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Feedback Stabilization of an $\ell = 0, 1, 2$ High-Beta Stellarator

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questions will remain unresolved until a definitive full-torus experiment is carried out.

In what follows, first the rationale of the high-beta stellarator is reviewed with a summary of the theory, including some recent progress. The experimental apparatus and diagnostics are described and the interpretation of the diagnostic measurements is discussed. This is done in some detail because certain plasma parameters, in particular beta, play crucial roles in the basic equilibrium theory. Then the loss mechanisms that complicate the experiment are identified and relevant measurements are discussed. Finally, experimental results concerning high-beta stellarator equilibrium and feedback stabilization are presented along with a theoretical interpretation.

II. REVIEW OF THE HIGH-BETA STELLARATOR CONCEPT

The high-beta stellarator can be understood as the natural configuration for a toroidal theta pinch. A theta-pinch plasma bent into a torus with no additional magnetic fields experiences an outward force caused by the magnetic field gradient. The selection of additional magnetic fields to produce a compensating inward force is based on stability considerations. In general, small additional magnetic fields applied to a highly compressed theta pinch cause instability in the gross transverse motion of the plasma.⁸ The least unstable configuration is produced by adding a helical $\ell = 1$ magnetic field $[B_r \sim \sin(\ell\theta - hz)]$. Combining $\ell = 1$ with $\ell = 0$ or $\ell = 2$ fields produces an inward directed interference force that balances the outward toroidal force. The gross motion can be stabilized by feedback⁸ or, if the plasma fills a sufficiently large fraction of the coil bore, by wall stabilization.⁹ In either case, a large-radius torus facilitates stabilization of the motion. Although the high-beta stellarator has no symmetry, it has the property of being arbitrarily close (as the toroidal radius approaches infinity) to a theta pinch.

A. Equilibrium

For the central problem of gross equilibrium and stability, the ideal magnetohydrodynamic equa-

tions, for the most part, describe the plasma behavior. In the sharp-boundary model, the basic theta pinch is characterized by a confining field B_0 , internal field $(1 - \beta)^{1/2}B_0$, density ρ , and radius a . The equilibrium consists in a balance of the outward toroidal force per unit length (divided by the mass per unit length),

$$F_R = \frac{\beta v_A^2}{R}, \quad (1)$$

where R is the radius of the torus, with interference forces given by

$$F_{1,0} = \frac{-h\beta v_A^2}{2} \frac{3 - 2\beta}{(1 - \beta)(2 - \beta)} b_0 b_1, \quad (2)$$

$$F_{1,2} = \frac{2\beta v_A^2}{a} \frac{b_1 b_2}{2 - \beta},$$

where b_0 gives the modulation of the on-axis vacuum magnetic field, $B_z(0) \sim 1 + b_0 \cos(hz)$, and b_ℓ for $\ell \geq 1$ gives the ℓ -helical Fourier component of the radial vacuum magnetic field at $r = a$, $B_r(a)/B_0 = \sum b_\ell \sin(\ell\theta - hz)$. Maxwell's equations are taken in their rationalized mks form so that $v_A^2 = B_0^2/(\mu_0\rho)$.

Recently, progress toward solving the equilibrium problem has been obtained by solving the magnetohydrodynamic equations in three dimensions numerically.^{10,11} The computations, in agreement with earlier experimental results, show the existence of equilibrium roughly consistent with the sharp-boundary theory. In addition, the computations agree with a diffuse-profile analytic theory,¹² which uses the approximation that the configuration is very close to a theta pinch. The analytic theory results in the interference force expressions given by Eq. (2), modified with profile- and beta-dependent factors. These factors, assuming $ha \ll 1$, and a Gaussian profile are listed in Table I (obtained from Ref. 12).

B. Gross Mode

The high-beta stellarator equilibrium is magnetohydrodynamically unstable for long-wavelength $m = 1$ perturbations that are basically gross transverse shifts. The instability is closely

TABLE I

DIFFUSE-PROFILE CORRECTION FACTORS

β^a	$f_{1,0}$	$f_{1,2}$
0.9	0.312	0.769
0.8	0.472	0.794
0.7	0.585	0.820
0.6	0.676	0.847
0.5	0.746	0.877
0.4	0.813	0.901
0.3	0.870	0.926
0.2	0.917	0.952
0.1	0.962	0.980

^a $\beta = 2\mu_0 p/B_0^2$, where p is the on-axis plasma pressure.

related to the interference force, and the instability force can be expressed in terms of interference forces by transforming the magnetic field to a coordinate system centered on the plasma.⁸ The growth rate for an $\ell = 0, 1, 2$ system is given by

$$\gamma^2 = \frac{h^2 T_{tot}}{M} \left[f_{1,0} \frac{3 - 2\beta}{(2 - \beta)(1 - \beta)} \left(\frac{b_0^2}{2} + b_1^2 \mp \frac{b_0 b_2}{ha} \right) + \frac{f_{1,2}}{2 - \beta} \left(b_1^2 + \frac{4b_2^2}{h^2 a^2} \mp \frac{b_0 b_2}{ha} \right) \right], \quad (3)$$

where $f_{1,0}$ and $f_{1,2}$ are the diffuse-profile correction factors for the $F_{1,0}$ and $F_{1,2}$ forces, T_{tot} is the profile-averaged total temperature $T_e + T_i$, and M is the ion mass. The $\mp$ signs refer to the fact that the growth rate is different for motion in the plane of the torus ($-$) and perpendicular to it ($+$). The quantities $f_{1,0}$ and $f_{1,2}$ are given in Table I.

