1735

179
12-22-80
JWH

MASTER

(1)

DECEMBER 1980

Dr. 2152

PPPL-1735

UC-20G

P764

INTEGRAL-EQUATION FORMULATION FOR
DRIFT EIGENMODES IN CYLINDRICALLY
SYMMETRIC SYSTEMS

BY

R. LINSKER

**PLASMA PHYSICS
LABORATORY**

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED.

**PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY**

This work was supported by the U.S. Department of Energy
Contract No. DE-AC02-76-CH0 3073. Reproduction, transla-
tion, publication, use and disposal, in whole or in part,
by or for the United States government is permitted.

Integral-Equation Formulation for Drift Eigenmodes in Cylindrically Symmetric Systems

Ralph Linsker

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08544

DISCLAIMER

A method for solving the integral eigenmode equation for drift waves in cylindrical (or slab) geometry is presented. A leading-order kinematic effect that has been noted in the past, but incorrectly ignored in recent integral-equation calculations, is incorporated. The present method also allows electrons to be treated with a physical mass ratio (unlike earlier work that is restricted to artificially small m_i/m_e owing to resolution limitations). Results for the universal mode and for the ion-temperature-gradient driven mode are presented. The kinematic effect qualitatively changes the spectrum of the ion mode, and a new "second region of instability" for $k_{\perp} \rho_i > 1$ is found.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

I. INTRODUCTION

This paper discusses some features of drift instabilities that emerge from a new integral-equation formulation. The work was originally motivated by application to cylindrical reversed-field-pinch systems, in which shear may be strong (L_s/L_n of order unity). The mode width for both electron and ion drift modes is determined by the ion Landau damping distance, which can be as small as a few ion gyroradii when $(\omega/\omega_i^*) (L_s/L_n)$ is of order unity. Modes this narrow occur (for $T_e \sim T_i$) for the electron drift mode when $L_s \sim L_n$, and for the ion drift mode (where typically ω/ω_i^* is a few percent) even if shear is weak (e.g., $L_s/L_n \sim 20-40$, typical tokamak values).

The stability of drift waves in sheared-slab (and, by extension, cylindrical) geometry has been studied extensively¹ using a second-order differential eigenmode equation, derived by assuming the radial wavenumber to be small ($|\rho_i^2 \partial_x^2| \ll 1$). When the mode width is a few ion gyroradii, this condition is not fulfilled, and an integral-equation formulation is mandatory.

In the present work, the perturbed distribution function is calculated by an explicit integration over unperturbed orbits, and quasineutrality is invoked to obtain an integral equation for the perturbed potential $\phi(x)$, where x is the radial distance from the mode-rational surface. This equation is solved by a Ritz method, in which $\phi(x)$ is approximated by a truncated Fourier expansion, and the resulting matrix equation is solved for the eigenmode frequency. (Periodic boundary conditions, required for the Fourier expansion, are imposed in a manner to be discussed in the next section.)

An integral formulation has been applied to the sheared-slab study of ion drift modes with adiabatic electron response, and the universal mode with

artificially small m_i/m_e , by Tang, Rewoldt, and Frieman.² The present work differs in several respects. First, Tang *et al.*² calculate the perturbed ion distribution function by the usual gyrokinetic method.³ This method is incorrect, because it does not consistently retain all terms of leading order in gyroradius/equilibrium "scale length. It omits a shear-related term of kinematic origin, discussed earlier by Bakshi *et al.*⁴ and implicit in the work of others^{5,6} (see also Linsker^{7,8}). The integral equation to be solved is therefore different from that considered in Ref. 2.

This kinematic effect arises from the fact that an explicit integration over the unperturbed orbit in cylindrical or slab geometry (with the ordering given in Section II) yields the resonant denominator $\omega - k_{\parallel}(x_{gc})v_{\parallel} - k_{\perp}v_d$, where v_d is the usual magnetic drift and x_{gc} is the guiding-center radial position. In the usual treatment of the problem one incorrectly evaluates $k_{\parallel}$ at x , the position at which the perturbed distribution function is to be calculated. The additional term in the resonant denominator is therefore $(-k_{\perp}v_{\perp}v_{\parallel} \cos \alpha)/(\Omega_c L_s)$, where α = gyrophase and L_s = shear length. Note that for $|x| \lesssim$ gyroradius, this correction term is of the same (leading) order as $k_{\parallel}(x)v_{\parallel}$ itself.

