected by its coupling to the electron. Figure A-1 shows the familiar energy-level diagram of the ground-state hydrogen atom in an external magnetic field.

It is useful in this connection to remember that the magnetic field which the electron causes at the site of the proton is about 1.74×10^5

Figure A-1: Energy level diagram of the hydrogen atom in a magnetic field.

gauss (for the hydrogen ground state, while the field of the proton at the site of the electron is a few hundred gauss. The region, $B/B_c > 1$, is often referred to as the region of "strong" field. In this region the energy of the atom changes with applied field strength primarily because the field energy of the electron magnetic moment, μ_B , changes. This energy simply is $W = \pm \mu_B B$, where the magnetic field B is in gauss and $\mu_p = -0.927 \times 10^{-20}$ erg/G is the Bohr magneton.

In presenting the energy-level diagram (Fig. A-1) it is customary to measure the strength of the external field in terms of the "critical field" B_c ;

$$\chi = B/B_c = (g_I - g_J)\mu_B B/\Delta W \quad (A-1)$$

where

$$g_I = -3.04 \times 10^{-3} \quad (\text{proton})$$

$$g_J = 2.002 \quad (\text{electron})$$

and the zero-field splitting is

$$\Delta W = h\nu = h \times 1420.4 \text{ MHz} = 5.8 \times 10^{-6} \text{ ev.}$$

Hence the external field for hydrogen ground state is

$$\chi = B/507 \text{ G} \quad (A-2)$$

The diagram in Fig. A-1 applies quantitatively also to another case of practical importance, namely the metastable $2S_{1/2}$ excited state of hydrogen. For the S state with principal quantum number $n = 2$, the hyperfine interaction is lower by a factor 2^3 , so that $\Delta W = 0.725 \times 10^{-6}$ ev, and

$$\chi = B/63.4 \text{ G} . \quad (\text{A-3})$$

The principle of all polarized ion sources is based on the preparation of hydrogen (or deuterium) atoms in such a way that the four (or six for deuterium) states are not equally populated. This can be accomplished by several methods. One of these methods is the use of the properties of the $2S_{1/2}$ metastable state of hydrogen, which are such that the $m_j = -\frac{1}{2}$ component can readily be made to decay to the ground state, leaving metastable atoms only in the $m_j = +\frac{1}{2}$ states.

A.2 Lamb-Shift Polarized Ion Source

Proposals to produce polarized protons by taking advantage of the unique properties of the $2S_{1/2}$ excited state of the hydrogen atom were first advanced by Zavoiskii³⁾ and by Mandansky and Owen.⁴⁾ The level in question is shown in Fig. A-2. In the absence of an external field, the $2P_{1/2}$ state is separated from the $2S_{1/2}$ state by Lamb-Shift 1058 MHz^{5,6)} which corresponds to 4.4×10^{-6} ev. The $2P_{1/2}$ state decays rapidly ($\tau = 1.6$ nsec) by dipole radiation to the ground state.

HYDROGEN BREIT-RABI DIAGRAM

Figure A-2: Breit-Rabi diagram of hydrogen atom for $n = 2$ (principal quantum number) showing the splitting of $2s_{1/2}$ and $2p_{1/2}$ states by the Lamb-shift 1058. MHz.

On the other hand, the $2S_{1/2}$ state is a metastable state with a lifetime of about 0.1 sec.

The application of the metastable hydrogen atom to the polarized ion sources requires the following steps:

- a) preparation of atomic hydrogen with as large a fraction as possible in the $2S$ state;
- b) selective quenching of the β state only, by applying suitable electric and magnetic fields for the proper duration;
- c) selective ionization, i.e., ionization of the remaining metastable atoms only, without ionizing the (unpolarized) ground-state atoms. The degree to which the ionization process must be selective depends, of course, on the fraction of atoms in the ground state.

A Lamb-shift polarized ion source capable of producing pure state of protons and deuterons have been constructed for use with the LAMPF linear accelerator (Fig. A-3). In this source, Cesium vapor is used to convert 500 ev protons or 1000 ev deuterons into metastable atoms (i.e., by electron pick-up from the Cesium atoms, and leaving a portion, optimally 43% of the neutral atoms formed are in the metastable state). These metastable hydrogen atoms are then polarized by the Lamb-shift source.

The mixture of $1S_{1/2}$ and $2S_{1/2}$ atoms which remains after deflection then enters the spin filter region. The spin filter consists of a cylindrical RF cavity (with four quadrants of an overall length of 25 cm). The cavity is operated at 1608 MHz (with the oscillation mode TM_{010}). The RF cavity is located in a solenoid-produced axial magnetic field of about 540 G. The spin

Figure A-3: A schematic layout of LAMPF Lamb-shift polarized ion source.

filter^{7,8)} transmits only those metastable atoms whose nuclei have the particular magnetic quantum number selected, with the remainder being quenched to the ground state.

The beam, now comprised of about 90% atomic hydrogen in the ground state and 10% atomic hydrogen in the polarized metastable state, then enters a solenoid containing Argon gas and an axial magnetic field of about 10 G. In the Argon cell the metastable hydrogen atoms are selectively ionized to form negative polarized hydrogen ions (P^-). A small percentage of ground state hydrogen atoms also ionize in the argon cell and form unpolarized negative hydrogen ions (H^-). For this reason, the beam cannot be polarized with 100% efficiency.

