Whispering-gallery mirrors for short-wavelength laser cavities: Shapes and tolerances

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CONF-9107115--78

DE92 040667

Abstract

The whispering-gallery mirrors desired for use in short-wavelength laser cavities are seen to be highly aspheric and very different from the shapes encountered in conventional optics. Fabrication tolerances are established by examining the effects of various surface imperfections. The mirrors are found to be relatively insensitive to figure errors. The requirements on surface finish are shown to be fairly strict, though less severe than with normal-incidence optics.

Introduction

Whispering-gallery optics is based on the principle of using a concave surface to deflect light through a large total angle, by means of a series of many glancing-incidence reflections. Because whispering-gallery mirrors (WGM's) offer potentially high reflectivities at short wavelengths, they have been suggested for use in x-ray laser cavities¹⁻³. Unfortunately, the simplest resonator geometries --- those employing cylindrical or spherical mirrors --- have cavity modes whose behavior is poorly suited for use with typical x-ray laser amplifiers.

As a beam propagates along the surface of a WGM, its transverse structure evolves in a manner determined by the local curvatures of the surface. This evolution is highly astigmatic, which is to say that the beam profile in the direction normal to the surface behaves very differently from that in the direction tangential to the surface. In and of itself, the fact that the normal and tangential structures evolve differently is not necessarily a major issue. The real problem is in the behavior of the normal structure: for mirrors of any reasonable size, a round geometry leads to unacceptably large beam divergence at the mirror exit⁴.

The beam divergence problem can be ameliorated by using an elongated beam path⁵. Selection of a particular shape for the beam path, however, only partially determines the required mirror shape. At each point along the beam path, the mirror surface looks locally toroidal, with two different principal curvatures for the directions along the beam and orthogonal to the beam. The desired beam path fixes the curvature along the beam, but the remaining curvature is determined by considerations of how the tangential beam structure should evolve. The resulting aspheric mirror shapes, although simple to describe in terms of their curvatures, are not at all like those encountered in conventional optics, and their construction appears to present a significant challenge.

The present paper, which attempts to outline the nature of this challenge, is divided into two parts. The first illustrates the elongated mirror geometries of interest by way of a specific example suggested by theory. The second addresses some of the fabrication tolerances involved, describing the effects of various types of surface imperfections.

Mirror Geometries

The shape of the beam path

An important issue in the design of WGM's is that of controlling beam divergence at the mirror exit⁴. The trouble here relates to the beam structure in the direction normal to the mirror surface. A full discussion of how best to deal with this situation lies outside the scope of the present paper, but it does seem that the problem can be greatly eased by using an elongated beam path rather than the more obvious circular path. It remains an open question as to which mirror shapes are optimal in the sense of minimizing the effects of beam divergence.

A theory to describe beam propagation in whispering galleries of non-uniform curvature has been developed⁶, and although it is primarily a tool for analysis rather than for synthesis, this theory has suggested a particular family of elongated curves that seem quite promising. These are most conveniently specified in terms of their local radius of curvature R : the family of interest

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FG 02-87ER/4572

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Figure 1. Outline of a typical elongated beam path.

is characterized by the form $R(s) = (as^2 + bs + c)^{3/2}$, in which a, b , and c are constants, and where the coordinate s denotes arc-length along the curve as measured from some fixed reference point. An example of such a curve is shown in Figure 1.

The overall shape of the mirror

Specification of the beam path only partially determines the mirror shape. Each part of the mirror can be regarded as locally toroidal, with two different principal curvatures in the directions along the beam and perpendicular to the beam. The desired beam path fixes the former, but the remaining curvature must be chosen on the basis of how the tangential transverse beam structure should behave as the beam propagates along the mirror.

The evolution of the tangential transverse beam structure is determined by the Gaussian curvature of the surface, *i.e.* the product of the two principal curvatures. The simplest case to deal with is a mirror surface for which the Gaussian curvature is constant. To achieve this, the transverse curvature must be inversely proportional to the curvature of the beam path. This requirement leads to mirror shapes of the sort shown in Figure 2. Notice that the transverse curvature is large near the mirror's entrance or exit and small near the apex.

Figure 2. View of a mirror surface based on the beam path shown in Figure 1.

Fabrication Tolerances

The goal of fabrication is to produce a physical realization that resembles the idealized design as closely as possible --- or at least as closely as is necessary for proper performance. While the realized surface can depart from the ideal surface in an innumerable variety of ways, it is useful to distinguish a few commonly occurring types of imperfections^{7,8}. First, the overall, large-scale shape of the surface might differ from the ideal; this is usually called a figure error. Second, there may be periodic tool marks left as the result of a machining process. Third, depending on the polishing process, there might be a random "waviness" or "orange-peel" structure. Fourth, there will usually be some fine-scale roughness. These different kinds of features have different effects on mirror performance. To get a feeling for the physics here, we will work with a simplified model in which we assume the mirror to be round and in which we concern ourselves only with the beam structure in the direction normal to the surface.