In Ref. 2, the mode $\phi(x)$ is expanded in Hermite polynomials (damped by a Gaussian), rather than the Fourier harmonics used here. Tang *et al.*² note that their method is well suited to a problem in which only one radial length scale appears (e.g., the ion Landau damping distance, x_i , if the electron response is taken to be adiabatic), but is not applicable to a situation in which two widely-separated length scales must be resolved. This difficulty arises when the electron dynamics must be included; the scales are then x_i and x_e , with $x_e/x_i \sim (m_e/m_i)^{1/2}$ (for $T_e \sim T_i$). Tang *et al.*² report that the smallest ratio of length scales for which their method is valid

is $x_e/x_i \sim N^{-1/2}$ for N basis functions (they use $N \sim 120$). In contrast, N Fourier basis functions provide spatial resolution that goes as N^{-1} . The present method, unlike that of Ref. 2, therefore can be used to study modes with full electron dynamics and physical mass ratios. The inclusion of full electron dynamics is known to be crucial for the case of the universal mode¹; we find also that both the electron dynamics and the kinematic term⁴⁻⁸ discussed above have an important effect on the ion drift mode.

Details of the present method are given in Section II for the case of a collisionless $\beta = 0$ plasma with cylindrical symmetry. Effects due to the density gradient, ion and electron temperature gradients, and electron current (modelled by a shifted Maxwellian) are incorporated. (The inclusion of magnetic drift effects is detailed in Ref. 8, to be denoted as Paper A. These effects are not considered here, for the sake of simplicity. In other words, the cylindrical case treated here is fully equivalent to a slab model.) Section III presents numerical results for the cases of the universal mode and the (ion temperature gradient driven) ion drift mode, including a comparison of the usual differential approximation, the integral equation considered by Tang et al.,² and the integral equation treated here. In particular, the correct ion drift mode spectrum is found in certain cases to have a new unstable region for $k_{\perp} \rho_i > 1$. Conclusions are given in Section IV.

II. INTEGRAL EQUATION

Consider a cylindrically symmetric collisionless plasma with $\beta = 0$, zero equilibrium electric field, and equilibrium magnetic field $B(r) = [0, B_\theta(r), B_z(r)]$ with pitch $P(r) = rB_z/B_\theta$. The shear length $L_s = - (r^2 + P^2)/(rP')$ corresponds to the usual shear length in the slab model. Choose the unperturbed distribution function to be

$$f_{0j} = n_{0j} \left(\frac{m_j}{2\pi T_j} \right)^{3/2} \exp \left[\frac{m_j}{2T_j} (-v^2 + 2u_{j\parallel} v_{\parallel} - u_{j\parallel}^2) \right] \quad (1)$$

($j = e, i$) with $u_i = 0$, where n_{0j} , T_j , and the parallel electron drift speed u_e are functions of guiding-center position. The potential perturbation is chosen to be of the form

$$\phi(\vec{r}, t) = \phi(x) \exp[i(k_\theta \theta + k_z z - \omega t)] \quad (2)$$

where x is the radial distance from the mode-rational surface in units of $\frac{1}{2} \rho_i = \Omega^{-1} (\gamma T_i / m_i)^{1/2}$. The ordering is $\epsilon = \rho_i / r$, $\rho_i /$ (all equilibrium gradient scale lengths) $\sim 0(\epsilon)$, $\omega / \Omega \sim 0(\epsilon)$, $\rho_i \sim 0(1)$, and $k_\parallel \rho_i \sim 0(\epsilon)$.

The derivation of the perturbed electron density n_{1e} is straightforward (we neglect nonzero electron gyroradius effects). The derivation of n_{1i} , including ion magnetic drifts and the kinematic term neglected in the usual gyrokinetic treatment, is given in Paper A. Here only the results will be given, and ion magnetic drift effects will be omitted for simplicity.

