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EIGENMODES IN TOKAMAKS

BY

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Electrostatic Drift Wave Eigenmodes in Tokamaks

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Abstract

The eigenmode equation governing both the toroidicity-induced and the ion temperature gradient driven modes of arbitrary wavelengths in tokamaks is derived by employing the ballooning mode formalism. For the toroidicity-induced mode (electron driftwave), the ion magnetic drift resonances are destabilizing and can have a significant effect when the temperature ratio, T_e/T_i , is of order unity. The ion temperature gradient driven mode (ion drift wave), which is unstable in a shear slab geometry, is found to be further destabilized by the ion magnetic drifts, resulting in a lower critical ion temperature gradient (n_1).

DISCLAIMER

I. Introduction

The stability of electron and ion drift waves in a sheared slab geometry has been investigated extensively [1-4]. For electron drift waves, the shear stabilization has been found to relate to the anti-well potential in which wave energy convects away from mode rational surface [5]. However, in a toroidal plasma the toroidal coupling effects due to ion magnetic drifts can create local potential wells and nullify the shear damping effects. The electron drift waves are trapped in these local potential wells and form marginally stable toroidicity-induced (TI) eigenmodes [6,7]. The toroidicity-induced modes are then shown to be destabilized by the electron dissipations [8-10].

Recently the use of neutral beams in PLT and PDX experiments has produced steep ion temperature profiles. Interest on stability of ion drift waves excited by ion temperature gradient ($\eta_i \equiv d \ln T_i / d \ln N_i$) is renewed [2]. These η_i modes are drift waves trapped in a complex local potential well created by large ion temperature gradient. Full kinetic treatment of the η_i modes using both differential and integral formulations in a sheared slab geometry has been performed to yield a stability threshold of η_i to be of order one [3,4]. In a toroidal geometry, the η_i modes have also been investigated in the limits of long perpendicular wavelengths, small ion magnetic drift frequency ω_{Di} and transit frequency $k_{\parallel} v_i$ to predict a critical $\eta_i = -1.0$ [11]. However, it has been pointed out in slab model calculations that ion transit resonances must be kept to determine the correct critical η_i [3,4]. In a toroidal geometry, the magnetic drift term appears in the resonant denominator together with the ion transit term. Because of the ordering $\omega_{Di} \gg k_{\parallel} v_i$, one would expect the magnetic drift term dominates the resonance behavior and determines the critical η_i .

The major drawback of these earlier works of drift waves in toroidal plasmas [6-11] is that the ion magnetic drift frequency, ω_{Di} , is assumed to be small compared with the mode frequency, ω , and is treated perturbatively without including resonances. This assumption is not always valid and it is the purpose of this paper to remove this limitation and to study the stability of both the electron and the ion drift waves. A systematic derivation of the eigenmode equation for studying these drift waves of arbitrary wavelengths in toroidal geometry is presented in Section II. We have made use of the high n ballooning mode formalism [12-15] to reduce the original two dimensional eigenmode equation into a one dimensional eigenmode equation along the field lines. Results from a shooting code are presented in Section III for the η_i modes. The ion magnetic drifts have a destabilizing effect on the η_i modes and the critical η_i is found to be smaller than that in a sheared slab geometry. In Section IV, we present the results for the toroidicity-induced (TI) modes. We find that the ion magnetic drift resonances can destabilize the TI modes for finite temperature ratio, $\tau = T_e/T_i$. A brief summary and general comments are included in Section V.