In addition, by reversing the magnetic fields in the spin filter and argon cell, the spin reversal is accomplished. That is, the two states of beam polarization are obtained, one parallel to the beam direction and one anti-parallel to the beam direction.

After passing through the argon cell, the beam is focused and accelerated to 750 keV. As mentioned in Section 2.1, a spin precessor system is then used to set the orientation of the beam to any desired direction for on-line experiments.

A.3 The Quench Ratio

The spin filter, as mentioned in previous section, has the property of quenching all of the metastable atoms except those in the selected state. This is a resonance phenomenon that occurs when the applied magnetic field has the correct value for the RF frequency used (1608 MHz) and the state desired. The resonances occur at about 540 G

for state-1 and about 605 G for state-2 hydrogen atoms. Figure A-4 shows the ion current versus magnetic field.

Figure A-4

With the aid of the spin filter, the expected polarization should be 1 for the peak contribution $(I - I_0)$ and zero for the background contribution (I_0) . Thus the overall beam polarization, P , is a weighted average

$$P = \frac{(I - I_0)(1) - I_0(0)}{I} = 1 - \frac{I_0}{I} \quad (\text{A-4})$$

where the ratio of beam intensities, I/I_0 , known as the "quench ratio Q ," hence

$$P = 1 - \frac{1}{Q} \quad (\text{A-5})$$

If quenching is done just after the cesium cell, the quenched beam would be unpolarized. Thus, the effect must arise from the metastable

components ($m_J = \pm 1/2$) of the atomic beam in the region between the cesium cell and spin filter, where the presence of residual argon is likely. This effect may be described as follows: when metastable atoms leave the cesium cell, one can assume an equal number of α -state ($m_J = 1/2$) and β -state ($m_J = -1/2$) metastable atoms. As these atoms move toward the spin filter, some of them are converted to ions by collision with the residual gas. Because of the electrostatic deflection in this region, these ions will be effectively moved. As the magnetic field goes up (toward ~ 575 gauss), the β -state atoms will be highly quenched to the ground state, and stay as neutral atoms, and so will remain in the atomic beam. While the α -state atoms will continue to be converted to negatives and lost. Thus the ground-state part of the atomic beam will reach the spin filter with a non-uniform population distribution.

The consequence of this mechanism leads the quenched beam to be somehow polarized, that is, $P_Q \approx -0.03$, where the minus sign denotes that the polarization is opposite to the one which was produced by $m_I = 1/2$ selection. Therefore, the polarization of the filtered beam is given by

$$P = 1 - 1/Q + P_Q/Q, \quad (\text{A-6})$$

since $P_Q \approx -0.03$, thus

$$P \approx 1 - \frac{1.03}{Q}. \quad (\text{A-7})$$

This quenching can be done quickly and repeated at fixed intervals. Signals from the ion source identical to the normal reverse signals are supplied to the experimental areas. If some beam intensity monitor is gated with both polarized and quenched signals, the beam polarization can be extracted. Absolute beam polarization determination for the LAMPF Lamb-shift source was done by the quench-ratio technique.⁹⁾ One of the major assumption was that there must be no depolarization between the spin filter and the experimental target. Since the quench-ratio method measures the polarization as it existed in the spin filter subsequent depolarization would not affect the measured quench ratio and would therefore lead to error.

A calculation¹⁰⁾ indicates that beam depolarization after acceleration through the LAMPF accelerator should be the order of 0.1%, and the investigations are consistent with this estimate.

