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DAMPING OF THE ORDINARY MODE IN TOKAMA PLASMAS

By

I. Fidone, R.L. Meyer and G. Granata

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Quasi-Linear Saturation Effects on Electron-Cyclotron-Wave
Damping of the Ordinary Mode in Tokamak Plasmas

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Abstract

Quasi-linear theory of electron cyclotron wave absorption for the ordinary mode propagating normal to the tokamak magnetic field is investigated. A two-dimensional initial value code has been developed to investigate the distortion of the electron momentum distribution from an initial Maxwellian. It is shown that for powers of interest for electron heating in future tokamaks, Coulomb electron collisions are not sufficient to avoid quasi-linear flattening of the electron distribution and therefore are not sufficient to avoid a reduction in the damping rate as compared to the linear case.

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I. INTRODUCTION

Electron cyclotron wave absorption is presently considered as a potential method for additional heating in tokamak plasmas. A series of theoretical¹ and experimental² investigations have demonstrated the attractiveness of this method of electron heating and have instilled confidence on the future applications of the electron cyclotron resonance plasma heating. However, it is worth pointing out that the experiments to date have been performed with relatively modest amount of wave power ($P_0 \leq 0.5$ MW), whereas in future applications of electron heating in large tokamaks the electron component will interact with radiation powers in the range of several MW. Therefore, a problem of great relevance for the future applications of electron cyclotron resonance absorption is that of the power dependence of the wave damping. In order to investigate this problem, the theory must go beyond the linear approximation used in Ref. 1. We assume that the standard quasi-linear equation gives the correct description of the electron-wave interaction. The power dependence of the damping rate is understood from the following qualitative argument. The direct result of electron cyclotron wave absorption is an increase of the electron velocity in the direction normal to the tokamak magnetic field B_t . Thus, the electron momentum distribution tends to flatten in the perpendicular direction and the decrease in the slope is proportional to the wave power P_0 . For sufficiently large values of P_0 , the time scale for an appreciable change in the slope of the electron distribution is less than the electron collision time. Therefore, a kinetic steady state is attained in which the perpendicular diffusion due to Coulomb collisions is balanced by a quasi-linear diffusion with a reduced rate. Since the wave damping rate is proportional to the slope of the electron momentum distribution, one expects that a decrease in the

slope will result in a reduction of the wave damping. One finds that the contrasting effects of wave absorption and electron collisions on the momentum distribution function are characterized by the parameter $q \approx P_0/v_e$, the ratio of the quasi-linear and collisional diffusion coefficients, where v_e is the electron collision frequency. For $q \ll 1$, the electron distribution remains close to a Maxwellian. In this case, the linear theory holds, and wave absorption proceeds at a rate which is independent of P_0 . For $q \gg 1$, the quasi-linear flattening of the momentum distribution persists in spite of electron collisions, and wave absorption is much reduced compared to the linear case. It is worth noting that in the case of small powers ($q \ll 1$), electron heating proceeds also at a relatively slow rate. Therefore, in the two extreme cases, namely $q \ll 1$ and $q \gg 1$, one obtains the same slow heating at approximately comparable rates. We show that the best compromise between quasi-linear effects and heating efficiency is the case of $q \approx 1$.

In this work, we investigate the quasi-linear kinetic theory of electron cyclotron wave absorption for the most attractive wave launching, namely, the ordinary mode coming from the low magnetic field side of the tokamak and propagating in the quasi-perpendicular direction. We assume that the wave-packet angular spread in the direction parallel to $\vec{B}_e$ is a few degrees and that $\omega_p^2 \ll \omega_e^2$, where ω_p and ω_e are the plasma and cyclotron frequencies, respectively. In this case, the wave beam pattern remains well collimated from the antenna to the absorbing region where the wave frequency $\omega \approx \omega_e$. Releasing the two restrictions of small beam spread and small values of ω_p^2/ω_e^2 , would mean to take into account ray tracing in the kinetic equation. This general case is very difficult and is not considered here. The investigation of quasi-linear wave absorption is restricted to the kinetic stage, that is, we disregard spatial diffusion of the thermal energy and relaxation to the ion

component. These processes will occur on a time scale much longer than the characteristic time for which a kinetic steady state is attained (a few v_e^{-1}). In other words, we investigate the properties of the source term of electron heating to be used in the evolution code describing the macroscopic changes of the entire plasma. The interest of our investigation is that the properties of the energy source will necessarily determine the long time evolution of the electron temperature and can then be used in the process of optimization. The solution of the governing kinetic equation is obtained numerically using a two-dimensional code in polar coordinates, as discussed in Ref. 3. We solve a full non-linear and self-consistent problem, namely, the wave amplitude is evaluated at any instant and at any point in space according to the actual damping. This is necessary since wave damping is a very sensitive function of the slope of the local momentum distribution because the resonance condition relates the electron momentum to the location of wave energy deposition.