C. $m \geq 2$ Modes

Perturbations with $\xi_r \sim \cos(m\theta)$ for $m \geq 2$ are also magnetohydrodynamically unstable.⁸ These modes are not observed for experiments with high-temperature plasmas, as has been explained in terms of finite-Larmor-radius effects. The stability criterion for $h a b_0$ and $b_2 \ll b_1$, using the sharp-boundary model^{9,13,14} is

$$b_1^2 \geq \frac{e_0^2 v^2 M}{e^2 n a^2} \left(\frac{T_i}{T_{tot}} \right)^2 \frac{(2 - \beta)^3}{16\beta(1 - \beta)}, \quad (4)$$

where n is the electron density, v the velocity of light, and e the electron charge. Recent experimental work shows that this criterion is much too pessimistic.¹⁵ This can be explained in part by the fact that the magnetohydrodynamic growth rate for $m \geq 2$ modes is considerably less for a diffuse- rather than sharp-boundary profile.^{16,17}

D. Internal $m = 1$ Modes

Ideal magnetohydrodynamic theory applied to diffuse-profile plasmas predicts unstable internal $m = 1$ modes in addition to the gross mode.^{18,19} The growth rate for these modes is given by¹⁹

$$\gamma \approx h v_A b_1 \frac{ha}{n + 1} \sqrt{\frac{1 - \beta}{\beta}}, \quad (5)$$

where $n \geq 1$ is the number of radial nodes of the mode. This growth rate is a great deal smaller than that for the gross mode, and thus the theory has not yet been subjected to experimental test (for the present experiment $1/\gamma_{n=1} \cong 180 \mu s$). Finite-Larmor-radius effects, which might have an important stabilizing effect on these modes, have not yet been calculated.

E. Reactor Scaling

As a first approximation, the additional energy balance considerations implied by feedback stabilization can be conceptually separated from those for any theta-pinch reactor, using shock and compression or some other form of heating. With feedback stabilization, the additional constraint is the requirement that the average feedback power be much less than the fusion power output.

The average feedback power is obtained from the fluctuation level of the plasma motion with feedback. This fluctuation level determines the feedback $\ell = 2$ magnetic field b_2 , hence the inductive energy W , which is given by

$$W = \frac{\pi^2 R r^4 b_2^2 B_0^2}{\mu_0 a^2 [1 - (r/b)^4]} \quad (6)$$

where r is the radius of the $\ell = 2$ winding and b is the conducting wall radius. A circuit Q is defined to specify the losses over a time $1/\gamma$, where γ is the long-wavelength $m = 1$ growth rate. The feedback power is then $\gamma W/Q$.

The wave number h and the magnetic field quantities b_0 , b_1 , and b_2 are chosen for a given toroidal radius R and line density na^2 to minimize the feedback power, with the equilibrium relation and Eq. (4) as constraints. As a result, b_1 in the optimum configuration is as large as is allowed by Eq. (4). The dependence on h is weak as long as nR is sufficiently small. The feedback power scales as $R^4 b_1^{-7} Q^{-1}$, which shows its strong dependence on the exact form of the $m \geq 2$ stability criterion. Detailed studies²⁰⁻²³ show that efficient, nonlinear feedback systems permit reactor radii of about 100 m, even if the sharp-boundary criterion, Eq. (4), is used. Because of the R^4 scaling, the average feedback power can always be made small for sufficiently large radius. The feedback power capability necessary for the maximum feedback force to be a given fraction of the toroidal force scales as R^{-2} .

These scaling laws are based on the application of small- δ theory even when δ_1 becomes large as the toroidal radius $R \rightarrow \infty$; however, for radii of interest for a reactor, $\delta_1 \lesssim 1$.

III. DESCRIPTION OF THE EXPERIMENT

A. Theta Pinch

For the feedback stabilization experiments, a 120° sector of a 4-m-radius toroidal theta pinch was driven from the Scyllac capacitor bank with one-half of the capacitors per unit length disconnected (that is, one-sixth of the full toroidal bank was used). Parameters of the basic theta pinch are given in Table II.

For the data discussed here, the main-bank voltage was 40 kV. The magnetic field waveform is shown in Fig. 1. A quartz discharge tube was used with deuterium gas fill. The filling pressure for most of the data was 20 mtorr. Pre-ionization was accomplished by a ringing theta-pinch discharge about 20

TABLE II

THETA-PINCH PARAMETERS

Major radius: 4 m
Length: 8.3 m
Average coil bore: 24 cm
Discharge-tube i.d.: 8.8 cm

Main-bank capacitance: 971 μ F
Source inductance: 1 nH
Charging voltage: 40 kV
Risettime: 4 μ s

Magnetic field: 13.7 kG
L/R decay time: 150 μ s

Pre-ionization bank capacitance: 6.6 μ F
Source inductance: 24 nH
Charging voltage: 50 kV

Fig. 1.

Magnetic field waveform at the locations $hz = 0$ and $hz = \pi$ in a wavelength.

μ s before the main bank was fired. This theta-pinch operating regime was similar to that of other experiments, in particular, the Culham 8-m theta pinch.²⁴

Relative to the Scyllac full-torus configuration,² the feedback experiments involved a reduction by one-half in the main-bank capacitance and an increase from 14.2 to 24 cm in the coil bore. This reduced the main field from 35 kG in the full torus to 13.7 kG, and similarly reduced the total temperature from ~ 1.2 keV to ~ 200 eV. As a result, the

feedback system power capability was matched to the plasma characteristics in the following way: (1) the $m = 1$ instability growth rate γ satisfied the criterion $\gamma\tau \lesssim 1/2$ (necessary for stability with feedback), where τ is the feedback-system delay plus risetime, and (2) the maximum feedback force represented a reasonable fraction ($\sim 1/4$) of the toroidal force.

B. High-Beta Stellarator Configuration

The inner surface of the theta-pinch compression coil was shaped to provide precisely known $\ell = 1$, 0 field quantities b_1 and b_0 . Templates, designed by a computer calculation of the flux surface and made by a numerically controlled milling machine, were used with a profile tracing lathe to machine the aluminum coil blocks.

The wall shape was an exact flux surface of the vacuum magnetic field $B = B_0[(1 - r/R \cos \theta)e_z + v\phi]$, where

$$\phi = \frac{2b_1}{h} I_1(hr) \sin(\theta - hz) + \frac{b_0}{h} I_0(hr) \sin(hz) + b_v r \sin \theta, \quad (7)$$

$b_1 = 0.0946$, $b_0 = 0.147$, and $b_v = 0.0035$. The helical field wave number h was 0.1 cm^{-1} . A toroidally curved, cylindrical coordinate system centered on the minor axis of the torus (the axis of the discharge tube) is used. The $\theta = 0$ (or x -) direction is outward from the center of the torus and the $\theta = \pi/2$ (or y -) direction is up. The vertical field magnitude, given by $b_v = (1/4)b_2b_1$, resulted in vacuum flux surfaces that were centered in an average sense on the axis of the discharge tube.