Define n_j^* by

$$n_{1j} = - \frac{e_j n_{0j}}{T_j} \left[\phi(x) + \left(1 - \frac{\omega_{*j}}{\omega} \right) n_j^*(x) \right] \quad (3)$$

where $\omega_{*j} = k_{\perp} T_j / (e_j B L_n)$. Quasineutrality implies

$$0 = (\tau + 1)\phi(x) + (1 - \omega_{*e}/\omega)n_e^-(x) + (\tau + \omega_{*e}/\omega)n_i^-(x), \quad (4)$$

where $\tau = T_e/T_i$. I now choose to solve not Eq. (4), but the equation [denoted (4)'] obtained by replacing n_j^- by $n_j'^-$, where $n_j'^-(x) = n_j^-(x)$ for $-x_0 < x < x_0$, and $n_j'^-$ is defined to be periodic with period $2x_0$ outside this domain. This is a valid strategy provided the solution $\phi(x)$ (for $|x| < x_0$) is in fact insensitive to x_0 . In practice, x_0 must be large enough that the correct $\phi(x)$ is negligible for $|x| > x_0$.

The solution of the periodic equation (4)' is written as

$$\phi(x) = \sum_{\ell=-\infty}^{\infty} \hat{\phi}_{\ell} \exp(ik_{\ell}x), \quad k_{\ell} = \ell\pi/x_0. \quad (5)$$

We have

$$n_j^-(x) = \sum_{m=-\infty}^{\infty} \hat{n}_j^-(m) \exp(ik_m x),$$

$$\hat{n}_j^-(m) = (2x_0)^{-1} \int_{-x_0}^{x_0} dx \exp(-ik_m x) n_j^-(x); \quad j = e, i. \quad (6)$$

Define $\alpha_{m\ell}^{(j)}$ ($j = e, i$) by

$$\hat{n}_j^-(m) = \sum_{\ell=-\infty}^{\infty} \alpha_{m\ell}^{(j)} \hat{\phi}_{\ell}. \quad (7)$$

Then operating on Eq. (4)' with

$$(2x_0)^{-1} \int_{-x_0}^{x_0} dx \exp(-ik_m x) \quad (8)$$

yields

$$0 = \sum_{l=-\infty}^{\infty} L_{ml} \hat{\phi}_l, \quad (9)$$

$$L_{ml} = (\tau + 1) \delta_{ml} + (1 - \omega_{*e}/\omega) \alpha_{ml}^{(e)} + (\tau + \omega_{*e}/\omega) \alpha_{ml}^{(i)}.$$

For the electrons,

$$(1 - \omega_{*e}/\omega) \alpha_{ml}^{(e)} = (2x_0)^{-1} \int_{-x_0}^{x_0} dx \exp[i(k_l - k_m)x] \left\{ \left[1 - \frac{\omega_{*e}}{\omega} \left(1 - \frac{\eta_e}{2} \right) \right] \zeta Z(\zeta) - \frac{\omega_{*e}}{\omega} \zeta [1 + \zeta Z(\zeta)] [\eta_e \zeta + 2\kappa \operatorname{sgn} k_{\parallel}] \right\} \quad (10)$$

where

$$\eta_j = d \ln T_j / d \ln n_0 \quad (j = e, i), \quad \omega' = \omega - k_{\parallel} u_e,$$

$$\zeta = \frac{x_e}{|x|} - \left(\frac{m_e}{2T_e} \right)^{1/2} u_e \operatorname{sgn} k_{\parallel} = \frac{\omega'}{|k_{\parallel}|} \left(\frac{m_e}{2T_e} \right)^{1/2},$$

$$x_e = (1/2) \left(\frac{m_e}{m_i} \right)^{1/2} \frac{\omega}{|\omega_{*e}|} \left| \frac{L_s}{L_n} \right|,$$

$$L_n^{-1} = d \ln n_0 / dx,$$

and

$$\kappa = \left(\frac{m_e}{2T_e} \right)^{1/2} L_n du_e / dr. \quad (11)$$

For the ions,

$$\alpha_{m\ell}^{(i)} = (2x_0)^{-1} \Gamma_0(b) \int_{-x_0}^{x_0} dx \exp[i(k_\ell - k_m)x] \int_{-\infty}^{\infty} dx' E(k_\ell, x', x), \quad (12)$$

where

$$E = \zeta^{-1} Z(\zeta^{-1}) F_1(k_\ell, x' - x) + \zeta^{-2} [1 + \zeta^{-1} Z(\zeta^{-1})] F_2(k_\ell, x' - x), \quad (13)$$

$$F_{1,2} = (2\pi)^{-1} \int_{-\infty}^{\infty} dk' \exp(-b_a) I_0(b_g) [\Gamma_0(b)]^{-1} \exp[i(k_\ell - k')(x' - x)] G_{1,2}(k_\ell, k'), \quad (14)$$

$$G_1(k_\ell, k') = 1 - \frac{\omega_{*e} \eta_i}{(\tau\omega + \omega_{*e})} [1/2 + b_a - b_g \frac{I_1(b_g)}{I_0(b_g)}],$$

$$G_2 = \frac{\omega_{*e} \eta_i}{(\tau\omega + \omega_{*e})}. \quad (15)$$

[F_n and G_n are the same functions discussed in Paper A.]