II. Formulation

Let us consider electrostatic drift waves in an axisymmetric large aspect ratio torus with concentric, circular magnetic surfaces. We use a (r, θ, ζ) coordinate system, where r is the minor radius, θ is the poloidal angle, and ζ is the toroidal angle. The equilibrium magnetic field is given by $\vec{B} = B_0 [1 - (r/R) \cos\theta] [\hat{\zeta} + (r/qR)\hat{\theta}]$. The lowest order (in ρ/L) equilibrium distribution function is taken to be a local Maxwellian distribution, $F_M(v^2, r) = N_0(r) [m/2\pi T(r)]^{3/2} \exp[-mv^2/2T(r)]$ with ρ being the particle Larmor radius and L

the macroscopic scale length. The electrostatic potential ϕ and the nonadiabatic part of the perturbed distribution function H can be written in the form

$$(\phi, H) = [\phi(r, \theta), H(r, \theta)] \exp[i(m\theta - n\tau - \omega t)]. \quad (1)$$

The θ variations of $\phi(r, \theta)$ are much slower than that of $\exp(im\theta)$. The ion gyrokinetic equation is given by

$$\begin{aligned} & [\omega - (v_{\perp}/qR) (s - i \frac{\partial}{\partial \theta}) + \omega_{Di}] H(s, \theta) \\ & = (eF_M/T)_i \{ \omega - \omega_{*i} [1 + \eta_i (\hat{v}^2 - 3/2)] \} J_0 \left(\frac{k_{\perp} v_{\perp}}{\Omega} \right) \phi(s, \theta) \end{aligned} \quad (2)$$

where $s = (r - r_0)/\Delta r_s$, r_0 is the minor radius of the reference mode rational surface with $m = nq(r_0)$, $\Delta r_s = 1/k_{\theta} \hat{s}$, $k_{\theta} = m/r_0$, $\hat{s} = (rq'/q)$ at $r = r_0$,

$\hat{k}_{\perp} = -i (\partial/\partial r \partial/\partial \theta + \hat{r} \partial/\partial r)$, $\eta_i = (d \ln T_i / d \ln N_i)$, $\omega_{Di} = 2 (r_n/r) \omega_{*e}$, $(\hat{v}_{\parallel}^2 + \hat{v}_{\perp}^2/2) (\cos \theta - i s \sin \theta \frac{\partial}{\partial s})$, $\tau = T_e/T_i$, $r_n = r_n/R$, r_n is the density scale length, R is the major radius, q is the safety factor, $\omega_{*e,i}$ is the electron (ion) diamagnetic frequency, $\hat{v} = v/v_i$, and $v_{\parallel}$ and $v_{\perp}$ are the parallel and perpendicular velocities respectively. Note that in Eq. (2) we have assumed that $k_{\parallel} v_{\parallel} \ll \omega$ so that the ion parallel velocities can be considered to be independent of θ along the field line. The ion density perturbation is given by $\delta \tilde{n}_i = -e\phi/T_i + \int H J_0 d^3 v$. For the electron density perturbation we will retain only the adiabatic electron response and ignore any electron dissipations, temperature-gradient and trapped particle effects.

Low frequency microinstabilities such as drift waves and ballooning modes in toroidal plasmas have very short wavelengths perpendicular to the sheared magnetic field $\vec{B}$ compared with the parallel wavelengths, $k_{\parallel}/k_{\perp} \ll 1$. However, in solving Eq. (2), a formal WKB type expansion in lowest order in $k_{\parallel}/k_{\perp}$ would

lead to an aperiodic lowest order equation in θ , since the field lines do not in general close on themselves. In order to resolve the problem of satisfying the periodicity constraint in θ , the ballooning representation [12] is introduced by representing ϕ and h as an infinite sum of aperiodic functions which add up to give a periodic function, i.e.,

$$\begin{matrix} \phi \\ h \end{matrix} (s, \theta) = \sum_{p=-\infty}^{\infty} \begin{matrix} \hat{\phi} \\ \hat{h} \end{matrix} (s, \theta - 2\pi p) \quad (3)$$

where $\hat{\phi}$ and $\hat{h}$ extend from $-\infty$ to ∞ in θ . To ensure convergence of this sum, $\hat{\phi}$ and $\hat{h}$ must vanish sufficiently fast as $\theta \rightarrow \pm \infty$. Since the coefficients in Eq. (2) are periodic in θ , Eq. (2) can be satisfied by $\hat{\phi}$ and $\hat{h}$ over an infinite range in θ with no periodicity constraint [14]; i.e.,

$$\begin{aligned} & [\omega - (v_y/qR) (s - i a/2) + \omega_{Di}] \hat{h}(s, \theta) \\ &= (eF_M/T)_i \{ \omega - \omega_{*i} [1 + \eta_i (v^2 - 3/2)] \} \int_0^{\hat{\phi}} \hat{\phi}(s, \theta). \end{aligned} \quad (4)$$