The actual shape of the coil wall can be approximated by

$$r = b[1 + \Delta_1 \cos(\theta - hz) + \Delta_0 \cos(hz)], \quad (8)$$

where $b = 12 \text{ cm}$, $\Delta_1 = 0.124$, and $\Delta_0 = -0.094$. The coordinate system to which Eq. (8) refers is shifted outward 2.5 cm relative to the coordinate system used in Eq. (7). The bumpy and helical deforma-

Fig. 2.

Layout of the experiment, showing the position-detector locations and the wavelength-numbering scheme.

tions of the wall produce greater corrugation of the inner wall surface than of the outer, as shown in Fig. 2.

The feedback experiments were conducted in three distinct phases with different auxiliary windings and discharge-tube configurations: (1) straight (toroidally curved) discharge tube with no additional windings, (2) straight discharge tube with $\ell = 2$ feedback windings and $\ell = 0$ equilibrium windings, and (3) helical discharge tube with $\ell = 2$ feedback windings and $\ell = 2$ equilibrium windings. During the experiments with configuration (3), 6-cm-diam quartz cylinders were inserted about 15 cm into the sector ends to provide a better defined and possibly stabilizing end configuration. These end plugs had no significant effect but were left in place.

For simplicity, the results reported are from the final configuration, which also used a more flexible circuit for driving the equilibrium windings. The helical discharge tube substantially eliminated the initial helical oscillations of the plasma with no apparent deleterious effects, as had been observed previously in experiments at Garching.^{2a} The experimental results from configurations (2) and (3) showed that $\ell = 2$ windings are advantageous for fine tuning the equilibrium because they require less driving current, as expected from theory. Configuration (1) was studied as a comparison case to see if the presence of auxiliary windings changed the plasma behavior. This was not true.

The layout of the final configuration is shown in Fig. 2. The discharge tube was a quartz cylinder formed into a toroidally curved helix of 1.4-cm. radius. The discharge tube was fabricated in a specially built quartz bending machine similar to that used at Garching.

The auxiliary $\ell = 2$ windings for adjustment of the equilibrium consisted of a single wire wound on the discharge tube (but over the feedback windings, which are discussed later) into a fourfold $\ell = 2$ pattern with leads brought out at one-wavelength intervals. The winding radius was $r = 5.8$ cm, giving an $\ell = 2$ field

$$b_2 = 0.8 \frac{aJ}{r^2 B_0}, \quad (9)$$

where a and r are in centimeters, B_0 is in gauss, and J is the current in amps.

Separate sets of windings were spaced to produce $F_{1,2}$ forces in the horizontal and vertical directions. The horizontal and vertical windings in wavelengths 1-4, 5-8, and 9-12 (see Fig. 2), were driven independently by six circuits of the Jones-Weibel type.²⁸ The circuits provided a step waveform with an adjustable final slope. Each circuit was supplied by two independently variable voltage sources, which separately controlled the step amplitude and the final slope.

C. Feedback System

The feedback algorithm gave local feedback with velocity control, that is, $B_{\ell=2}^{FB}(z) \sim \xi(z) + \tau_1 \dot{\xi}(z)$, where $\xi(z)$ is the plasma displacement (as determined by the feedback position detectors) at longitudinal location z and τ_1 is a velocity gain parameter. The feedback system consisted of plasma position detectors, intermediate amplifiers, power amplifier modules, and $\ell = 2$ feedback windings.

The five position-detector stations had two position detectors viewing the plasma orthogonally at angles of $\pm 45^\circ$ from the plane of the torus. Each position detector consisted of a 16-segment photodiode with a peak-sensing electronic circuit, which produced a voltage proportional to position.²⁷ The position-detector output was fed to an inter-

mediate amplifier, which formed the derivative, applied separate position and velocity gains, then distributed signals to nearby power amplifier modules. There were actually 10 independent feedback systems corresponding to the 10 position detectors controlling the $\ell = 2$ currents in their vicinity.

The power amplifiers used two triode vacuum tubes (Machlett 8618*) in push-pull in the final stage. The plate voltage was normally 30 kV. Typical plate currents of about 600 A, much higher than the rated output of the tube, were obtained by operating the filaments at higher-than-rated voltage. The high currents resulted in some problems with internal arcing, but generally only a few tubes arced during a shot. Tube-arcing produced a level of nonreproducibility but did not dominate the plasma behavior.

The push-pull output was coupled through a 9:7 transformer to a four-turn $\ell = 2$ winding. The feedback $\ell = 2$ winding was wound directly on the discharge tube and in cross section consisted of 16 wires spaced to approximate a $\cos(2\theta - \text{hz})$ current distribution. The $B_{\ell=2}$ field is given by Eq. (9) with $r = 5.65$ cm and with an additional factor of π . The factor is due to the feedback winding having four turns and to the sinusoidal spacing of the turns, which reduced the field by $\pi/4$.

The $\ell = 2$ windings were spaced to produce orthogonal forces acting in the directions of the plasma displacements detected by the position detectors. Two modules drove the two orthogonal force windings in each wavelength. There were 26 modules in all, corresponding to 13 wavelengths. The position detectors controlled the modules in each wavelength in the following way (see Fig. 2): PD₁, wavelengths 1-2; PD₂, wavelengths 3-4; PD₃, wavelengths 5-8; PD₄, wavelengths 9-10; and PD₅, wavelengths 11-13.

D. Diagnostics

Several of the important measurements utilized the plasma radiation in the visible range. These were measurements with feedback position detectors, streak cameras, and two 20-channel luminosity devices. The position-detector measurements indicated that the plasma luminosity was mainly due

*Machlett Laboratories Inc., Stamford, CT 06907.

to bremsstrahlung. Position detectors used a 16-segment photodiode and provided, along with position information, a signal proportional to the maximum current from the 16 segments, thereby giving a measure of the maximum luminosity across the plasma profile. Using the known geometrical acceptance and spectral response of the detector (blue filters were used to limit the spectral range from 3600 to 5800 Å), one could infer the absolute luminosity. Figure 3 shows data for a shot that had good confinement using feedback. For comparison, bremsstrahlung is calculated²⁸ to give about 15 μA ($n_e = 2 \times 10^{16}$, $T_e = 100 \text{ eV}$, $a = 1 \text{ cm}$) with an uncertainty factor of about 2. The discharge clean-up effect is important in this comparison because the light intensity was 4-6 times greater initially than after a large number of shots. The feedback position detectors measured the position of the peak light intensity with an accuracy of about $\pm 1.5 \text{ mm}$ over a range of $\pm 2 \text{ cm}$, with a maximum range of $\pm 2.5 \text{ cm}$.