APPENDIX B

B.1 Test File

```

761.64, 48.2
GAT, 101, 10936, 11069, ; 1 ; CKSM X1 FRONT
GAT, 102, 10779, 10916, ; 2 ; CKSM X2 FRONT
GAT, 103, 10833, 10986, ; 3 ; CKSM X3 FRONT
GAT, 104, 10755, 10964, ; 4 ; CKSM X4 BACK
GAT, 105, 10840, 11030, ; 5 ; CKSM X5 BACK
GAT, 106, 10882, 11063, ; 6 ; CKSM X6 BACK
GAT, 108, 10852, 11038, ; 7 ; CKSM Y1 FRONT
GAT, 109, 10835, 11002, ; 8 ; CKSM Y2 FRONT
GAT, 110, 10872, 11034, ; 9 ; CKSM Y3 FRONT
GAT, 111, 10866, 11063, ; 10 ; CKSM Y4 BACK
GAT, 112, 10868, 11087, ; 11 ; CKSM Y5 BACK
GAT, 113, 10862, 11081, ; 12 ; CKSM Y6 BACK
GAT, 107, 10199, 10833, ; 13 ; CKSM X7
GAT, 114, 10012, 10612, ; 14 ; CKSM Y7
GAT, 200, 9990, 10010, ; 15 ; RES X2 +/- 1MM
GAT, 203, 9990, 10010, ; 16 ; RES X5
GAT, 206, 9990, 10010, ; 17 ; RES Y2
GAT, 209, 9990, 10010, ; 18 ; RES Y5
GAT, 212, 10004, 10100, ; 19 ; SIN THETA FOR ANGLES >=2.97 DEG
IGAT, 1, ; 20 ; 225 IS XYT, X UPSTREAM
IGAT, 2, ; 21 ; 227 IS XYT, Y UPSTREAM
BOX, 1, ; 22 ; SPARE
GAT, 32, 0, 2048, ; 23 ; TDC X1 UP FRONT
GAT, 33, 0, 2048, ; 24 ; TDC X1 DN FRONT
GAT, 34, 0, 2048, ; 25 ; TDC X2 UP FRONT
GAT, 35, 0, 2048, ; 26 ; TDC X2 DN FRONT
GAT, 36, 0, 2048, ; 27 ; TDC X3 UP FRONT
GAT, 37, 0, 2048, ; 28 ; TDC X3 DN FRONT
GAT, 38, 0, 2048, ; 29 ; TDC X4 UP BACK
GAT, 39, 0, 2048, ; 30 ; TDC X4 DN BACK
GAT, 40, 0, 2048, ; 31 ; TDC X5 UP "
GAT, 41, 0, 2048, ; 32 ; TDC X5 DN "
GAT, 42, 0, 2048, ; 33 ; TDC X6 UP "
GAT, 43, 0, 2048, ; 34 ; TDC X6 DN "
GAT, 44, 0, 2300, ; 35 ; TDC X7 UP
GAT, 45, 0, 2300, ; 36 ; TDC X7 DN
GAT, 46, 0, 2048, ; 37 ; TDC Y1 UP FRONT
GAT, 47, 0, 2048, ; 38 ; TDC Y1 DN FRONT
GAT, 48, 0, 2048, ; 39 ; TDC Y2 UP FRONT
GAT, 49, 0, 2048, ; 40 ; TDC Y2 DN FRONT
BIT, 1, 8, ; 41 ; P
GAT, 221, 9922, 10125, ; 42 ; DELX
GAT, 222, 9887, 10027, ; 43 ; DELY
BIT, 1, 0, ; 44 ; NORMAL
BIT, 1, 1, ; 45 ; REVERSE
BIT, 1, 12, ; 46 ; FAST CLEAR=SET
AND, 1, 2, 3, 7, 8, 9, 15, 17, 20, 21, 44, ; 47 ; GOOD UPSTREAM. NOR
AND, 1, 2, 3, 7, 8, 9, 15, 17, 20, 21, 45, ; 48 ; GOOD UPSTREAM. REV
AND, 1, 2, 3, 4, 5, 6, ; 49 ; GOOD CKSMS X
AND, 7, 8, 9, 10, 11, 12, ; 50 ; GOOD CHSMS Y
AND, 49, 30, ; 51 ; GOOD CHKSUMS
AND, 15, 16, 17, 18, ; 52 ; GOOD RESIDUALS
AND, 51, 52, ; 53 ; GOOD CKSM + RES
AND, 53, 46, ; 54 ; GOOD CKSM RES FAST CLEAR
AND, 50, 1, 3, 4, 5, 6, ; 55 ; EFF X2
AND, 50, 1, 2, 4, 5, 6, ; 56 ; EFF X3
AND, 50, 1, 2, 3, 5, 6, ; 57 ; EFF X4
AND, 50, 1, 2, 3, 4, 6, ; 58 ; EFF X5