In Sec. II of this paper, we discuss the linear theory of the ordinary mode damping for propagation near perpendicular to $\vec{B}_t$. We also formulate the quasi-linear equation for a narrow wave packet in a tenuous plasma. In Sec. III, we present the numerical results for the wave spatial damping for several values of the wave power, the initial electron temperature, and the beam angular spread. The conclusions and the implications of the results obtained are presented in Sec. IV.

II. THEORETICAL MODEL FOR THE ORDINARY MODE

We consider a narrow wave packet propagating in the tokamak equatorial plane in the direction normal to $\vec{B}_t$. We also assume that $\omega_p^2/\omega_e^2 \ll 1$ and neglect refraction and lateral deviations of the wave beam. The only angular

variation of relevance is in the parallel direction and is characterized by the parallel index $N_{\parallel} = \sin\psi$, where ψ is the angle between each single wave in the wave packet and the normal to $\vec{B}_t$. For a symmetric wave packet with respect to the normal direction, one has $\bar{N}_{\parallel} \approx 0$ and $\Delta N_{\parallel} \approx \Delta\psi =$ a few degrees, where $\bar{N}_{\parallel}$ is the average value of $N_{\parallel}$ and $\Delta N_{\parallel}$ is the half-width of the angular spread. The linear theory⁴ of the ordinary mode damping yields for each value of $N_{\parallel}$ ($|N_{\parallel}| \lesssim \Delta N_{\parallel}$).

$$k_i = (aA) k_r \left(\frac{\pi \omega_p^2}{4\omega_e^2} \right) \left\{ \mu R^{7/2} / N_{\parallel}^{3/2} (1 - N_{\parallel}^2)^{5/2} \right\} \left\{ I_{3/2}(\xi) \right. \\ \times (1 + N_{\parallel}^2 \omega_e^2 / R^2 \omega^2) \\ \left. - 2 I_{5/2}(\xi) (2/\xi + N_{\parallel} \omega_e / R\omega) \right\} \exp \left\{ \mu \left(1 - \frac{\omega_e / \omega}{1 - N_{\parallel}^2} \right) \right\}, \quad (1)$$

where $k_r = (\omega/c) (1 - \omega_p^2/\omega^2)^{1/2}$ and k_i are the real and imaginary parts of the propagation vector in the x direction, normal to $\vec{B}_t = B_t \vec{e}_z$, $\mu \approx mc^2/T_e \gg 1$, $R = (\omega_e^2/\omega^2 - 1 + N_{\parallel}^2)^{1/2}$, $\xi = N_{\parallel} R \mu / (1 - N_{\parallel}^2)$, $A = (\pi/2\mu)^{1/2} / \exp(\mu) K_2(\mu) \approx 1$, $a = (1/2) [1 + (\omega_e^2/\omega^2 - 1 + N_{\parallel}^2) / |R|^2]$, T_e is the electron temperature, c is the speed of light, and m is the electron rest mass. The presence of the step function a in Eq. (1) shows that wave absorption occurs only for $\omega_e^2/\omega^2 > 1 - N_{\parallel}^2$. For $N_{\parallel}^2 = 0$, Eq. (1) yields the well-known result⁵

$$k_i = a k_r (2\pi^{1/2}/15) \left(\frac{\omega_p^2}{\omega_e^2} \right) [\mu(\omega_e/\omega - 1)]^{5/2} \exp \{ \mu(1 - \omega_e/\omega) \}. \quad (2)$$