The streak cameras operated in 10 different slit locations around the sector and gave basic information about the plasma containment. A mirror arrangement was used occasionally to superimpose two views of the plasma at a one-half-wavelength interval. The resulting streak photograph directly showed the behavior in time of the helical deformation of the plasma shape.

Fig. 3.

Position-detector photodiode current vs time as a measure of the plasma luminosity. The main bank was fired at 4 μs on this time scale.

The luminosity apparatus consisted of 20 photomultiplier tubes viewing the plasma along parallel chords through the discharge tube to give the luminosity profile as a function of time. The photocathode response (S4) determined the spectral range. The luminosity profiles were used to obtain density profiles by assuming that luminosity is $\sim n^2$. Figure 4 shows luminosity profiles for the same shot as that in Fig. 3. Luminosity measurements were made in two locations, one-half wavelength apart, where the plasma radius was locally maximum and minimum because of the $\ell = 0$ field.

The electron temperature was measured using the 90° scattering of 6943-Å ruby laser light. The incident laser energy was about 5 J in a 30-ns pulse. The scattered light was detected with a multigrating polychromator.²⁹ The Thomson scattering apparatus allowed the determination of the electron temperature in a volume $\sim 1.4 \text{ cm}$ high by 3 mm diam at the time of the laser pulse. Temperature measurements were difficult at late times when the

Fig. 4.

Luminosity profiles measured by a 20-channel phototube array. The time sequence runs from top to bottom, left to right. The time increment is 0.8 μs .

Fig. 5.

Typical Thomson scattering data taken 2.5 μ s after main-bank time.

plasma position was uncertain. Typical Thomson scattering data are shown in Fig. 5.

The electron area density was measured using a coupled cavity 3.39- μ m He-Ne laser, which had a ~ 2 -mm-diam beam that passed vertically through the center of the discharge tube. A single fringe produced by the rotating mirror corresponded to 3.28×10^{16} electrons/cm² and the error, based on the scatter of vacuum data, was less than $\pm 1/30$ fringe. The data were recorded over a 40- μ s interval so that both pre-ionized and final plasma phases could be observed. The fringe rate was 3 MHz. Although a fringe was sometimes lost while passing through the implosion-compression phase, this was easily detected and corrected for. In those (not infrequent) cases where plasma crossed the beam, the plasma radius measured from the luminosity profile could be used to obtain the plasma density by the relation

$$\sqrt{\pi} a n_{e0} = \int n_e dl, \quad (10)$$

where the electron density is assumed to be $n_e(r) = n_{e0} \exp[-(r/a)^2]$.

An example of coupled cavity data is shown in Fig. 6. From position-detector information, the peak at $\sim 15 \mu$ s after the implosion corresponds to the plasma crossing the laser beam and therefore can be used to determine the density. The pre-ionization level is obtained by assuming that the pre-ionized plasma uniformly fills the discharge tube.

Fig. 6.

Typical data from the coupled cavity interferometer. The pre-ionization bank is fired at 6 μ s and the main bank at 22 μ s.

Vacuum-balanced loop and magnetic probe systems were used to measure the magnetic flux excluded by the plasma. The excluded flux in conjunction with the plasma radius, as determined by the luminosity apparatus, allowed a determination of β . For a Gaussian pressure profile $\{p \sim \exp[-(r/a)^2]\}$, and defining β in terms of the magnetic field at the same longitudinal location far outside the plasma, the excluded flux is given by

$$\Delta\phi = 2\pi a^2 B [\log(1 + \sqrt{1 - \beta}) - \sqrt{1 - \beta} + 0.307] \quad (11)$$

As $\Delta\phi$, B , and a are known, this equation can be solved for β .

Two loop-probe systems, located at $hz = 0$ and $hz = \pi$, were used to study the $\ell = 0$ response of the plasma. The excluded flux can be written as $\Delta\phi = \pi r^2 B - \phi_{in}$, where r is the plasma radius and ϕ_{in} is the flux inside the plasma. The flux in the plasma is a constant along the length of the plasma; thus the difference in excluded flux gives information about the plasma radius. In the small- δ , sharp-boundary model,

$$\begin{aligned}\Delta\phi\Big|_{hz=\pi} - \Delta\phi\Big|_{hz=0} &= 4\pi a^2 B_0 (\delta_0 + b_0/2) \\ &= 4\pi a^2 B_0 (\delta_0 - \delta_{0,vac}) \quad , \quad (12)\end{aligned}$$

where B_0 is the average field, a is the average radius, and δ_0 is the relative bumpiness, $r = a[1 + \delta_0 \cos(hz) + \dots]$. This relationship is used in Sec. V.C to determine the transient behavior of δ_0 .

In equilibrium, the ratio of the excluded flux at the field maximum ($hz = 0$) and field minimum ($hz = \pi$) locations is a function only of β , b_0 , and the pressure profile and thus gives information about β . The model described in Appendix A was used to calculate this ratio versus β for a Gaussian profile using the value of b_0 for the experiment. Results are given in Table III. Large values of the ratio are associated with very high beta values near the pressure balance limit $(1 - b_0)^2$ (see Appendix A). The quantity β_0 listed in Table III is the beta value before the application of the $l = 0$ field.

IV. PLASMA PARAMETERS

A. Temperature, Density, and Beta

After discharge-tube clean-up (several hundreds of shots), the measured plasma parameters were

fairly reproducible. Table IV gives measured electron temperatures from Thomson scattering, excluded flux, electron area density ($\int n_e dl$) from the coupled cavity interferometer, plasma radius from the 20-channel luminosity apparatus, and magnetic field. The measurement apparatus were located as follows (see Fig. 2): Thomson scattering between wavelengths 4 and 5, excluded flux and luminosity between wavelengths 7 and 8, and coupled cavity interferometer between wavelengths 2 and 3. The location in a wavelength was $hz = \pi$, where the magnetic field was locally minimum and the plasma radius maximum. The electron temperature measurements were made 2.5 μs after main-bank time when the magnetic field was about 11.5 kG. A few late-time temperature measurements (discussed subsequently) do not indicate a significant drop in temperature. All the other measurements were made at the times listed in column 2 of Table IV, when $\int n_e dl$ from the coupled cavity was at a maximum, corresponding to the plasma being centered on the coupled cavity laser beam.