```


B.1 Test File (continued)

```

AND, 50, 1, 2, 3, 4, 5,      ; 59      ; EFF X6
AND, 50, 2, 3, 4, 5, 6,      ; 60      ; EFF X1
AND, 49, 8, 9, 10, 11, 12,   ; 61      ; EFF Y1
AND, 49, 7, 9, 10, 11, 12,   ; 62      ; EFF Y2
AND, 49, 7, 8, 10, 11, 12,   ; 63      ; EFF Y3
AND, 49, 7, 8, 9, 11, 12,    ; 64      ; EFF Y4
AND, 49, 7, 8, 9, 10, 12,    ; 65      ; EFF Y4
AND, 49, 7, 8, 9, 11, 10,    ; 66      ; EFF Y4
AND, 51, 13,                  ; 67      ; EFF Y7
AND, 51, 14,                  ; 68      ; EFF X7
AND, 51, 14, 13,              ; 69      ; ALL CHKSMS OK
AND, -46, 53,                 ; 70      ; NO CLEAR +GOOD CHK+RES
IDR, -1, 1,                   ; 71      ; EVENT COUNTER 1ST LOOP
AND, 20, 21, 51, 52,          ; 72      ; GOOD EVENT 1ST LOOP, NO CKSM XY7
AND, 20, 21, 69, 52,          ; 73      ; FAST CLEAR DELX
AND, 21, 69, 52,              ; 74      ; FAST CLEAR DELY
AND, 52, 69,                  ; 75      ; ALL CKSM + RES
END                             ; END OF FIRST LOOP
GAT, 270, 7100, 12830,        ; 76      ; XCLOSE
GAT, 271, 7100, 12830,        ; 77      ; YCLOSE
GAT, 234, 12500, 15000,       ; 78      ; ABSXCL GREATER THAN 25CM
GAT, 272, 8412, 11588,        ; 79      ; ZCLOSE +/- 6.25 IN. BOTH SIDES OF CARBON
GAT, 273, 9999, 10049,        ; 80      ; DCLOSE < 5 MILLIMETERS
GAT, 274, 9999, 10000,        ; 81      ; CONE TEST
GAT, 278, 7150, 12800,        ; 82      ; XSB
GAT, 280, 7150, 12800,        ; 83      ; XSBA
GAT, 279, 7150, 12800,        ; 84      ; YSB
GAT, 281, 7150, 12800,        ; 85      ; YSBA
IGAT, 3,                      ; 86      ; 238 IS DTHETA
IGAT, 4,                      ; 87      ; 239 IS DPHI
GAT, 235, 12800, 15000,       ; 88      ; ABSYCL GREATER THAN 28 CM
IGAT, 5,                      ; 89      ; TDC SC
GAT, 284, 9899, 9929,         ; 90      ; COS PHI RIGHT
GAT, 279, 10000, 15000,       ; 91      ; YSB GT 10000 FOR TOP SC
GAT, 279, 0, 9999,            ; 92      ; YSB LT 10000 FO LOWER SC
GAT, 235, 12500, 15000,       ; 93      ; ABSYCL GREATER THAN 25 CM
AND, 93, 78,                  ; 94      ; FOUR CORNERS
AND, 72, 19,                  ; 95      ; GOOD SCATTERED
AND, 72, -19,                 ; 96      ; GOOD UNSCATTERED
IDR, 1, -1,                   ; 97      ; EV COUNTER
AND, 79, 86, 87, 76, 77, -94, 95, 89, 41, ; 98 ; X, Y, ZCLOSE, DTH, DPHI, 4CCORNER, TDC SC, P
AND, 82, 83, 84, 85, 98,      ; 99      ; GOOD EV WITH LARGE ACC.
AND, 80, 99,                   ; 100     ; GOOD EV EXCLUDE CORNERS
AND, -54, 100,                 ; 101     ; FAST CLEAR NOT SET (SIMULATE FC REJECT)
AND, 54, 100,                  ; 102     ; FAST CLEAR SET
AND, 86, 89,                   ; 103     ; CUT ON TDC SC * DTHETA
AND, 87, 89,                   ; 104     ; DPHI * CUT ON TDC SC
AND, 91, 72,                   ; 105     ; GOOD EV AND YSB GT 10000
AND, 92, 72,                   ; 106     ; GOOD EV AND YSB LT 10000
END
FIN

```

APPENDIX B

B.2 Histogram File

```

; POLGEN. HST
FILE=POL1. DSP
FREE ALL
H1=212, 10000, 10050, 1      ; SIN THETA
T1=53
MARK=10004
H2=212, 10000, 10050, 1      ; SIN THETA (T100)
T2=100
H3=263, 6858, 13142, 157     ; PHI
T3=100
H4=225, 7500, 12500, 50      ; XT HYDR
T4=53
H5=227, 7500, 12500, 50      ; YT HYDR
T5=53
H6=278, 6500, 13500, 50      ; XSB
T6=72
H7=279, 6500, 13500, 50      ; YSB
T7=72
H8=273, 10000, 10200, 1      ; DCLOSE
T8=72
H9=238, 9900, 10100, 2      ; DTHETA
T9=104
H10=239, 9900, 10100, 2      ; DPHI
T10=103
H11=270, 6500, 13500, 50     ; XCLOSE
T11=100
H12=271, 6500, 13500, 50     ; YCLOSE
T12=100
H13=272, 6500, 13500, 50     ; ZCLOSE
T13=95
MARK=8412, 11588
H14=200, 9900, 10100, 2      ; RES X2
T14=51
MARK=9990, 10010
H15=203, 9900, 10100, 2      ; RES X5
T15=51
MARK=9990, 10010
H16=206, 9900, 10100, 2      ; RES Y2
T16=51
MARK=9990, 10010
H17=209, 9900, 10100, 2      ; RES Y5
T17=51
MARK=9994, 10010
H18=30, 0, 2000, 50          ; TDC SB1
T18=96
H19=31, 0, 2000, 50          ; TDC SB2
T19=96
H20=75, 0, 1000, 5           ; TDC SC
T20=72
H21=76, 0, 1000, 5           ; CORRECTED TDC SC
T21=72
H22=233, 9850, 10150, 1      ; TRUE SCATTERING ANGLE NORM.
T22=47
MARK=9934, 9978, 10022, 10066
H23=233, 9850, 10150, 1      ; TRUE SCATTERING ANGLE REV.
T23=48
MARK=9934, 9978, 10022, 10066

```

APPENDIX C

C.1 Anode Wire Determination

A crude anode position AA is given by the time difference between the two ends of the delay line (see Fig. 2-13)

$$AA = T_+ - T_-$$

where as mentioned earlier (Section 2.4.4)

$$T_+ = DTIM + nD + R_1$$

$$T_- = DTIM + (N-n)D + R_2,$$

hence

$$T_+ - T_- = 2nD - ND + \text{Const.}$$

where D delay per wire, n the wire number, and N the number of wires. In practice, this is not the case, therefore a more precise calculation can be made by using an empirical correction formula of the following form

$$AP = R1 + R2 \cdot AA + R3 \cdot AA^2 + R4 \cdot AA^3$$

where the R quantities are the constants and can be determined, to give the anode position (AP) equal to an integral multiples of wire spacing in centimeter.