Equation (1) is valid for finite but small values of $N_{\parallel}$. It is instructive to investigate linear wave damping using Eq. (1) for several values of $N_{\parallel}$. We choose the following spatial dependencies for the electron density, temperature, and toroidal magnetic field, $n_e(r) = n_e(0) (1 - r^2/a_0^2)$, $T_e(r) = T_e(0) (1 - r^2/a_0^2)^2$, $B_t(x) = B_t(0)/(1 + x/R_0)$, where, $a_0 = 40$ cm, $R_0 = 150$ cm, $B_t(0) = 32$ kG. We also assume that $\omega = \omega_e$ at $x = x_e = -0.5$ cm. In Fig. 1, we show k_i versus x for $n_e(0) = 3 \times 10^{13}$ cm $^{-3}$ and $T_e(0) = 1$ keV for $\psi = 0$ and $\psi = 2.5^\circ$. It appears that k_i is a sensitive function of $N_{\parallel}$. However, the energy deposition is less sensitive to the precise value of $N_{\parallel}$. This is seen by plotting $\eta(x) = 1 - \exp(-\int_0^x 2 k_i dx')$ for the conditions of Fig. 1. As seen in Fig. 2, there is a small deviation between $\eta(\psi = 0)$ and $\eta(\psi = 2.5^\circ)$, and we may assume that for small values of ψ ($|\psi| \lesssim 3^\circ$) $\exp[-\int_0^x 2 k_i(N_{\parallel}) dx'] \approx \exp[-\int_0^x 2 k_i(0) dx']$. This approximation will be used also in the quasi-linear theory. The spatial range Δx in which the wave energy is practically deposited is determined from the approximate relation $\eta(x^*) \approx 1$. From Fig. 2, we obtain $x^* \approx 2$ cm, thus, $\Delta x = |x^* + 0.5| = 1.5$ cm. Note that wave propagation occurs in the x direction, and, therefore, the wave energy is initially deposited in the elementary cylinder of cross-section σ and height Δx . However, in view of the fast electron motion over the magnetic surfaces (here approximated by cylinders of surface $S = 4\pi^2 r R_0$), the effective region of energy deposition is a cylinder of volume $V \approx \pi r^{*2} (2\pi R_0)$, where $r^{*2} \approx x^{*2} - x_e^2$. The spatial deposition is then formally described by $\eta(r) = 1 - \exp[-\int_0^r 2 k_i(r') dr']$, where in the definition of k_i we use $B_t(r) = B_t(0)/(1-r/R_0)$, and $0 \leq r \leq a_0$. It is ordinarily assumed that the fraction of wave energy absorbed by the electrons is rapidly converted into electron temperature. That is, electron collisions are sufficiently frequent so to randomize the deformation in the momentum distribution caused by wave

absorption and are then able to restore continuously the Maxwell form of the electron distribution. At any instant, wave absorption proceeds at the rate determined by Eqs. (1) and (2) with a time increasing T_e . So far the proof of this fundamental assumption has been restricted to the following qualitative argument. For $N_{\parallel} = 0$, resonance between electrons and the wave occurs for

$$(p/mc)^2 \approx \omega_e^2/\omega^2 - 1 \approx 2 (\omega_e/\omega - 1), \quad (3)$$

where p is the electron momentum. Assuming that a large fraction of the wave energy is absorbed in the region where $\mu(\omega_e/\omega - 1) \approx 1$, from Eq. (3) one obtains $w^2 \lesssim w_0^2 = 2\mu (\omega_e/\omega - 1) \approx 2$, where $\vec{w} = \vec{p}/(m T_e)^{1/2}$. This seems to suggest that most of the wave energy is absorbed by the bulk of electrons and, assuming that for the electron body Coulomb collisions are a very efficient randomizing mechanism, one concludes that the electron momentum distribution remains Maxwellian with a time increasing temperature. This qualitative argument has no general validity. First of all, the randomizing efficiency of Coulomb collisions decreases with increasing wave power and electron temperature. Secondly, even a small angular spread of a few degrees in the wave beam will change Eq. (3) by an appreciable amount. For finite $N_{\parallel}$, the resonance condition is

$$w^2 \approx w_0^2 + 2\mu^{1/2} N_{\parallel} w_{\parallel}, \quad (4)$$

where $w_{\parallel} = \vec{w} \cdot \vec{e}_z$. For $\psi = 2.5^\circ$, $N_{\parallel} = 4 \times 10^{-2}$ and for $\mu = 511$ one has $2\mu^{1/2} N_{\parallel} = 1.8$. We see that for a wave packet of finite spread, electrons with velocity above the thermal speed will contribute to wave absorption. For these electrons, the collision rate decreases as w^{-3} , and the parameter p

teds to be greater than unity. The effect of beam spread on the extent of the region in the momentum space in which the distribution function is perturbed by wave absorption was first discussed in Ref. 4. For sufficiently small values of P_0 one expects Coulomb collisions to be an efficient randomizing process, and wave absorption and electron heating are determined by Eq. (1). An interesting property of the linear wave absorption is the temperature independence of the maximum value of k_i . This is readily seen from Eq. (2). We now investigate the range of validity of the linear theory by solving the quasi-linear kinetic equation. The quasi-linear equation for the ordinary mode and small values of $N_{||}$ can be obtained from the general formulation given in Ref. 4. It is instructive, however, to give a simple derivation using the definition of diffusion coefficient. For a narrow wave packet, we assume that the wave field is given by $E = \vec{e}_z E(x, z, t)$, where

$$E(x, z, t) = \frac{1}{2} \int_{-\infty}^{\infty} dN_{||} E(N_{||}) \exp(i \int^x k dx' + ik_{||} z - i\omega t) + c.c.,$$

where c.c. denotes the complex conjugate, $k_{||} = (\omega/c) N_{||}$, $k = k_r + ik_i$, and $E(N_{||})$ is non-zero in a small range $\Delta N_{||}$ near $N_{||} = 0$. We take

$$|E(N_{||})|^2 = |E_0|^2 \exp[-(N_{||}/\Delta N_{||})^2]/\pi^{1/2}(\Delta N_{||}),$$