The average electron temperature for the shots listed in Table IV is $T_e = 110 \pm 5$ eV. The average central electron density obtained from the radius and $\int n_e dl$ using Eq. (10) is $n_{e0} = (2.0 \pm 0.2) \times 10^{16}$ cm $^{-3}$. Assuming $T_i = T_e$ and $n_i = n_e$, beta values somewhat greater than 1 are obtained (column 9 of Table IV). The beta values from excluded flux and radius, derived using Eq. (11), also tend to be slightly greater than 1 (column 10).

Because the equilibration time between ions and electrons is about 3 μs , the conclusion from Table IV is that $T_i \cong T_e \cong 100$ eV and $\beta|_{hz=\pi} \cong 1$. The fact that beta obtained from excluded flux and luminosity measurements is greater than 1 is probably explained by the non-Gaussian radial profile at $hz = \pi$ (see Appendix A). The discrepancy of the beta value obtained from coupled cavity and Thomson scattering cannot be corrected by using a non-Gaussian profile and is not understood. The average beta is defined by referring the pressure to the average field B_0 . Because $B|_{hz=\pi} = 0.853B_0$ and p is a constant (neglecting field-line curvature effects), $\beta_{av} = 2\mu_0 p/B_0^2 \cong 0.7$. This value of β is also consistent with less precise excluded flux and radius data for the field maximum region $hz = 0$. The ratio between excluded flux at $hz = \pi$ and $hz = 0$ is greater than 3 at 10 μs , which gives a similar value of β using Table III.

TABLE III

CALCULATED EXCLUDED FLUX RATIOS

β^a	β_0	$\Delta\phi_{max}/\Delta\phi_{min}$
0.73	0.95	4.82
0.72	0.9	4.70
0.69	0.8	2.86
0.64	0.7	2.51
0.56	0.6	2.29
0.47	0.5	2.15
0.38	0.4	2.04
0.29	0.3	1.97
0.19	0.2	1.90
0.10	0.1	1.84

^a $\beta = 2\mu_0 p/B_0^2$, where p is the on-axis plasma pressure.

TABLE IV
MEASURED PLASMA PARAMETERS^a

Shot	t (μ s)	B (kG)	a (cm)	$\int n \, dl$ ($\times 10^{16} \text{ cm}^{-2}$)	$\Delta\phi$ (kG $\cdot$ cm ²)	T _e (eV)	n ₀ ($\times 10^{16} \text{ cm}^{-3}$)	β_{cc}^b	β_{ex}^b
5370	16	11	1.1	3.9	34	109	2	1.3	1.1
5405	15	11	1.3	3.8	38	120	1.65	1.1	1
5466	14	11	1.2	3.7	36	103	1.75	1.1	1.1
5502	12	10.5	1.2	4.4	32	110	2.1	1.5	1
5503	10	11.5	1.1	3.7	36	110	1.9	1.1	1.15
5505	10	11.5	1.1	4.0	36	112	2.05	1.2	1.15
5506	10	11.5	1	4.0	35	109	2.25	1.35	1.2
5511	10	11	1.05	4.3	34	105	2.3	1.5	1.2

^ahz = π , B = 0.853B₀.

^b $\beta \equiv 2\mu_0 p/B^2$, where p is the on-axis plasma pressure and B the field far outside the plasma.

The coupled cavity interferometer also provided a measurement of the plasma density outside the main column. For shots where the plasma was far off-axis, as determined by position detectors, the quantity $\int n_0 \, dl$ was assumed to be a sum of two terms: (1) the contribution from the tail of the main plasma column, which is assumed Gaussian, and (2) a contribution from a uniform density background plasma assumed to fill the discharge tube. The background density thus determined was about 10^{14} cm^{-3} .

B. Loss Mechanisms

Because of the strong dependence of the high-beta stellarator equilibrium on β , it is important to determine the decay time for β . By the use of excluded flux and luminosity radius data, β was determined directly as a function of time, but with a rather large error. Data for a shot with a long confinement time using feedback are shown in Fig. 7. The theoretical curves were calculated from a model, described in Appendix A and discussed later.

The characteristic time describing particle loss from a theta pinch is L/v_i , where L is the half-length and $v_i = \sqrt{2T_i/M}$ is the ion thermal velocity. That time is about 40 μ s. As the plasma is collisional (mean free path on axis $\sim 1 \text{ cm}$), end loss occurs by

the inward propagation of a rarefaction wave.^{30,31} The propagation time to the center of the sector is given by $(\beta/2 + 1/\gamma)^{1/2} L/v_i$ from Ref. 31, where γ is the ratio of specific heats, or about 40 μ s for this experiment.

A few temperature measurements at late times were carried out. The results ($T_e \cong 90 \text{ eV}$ at 30 μ s) indicate only a small temperature decrease with time. The classical cross-field ion thermal conduction time is about 2 μ s, much shorter than the longitudinal thermal conduction time (using on-axis plasma conditions), so one would expect the plasma temperature profile to be flat. Thermal conduction to the ends gives a cooling time $\sim 5nL^2/\chi$ (Refs. 32 and 33), where χ is the electron axial heat conductivity and n is the average electron density over the region where the ionic cross-field heat conductivity maintains a uniform temperature. Assuming a 2-cm radius for the high-temperature region, the classical end-cooling time is about 250 μ s. More rapid cooling is expected upon the arrival of the rarefaction wave from the ends, which is predicted at about 30 μ s at the location of the Thomson scattering apparatus.

The plasma resistivity leads to a significant decrease of beta during the experiment. For $T_e = 100 \text{ eV}$ and $Z = 1$, the classical resistivity value is $\eta/\mu_0 \cong 0.01 \text{ cm}^2/\mu\text{s}$. The characteristic time for decay of the central beta in a theta pinch is given by $a^2\mu_0/\eta$ (Ref. 34) or $\sim 100 \mu\text{s}$ for $a = 1 \text{ cm}$.

Fig. 7.

Plasma β and radius vs time at field maximum ($hz = 0$) and minimum ($hz = \pi$) locations.

Radiation energy losses are small. Radiation losses 300 times greater than bremsstrahlung would be necessary for a 100- μ s cooling time. Also, assuming that oxygen is the principal impurity, calculations^{35,36} show that the impurity level would have to be more than 5% to cool the plasma in 100 μ s. The impurity levels in similar experiments^{7,24} were measured to be a fraction of 1%.