From the anode position AP, one can derive the wire number, NAP, and the Truncated Anode Position (TAP) by the following relations

$$NAP = AP/GAP + 50.5$$

$$TAP = (NAP - 50)GAP$$

where 50.5 is an offset, GAP is a wire spacing, and TAP is then the nearest integer to AP. The Truncated Anode Position (TAP) and Anode Position (AP) are equal, if AP falls on integral multiples of wire spacing (GAP). Figure C-1 shows AP over the whole range of a chamber.

C.2 Drift Time Determination

All wire chambers in the polarimeter system require the conversion of delay line time differences and drift time into spatial positions. Therefore, once the correct wire has been identified, the Drift Time (DTIM) could be determined. As mentioned in Section 2.4.1, a mixture of Argon, Isobutane, and Freon gases serves as an ionization gas. When a charged particle passes through the chambers, it ionizes the gas inside the chamber. The freed electrons gradually drift toward the anode wire, colliding with atoms along the way, and soon a large number of electrons (avalanche) reach the anode, generating a signal pulse. The generated pulse travels to each end of the delay line. At the same time a large number of positive ions are created

Figure C-1: MWDC's anode position spectrum.

and travel toward the cathode wire. Thus, at the time of an avalanche results two pulses , one pulse is collected on the anode wire while the other is collected on the cathode wire (i.e. the summed (O+E) pulse output). The drift time then can be determined by comparing the time of arrival ofse pulses with that of a scintillator trigger. A single drift time spectrum is shown in Fig. C-2.

In order to transform the drift time into a position, one must consider the similarity of all drift time spectra. But mainly because of differences in TDC calibration constants, we may not expect to have identical spectra. Therefore the following correction can be made to produce a set of nearly identical drift time spectra.

1. A linear correction to the drift time (DTIM) as a function of anode position (AP) is made in the form

$$DTIM = DTIM + R5 \cdot AP$$

where R5 is a constant, which is found by plotting the drift time versus anode position, then adjusting the value of R5 until the line formed is horizontal.

2. The next correction is applied to eliminate the different calibration constants of the TDCs

$$DTIM = (DTIM) \cdot R6$$

R6 is calculated by measuring each TDC calibration constant and normalizing to one arbitrarily chosen TDC.

Figure C-2: A single channel corrected drift time spectrum.

3. The final correction is made such that all drift time spectra (12 spectra) are similar and start at the same channel, thus reducing the number of look-up file to only one.

Once these corrections have been made, a simple look-up file can be made to transform the time into a distance. Figure C-3 shows a drift position (DPOS) spectrum.

C.3 Left-Right Ambiguity Determination

After the drift position has been determined, the side of the wire on which the event occurred must be identified. As mentioned earlier (section 2.4.1), this is done with odd-even ambiguity method. Figure 2-16 shows the O-E spectrum, with two peaks. An offset, can be determined such that half of the events have a positive value and half a negative value. The correct sign of the drift position can be determined from the following formula

$$(DPOS)_c = (-1)^n \cdot \text{sign}(ADC) \cdot (DPOS)_i ,$$

where the factor $(-1)^n$ gives a positive value for a trajectory, which corresponds to the left side of wire $(n+1)$ and so to the right side of wire n , $(DPOS)_i$ is the initial drift position and $(DPOS)_c$ is the signed drift position. Once the correct sign of drift position is determined, the final position in each x (or y) plane is therefore given by

$$X(Y)_f = TAP + (DPOS)_c - R7$$

Figure C-3: Drift position spectrum.

where $R7$ adjusts for the alignment of the chamber. Scatter plots of $X(Y)_f$ in one chamber versus to $X(Y)_f$ in the other chamber, are then used to determine the correct O-E phase of the chambers. Figure C-4 shows three cases of the above comparison. For the case of both chambers have correct O-E phase gives a sharp straight line.

Figure C-4: Typical dot plot of two adjacent parallel planes, a) one O-E phase correct and one incorrect, b) both O-E phases correct, c) both O-E phases incorrect.

APPENDIX D

As mentioned earlier, once an event is triggered (when SF and SB both fire), the analyzer will then fit a straight line through three front chambers (C1-C3) called the incident ray, and a line through three back chambers (C4-C6) called final ray (Fig. D-1). Using the information of these two rays, a polar and azimuthal scattering angles (θ_c and ϕ_c) along with others require quantities can be calculated.