where

$$|E_0|^2 = \int_{-\infty}^{\infty} dN_{||} |E(N_{||})|^2.$$

The quasi-linear diffusion equation is of the general form

$$\partial f / \partial t = p_{\perp}^{-1} (\partial / \partial p_{\perp}) p_{\perp} D_{rf} \partial f / \partial p_{\perp} + (\partial t / \partial t)_{coll}, \quad (5)$$

where $f = f(p_{\perp}, p_{\parallel}, r, t)$ is the electron momentum distribution averaged over the momentum gyration phase ϕ , and over the magnetic surface S at r , and $D_{rf} = (\sigma/S) (\Delta p_{\perp})^2 / 2\Delta t$, where $\Delta p_{\perp}$ is the increment of $p_{\perp}$ resulting from wave absorption in the time interval Δt . Note that the previous definition of D_{rf} is valid if $\sigma^{1/2}$, which characterizes the linear dimension of the beam cross section, is much less than r . For $r \sim \sigma^{1/2}$, the surface S is determined by the actual shape of the beam cross section. The determination of S for $r \sim \sigma$ is a difficult problem and we here adopt a crude estimation, namely, we assume that $S = 4\pi^2 \rho(r) R_0$, where $\rho(r) = r$ for $r > \sigma^{1/2}$ and $\rho(r) = \sigma^{1/2}$ for $r \leq \sigma^{1/2}$. In the numerical computations, we take $\sigma^{1/2} = 2$ cm.

The collision operator is in general given by

$$\begin{aligned} (\partial f / \partial t)_{coll} = \sum_{\beta=1,2} (2\pi A_{\beta} e^4 n_e \hat{z}_{\beta}) [& (\partial_{\perp}^2 f \partial_{\perp}^2 + p_{\perp}^{-2} \partial_{\perp} f \partial_{\perp} \\ & + 2 \partial_{\perp} \partial_{\parallel} f \partial_{\perp} \partial_{\parallel} + \partial_{\parallel}^2 f \partial_{\parallel}^2) r_{\beta} \end{aligned}$$

$$+ \{1 - m/m_{\beta}\} (\partial_{\perp} f \partial_{\perp} + \partial_{\parallel} f \partial_{\parallel}) (\partial_{\perp}^2 + p_{\perp}^{-1} \partial_{\perp} + \partial_{\parallel}^2) r_{\beta} + 4\pi (m_{\beta}/m)^2 f_{\beta} (m_{\beta} \vec{p}/m)],$$

where $\hat{z}_\beta = n_\beta z_\beta^2 / n_e$, z_β is the charge number, Λ_β is the Coulomb logarithm, $-e$ is the electron charge, $\partial_\perp = \partial / \partial p_\perp$, $\partial_\parallel = \partial / \partial p_\parallel$, and

$$\Gamma_\beta = \int d\vec{p}' f_\beta(\vec{p}') |\vec{p} - (m/m_\beta) \vec{p}'|. \quad (6)$$

Now

$$d\vec{p}/dt = -e \vec{p} \times (B_x \vec{e}_z + B_y \vec{e}_y) / m\gamma c - eE\vec{e}_z,$$

where $\gamma = (1 + p^2/m^2c^2)^{1/2}$, B_y is the y component of the wave magnetic vector, and $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$ are the unit vectors of an orthogonal system. For the Fourier component of B_y , one has $B_y(N_\parallel) = -N_r E(N_\parallel)$, where $N_r = (1 - \omega_p^2/\omega^2)^{1/2}$. In first approximation, $p_\parallel = \text{constant}$, $\gamma = \text{constant}$, and

$$\Delta p_x \approx -i [e N_r p_\parallel E(N_\parallel) / 2 m\gamma c] / (\omega - \omega_e / \gamma - k_\parallel p_\parallel / m\gamma).$$

Noting that

$$|\omega - \omega_e / \gamma - k_\parallel p_\parallel / m\gamma|^{-2} = (\pi / \omega_i) \delta(\omega - \omega_e / \gamma - k_\parallel p_\parallel / m\gamma),$$

where $\omega_i \rightarrow 0$ is the imaginary part of ω , we obtain

$$\omega_i |\Delta p_x|^2 = \omega_i |\Delta p_y|^2 = \pi (e N_r p_\parallel / 2mc)^2 |E(N_\parallel)|^2 \delta(\omega - \omega_e / \gamma - k_\parallel p_\parallel / m\gamma),$$

and

$$(\Delta p_{\perp})^2 / \Delta t = \omega_i (\Delta p_{\perp})^2 = \int_{-\infty}^{\infty} dN_{\parallel} \omega_i |\Delta p_x|^2 \exp \left[- \int_0^r 2 k_i(r') dr' \right].$$