Figure 8 shows excluded flux data for a shot with long confinement time using feedback. The excluded flux in the field minimum region, $hz = \pi$, decays substantially during the experiment. The erratic variation of the data, especially for $hz = 0$, is due to pickup on the small excluded flux signal from

Fig. 8.

Excluded flux vs time at field maximum ($hz = 0$) and minimum ($hz = \pi$) locations.

nearby feedback windings (most of the pickup was eliminated by subtraction). The average character of the data was quite reproducible.

The theoretical curves in Figs. 7 and 8 come from a model calculation, described in Appendix A. The calculation included classical radial diffusion and heat conductivity and the fall-off of the main field. The excluded flux measurements show a decrease below the calculated value beginning at about 20 μ s. The model calculation cannot be made to agree with the measurements by increasing the resistivity because diffusion does not decrease the excluded flux in a theta pinch. Beta is reduced by diffusion, which reduces the ratio between excluded flux at the field minimum and maximum locations (see Table III); however, an increase in the excluded flux in the field maximum region would be involved.

Good agreement between measurement and calculation was obtained by adding particle loss to the model calculation. Particle loss might be due to dispersive effects that smear out the arrival time of the rarefaction wave (predicted at 35 μ s). End loss is the most likely explanation of the excluded flux behavior.

Although not a loss mechanism, the open ends of the sector could affect the confinement by generating propagating $m = 1$ displacements caused by lack of equilibrium in the end regions. Such an

effect would depend on the complicated and unknown physics of the end region. Observation of propagating $m = 1$ waves in Scyllac is discussed in Ref. 37; the propagation time from the ends of the sector to the center was approximately $20 \mu\text{s}$.

Another phenomenon that may play a role in a sector experiment is the shorting out of the plasma's radial electric field by the ends.³⁹ The shorting currents propagate into the plasma at the Alfvén speed and would cause the plasma to rotate. The Alfvén propagation time from the ends of the sector to the center is about $20 \mu\text{s}$.

V. TOROIDAL EQUILIBRIUM

A. Experimental Results

To study the toroidal equilibrium, auxiliary $\ell = 2$ windings were driven with different current waveforms, and the plasma trajectory was observed using streak cameras and position detectors. The current waveform was essentially a step with variable amplitude and starting time. For these experiments, equilibrium was defined as a situation where a small change in the amplitude of the $\ell = 2$ field changed the plasma trajectory from toroidally inward to toroidally outward.

The results are shown in Fig. 9 and summarized in Table V(a) for three cases that illustrate the important experimental variables. The average plasma trajectory for the three centermost position detectors and the average $\ell = 2$ magnetic field divided by the main field are plotted for two shots that showed inward and outward motion. The filling pressure, which affects β , and the time of initiation of the $\ell = 2$ field were varied. The $\ell = 2$ fields were turned on after the main field to insure that initial transients (discussed in Secs. V.B-D) had largely died out. The three cases are: (1) 20-mtorr filling pressure with the $\ell = 2$ field turned on late (the standard operating condition), (2) 20-mtorr filling pressure with the $\ell = 2$ field turned on early, and (3) 10-mtorr filling pressure with the $\ell = 2$ field turned on early. Another observation, listed in Table V(a), was of apparent equilibrium with no additional fields. This behavior occurred after more than 1000 shots for which the discharge tube was continuously under vacuum.

Fig. 9.
Plasma trajectories in the horizontal plane and normalized $\ell = 2$ field waveform for pairs of shots showing inward and outward motion. The effects of variation of the filling pressure and time of initiation of the $\ell = 2$ field are shown. Main-bank time is $4 \mu\text{s}$.

TABLE V
TOROIDAL EQUILIBRIUM

(a) EXPERIMENTAL PARAMETERS

Case	Shot	P_0 (mtorr)	$-b_2$	Time	ξ (cm)
1	5255, 5256	20	0.009	late	0.25
2	5333, 5337	20	0.0038	early	-0.5
3	5311, 5314	10	0.01	early	0.25
4	3942	20	0	---	---

(b) EFFECTIVE FIELDS AND THEORETICAL β

Case	b_{-1}	b_0	b_1	$-b_2$	β
1	-0.9×10^{-3}	0.145	0.0914	0.89×10^{-2}	0.65
2	1.8×10^{-3}	0.152	0.0982	0.39×10^{-2}	0.76
3	-0.9×10^{-3}	0.145	0.0912	1.0×10^{-2}	0.59
4	0	0.147	0.0946	0	~ 0.93

These data were analyzed using small- δ , diffuse-profile theory.⁹ Table V(b) gives the helical fields transformed to a coordinate system centered on the plasma with the shift ξ taken into account. The transformation is made to correct for the instability force caused by different equilibrium positions. Also listed are the β values giving equilibrium according to theory. The inward force transient due to δ_0 (discussed in Sec. V.C) occurring at early times is probably of importance here and would result in lower β values in cases (2) and (3) for a true equilibrium situation. The measured value of β at early times was $\beta_{exp} = 0.7 \pm 0.1$ for all cases, in good agreement with the theoretical value except for case (4). The value of β obtained for case (4) exceeds the limiting value $(1 - b_0)^2$, and the small- δ theory is obviously no longer applicable.

Windings producing a vertical force were also driven, and the currents were adjusted for minimum motion in the vertical plane. The data in Fig. 9 refer only to the horizontal windings. The current in the vertical windings was a step waveform beginning at main-bank time. The optimum field was about the same in the three cases, with $b_2 \cong 0.002$. The need for such a field would be evident, for example, if the discharge tube were slightly out of position vertically.

The plasma trajectories in Fig. 9 could be fit by exponentials and the $m = 1$ growth rate determined empirically, but the error estimate would be large. The trajectories shown in Fig. 9a are fit well by use of the theoretical growth rate from Eq. (3), which gives $\gamma_H = 0.32 \mu s^{-1}$. The growth rate for motion in the vertical plane from Eq. (3) is $\gamma_V = 0.17 \mu s^{-1}$. The corresponding values from sharp-boundary theory are $\gamma_H = 0.38 \mu s^{-1}$ and $\gamma_V = 0.19 \mu s^{-1}$.