By assuming each ray as represented by its projection onto the X-Z and Y-Z planes, one gives the following four equations:¹⁾

$$\begin{aligned} X_i &= AX_i Z_i + BX_i & X_f &= AX_f Z_f + BX_f \\ (D-1) \end{aligned}$$

$$\begin{aligned} Y_i &= AY_i Z_i + BY_i & Y_f &= AY_f Z_f + BY_f \end{aligned}$$

In general these two rays will not intersect, therefore we must define the mid point of the shortest segment (a segment perpendicular to both rays) joining the two rays as the point of intersection or scatter.

Let us write these two rays in a vector form, such as

$$\vec{I} = (AX_i, AY_i, 1), \quad \vec{F} = (AX_f, AY_f, 1) \quad (D-2)$$

then, the segment is parallel to $\vec{I} \times \vec{F}$ (Fig. D-1).

Let the end points of this segment be (x_i, y_i, z_i) and (x_f, y_f, z_f) .

Figure D-1: Polarimeter coordinate system.

Then

$$\begin{aligned}
 (x_f - x_i, y_f - y_i, z_f - z_i) &= t(\vec{I} \times \vec{F}) \\
 &= t[(AY_i - AY_f), (AX_f - AX_i), (AX_i AY_f - AY_i AX_f)]
 \end{aligned} \tag{D-3}$$

where t is a constant.

Using Eqs. (D-1) and (D-3), one gives

$$x_f = AX_i z_i + BX_i + t(AY_i - AY_f) \tag{D-4}$$

$$y_f = AY_i z_i + BY_i + t(AX_f - AX_i)$$

$$z_f = z_i + t(AX_i AY_f - AY_i AX_f)$$

and

$$x_f = AX_f [z_i + t(AX_i AY_f - AY_i AX_f)] + BX_f \tag{D-5}$$

$$y_f = AY_f [z_i + t(AX_i AY_f - AY_i AX_f)] + BY_f$$

Equating the two values of x_f and y_f leaves two equations and two unknown, t and z_i . Solving for z_i gives

$$z_i = \frac{CD - BE}{C^2 - AB} \tag{D-6}$$

where

$$A = \vec{I} \cdot \vec{I} = AX_i^2 + AY_i^2 + 1$$

$$B = \vec{F} \cdot \vec{F} = AX_f^2 + AY_f^2 + 1$$

$$C = \vec{I} \cdot \vec{F} = AX_1AX_f + AY_1AY_f + 1 \quad (D-7)$$

$$D = AX_f(BX_f - BX_1) + AY_f(BY_f - BY_1)$$

$$E = AX_1(BX_f - BX_1) + AY_1(BY_f - BY_1)$$

Similarly,

$$z_f = \frac{AD - CE}{C^2 - AB} \quad (D-8)$$

with given z_1 , z_f we can solve for x_1 , y_1 , x_f and y_f to give

$$X = \frac{x_1 + x_f}{2}, Y = \frac{y_1 + y_f}{2}, \text{ and } Z = \frac{z_1 + z_f}{2} \quad (D-9)$$

Therefore the distance between the two rays (D_{close}) along with other quantities such as X_{close} , Y_{close} and Z_{close} can be obtained. Thus

$$D_{\text{close}} = [(x_1 - x_f)^2 + (y_1 - y_f)^2 + (z_1 - z_f)^2]^{1/2} \quad (D-10)$$

$$X_{\text{close}} = X = (AX_1z_1 + BX_1 + AX_fz_f + BX_f)/2$$

$$Y_{\text{close}} = Y = (AY_1z_1 + BY_1 + AY_fz_f + BY_f)/2$$

$$Z_{\text{close}} = Z = \frac{1}{2}(z_1 + z_f) \quad .$$

The scattering angle (θ_c) can easily be calculated by the following relation

$$\vec{I} \cdot \vec{F} = |\vec{I}||\vec{F}|\cos(\theta_c)$$

or

$$\theta_c = \cos^{-1} [C / \sqrt{AB}] \quad . \quad (D-11)$$

The calculation of the azimuthal angle, ϕ_c , is somehow complicated. There are a number of ways to calculate ϕ_c , but none of which are entirely correct for all or any experimental situations. The following presents a method that was used in this experiment.

For this method, one needs the coordinate system to be defined. Figure D-2 shows such a coordinate system (the unit vector $\hat{z}'$ is parallel to the incident ray, $\vec{I}$).

Figure D-2

It is easy to see that

$$\hat{x}' \cdot \hat{f} = \sin\theta \cos\phi$$

$$\hat{y}' \cdot \hat{f} = \sin\theta \sin\phi \quad ,$$

yielding

$$\phi = \tan^{-1}(\hat{y}' \cdot \hat{f}/\hat{x}' \cdot \hat{f}) \quad . \quad (D-12)$$

Given that

$$z' = (AX_1/\sqrt{A}, AY_1/\sqrt{A}, 1/\sqrt{A})$$

and

$$\hat{f} = (AX_f/\sqrt{B}, AY_f/\sqrt{B}, 1/\sqrt{B}) \quad .$$

After some algebra, one gives ϕ_c to be

$$\phi_c = \tan^{-1} \left\{ \frac{(AY_f + AY_f AX_1^2 - AX_1 AY_1 AX_f - AY_1)/\sqrt{A}}{AX_f - AX_1} \right\} \quad (D-13)$$