A convenient way of understanding beam propagation in a round whispering gallery is to regard the WGM as a multi-mode waveguide: beam confinement is provided on one side by reflection from the mirror surface, and on the other side as a result of the curvature. The low-order waveguide modes cling very tightly to the mirror surface and correspond to rays that strike the surface at very shallow grazing angles; conversely, the high-order modes stray farther from the surface and correspond to rays striking the surface at larger grazing angles. Letting the coordinate n denote distance away from the mirror surface, and letting

the coordinate s denote distance along the surface in the direction of beam propagation, the waveguide modes take the form⁹

$$F_j(n, s) = \text{Ai}(n/N + a_j) \cdot \exp(ik_j s), \quad j = 1, 2, 3, \dots,$$

in which $\text{Ai}()$ denotes an Airy function and a_j its j th zero. The radial length scale N , related to how far the low-order modes extend from the surface, depends on the wavelength λ and the mirror radius R :

$$N = (\lambda^2 R / 8\pi^2)^{1/3}.$$

The propagation constant k_j is given approximately by

$$k_j \approx \frac{2\pi}{\lambda} + a_j \left[\frac{\pi}{\lambda R^2} \right]^{1/3} = \frac{2\pi}{\lambda} + \frac{a_j}{D}$$

where $D \equiv (\lambda R^2 / \pi)^{1/3}$; this longitudinal length scale D will be important in what follows. As an indicator of the actual dimensions typically involved here, a wavelength of 100 angstroms and a mirror radius of 10 cm leads to a radial length scale $N \approx 0.5 \mu\text{m}$ and a longitudinal length scale $D \approx 317 \mu\text{m}$.

With a perfect mirror surface, these waveguide modes propagate independently of one another: a beam initially launched in one mode will remain in that same mode as it travels along the surface. However, when the actual surface departs from the ideal, the modes become coupled, and a beam initially in a single mode will end up with its part of its power scattered into a number of other modes. The amount of power coupled from one mode to another depends on the nature of the surface imperfection.

Let $h(s)$ denote the deviation of the actual surface from the ideal. If $h(s)$ is small everywhere as compared to N , the effects of surface errors can be computed by perturbation theory. The crucial result is that an imperfection leads to a coupling between two modes whenever $h(s)$ contains spatial frequencies resonant with the difference in the propagation constants of the two modes. Suppose one mode l is initially excited with unit amplitude at the mirror entrance; after this beam has traveled a length L along the mirror surface, the amplitude coupled to another mode m is given, in the limit of small net coupling, by

$$c(l \rightarrow m) = -i(-1)^{l+m} \int_0^L \frac{ds}{D} \frac{h(s)}{N} \exp[i(a_l - a_m)s/D].$$

Note that $c(l \rightarrow m)$ is essentially a Fourier transform of $h(s)$ evaluated at the spatial frequency $k_l - k_m$. Since the different varieties of surface imperfections are characterized by different spatial-frequency spectra^{7,8}, they have different effects on beam propagation.

Figure errors

Unlike conventional optics, whispering-gallery mirrors are fairly insensitive to figure errors. This is because the length scales associated with figure are very much longer than the length scale D associated with mode coupling.

As an explicit example, suppose the mirror is slightly out of round, with a deviation $h(s) = H \sin(\pi s/L)$. The strength of the mode coupling, as measured by the fraction of the power coupled out of the desired mode, is then something on the order of $(H/N)^2 \cdot (D/L)^2$, even when summed over all final modes. Now, for our perturbation theory to be valid we have assumed $H \ll N$; moreover, we have $D \ll L$, so the coupling is definitely quite small. In fact, the sensitivity to figure error is probably even less than might be suggested by our requirement here that $h(s)$ be small in comparison with N ; even when this condition is violated, the modes will tend to adapt adiabatically to figure errors so long as they occur over length scales large in comparison with D .

Periodic Tool Marks

This type of surface imperfection is potentially very serious, for it can act as a diffraction grating that very efficiently couples power from one mode to another. The length scales involved --- typically a few hundred microns --- are such that the grating period could be just right to resonate with $k_l - k_m$, and the long-range order of the tool marks could allow for a very long coherent interaction length.