Now we can use $\varepsilon \equiv 1/n \rightarrow 0$ ($k_{\parallel}/k_{\perp}$) as an expansion parameter to develop an asymptotic solution of Eq. (4). $\hat{h}$ and $\hat{\phi}$ can be expressed by the eikonal representation

$$\begin{matrix} \hat{h} \\ \hat{\phi} \end{matrix} = \begin{matrix} h \\ \phi \end{matrix} (s, \vartheta, \varepsilon) \exp[iS(\theta, s)/\varepsilon]. \quad (5)$$

In Eq. (5), S describes the rapid cross field variations and $\vec{B} \cdot \nabla S = 0$. In the axisymmetric case S can be expressed as

$$S(\theta, s) = (m/n - q) \theta + \int k_q(s) d\vartheta$$

$$= -\Delta r_s [4's\theta - \int k_q q' ds], \quad (6)$$

where $k_q(s)/\epsilon$ is the radial wavenumber in the (θ, q) space. We can expand h , ϕ and ω in powers of ϵ : $h = h^{(0)} + \epsilon h^{(1)} + \dots$, $\phi = \phi^{(0)} + \epsilon \phi^{(1)} + \dots$, and $\omega = \omega^{(0)} + \epsilon \omega^{(1)} + \dots$. The zeroth order of Eq. (4) is then given by

$$\begin{aligned} \omega^{(0)} &= i \left[\omega_{ti} \hat{v}_\parallel^2 / 2 + \omega_{Di} \right] h^{(0)} \\ &= (eF_M/T)_i \omega_{ti}^{(0)} \sim \omega_{ti} [1 + \eta_i (\hat{v}_\parallel^2 - 3/2)]^{1/2} J_0(b_i^{1/2}) \phi^{(0)} \equiv G_i \end{aligned} \quad (7)$$

where $\omega_{ti} = v_i/qR$, $\omega_{Di} = 2\omega_{*e} (\epsilon_n/\tau) (\hat{v}_\parallel^2 + \hat{v}_\perp^2/2) [\cos\theta + \hat{s}(\theta - k_q)\sin\theta]$, and $b_i = (k_q \hat{v}_\perp)^2 (1 + \hat{s}^2 \hat{v}_\perp^2)$. The fact that k_q occurs in Eq. (7) only in the combination of $\theta - k_q$ implies that $\omega^{(0)}$ is a periodic function of k_q . We note that k_q plays a similar role to the parameter η_0 occurring in Ref. [14]. We learn from numerical experience that for up-down symmetric equilibrium the maxima of growth rates occur at $k_q = 2\pi N$ ($N = 0, \pm 1, \dots$). In this work we will concentrate on the lowest order equation, Eq. (7), and choose $k_q = 0$ to obtain the maximum growth rate. We also note that this choice of k_q yields perturbations centered at the outside of the torus. In the following we will drop the superscripts of $\omega^{(0)}$, $h^{(0)}$ and $\phi^{(0)}$ in Eq. (7).

In the limit $\omega_{ti} \ll (\omega + \omega_{Di})$, Eq. (7) can be solved to obtain the nonadiabatic ion distribution function

$$\begin{aligned} h &= \frac{G_i}{(\omega + \omega_{Di})} + \frac{i\omega_{ti} \hat{v}_\parallel \partial G_i / \partial \theta}{(\omega + \omega_{Di})^2} + \frac{(i\omega_{ti} \hat{v}_\parallel)^2 \partial^2 G_i / \partial \theta^2}{(\omega + \omega_{Di})^3} \\ &+ O\left(\frac{\omega_{ti}}{\omega}\right)^4. \end{aligned} \quad (8)$$