Using Eq. (5) and Poynting's theorem, one obtains

$$\begin{aligned} (\partial/\partial t) n_e \int d\vec{p} (p^2/2m) f &= (\sigma/S) \int_{-\infty}^{\infty} dN_{\parallel} 2 k_i (N_{\parallel} c/8\pi) |E(N_{\parallel})|^2 \\ &\times \exp \left\{ - \int_0^r 2 k_i dr' \right\} \\ &= (P_0/\pi^{1/2} S \Delta N_{\parallel}) \int_{-\infty}^{\infty} dN_{\parallel} 2 k_i \exp \left[- (N_{\parallel}/\Delta N_{\parallel})^2 \right] \exp \left(- \int_0^r 2 k_i dr' \right), \quad (7) \end{aligned}$$

where $P_0 = (N_{\parallel} c/8\pi) |E_0|^2 \sigma$ is the total power in the wave beam delivered by the antennas at frequency ω , and

$$2 k_i = -(\pi/4m^2 c^2) k_r (\omega_p^2/\omega_e^2) \int d\vec{p} p_{\perp}^2 p_{\parallel} (\partial f/\partial p_{\parallel}) \delta(\gamma - \omega_e/\omega - N_{\parallel} p_{\parallel}/mc). \quad (8)$$

Note that we refer to a single beam of power P_0 , but the analysis is also applicable to the case of several beams of power P_i at the same frequency with $P_0 = \sum_j P_j$. For

$$f = f_m = \left[\mu/4\pi(r')^3 K_2(\mu) \right] \exp(-\mu\gamma),$$

Eq. (8) reduces to Eq. (1). Letting $\tau = t/t_e$, where

$$t_e^{-1} = 2\pi n_e e^4 \Lambda_e/m^{1/2} T_e^{3/2}; \quad \Lambda_e = 24 - \ln[n_e^{1/2}/T_e(\text{eV})],$$

where T_e is the electron temperature at $t = 0$, the kinetic equation becomes

$$\partial f / \partial \tau = w_{\perp}^{-1} (\partial / \partial w_{\perp}) w_{\perp} D \partial f / \partial w_{\perp} + t_e (\partial f / \partial t)_{\text{coll}}, \quad (9)$$

where $(\partial f / \partial t)_{\text{coll}}$ is written in terms of the normalized variable $\vec{w}$, and

$$D = (1 - \omega_p^2 / \omega^2)^{1/2} [10^{30} P_0 w_{\parallel} (T_e / mc^2) / \rho(r) R_0 \Delta N_{\parallel} n_e \omega_{\perp}^{\Lambda} \omega_e] \\ \times \exp \{ -(N_{\parallel} / \Delta N_{\parallel})^2 \} \exp \left\{ - \int_0^r 2 k_i(r', N_{\parallel}) dr' \right\}, \quad (10)$$

where P_0 is in MW and

$$N_{\parallel} = \mu^{1/2} [(1 + w^2 / \mu)^{1/2} - \omega_e(r) / \omega] / w_{\parallel}. \quad (11)$$

Note that this value of $N_{\parallel}$ enters also in the expression of k_i [Eq. (8)]. In Eq. (10), $\delta(r, N_{\parallel}) = \delta(r, w_{\parallel}, w_{\perp}) = \exp \left[- \int_0^r 2 k_i(r', N_{\parallel}) dr' \right]$ is a complex function of $w_{\parallel}$ and $w_{\perp}$. The presence of $\exp[-(N_{\parallel} / \Delta N_{\parallel})^2]$, however, restricts the range of values of $w_{\parallel}$ and $w_{\perp}$ to those for which $|N_{\parallel}| \lesssim \Delta N_{\parallel}$. The linear analysis has shown that, although k_i varies appreciably with $N_{\parallel}$, $\delta(r, N_{\parallel}) \approx \delta(r, N_{\parallel} = 0)$ for $|\psi| \lesssim 3^\circ$. We assume that this also occurs in the non-linear case, and in Eq. (10) we substitute $k_i(r', N_{\parallel})$ by $k_i(r', 0) = k_i(r')$ obtained from Eq. (8) by letting $N_{\parallel} = 0$. On the other hand, the exact value of $N_{\parallel}$, as given by Eq. (11), is used in the evaluation of the critical factor $\exp[-(N_{\parallel} / \Delta N_{\parallel})^2]$. A second simplification is made in the collision term. The quasi steady state in which we are interested is generally attained in a time interval of the order of t_e , that is, the balance between the two competitive effects described by the two terms in the right