For the $\phi + \pi$ test, the calculation of a quantity that is related to acceptance of the back scintillator is required. Let us define the vector $\vec{A}$ with scattering angles θ_c and ϕ_c as follows :

$$\vec{A} = (AX, AY, AZ) \quad ,$$

and the alternate vector $(AXA, AYA, 1)$ with the same θ_c but $\phi'_c = \phi_c + \pi$, where

$$AXA = \frac{AX}{AZ}$$

and

$$AYA = \frac{AY}{AZ} \quad .$$

By solving the following two equations

$$\vec{A} \cdot \vec{I} = \vec{F} \cdot \vec{I}$$

$$\vec{A} \times \vec{I} = - (\vec{F} \times \vec{I}) \quad .$$

The following three equations can be obtained

$$AY_1AZ - AY = AY_f - AY_1$$

$$AX - AX_1AX = AX_1 - AX_f$$

$$AX_fAX + AY_1AY + AZ = AX_1AX_f + AY_1AY_f + 1 \quad .$$

By solving simultaneously, one gives:

$$AXA = \frac{AX}{AZ} = [2AX_1(1 + AY_fAY_1) + AX_f(AX_1^2 - AY_1^2 - 1)]/DN$$

$$AYA = \frac{AY}{AZ} = [2AY_1(1 + AX_1AX_f) - AY_f(AX_1^2 - AY_1^2 + 1)]/DN$$

where

$$DN = B^2 - (AY_1 - AY_f)^2 - (AX_f - AX_1)^2$$

The true scattering angle at the target (LD_2) can also be calculated as follows:

Let the distance from LD_2 to carbon be ZDC, and the scattering angles θ_s , if we define

$$XDIST = ZDC \cdot \sin(\theta_s) + X_{close} \cdot \cos(\theta_s)$$

$$YDIST = Y_{close}$$

$$ZDIST = ZDC \cdot \cos(\theta_s) - X_{close} \sin(\theta_s)$$

$$DIST = (XDIST^2 + YDIST^2)^{1/2}$$

then

$$\tan(\theta_s) = \frac{DIST}{ZDIST}$$

or

$$\theta_s = \tan^{-1} \left(\frac{DIST}{ZDIST} \right)$$

and

$$\phi_s = \tan^{-1} \left(\frac{YDIST}{XDIST} \right)$$

where the ϕ_s is the azimuthal scattering angle at the target.

APPENDIX E

As mentioned in Ch. 1, the knowledge of the density matrix ρ is sufficient in order to obtain the average value $\langle \Omega \rangle$ of any spin operator (Eq. 1-9) ,

$$\langle \Omega \rangle \text{Tr} \rho = \text{Tr}(\rho \Omega) \quad (\text{E-1})$$

Since the density matrix ρ is a 4×4 Hermitian matrix, it is determined by 16 real numbers which may be expressed as the average values of a complete set of 16 Hermitian operators Ω^μ in the spin space. The density matrix ρ_i for the initial state of two-nucleon systems is then given by

$$\rho_i = \sum_{\mu=1}^{16} c_\mu \Omega^\mu \quad (\text{E-2})$$

$$c_\mu = \frac{\text{Tr}(\rho_i \Omega^\mu)}{\text{Tr}(\Omega^\mu \Omega^\mu)} = \frac{1}{4} \text{Tr}(\rho_i) \langle \Omega^\mu \rangle \quad (\text{E-3})$$

where

$$\Omega^\mu \equiv \sigma_\alpha^1 \sigma_\beta^2 \quad [\mu = (\alpha, \beta) = 1, 2, \dots, 16], \quad (\text{E-4})$$

and obeys the following orthogonality relations

$$\text{Tr}(\Omega^\mu \Omega^\nu) = 4\delta_{\mu\nu} \quad . \quad (\text{E-5})$$

Let us rewrite Eq. (E-1) in the following form

$$\langle \Omega^\mu \rangle_f = \frac{\text{Tr}(\rho_f \Omega^\mu)}{\text{Tr} \rho_f} \quad (\text{E-6})$$

By substituting Eq. (1-7) into Eq. (E-6), one gets

$$\langle \Omega^\mu \rangle_f = \frac{\text{Tr}(M \rho_i M^\dagger \Omega^\mu)}{\text{Tr}(M \rho_i M^\dagger)} \quad (\text{E-7})$$

Using Eqs. (E-2), (E-3), and (E-7) we get, therefore

$$\langle \Omega^\mu \rangle_f I = \frac{1}{4} \sum_{v=1}^{16} \langle \Omega^v \rangle_i \text{Tr}(M \Omega^v M^\dagger \Omega^\mu) \quad (\text{E-8})$$

where

$$I = \frac{\text{Tr}(M \rho_i M^\dagger)}{\text{Tr} \rho_i} = \frac{1}{4} \sum_{v=1}^{16} \langle \Omega^v \rangle_i \text{Tr}(M \Omega^v M^\dagger) \quad (\text{E-9})$$

is the differential cross section.