Notice that the ion magnetic drift term now is retained in the resonant denominator instead of in an expansion. Significant departures from the previous perturbation results (in ω_{pi}/ω) [6,7] are expected when the resonance does occur. We must point out that we have modified the resonant poles by expanding Eq. (7) in terms of $\omega_{ti}/(\omega + \omega_{pi})$. This can be justified if $|\omega_{ti} \frac{\hat{v}_r}{r} \frac{1}{\partial s} h| \ll |(s + \omega_{pi}) h|$. In order to carry out the velocity integration of h , we will make the approximation with

$$\omega_{pi} = 2 \omega_{te} (r_{\perp}/r) (2\hat{v}_{\parallel}^2) (\cos \theta + \hat{s} \theta \sin \theta). \quad (9)$$

Then, the ion nonadiabatic density response can be expressed by the plasma dispersion function. Otherwise, the double integral involving $v_{\parallel}$ and $v_{\perp}$ will have to be performed numerically [16]. This approximation deforms the resonance surface in velocity space to a plane without affecting the basic resonance effects.

From the quasi-neutrality condition, we obtain a second order differential equation

$$\left[\frac{\partial^2}{\partial s^2} + Q \right] \psi = 0 \quad (10)$$

where

$$Q = (1 + \tau - \hat{D}_0) / (\omega_{ti}^2 \hat{D}_3), \quad (11)$$

$$\hat{D}_0 = [\tau R + 1 - \eta_i/2 + \eta_i b(\Gamma_1 - \Gamma_0)/\Gamma_0] D_0 + \eta_i D_1,$$

$$\hat{D}_3 = [\tau R + 1 + 2 \eta_i + \eta_i b(\Gamma_1 - \Gamma_0)/\Gamma_0] D_3$$

$$= \eta_i \Gamma_0 (\tau/4 \epsilon_n f_\theta)^3 \epsilon_i^2 \frac{\partial G_3}{\partial \epsilon_i^2},$$

$$D_0 = \Gamma_0 (\tau/4 \epsilon_n f_\theta) G_0, \quad D_1 = \Gamma_0 (\tau/4 \epsilon_n f_\theta) G_1,$$

$$D_3 = \Gamma_0 (\tau/4 \epsilon_n f_\theta)^3 G_3,$$

$$G_0 = \frac{1}{\epsilon_i^2} \int_{-\infty}^{\infty} \frac{\exp(-y^2)}{(y^2 - \epsilon_i^2)} dy = Z(\epsilon_i)/\epsilon_i,$$

$$G_1 = \frac{1}{\epsilon_i^2} \int_{-\infty}^{\infty} \frac{\exp(-y^2)}{(y^2 - \epsilon_i^2)} y^2 dy = 1 + \epsilon_i Z(\epsilon_i),$$

$$G_3 = \frac{1}{\epsilon_i^2} \int_{-\infty}^{\infty} \frac{\exp(-y^2)}{(y^2 - \epsilon_i^2)^3} y^2 dy = \frac{1}{2} \frac{3}{\epsilon_i^2} (1 + \frac{3}{4 \epsilon_i^2} (G_1)),$$

$$\Gamma_{0,1} = \frac{\Gamma_0}{\epsilon_n}, \quad \Gamma_{0,1} = I_{0,1} \exp(-b), \quad b = (k_{n,1}^2/2) (1 + \hat{s}^2 \epsilon^2),$$

$$\epsilon^2 = -(\tau/4 \epsilon_n f_\theta), \quad f_\theta = \cos \theta + \hat{s} \sin \theta,$$