B. Dynamical Aspects of the Equilibrium

The small- δ , sharp-boundary model^{8,9} provides a useful framework for discussion of the dynamical aspects of the toroidal equilibrium. The interference forces, expressed in terms of the instantaneous plasma shape and the applied helical fields, are given by

$$F_{0,1} = h v_A^2 (b_0 b_1 - \frac{h a}{2} b_0 \delta_1 + \delta_0 b_1) \quad ,$$

$$F_{1,2} = 2 v_A^2 / a (-b_1 b_2 + h a \delta_1 b_2) \quad , \quad (13)$$

where the plasma shape is

$$r = a [1 + \delta_0 \cos(hz) + \delta_1 \cos(\theta - hz) + \dots] \quad . \quad (14)$$

After the occurrence of initial transients excited by the theta-pinch implosion, the plasma deformations δ_0 and δ_1 assume the equilibrium values

$$\delta_0 = \frac{-b_0}{2(1 - \beta)} \quad ,$$

$$\delta_1 = \frac{2b_1}{h a (2 - \beta)} \quad . \quad (15)$$

When substituted into Eq. (13), the equilibrium plasma deformations lead to the expressions for the interference forces given by Eq. (2). During the transient phase, δ_0 and δ_1 oscillate about their equilibrium values with oscillation frequencies

$$\omega_0 \cong h v_A \sqrt{\beta(1 - \beta)} \quad ,$$

$$\omega_1 = \hbar v_A \sqrt{2 - \beta} \quad (16)$$

With the experimental values $\beta \cong 0.7$ and $v_A \cong 18$ cm/ μ s, these frequencies are $\omega_0 \cong 0.8 \mu\text{s}^{-1}$ and $\omega_1 \cong 2 \mu\text{s}^{-1}$.

C. Behavior of δ_0

The bumpiness δ_0 enters strongly into the expression for the $F_{0,1}$ interference force, Eq. (13). The most satisfactory measurement of δ_0 was obtained from the excluded flux measured simultaneously in the field maximum ($hz = 0$) and minimum ($hz = \pi$) regions. Using Eq. (12), the difference of the two excluded flux traces is related to the quantity $\delta_0 - \delta_{0,\text{vac}}$. Data for three filling pressures are shown in Fig. 10.

The initial oscillation expected from magnetohydrodynamic theory shows up most clearly at high filling pressures. The theoretical curve shown in Fig. 10a was calculated from a sharp-boundary incompressible model discussed in Appendix B. The transient behavior for lower filling pressures corresponds to stronger damping of the oscillation.

The separation between the excluded flux traces is linearly related to δ_0 ; therefore, using Eq. (13), this separation as a function of time gives a corresponding time dependence of the force acting on the plasma column. The force was determined experimentally by taking the second derivative of the plasma trajectory as measured by streak cameras. Results are shown in Fig. 11 (20-mtorr filling pressure, reverse magnetic field bias). Using different data smoothing procedures, the acceleration analysis always showed an initial inward force transient that occurred at about the time of maximum separation of the excluded flux traces and that was apparently due to the δ_0 transient. This force transient is important because it complicates the setup (using driven auxiliary windings) of a situation with good toroidal equilibrium at late times.

D. Behavior of $a\delta_1$

Although the helical plasma oscillations were suppressed by the use of a helical discharge tube, they

Fig. 10.

Excluded flux vs time at field maximum ($hz = 0$) and minimum ($hz = \pi$) locations for different filling pressures. A sinusoidal δ_0 oscillation, evident at high filling pressure, is damped more rapidly at lower filling pressures.

were still observable. The radius of the plasma helix, $a\delta_1$, was measured using a streak camera with two superimposed simultaneous views of the plasma one-half wavelength apart. Data are shown in Fig.

Fig. 11.

Measured acceleration of the plasma column from streak cameras. The inward force transient occurs at the time when δ_0 , measured by the excluded flux, is maximum.

12. The difference of the two trajectories gives twice the helical radius as a function of time. For comparison, Fig. 12 also contains a plot of the equilibrium value of $a\delta$, expected from small- δ , sharp-boundary theory, taking into account the coupling between the plasma position and the helical magnetic field. The two curves are in reasonable agreement for $\beta = 0.5$, although the measured beta value is somewhat larger. Numerical calculations with diffuse profiles⁴⁰ show a smaller helical radius than predicted with the sharp-boundary model, thereby agreeing better with the measurements.

The helical radius of the discharge tube was 1.4 cm, which is the value of $a\delta$, from sharp-boundary theory for $\beta = 0.65$. The observations are consistent with the plasma being initially at the center of the discharge tube and then moving to its equilibrium configuration. The initial helical oscillation had a frequency $\omega_1 \cong 1.9 \mu s^{-1}$, in good agreement with Eq. (16).

Fig. 12.

The helical radius of the plasma as measured by a streak camera simultaneously viewing the plasma at two locations one-half wavelength apart.

VI. FEEDBACK STABILIZATION

A. Feedback-System Parameters

The equation of motion of the gross motion mode with feedback can be written

$$(\ddot{\xi} - \gamma^2 \xi)_t = -\alpha \gamma^2 (\xi + \tau_1 \dot{\xi})_{t-\tau}, \quad (17)$$

where ξ is the sideways displacement of the plasma, γ is growth rate of the $m = 1$ instability, τ is the feedback-system delay, and α and τ_1 are feedback gain parameters. The product $\gamma\tau$ is crucial and must be less than about 1 if Eq. (17) is to have stable solutions.⁴¹ The range over which the gains α and τ_1 may be varied and still have stable motion shrinks to 0 as $\gamma\tau$ increases toward 1.

The feedback-system overall delay was determined by using a light-emitting diode to drive a position detector and measuring the $\ell = 2$ current and the diode current. The delay for a step input was about 0.5 μ s pure delay for up to 10% output current plus 1 μ s for 10 to 90% risetime. The delay to 50% of full output was about 1 μ s. The risetime was roughly twice LI/V , where L is the load inductance as seen by the output tubes (27 μ H), I is the maximum current, and V is the plate voltage. The delay was roughly the sum of delays in the position detector (30 ns), cabling (100 ns), intermediate amplifier (70 ns), and power module (280 ns).