Here $\Gamma_0(b) = \exp(-b) I_0(b)$, $b = (1/2)(k_\perp^2 + k_\ell^2)$, I_0 is the Bessel function, $b_a = (1/2)(b + b')$, $b_g = (b b')^{1/2}$, and $b' = (1/2)(k_\perp^2 + k'^2)$. Here $k_\perp$ is the perpendicular wavenumber in units of $2^{-1/2} \rho_i^{-1}$. Also $\zeta^{-1} = x_1/|x'|$, with

$$x_1 = \frac{\tau}{2} \frac{\omega}{|\omega_{*e}|} \left| \frac{L_s}{L_n} \right|. \quad (16)$$

A substantial simplification is now realized by replacing $\alpha_{m\ell}^{(i)}$ by

$$\alpha_{m\ell}^{(i)'} = (2x_0)^{-1} \exp(-b_a) I_0(b_g) \int_{-x_0}^{x_0} dx' \exp[i(k_\ell - k_m)x']$$

$$\{G_1(k_\ell, k_m) \zeta^{-2} Z(\zeta') + G_2 \zeta^{-2} [1 + \zeta^{-2} Z(\zeta')]\} . \quad (17)$$

This replacement results from deliberately introducing a nonphysical edge effect: The perturbed density $n_{1i}(x)$ will be computed as the sum of contributions not from all orbits (which would give $\alpha_{m\ell}^{(i)}$), but only from those orbits with guiding center lying within the box $[-x_0, x_0]$. This replacement does not affect the resulting mode shape or frequency as long as x_0 is large enough and $n_1 \rightarrow 0$ at the boundaries. (This was checked explicitly for the universal mode by solving the matrix problem using $\alpha_{m\ell}^{(i)}$ as well as $\alpha_{m\ell}^{(i)'}.$)

For comparison, if one were to neglect the kinematic effect, as is done in the usual gyrokinetic treatment,^{2,3} the following modifications would result. In Eq. (17) replace $\exp(-b_a) I_0(b_g)$ by $\Gamma_0(b)$, and the bracket $[1/2 + b_a - b_g I_1(b_g)/I_0(b_g)]$ in G_1 [Eq. (15)] by $[1/2 + b - b I_1(b)/I_0(b)]$. With these changes, Eq. (17) would embody the ion physics treated in Tang et al.²

If one not only neglects the kinematic effect, but also makes the long-radial-wavelength approximation (used to derive the usual second-order differential eigenmode equation) one must expand $\Gamma_0(b)$ and $\Gamma_1(b) = \exp(-b) I_1(b)$ through second order in k_ℓ . [See e.g., Ref. 2 following Eq. (29).] Using the integral equation apparatus to solve the differential eigenmode equation serves as a partial check of the present method.

Returning to the general case, the sum in Eq. (9) is truncated and written as

$$\sum_{l=-l_0}^{l_0} L_{ml} \hat{\phi}_l = \lambda(\omega) \hat{\phi}_m, \text{ for all } |m| \leq l_0. \quad (18)$$

The Fourier integrals required to evaluate $\alpha_{ml}^{(i)}$ and $\alpha_{ml}^{(e)}$ are performed using a fast-Fourier transform code capable of resolving the detailed electron dynamics. An ω is sought such that $\lambda(\omega) = 0$ for an eigenmode $\{\hat{\phi}_m\}$ that is physically meaningful [i.e., such that $\phi(x)$ is insensitive to the choice of period $2x_0$ and (for the drift modes of present interest) becomes negligible near the boundary $|x| = x_0$].