$\epsilon = \omega/\omega_{pe}$, Z is the plasma dispersion function, and I_0 and I_1 are the modified Bessel functions of order zero and one, respectively. Note that to obtain the functions G_0 , G_1 , G_3 we must perform proper analytic continuation of the integration contour as well as ϵ in the complex v_y space. Eq. (10) is the eigenmode equation governing both the toroidicity-induced (TI) and the ion temperature gradient driven modes of arbitrary wavelengths, and is now ready to be solved as an eigenvalue equation for ω under the boundary condition that ϵ decay asymptotically as $|\omega| \rightarrow \infty$. We will solve Eq. (10) with a numerical shooting method. Since Eq. (9) is derived on the assumption that $|\omega_{t1}/(\omega + \omega_{D1})| \ll 1$, its solutions can be justified only if $|P(A)| \ll 1$ is satisfied, where

$$P(\theta) = \frac{\omega_{ti}^2 D_3^2 \lambda^2 \phi / \lambda \theta^2}{D_0 \phi} = \frac{\omega_{ti}^2 D_3^2}{D_0} \quad (12)$$

In the limits of $\omega_{Di}/\omega \ll 1$ and $b \ll 1$, we can expand Γ_0 , Γ_1 , and $Z(\zeta)$ to obtain the eigenmode equation [6,7,10,11]

$$\frac{\omega^2}{\omega_{pe}^2} + \frac{\omega_{ti}^2}{\omega_{pe}^2} [b_{ei}(1 + s^2 \omega_{pe}^2) + \frac{\tau(\tau - 1)}{\tau + 1 + \epsilon_1}] + \frac{2\omega_{ti}}{\omega_{pe}} (\cos \theta + s^2 \sin \theta) = 0 \quad (13)$$

where $\omega_{pe}^2 = q^2 B_0^2 / \mu_0 n^2$ and $b_{ei} = (k q^2 B_0^2 / 2)$. Eq. (13) has been previously studied for the toroidicity induced modes with $\omega_{ti} = 0$ [6,7]. However, Eq. (13) is not appropriate for studying the ion temperature gradient driven modes because it is essential to retain magnetic drift resonances in determining the critical τ_i .

III. Ion Temperature Gradient Driven Modes

As described in the previous section, the eigenmode equation, Eq. (10), is obtained by assuming the electron density response to be adiabatic. The reason is that the nonadiabatic electron response is unimportant for the ion temperature gradient driven modes. This fact has been demonstrated in the study of these eigenmodes in a sheared slab geometry [3,4]. In the following, we will concentrate on the most unstable even eigenmodes. The odd eigenmodes are more stable and will not be discussed here.

Figure 1 shows (a) the eigenfunction $\phi(\theta)$, (b) potential $-Q(\theta)$, and (c) $P(\theta)$ defined in Eq. (11) of a typical case (with parameters: $\tau = 2.0$, $\epsilon_n =$

0.1, $b_0 = 1.0$, $q = 2.0$, $\hat{s} = 1.0$, $n_i = 3.0$). The eigenfrequency is $\Omega = -0.303 + 0.166i$. We note that the condition $|P(\theta)| \ll 1$ is satisfied except where the eigenfunction is asymptotically small and, therefore, our solution is a legitimate one. The eigenfunction ϕ is localized around the origin by the local complex well structure of $-Q$ which is created by the combined effects of ion temperature gradient and magnetic drifts. The dependence of the eigenfrequency, Ω , on n_i is displayed in Fig. (2) for the parameters: $\tau = 2.0$, $\epsilon_n = 0.1$, $b_0 = 1.0$, $q = 2.0$, and $\hat{s} = 1.0$. Numerical eigenfrequencies from solving Eq. (2) [curves (a)] are compared with the results from the initial value code [12] of solving Eq. (7). Curves (b) and (c) are results from initial value code with and without the ion magnetic drift term respectively by employing the approximate ω_{pi} defined by Eq. (9). Without the ion magnetic drift term, Eq. (7) is a gyrokinetic equation in a sheared slab geometry. From curves (a) and (b) we see that both the initial and eigenvalue codes give comparable results and the effect of neglecting ion transit resonance in the eigenvalue code is negligible. Comparison between curves (a) and (c) indicates that the critical n_i in toroidal geometry ($n_i \approx 1.3$) is smaller than that in slab geometry ($n_i \approx 1.6$). The toroidal effects due to ion magnetic drifts introduce a negative frequency shift and further destabilize the n_i modes. Finally, to see the effect of the approximate ω_{pi} , we have also included the results of the initial value code with exact ω_{pi} [curves (d)] for comparison. Curves (d) show a lower critical n_i (≈ 0.9) than curves (a) ($n_i \approx 1.3$). This means that we may have to employ exact ω_{pi} term to obtain correct critical n_i for most of the parameter regimes. However, the parameter dependence of the n_i modes due to the eigenvalue analysis presented here is still very good.