hand side of Eq. (9) is obtained for $\tau \approx 1$. During this interval the electron temperature has changed little from the initial value (20-30%). This suggests that Γ_e given by Eq. (6) is close to the value obtained by letting $f_e = f = f_m$ (for $\beta = 1$ the assumption is always fulfilled), where f_m is the initial Maxwellian distribution. Using this assumption in Eq. (6), the collision term becomes linear. This approximation is no longer valid for $\tau > 1-2$ and the linearized collision term is not adequate, in general, for investigating electron heating for $\tau \gg 1$. The approximation is, however, adequate for investigating the quasi-linear wave absorption decrease since this is a relatively fast process. The two simplifications utilized in our numerical code, namely, the spatial distribution of the wave energy described by $\delta(r, N_{\parallel} = 0)$ and the linearization of the collision operator, are dictated by the excessive time consuming kinetic code. However, in our numerical study, nothing essential has been neglected. In a series of checks, the results of the simplified code were found to be in good agreement with those of the code employing the full non-linear collision term and the exact $N_{\parallel}$ dependence of $\delta(r, N_{\parallel})$. As already remarked, for a wave packet of finite half-width, a fraction of the wave energy is absorbed by electrons with velocity above the thermal speed. For these electrons, $w > 1$ and $\delta_{\perp}^2 \Gamma_e \approx w_{\parallel}^2/w^3$, and the parameter q is given by

$$q = \left(1 - w_{\parallel}^2/w^2\right)^{1/2} 10^{30} P_0 w^3 (T_e/mc^2)/\rho(r) R_0 \Delta N_{\parallel} n_e \omega_e w_{\parallel}.$$

For $\rho(r) = 2$ cm, $n_e(0) = 3 \times 10^{13}$ cm $^{-3}$, $\Delta N_{\parallel} = 0.05$ ($\Delta\psi = 3^\circ$), $\omega \approx \omega_e(0)$, one obtains $q \approx 0.7 P_0 T_e w^3/w_{\parallel}$, where T_e is in keV. For values of interest for electron heating, p is easily greater than unity, and quasi-linear effect will be predominant.

III. NUMERICAL RESULTS

We have developed a two-dimensional initial value code following the alternating-direction implicit scheme discussed in Ref. 3 to investigate the time evolution of the distribution function from an initial Maxwellian distribution at temperature T_e . In all cases $T_i = T_e$ and $\hat{Z}_i = 1$ was assumed. Specifically, f was computed as a function of r and τ for several values of P_0 , T_e , and $\Delta N_{\parallel}$ in the range $0 \leq w \leq 6$. Simultaneously $k_i(r, \tau)$ and $\eta = 1 - \exp(-\int_0^{\tau} 2 k_i dr')$ was evaluated for $N_{\parallel} = 0$ according to Eq. (8). Equation (9) was integrated by using a 40×40 grid in the (w, θ) space. The time step was 0.025 from $\tau = 0$ to $\tau \approx 1-2$. This interval of time was generally sufficient to obtain a dynamical equilibrium where the quasi-linear diffusion is balanced by the collisional one. This stage is not a true steady state but one in which the distribution function and k_i are changing very slowly compared to the initial variation during the interval $\tau \approx 1-2$. The numerical procedure started with $P_0 \ll 1$ in order to check that $(\partial f / \partial t)_{\text{coll}}$ remains close to zero for a Maxwellian distribution. For increasing power ($P_0 > 0.5$), quasi-linear effects start to play a role. The process begins with a flattening of the distribution function in $w_{\perp}$, i.e., $|\partial f / \partial w_{\perp}|_{\tau > 0} < |\partial f / \partial w_{\perp}|_{\tau = 0}$. The region in the $w_{\perp}$ space where the flattening occurs is correlated to the position r by the relation $|N_{\parallel}(w_{\perp}, w_{\parallel}, r)| \leq (\Delta N_{\parallel})$. The adjustment in the slope results in a decrease of k_i . The new value of k_i is used to evaluate the spatial deposition of the wave energy. This process is continued until the reduced slope of f will make the two terms in the right hand side of Eq. (9) of comparable magnitude. This occurs typically at $\tau \approx 1-2$. For later times, the time variation of f and k_i becomes much smaller than the initial rate and is due to slow heating of the electron system.