When $\phi(x)$ is of definite parity P_ϕ (as is the case when the electron current u_e and its radial derivative both vanish) it is more efficient to replace Eq. (18) by

$$\sum_{l=0}^{l_0} [L_{ml} + (1 - \delta_{l0}) P_\phi L_{m, -l}] = \lambda(\omega) \hat{\phi}_m, \quad 0 \leq m \leq l_0. \quad (19)$$

The formulation presented here for the electrostatic case can be extended without any fundamental changes to the $\beta \neq 0$ case.⁷ Quasineutrality and the parallel and perpendicular components of Ampere's law yield three integral equations, leading to an equation of the form of Eq. (18) in which $\{\hat{\phi}_l\}$ then denotes the Fourier coefficients of the scalar potential ϕ and two independent components $A_\parallel$ and $A_\perp$ of the vector potential.

III. RESULTS

A. Electron drift mode.

As noted above, the present method (unlike that of Ref. 2) is capable of resolving electron Z-function structure for physical mass ratios (e.g.,

$m_i/m_e = 1836$). The universal mode (without temperature gradients or electron current) is well known to be absolutely stable for arbitrarily weak shear, at least when the kinematic term⁴⁻⁸ is neglected. This neglect is legitimate for the electron mode when $L_s \gg L_n$; therefore it is not surprising that the present code (including the kinematic term) finds this mode to be stable for all L_s/L_n studied. For strong shear (e.g., $L_s/L_n \sim 6$) the eigenmode shape is substantially altered within $\sim \rho_i$ of the mode-rational surface (recall that the mode itself is only a few ion gyroradii in radial extent for this case). The mode is strongly stabilized for $L_s/L_n = 6$ (for $T_e = T_i$, $k_\perp = 1$, we find $\omega/|\omega_{*e}| = 0.354 - 0.147i$). The eigenfrequency is quite insensitive to the presence of the kinematic term even for strong shear. (In the absence of the kinematic term, the above parameters yield $\omega/|\omega_{*e}| = 0.350 - 0.164i$, i.e., only a 10% shift in the damping rate.)

B. Ion drift mode.

For waves travelling in the ion diamagnetic direction, with the ion temperature gradient $\eta_i \neq 0$, the present method reproduces results of Tang *et al.*² for adiabatic electrons when the kinematic term is omitted. Including the kinematic term substantially alters the eigenmode shape for $|x| < \rho_i$, and also changes the $\omega(k_\perp)$ spectrum qualitatively and quantitatively.

Consider the weakly sheared case $L_s/L_n = 40$, with $T_e = T_i$ and $\eta_i = 2$. (Treat the electrons adiabatically; $\alpha_{m\ell}^{(e)} = 0$.) Figures 1 and 2 show the growth rate $\omega_i \tau_s$ and frequency ($-\omega_r \tau_s$) for two ion drift modes. [$\tau_s = (T_e/2T_i)^{1/2} |k_\perp/\omega_{*e}|$.] Mode I passes continuously to the dashed curve when the kinematic term is "turned off."⁹ Mode II is a new set of unstable solutions of the dispersion relation, important when $k_\perp > 1$. (The growth rate

decreases slowly for $k_{\perp} > 5$. Also note that only even ϕ solutions are presented here.)

Figures 3(a) and 3(b) show modes I and II for $k_{\perp} = 1$ (including the correct kinematic term). Figure 3(c) shows the eigenmode (corresponding to mode I) when the kinematic term is omitted. In both cases we normalize to $\phi(x=0) = 1$. The eigenmode is symmetric in x for all cases presented here. Note that the mode acquires additional structure near $x = 0$ when the kinematic effect is (incorrectly) neglected.

As might be expected, when $k_{\perp} = 1$ the eigenmode develops additional nodes as one passes from mode I to the less unstable mode II. The well-studied solutions of the differential drift eigenmode equation also have the property that as one increases the number of nodes, the modes become increasingly stable. In the present case, as $k_{\perp}$ increases, mode I acquires additional structure (and becomes less unstable), while mode II becomes simpler in structure (and more unstable). For example, at $k_{\perp} = 2.8$, mode II has the qualitative shape shown in Fig. 3(a), and mode I has that shown in Fig. 3(b). The region of increased instability due to mode II has no counterpart when the kinematic term is omitted. In that case, the eigenmodes generally develop increasing radial structure and rapidly become stable as $k_{\perp}$ is increased beyond ~ 1 .

Figures 4 and 5 show growth rate and real frequency versus $k_{\perp}$ for $L_S/L_n = 20$, $T_e = T_i$, and $n_i = 2$. Here only the mode labelled 'I' is still unstable. The solution (most unstable mode) of the integral equation with the kinematic term omitted is again shown as the dashed curve.