We have examined the evolution of the n_i modes as the parameters, ϵ_n

and $\hat{s}$, are varied. The fixed parameters are $q = 2.0$, $\tau = 2.0$, $b_\theta = 0.25$, and $n_i = 3.0$. In Fig. 3(a) (with $\hat{s} = 1.0$) it is seen that the maximum growth rate, Ω_i , occurs at $c_n = 0.12$, and that the real frequency, Ω_r , becomes more negative as c_n is increased. This is because c_n measures not only the strength of the magnetic drift but the strength of magnetic shear ($L_s/L_n \equiv q/c_n \hat{s}$) as well. Fig. 3(b) (with $c_n = 0.2$) shows that the stabilizing effect of $\hat{s}$ takes place only for $\hat{s} > 1/2$ and Ω_r becomes more negative as $\hat{s}$ increases. The dependence of the eigenfrequency, Ω , on the azimuthal wavenumber (b_θ) and the temperature ratio ($\tau = T_e/T_i$) has also been investigated and is shown in Fig. 4 with the fixed parameters: $q = 2.0$, $\hat{s} = 0.5$, $c_n = 0.2$, and $n_i = 3.0$. With $\tau = 1$, Fig. 4(a) shows that the maximum growth rate, Ω_i , occurs at $b_\theta = 0.1$, and the real frequency Ω_r decreases as b_θ increases and reaches the minimum at $b_\theta = 0.6$. It is important to point out that the solution is not valid for $b_\theta \lesssim 0.01$, because the underlying assumption for obtaining Eq. (10), $|P(\theta)| \ll 1$, is violated. Fig. 4(b) shows the τ dependence of Ω with $b_\theta = 0.25$. We see that the η_i mode is unstable only for $0.7 \lesssim \tau \lesssim 9.0$. Its growth rate peaks at $\tau \approx 2$ and its real frequency reaches the maximum at $\tau \approx 8$.

IV. Toroidicity-Induced Modes

The toroidicity-induced modes have previously been studied [6,7] by employing Eq. (13) derived in the limits of $b_0 \ll 1$ and $\omega_{Di} \ll \omega$. Due to nonuniform toroidal magnetic field, the ion magnetic drifts can cause coupling between the eigenmodes which centered on different mode rational surfaces. In θ space, the toroidal coupling effects can form local potential wells, which inhibit convection of wave energy and form marginally stable toroidicity-induced modes. However, for larger azimuthal wavenumber ($b_0 \sim 1$) or large magnetic drift frequency ($\omega_{Di} \sim \omega$), the eigenmode equation, Eq. (13), breaks down, and it becomes necessary to solve Eq. (10) in order to determine the behavior of these modes.

As noted in Section II, Eq. (10) is derived by neglecting the nonadiabatic electron response, so that we can concentrate on the ion dynamics. In addition, it has been shown that nonadiabatic electron response can be treated perturbatively for the toroidicity-induced modes [8-10]. Figure 5 shows (a) the eigenfunction $\phi(\theta)$, (b) potential $-Q(\theta)$, and (c) $P(\theta)$ of a typical case with parameters: $r = 1.0$, $\epsilon_n = 0.1$, $\hat{s} = 1.0$, $b_0 = 0.1$, $q = 1.0$, $\eta_i = 0$. The eigenfrequency is $\Omega = 0.36 + 0.009i$. We have chosen η_i so that we can compare with the previous results [6,7]. Note that the condition $|P(\theta)| \ll 1$ is satisfied and the eigenfunction ϕ is localized around $\theta = \pi/2$ by the off-center potential well of $-Q$ due to large shear ($\hat{s} > 1/2$). The mode is weakly unstable due to ion magnetic drift resonances, while in the previous perturbative treatment of ω_{Di} term, Eq. (13), it experiences weak shear damping through tunneling leakages. The ion temperature gradient (η_i) will reduce the real frequency but has negligible effects on the growth rate of the toroidicity-induced mode.