To see just how severe this effect can be, suppose that the resonance is perfect and the tool marks have an RMS height δ , so that $h(s) = \sqrt{2} \delta \sin[(a_l - a_m)s/D]$. The fraction of power coupled out of the initial mode over an interaction length L is then $|c(l \rightarrow m)|^2 = \delta^2 L^2 / 2N^2 D^2$. Notice that the coupled power varies as δ^2 , which is characteristic of all types of scatter, and also

as L^2 , which is a signature of the interaction's assumed coherence. If we further assume that the grating remains coherent over an entire 180° deflection inside a WGM, and that the maximum power coupling must be restricted to at most 10%, we find that the grating height must be small enough that

$$\delta < \frac{\lambda}{4\pi\sqrt{5}} \approx \frac{\lambda}{28}.$$

With a wavelength of 100 angstroms, this translates to a maximal maximum allowable height of only 4 angstroms. Naturally, if the coherent interaction length over which resonance holds is smaller than assumed here, the restriction on δ becomes correspondingly less severe.

Waviness

This is a term used to describe random undulations in a surface that remain coherent with one another over just a few of their wavelengths. The surface length scales involved are typically on the order of a few hundred microns, so we are again in a situation where coupling can occur between nearby modes, whose propagation constants differ by something like $1/D$. Fortunately, though, the random nature of the surface prevents coherent coupling. A WGM is therefore less easily compromised by this type of surface feature than by a periodic tool marks.

Figure 3. Profile of a simulated wavy surface. The horizontal scale is in microns; the vertical scale indicates surface height in units of the RMS value δ .

We can estimate the effect by regarding the surface of length L as being composed of small regions of length l ; the interaction is assumed to be coherent within each region, but incoherent from one region to another. The net power coupling can then be approximated by summing the power scattered in each region. If the coherence length l is something comparable to D , say $l = \alpha D$, the fractional power coupled out of the mode is then roughly $|c|^2 \sim \alpha \delta^2 L / 2DN^2$. Again assuming a deflection of 180° and a 10% upper bound on scattered power, the allowable RMS waviness is on the order of

$$\delta \approx \frac{1}{20\sqrt{\alpha}} \lambda^{5/6} R^{1/6}.$$

Taking $\alpha \approx 1$, $\lambda = 100$ angstroms, and $R = 10$ cm, we find that about 70 angstroms of waviness can be tolerated.

Roughness

When a surface is polished mechanically, the residual fine scratch marks leave a random, rough pattern on the surface. The surface length scales observed here are small: one micron is a representative correlation length^{7,8}. Because the spatial frequencies in such a surface are so large as compared to $1/D$, the result of surface roughness is large-angle, diffuse scattering from the desired mode to a very large number of other modes. The power coupled into any one particular mode is small, but so many modes are accessible that the net power coupled away is substantial.

Figure 4. Profile of a simulated rough surface. The horizontal scale is in microns; the vertical scale indicates surface height in units of the RMS value δ .

The effects of surface roughness in WGM's were studied previously by Vinogradov *et. al.*¹⁰; they approached the problem in terms of bouncing plane waves rather than waveguide modes. We have reproduced their main result by means of the mode-coupling formalism, and although the calculations are too lengthy to present here, we can give the final answer. Assuming that the correlation length l of the roughness is much smaller than D , the power scattered in a 180° deflection is given by $(2\pi)^{5/2}\delta^2/\lambda^{3/2}l^{1/2}$. With a 10% upper bound on scattered power, we find a maximum allowable RMS surface roughness of

$$\delta \approx \frac{1}{31} \lambda^{3/4} l^{1/4}.$$

At a wavelength of 100 angstroms and with a roughness correlation length of $1 \mu\text{m}$, a surface finish of 10 angstroms is required.

Conclusion

We have seen that the mirror geometries desired for use in whispering-gallery laser resonators are very different from those encountered in conventional optics. We have also seen that while WGM's require a high-quality surface finish in order to avoid scattering losses, they are fairly insensitive to figure errors. These facts should be kept in mind when trying to find a suitable fabrication technique for these optics.

As a final comment, we point out that although the surface finish requirements for short-wavelength WGM's are strict, they are not so severe as those for normal-incidence optics. For this latter case, the fractional scattered power is given by $16\pi^2\delta^2/\lambda^2$. Requiring the scattered power be limited to 10% therefore demands an RMS surface finish of $\delta \approx \lambda/40$. At a wavelength of 100 angstroms, this corresponds to a finish better than 3 angstroms.

Acknowledgment

Part of this work was supported by the U.S. Department of Energy, Division of Advanced Energy Projects, under contract DEFG02/89CR14012.

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