For $L_S/L_n = 6$, $T_e = T_i$, and $n_i = 2$, Figures 6 and 7 show $\omega_i \tau_s$ and $(-\omega_r \tau_s)$ versus $k_{\perp}$ for the most unstable modes, comparing the full integral-equation solution including the kinematic term (KT), the

solution obtained by neglecting the kinematic term (NKT), and the solution of the conventional second-order differential approximation (DE). The curves are qualitatively similar here, but the growth rates change by more than a factor of two in 20 - 50% in passing from NKT to KT.

The dependence of growth rate on the shear parameter is indicated in Fig. 8 for $T_e = T_i$, $k_{\perp} = 1$, $\eta_i = 2$. There is no significant qualitative difference between the growth rate of the "mode-I" solution of the full integral equation (KT) and that of the corresponding solution of the integral equation without the kinematic term (NKT).

The critical η_i for instability is found to be ~ 1 , in qualitative agreement with Tang *et al.*² and earlier (differential-code) results. This is true of the new branch II (see Fig. 1) as well as the usual branch of the dispersion relation. (For example, when $k_{\perp} = 2.8$, the mode-II solution in Fig. 1 becomes stable for $\eta_i \sim 0.95$.)

Introducing a physical mass ratio (e.g., $m_i/m_e = 1836$) rather than treating the electrons adiabatically can make a substantial difference in the mode frequencies and growth rates. Tang *et al.*² state that the electron Z-function structure is unimportant for the ion drift eigenmodes; their conclusion was based however, on a study of the differential approximation for the case $\eta_i = \eta_e$ only. For adiabatic electrons with $L_s/L_n = 20$, $T_e/T_i = 2$, $\eta_i = 2$, and $k_{\perp} = 1$, the differential approximation used by Tang *et al.*² yields $\omega/|\omega_{*e}| = -0.044 + 0.042i$; for $m_i/m_e = 1836$ with $\eta_e = \eta_i = 2$, $\omega/|\omega_{*e}| = -0.050 + 0.039i$. We find that for $m_i/m_e = 1836$ with $\eta_i = 2$, $\eta_e = 0$, the differential approximation yields $\omega/|\omega_{*e}| = -0.0688 + 0.028i$. Thus the insensitivity to the nonadiabatic electron contribution claimed by Tang *et al.*² appears to be a fortuitous result of their parameter choice.

Solving the full integral equation (with the kinematic effect) for the

case $L_s/L_n = 40$, $T_e = T_i$, $n_i = 2$, $k_\perp = 1$, we find: for adiabatic electrons, $\omega/|\omega_{*e}| = -0.035 + 0.043i$; for $m_i/m_e = 1836$ with $n_e = 2$, $\omega/|\omega_{*e}| = -0.040 + 0.039i$; $n_e = 1$, $\omega/|\omega_{*e}| = -0.056 + 0.033i$; and $n_e = 0$, $\omega/|\omega_{*e}| = -0.070 + 0.022i$. Thus the nonadiabatic electron contribution can effect a factor-of-two change in both the real frequency and the growth rate.

IV. CONCLUSION

A method has been developed to solve the dispersion relation for drift eigenmodes in cylindrical geometry using an integral-equation formulation. The method differs from earlier work in that (1) it is capable of treating the electron contribution for realistic mass ratios, and (2) it correctly treats a leading-order kinematic effect that has been ignored in Tang et al.² (and in much prior work).

For the universal mode no new instability is found. The kinematic effect modifies the eigenmode shape significantly for $|x| \lesssim \rho_{i\perp}$, but has only a small effect on real frequency or growth rate. For the ion drift mode (with an ion temperature gradient and adiabatic electrons) the frequency (both real and imaginary parts) is changed substantially by the presence of the kinematic effect. A qualitative change in the $\omega(k_\perp)$ spectrum is also found even for weak shear. This change corresponds to a second region of increasing instability, for $k_\perp \rho_{i\perp} > 1$. It is also found, contrary to the conclusion of Tang et al.², that the nonadiabatic electron structure for physical m_i/m_e can alter the real frequency and growth rate by, e.g., a factor of two (depending on n_e) from their values in the adiabatic case.