The dependence of the eigenfrequency, Ω , on the magnetic shear, $\hat{s}$, is plotted in Fig. 6 for the parameters: $\epsilon_n = 0.1$, $b_\theta = 0.1$, $q = 1.0$, and $n_i = 0$. Three values of τ [(a) $\tau = 1$, (b) $\tau = 10$, and (c) $\tau = \infty$] are used to compare the eigenfrequency. In the cold ion limit ($\tau = \infty$), Eq. (13) is solved to obtain the eigenfrequency, and Ω_i is not shown in Fig. 6 because it is negative and small ($\sim 10^{-4}$). We also note that the real frequency is reduced and the maximum growth rate enhanced as τ decreases. For $\hat{s} < 1/2$, the eigenfunction becomes localized around the origin, corresponding to center potential well as observed in the previous studies [6,7]. Figure 7 displays the dependence of the eigenfrequency, Ω , on the toroidicity, ϵ_n , for fixed parameters: $b_\theta = 0.1$, $\hat{s} = q = \tau = 1.0$, and $n_i = 0$. The real frequency, Ω_r , decays weakly with ϵ_n . But the growth rate Ω_i rises with ϵ_n . This indicates that the instability is due to the magnetic drift, which is measured by ϵ_n . Figure 8 shows the dependence of the eigenfrequency, Ω , on the azimuthal wave number, b_θ , with the fixed parameters: $\epsilon_n = 0.1$, $\hat{s} = q = \tau = 1.0$, and $n_i = 0$. Both the real frequency and the growth rate decrease as b_θ increases, but reach plateau for $b_\theta > 0.5$.

V. Conclusions

In this paper we have employed the high n ballooning mode formalism and derived the eigenmode equation for drift waves of arbitrary wavelengths in toroidal plasmas. We do not assume the ion magnetic drift frequency, ω_{Di} , to be smaller than the mode frequency, so that the ion magnetic drift resonances are retained. In order to express the ion density perturbation in terms of the plasma dispersion, we have approximated $\omega_{Di} = 2 \omega_{*e} (\epsilon_n / \tau) (2v_\parallel^2) (\cos\theta +$

$\hat{s}\theta \sin\theta$). We also have neglected the transit ion resonance contribution. However, this only modified the resonance poles in complex v_y space and will have negligible effects if $|\omega_{ti}/(\omega + \omega_{pi})| \ll 1$, as shown in Sections III and IV. Both of the ion temperature gradient driven modes and the toroidicity-induced modes are studied by a shooting code with the parametric variations on $\hat{s}$, ϵ_n , b_θ , τ , and η_i . For ion drift waves (η_i modes) the eigenfunction ϕ is localized around the origin by the local complex potential well, which is created by the combined effects of ion temperature gradient and magnetic drifts. Comparing with the eigenmode solution in slab calculations, the ion magnetic drifts produce a negative frequency shift, further destabilize the modes, and result in a smaller critical η_i . These results have also been confirmed independently by solving the unexpanded gyrokinetic equation, Eq. (7), as an initial value problem.