For a polarized incident beam with polarization $\langle \vec{\sigma} \rangle_i = \vec{P}$, and unpolarized target nucleons, the polarization $\langle \vec{\sigma} \rangle_f = \vec{P}'$ of the scattered particles is from Eq. (E-8), given by

$$I P' = I \langle \sigma_\beta \rangle_f = \frac{1}{4} \text{Tr}(M M^\dagger \sigma_\beta) + \frac{1}{4} \sum_{\alpha=1}^3 \text{Tr}(M \sigma_\alpha M^\dagger \sigma_\beta) \langle \sigma_\alpha \rangle_i \quad (\text{E-10})$$

with

$$I = \frac{1}{4} \text{Tr}(M M^\dagger) + \frac{1}{4} \sum_{\alpha=1}^3 \text{Tr}(M \sigma_\alpha M^\dagger) \langle \sigma_\alpha \rangle_i \quad (\text{E-11})$$

where suffices α and β now are one of the $\hat{s}$, $\hat{n}$ and $\hat{l}$ (as defined by Eqs. 1-21 and 22), respectively. By noticing that $\langle \vec{\sigma} \rangle \cdot \hat{n}$ is a scalar while $\langle \vec{\sigma} \rangle \cdot \hat{s}$ is a pseudoscalar, we can rewrite Eq. (E-10) in the laboratory coordinate system as follows

$$I \vec{P}' = I_0 \{ (A p_l + R p_s) \hat{s}' + (P(\theta) + D p_n) \hat{n}' + (A' p_l + R' p_s) \hat{l}' \} \quad (E-12)$$

where

$$I_0 P(\theta) = \frac{1}{4} \text{Tr}(M M^\dagger \sigma_n) \quad (E-13a)$$

$$I_0 D = \frac{1}{4} \text{Tr}(M \sigma_n M^\dagger \sigma_n')$$
(E-13b)

$$I_0 R = \frac{1}{4} \text{Tr}(M \sigma_s M^\dagger \sigma_s') \quad (E-13c)$$

$$I_0 A = \frac{1}{4} \text{Tr}(M \sigma_l M^\dagger \sigma_s') \quad (E-13d)$$

$$I_0 R' = \frac{1}{4} \text{Tr}(M \sigma_s M^\dagger \sigma_l') \quad (E-13e)$$

$$I_0 A' = \frac{1}{4} \text{Tr}\{M \sigma_l M^\dagger \sigma_l'\} \quad (E-13f)$$

and

$$\langle \vec{\sigma} \rangle_i = \vec{P} = \begin{pmatrix} p_s \\ p_n \\ p_l \end{pmatrix}$$

Eq.(E-12) is the same as Eq. (1-20).

In order to prove Eq. (1-15), let us rewrite Eq. (1-3) in the following form

$$M = M_0 + M_x \sigma_x + M_y \sigma_y + M_z \sigma_z \quad (E - 14)$$

where

$$\begin{aligned} M_0 &= \alpha + \gamma \sigma_y' \\ M_x &= \delta \sigma_z' \\ M_y &= \gamma + \beta \sigma_y' \\ M_z &= \epsilon \sigma_x' \end{aligned} \quad (E - 15)$$

Thus, one can obtain

$$\text{Tr}(M \sigma_y M^\dagger) = 2\text{Re Tr}(M_0 M_y^\dagger) - 2\text{Im Tr}(M_x M_z^\dagger)$$

and

$$\text{Tr}(M M^\dagger \sigma_y) = 2\text{Re Tr}(M_0 M_y^\dagger) + 2\text{Im Tr}(M_x M_z^\dagger)$$

Since $\sigma_z \sigma_x = i \sigma_y$, then it follows from Eq. (E - 14) that

$$\text{Tr}(M_x M_z^\dagger) = 0.$$

Hence

$$\text{Tr}(M \sigma_y M^\dagger) = \text{Tr}(M^\dagger M \sigma_y) = 2\text{Re Tr}(M_0 M_y^\dagger). \quad (115)$$

ACKNOWLEDGEMENTS

I wish to express my deepest appreciation and thanks to my research advisors Professors George J. Igo and Charles A. Whitten, who suggested this experimental problem and provided guidance, support, and encouragement throughout the course of this work.

I would like to thank Dr. B. Aas and Dr. M. Bleszynski for their friendship and assistance on all aspects of this work and physics in general.

I would also like to thank Drs. M. Gazzaly, H. Hasai, J. McClelland, and G. Pauletta, and my fellow graduate students A. Azizi, M. Hajisaïd, and A. Wang for their friendship and valuable comments.

Special thanks go to Dr. B. Bonner and Dr. M. McNaughton for their assistance in preparation and performance of the experiment.

I wish to express my gratitude to the other members of my committee as well, Professor C. W. Wong, Professor W. E. Kastenberg, and Professor E. G. Straus.

I am especially grateful to Mrs. B. Blonsky, J. Bardin, and Ms. B. Yamadera for their kind assistance during my graduate study at UCLA. Thanks also go to Mrs. L. Cordova and Mrs. M. Eutsler for their kindness and friendly help, especially Mrs. L. Cordova for the preparation of the final manuscript.

I am also grateful to the staffs of both UCLA and LAMPF for providing assistance to me over the years.

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