In Fig. 3, we show k_i versus r for $P_0 = 1$, $T_e(0) = 1$ keV, and $\Delta N_{||} = 0.05$ ($\Delta\phi = 3^\circ$) for several values of τ . The fastest decrease of k_i occurs for $\tau < 1$. As $\tau \rightarrow 2$, k_i tends to a saturation value of 0.45 near the maximum. Figure 3 shows the damping of the central part of the wave packet ($N_{||} = 0$). A similar drop occurs for k_i ($N_{||} \neq 0$). Now, the spatial deposition of the wave-packet energy is proportional to $\sum_{N_{||}} k_i \delta$ and is therefore expected to be appreciably less than that given by the linear theory. Since $\delta(r, N_{||}) \approx \delta(r, 0)$, $\eta(r, N_{||}) = 1 - \delta(r, N_{||}) \approx 1 - \delta(r, 0) = \eta(r)$ and the fraction of the wave-packet energy deposited from $r = 0$ to r is approximately given by the damping of the central component. In Fig. 4, we present $\eta(r)$ for the conditions of Fig. 3. The decrease in the value of η for large values of r is less pronounced than that of the maximum value of k_i (50%). It is, however, an appreciable change from the linear case and demonstrates the relevance of the non-linear theory for investigations of wave damping for large wave powers. For increasing power the quasi-linear effect becomes more pronounced. This is shown in Fig. 5 where we plot $\eta(r)$ for the condition of Fig. 1, but $P_0 = 3$. We see that η is reduced by a factor of 2 from the linear case. We do not show k_i for this value of P_0 . It is very similar to that of Fig. 1, but the maximum value has dropped to 0.2 from the initial value 0.9. Clearly in this case the quasi-linear effect has changed the electron system from optical thick to optical thin, and most of the rf energy will not be absorbed in one transit. We discuss later the fate of that part of the wave energy which is not absorbed and likely is reflected back by the tokamak wall. It is also interesting to investigate the effect of varying $\Delta N_{||}$ and $T_e(0)$. As seen from Eq. (10), D increases by decreasing $\Delta N_{||}$. The quasi-linear effect is approximately the same as increasing P_0 . This is shown in Fig. 6 where $\Delta N_{||}$ has been reduced by 2 from the case shown in Fig. 4. We

find that the reduction in the saturated value of η is 45% compared to 50% shown in Fig. 5 for $\Delta N_{\parallel} = 0.05$ and $P_0 = 3$. At first glance, from Eq. (10) one expects the quasi-linear effect for increasing values of T_e is similar to that of increasing P_0 . In fact, the temperature dependence of D is rather complicated, and the effect of increasing T_e differs from that due to the increasing of P_0 by the same factor. This is shown in Fig. 7 where we present k_i vs r for $T_e = 3$ keV, $\Delta N_{\parallel} = 0.05$ and $P_0 = 1$. The maximum value decreases by approximately the same factor as in Fig. 3 where $T_e = 1$. For comparison, we note that the value of the maximum was 0.2 for $T_e = 1$ keV and $P_0 = 3$. Furthermore, the spatial extension of k_i for $T_e = 3$ keV is wider than that at $T_e = 1$ keV; thus, for large r , $\eta(r)$ is still close to unity even after the quasi-linear reduction. This is shown in Fig. 8. The quasi-linear effect is manifested for small values of r but the plasma remains optically thick. Note that the time scale is not the same as for $T_e = 1$ keV since $t_e(3 \text{ keV}) \approx 5 t_e(1 \text{ keV})$. This is relevant when investigating electron heating. It is likely that in a plasma with initial electron temperature of 3 keV, a good heating efficiency is obtained with $P_0 > 1$. In this case, wave absorption is again appreciably modified by quasi-linear effects. In Fig. 9, we show η vs r for the conditions of Fig. 7, but $P_0 = 3$. Here the wave beam is partially transmitted, and the heating process is expected to proceed at reduced rate. The general qualitative conclusion to be drawn from our investigation is that for powers $P_0 < 1$, quasi-linear effects are generally negligible, and electron heating proceeds at the linear rate ($\approx k_i P_0$). For $P_0 > 1$, quasi-linear effects may become important, k_i is less than the linear value, and $k_i P_0$ is roughly independent of P_0 .

Combining the two results, one concludes that the electron heating efficiency tends to remain relatively slow. This qualitative conclusion is

merely orientative. A definitive quantitative conclusion about the heating efficiency is only possible by linking the quasi-linear code to the plasma evolution code. So far we have considered wave propagation from the low magnetic field side. For $P_0 > 1$ a large fraction of the wave energy is not absorbed by the electrons in the first transit. The transmitted part is generally reflected by the tokamak wall, will again transit through the resonant region, and can then be absorbed. Assuming that the reflected wave packet is still polarized in the ordinary mode and propagates nearly normally to B_t , we expect that wave damping of the reflected wave packet will be reduced by approximately the same amount as that of the primary beam. The situation is complicated by the fact that the effects of the incident and reflected beams may overlap in space, and the correct way to handle this case is to take into account the two beams simultaneously. We do not consider this problem here. We only discuss an important difference between the two directions of propagation by investigating wave damping for propagation from the high magnetic field side in the absence of a beam coming from the opposite direction. The main difference between the two directions of propagation is that, for propagation from the high magnetic field side, most of the wave energy is absorbed by electrons with velocity above the thermal speed. This is readily seen from Eq. (3) which correlates the position where the wave energy is deposited to the momentum of the absorbing electrons. Since the wave is coming from the region of large values of $|\omega - \omega_e|$, it is expected that the fast electrons will first interact with the incoming wave. For fast electrons the quasi-linear flattening is more efficient than for thermal electrons, and the decrease of k_i for large values of r is very pronounced. This results in a reduced damping of the wave which then propagates toward the resonance and is essentially absorbed by electrons with energy close to the