We can also obtain approximate solutions for the η_i modes by expanding the potential, Q , around $\theta = 0$ with $Q(\theta) \approx Q(0) + (Q''(0)/2)\theta^2$. The eigenmode equation, Eq. (9), then reduces to

$$\frac{\partial^2}{\partial y^2} \phi + [Q(0)/(-Q''(0)/2)^{1/2} - y^2] \phi = 0, \quad (13)$$

with $\phi(y) = \exp(-y^2/2)$ as the lowest eigenstate. The corresponding dispersion relation is given by

$$Q(0)/(-Q''(0)/2)^{1/2} = 1. \quad (14)$$

In general, Eq. (14) is a complicated nonlinear equation and can only be analyzed numerically. But in the limit of $\hat{s}$, $|Q''(0)| \ll 1$, Eq. (14) can be

approximated by $Q(0) \approx 0$. We note that $Q(0)$ is independent of $\hat{s}$ and hence the eigenfrequency, Ω , is only weakly dependant on $\hat{s}$ for $\hat{s} < 1/2$ as shown in Fig. 3(b).

For electron drift waves (toroidicity-induced modes) our main conclusion is that the ion magnetic drifts can destabilize the TI eigenmodes and have a significant effect when the temperature ratio, τ , is of order unity. The eigenfrequency, Ω , decreases as b_θ increases and reaches a plateau at large azimuthal wavenumber, $b_\theta > 0.5$.

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Figure Captions

Fig. 1 Plot of (a) eigenfunction $\phi(\theta)$, (b) potential $-Q(\theta)$, and (c) $P(\theta)$ of the ion temperature gradient (η_i) driven modes for the parameters: $\tau = 2.0$, $\epsilon_n = 0.1$, $b_0 = 1.0$, $q = 2.0$, $\hat{s} = 1.0$, and $r_i = 3.0$. The eigenvalue is $\Omega = -0.31 + 0.17i$.

Fig. 2 Plot of real and imaginary parts of the η_i mode eigenfrequency Ω versus η_i with the parameters: $\tau = 2.0$, $\epsilon_n = 0.1$, $b_0 = 1.0$, $q = 2.0$, and $\hat{s} = 1.0$. Numerical solutions of eigenvalue equation (10) [curves (a)] are compared with the results from the initial value code of solving Eq. (7) with [curves (b)] and without [curves (c)] the approximate ion magnetic drift term, Eq. (9). Curves (d) show results of initial value code with the correct ion magnetic drift term.

Fig. 3 The dependence of the η_i mode eigenfrequency, Ω , on (a) ϵ_n for $\hat{s} = 1.0$ and (b) $\hat{s}$ for $\epsilon_n = 0.2$. The fixed parameters are $q = 2.0$, $\tau = 2.0$, $b_0 = 0.25$, and $r_i = 3.0$.

Fig. 4 The dependence of the η_i mode eigenfrequency, Ω , on (a) b_0 for $\tau = 1.0$ and (b) τ for $b_0 = 0.25$. The fixed parameters are $q = 2.0$, $\hat{s} = 0.5$, $\epsilon_n = 0.2$, and $r_i = 3.0$.

Fig. 5 Plot of (a) eigenfunction $\phi(\theta)$, (b) potential $-Q(\theta)$, and (c) $P(\theta)$ of the toroidicity-induced (TI) modes for the parameters: $\epsilon_n = b_0 = 0.1$, $\tau = \hat{s} = q = 1.0$, and $\eta_i = 0$. The eigenfrequency is $\Omega = -0.36 + 0.009i$.

Fig. 6 The dependence of the TI mode eigenfrequency, Ω , on $\hat{s}$ for three different values of τ ; (a) $\tau = 1.0$, (b) $\tau = 10.0$, and (c) $\tau = \infty$. The fixed parameters are $c_n = b_\theta = 0.1$, $q = 1.0$, and $\eta_i = 0$.

Fig. 7 The dependence of the TI mode eigenfrequency, Ω , on c_n for the fixed parameters: $b_\theta = 0.1$, $\hat{s} = q = \tau = 1.0$ and $\eta_i = 0$.

Fig. 8 The dependence of the TI mode eigenfrequency, Ω , on b_θ for the fixed parameters: $c_n = 0.1$, $\hat{s} = q = \tau = 1.0$, and $\eta_i = 0$.

Fig. 1.

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Fig. 2.

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Fig. 3.

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Fig. 4.

Fig. 5.

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Fig. 6.

Fig. 7.

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Fig. 8.