ACKNOWLEDGMENTS

The investigation of microinstabilities in the reversed field pinch configuration was suggested to me by Dr. F. W. Perkins. I am grateful to Drs. F. Perkins, A. Boozer, and G. Rewoldt for useful discussions.

This work was supported by the United States Department of Energy Contract No. DE-AC02-76-CH03073.

REFERENCES

- ¹W. Ross and S. M. Mahajan, Phys. Rev. Lett. 40, 324 (1978); K. T. Tsang, P. J. Catto, J. C. Whitson, and J. Smith, Phys. Rev. Lett. 40, 327 (1978); L. Chen, P. N. Guzdar, R. B. White, P. K. Kaw, and C. Oberman, Phys. Rev. Lett. 41, 649 (1978).
- ²W. M. Tang, G. Rewoldt, and E. A. Frieman, Phys. Fluids 23 (1980) (to be published).
- ³P. J. Catto, Plasma Physics 20, 719 (1978); and W. M. Tang, Nucl. Fusion 18, 1089 (1978)
- ⁴P. Bakshi, W. Bellew, G. Ganguli, and P. Satyanarayana, 1977 Sherwood Meeting; W. Bellew and P. Bakshi, Bull. Am. Phys. Soc. 22, 1089 (1977).
- ⁵W. E. Drummond and K. W. Gentle, Phys. Fluids 10, 2021 (1967).
- ⁶B. Coppi, M. N. Rosenbluth, and R. Z. Sagdeev, Phys. Fluids 10, 582 (1967); T. M. Antonsen and W. M. Manheimer, Phys. Fluids 21, 2295 (1978).
- ⁷R. Linsker, 1980 Sherwood Meeting paper 3B26 (Tucson, AZ; April 24-25, 1980); and Proceedings of the Reversed Field Pinch Theory Workshop (Los Alamos Scientific Laboratory, April 28-May 2, 1980), to be published.
- ⁸R. Linsker, "Incorrectness of the Usual Gyrokinetic Treatment in Cylindrically Symmetric Systems," Princeton University Plasma Physics Laboratory Report PPPL-1682 (1980).
- ⁹The kinematic term is "turned off" in a continuous manner to obtain this correspondence. In Eq. (9), we replace $\alpha^{(i)}$ by $\delta\alpha^{(i)}$ (with kinematic term) + $(1 - \delta)\alpha^{(i)}$ (without kinematic term) and let δ pass from one to zero.

Fig. 1. Growth rate $\omega_i \tau_s$ versus $k_\perp$ for the ion drift mode with $L_S/L_n = 40$, $T_e = T_i$, $n_i = 2$, and adiabatic electron response. Solid curves denote solutions of the full integral equation (including the kinematic term discussed in the text); the dashed curve is obtained by omitting the kinematic term. Here $\tau_s = (T_e/2T_i)^{1/2} |k_\perp/\omega_{*e}|$. (PPPL-802357)

Fig. 2. Same as Fig. 1, except that the real frequency ($-\omega_r \tau_s$) is plotted. (PPPL-802355)

Fig. 3. Ion drift eigenmodes for $k_{\perp} = 1$; other parameters as in Fig. 1. Modes I and II of Fig. 1 are plotted in (a) and (b) respectively. Omission of the kinematic term yields (c), corresponding to the 'dashed' mode of Fig. 1. [Solid and dashed curves denote real and imaginary parts of ϕ respectively.]

(PPPL-802358)

Fig. 4. Same as Fig. 1, except $L_S/L_n = 20$.

Fig. 5. Same as Fig. 4, except that the real frequency is plotted. (PPPL-802359)

Fig. 6. Growth rate $\omega_i \tau_s$ versus $k_\perp$ for the ion drift mode with $L_s/L_n = 6$, $T_e = T_i$, $\eta_i = 2$, and adiabatic electron response. The spectrum of the most unstable mode is plotted for the cases of the full integral equation (KT), the integral equation with no kinematic term (NKT), and the usual differential approximation (DE) (also omitting the kinematic term). (PPPL-802361)

Fig. 7. Same as Fig. 6, except that the real frequency is plotted.
(PPPL-802363)

Fig. 8. Growth rate versus $L_s L_n$ for $T_e = T_i$, $n_i = 2$, $k_i = 1$, and adiabatic electron response. KT denotes the most unstable mode including the correct kinematic term; NKT omits kinematic term.

(PPPL-802362)