thermal energy. The final result of this electron-wave interaction is illustrated in Fig. 10, where we show η vs r for $T_e = 1$ keV, $\Delta N_{\parallel} \approx 0.05$ and $P_0 = 3$. Comparing this result with the corresponding for propagation from the low magnetic field side (Fig. 5), it appears that in the case of Fig. 10 η saturates at slightly smaller value as compared to that of Fig. 5. However, in this case, the formation of the electron tail will occur. As expected, the tail effect increases with increasing T_e , as is shown in Fig. 11 for $T_e = 3$ keV. The activation of the electron tail will also mean that less energy is transferred to the electron body and will alter the time evolution of the mean electron kinetic energy E . At the end of the fast kinetic stage, which in the case of $P_0 = 3$ is situated near $\tau \approx 1.6$, we find that the values of E/T_e averaged over the region of absorption are 1.3 and 1.2 for the situations of Figs. 9 and 11, respectively.

IV. CONCLUSIONS

We have presented the first systematic study of the quasi-linear evolution of wave damping near the electron cyclotron frequency for large wave powers. We have investigated the propagation of a narrow wave packet polarized in the ordinary mode for propagation normal to the tokamak magnetic field. The total rf energy is assumed to be launched by a number of radiators (gyrotrons) at the same frequency. In view of the averaged effect over the magnetic surface, the result is as one gyrotron radiates the total power. The investigation is restricted to the early phase of the heating process, namely, the fast time quasi-linear deformation (flattening) of the electron distribution. In few collision times, a dynamical equilibrium takes place in which the wave damping rate is appreciably reduced from the linear ($\tau = 0$) case. For later times, the time evolution of the wave and plasma parameters proceeds at slow rate determined by the change of the electron temperature.

The main result of our investigation is that for powers $P_0 > 1$, the quasi-linear reduction of the wave damping will bring the product $k_i P_0$ to roughly the same value as for $P_0 < 1$, and this suggests that the electron heating rate ($\sim k_i P_0$) is not a linear function of P_0 , but tends to saturate for P_0 above 1 MW. An interesting problem then is that of determining the best sharing of the total power between several values of the wave frequency. This optimization problem can be investigated only by combining the quasi-linear code to the plasma evolution code. A second problem of paramount interest is the determination of the nature of the electron collisions. We have used the classical Coulomb collisions, but we are aware of the fact that so far this is just an assumption. We have considered a tenuous plasma ($\omega_p^2/\omega_e^2 \ll 1$). Equation (10) suggests that by increasing the electron density the quasi-linear effect decreases. This beneficial effect is more apparent than real. First of all, if we increase the density, refraction increases and the validity of the quasi-perpendicular propagation is no longer valid. A more general treatment is required where the effect of the neglected components of the wave field are to be taken into account. Secondly, electron heating of denser plasmas will ultimately require a larger value of P_0 .

In conclusion, we are not yet in the position to determine the best conditions for optimum electron heating, but the result of our investigation clearly shows that for powers of interest for electron heating in future tokamaks, Coulomb collisions are not sufficient to avoid quasi-linear effects and, therefore, the reduction of the damping rate.

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Figure Captions

Fig. 1 k_i vs x for $n_e(0) = 3 \times 10^{13} \text{ cm}^{-3}$, $T_e(0) = 1 \text{ keV}$ and $\psi = 0(a)$, $\psi = 2.5^\circ (b)$, as given by Eq. (1).

Fig. 2 $\eta(x)$ vs x for the conditions of Fig. 1 for propagation from the low magnetic field side.

Fig. 3 k_i vs r for $\psi = 0$, $T_e(0) = 1 \text{ keV}$, $P_0 = 1$, $\Delta N_{||} = 0.025$, and several values of τ resulting from the quasi-linear equation.

Fig. 4 η vs r for the conditions of Fig. 3.

Fig. 5 η vs r for the conditions of Fig. 3 but $P_0 = 3$.

Fig. 6 η vs r for the conditions of Fig. 3, but $\Delta N_{||} = 0.025$ ($\psi = 1.5^\circ$)

Fig. 7 k_i vs r for $\psi = 0$, $T_e(0) = 3 \text{ keV}$, $P_0 = 1$, and $\Delta N_{||} = 0.05$.

Fig. 8 η vs r for the conditions of Fig. 7.

Fig. 9 η vs r for the conditions of Fig. 7 but $P_0 = 3$.

Fig. 10 η vs r for $T_e(0) = 1 \text{ keV}$, $P_0 = 3$, and $\Delta N_{||} = 0.05$ for propagation from the high magnetic field side.

Fig. 11 η vs r for the conditions of Fig. 10 but $T_e(0) = 3 \text{ keV}$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

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