The velocity gain parameter τ_1 was determined by the calibration of the intermediate amplifier position and velocity gains and was quite well defined. From small- δ , sharp-boundary theory, the gain parameter α is given by

$$\alpha = \frac{v_A^2}{\gamma^2} \frac{2\beta}{2-\beta} b_1 \frac{G}{B_0 a}, \quad (18)$$

where $G = B_{\ell=2}/\xi$ is the feedback-system gain. G was poorly defined because the curves for module output current versus input voltage were not linear. A fitting procedure was used to obtain a definite value. The theoretical quantities entering into Eq. (18) are uncertain, but for definiteness the values $v_A = 18$ cm/ μ s, $\beta = 0.7$, and $\gamma = 0.3$ μ s⁻¹ are used.

The maximum, or saturation, current of the modules can be expressed as the displacement $\xi_{\max}$ for which the instability force equals the maximum feedback force and is given by

$$\xi_{\max} = \frac{h\nu_A^2}{\gamma^2} \beta b_1 b_2, \quad (19)$$

where b_2 is the maximum feedback field divided by the main field. Also of interest is the calculation of the fraction of the toroidal force represented by the maximum feedback force,

$$f = \frac{2R}{a} \frac{b_1 b_2}{2-\beta}. \quad (20)$$

For the standard operating conditions with a module plate voltage of 30 kV and a filament voltage of 8.8 V, the maximum current was about 600 A and the maximum feedback field was 58 G at $a = 1$ cm. For $\delta_1 = 1.4$, $B_0 = 13$ kG, and with the other parameters as given above, $\xi_{\max} \cong 1.5$ cm and $f \cong 0.25$.

B. Behavior with Feedback

Figure 13 is a comparison of position-detector traces for shots with and without feedback, together with the average currents for the modules driven by each position detector. For the same shots, Fig. 14 shows streak-camera photographs of plasma motion in the horizontal and vertical planes in the center of the sector.

The feedback gains were $\tau_1 = 4$ μ s, $G = 500$ A/cm $\times$ 0.1 G/A = 50 G/cm, and $\alpha = 1.4$. The vertical scales for plasma displacement and current in Fig. 13 are such that the two signals should be equal except for velocity gain. Arcing in the output tubes occurred in shot 5515 for the modules driving wavelength 10 at 26 μ s, wavelength 5 at 40 μ s, wavelength 11 at 45 μ s, and wavelengths 12 and 13 at 23 μ s. Arcing caused the current excursions beyond 1 kA on the PD₅ and PD₄ current traces in Fig. 13.

The feedback loop was closed 9 μ s after main-bank time and can be seen in Fig. 13 as the beginning of module output current. The optimum time

Fig. 13.

Plasma trajectories measured by feedback position detectors and the corresponding feedback current. The numbering scheme for position detectors is given in Fig. 2. Comparable shots with and without feedback are shown. Main-bank time is 4 μ s.

was not well defined. The principal observable effect was that when the loop was closed too early the feedback system reacted to the initial helical oscillations and produced violent current excursions, which increased the probability of tube-arcng. After a long time the plasma was moving far off-axis at great speed and feedback had little effect.

The average trajectories in the horizontal plane (for the three centermost position detectors) with and without feedback are shown in Fig. 15. Also shown is the average $\ell = 2$ field over the same central portion of the sector divided by the main field. The $\ell = 2$ field consists of the auxiliary equilibrium field, which is the same for both shots, and the feedback field. With feedback, the tendency of the plasma to move slowly outward is attributable to the decrease in β with time.

Confinement was poorer near the ends of the sector than in the central region. For shot 5515, streak

cameras were operated in wavelength 11, wavelength 10, wavelength 3, and wavelength 7 (see Fig. 2). Streak photographs showed a wall hit occurring at 28 μ s in wavelength 11 and a wall brush at about the same time in wavelength 10. The plasma motion near the ends was always more violent, with larger displacements and accelerations than in the central region. Because of the anomalous behavior of the ends, the experiment was defined to cover only the central region, consisting of wavelengths 3-10. In the central region and under optimum conditions, the confinement time with feedback was 10-20 μ s longer than without feedback.

The ultimate loss of confinement with feedback has not been modeled in detail, but appears to be related to the loss of β caused by resistive diffusion and end effects. The usual occurrence at late times of several tube arcs in the feedback modules degraded the performance of the system and complicated the situation.

C. Effect of Gains

The position and velocity gain settings did not affect the plasma behavior with feedback in a sensitive way, although there was some indication that large gain values were not optimum.

The results of reversing the velocity gain are given in Fig. 16. The plasma trajectory in the horizontal plane in the center of the sector is shown together with the feedback current. It is apparent that positive velocity gain effectively cancels delay, whereas negative velocity gain increases the delay and destabilizes the plasma.

The effect of varying α is shown in Fig. 17. Plasma trajectories in the vertical plane at the center of the sector are shown for $\alpha = 0, 1.4$, and 5.6 and $\tau_1 = 3 \mu$ s. High gain seems to increase the tendency for the column to oscillate.

Two mathematical models were used to simulate these data. The first model, described in Refs. 42 and 8, allowed only the long-wavelength longitudinal mode but treated the plasma dynamics realistically. The results are shown in Fig. 18. The $m = 1$ motion is stabilized and the amplitude damps out completely for both gain values. The second model, described in Appendix C, allowed for longitudinal modes but with a simplified dynamical

Fig. 14.

Streak-camera photographs of plasma behavior in the center of the sector with and without feedback.

equation. The results are given in Fig. 19, which shows the plasma trajectories at four equally spaced points around a torus. The longitudinal mode model agrees best with the data in that it reproduces the observed undamped behavior.

D. Longitudinal Mode Structure

The plasma motion with feedback often involved longitudinal mode structure. An example is shown in Fig. 20. The plasma position, as viewed by neighboring position detectors, was oscillating 180° out of phase. In general, the plasma motion with feedback tended to be oscillatory (although the oscillations were not necessarily simple) and did not show continuity from one position detector to the next.

This behavior can be explained by the feedback model including longitudinal modes that is

described in Appendix C. In the small-amplitude regime for this model, feedback drives some modes unstable. These have a short longitudinal wavelength and are stable without feedback. The delay of the feedback system is responsible for the instability in that it involves an $\omega\tau$ criterion similar to the $\gamma\tau < 1$ criterion for the long-wavelength mode. The mode amplitude is limited by saturation of the feedback amplifiers.

These effects are illustrated in Fig. 21, which shows the model calculations for feedback without amplifier saturation. For large α the existence of an instability is proven by its visible growth (as in Fig. 21). For small α (for example, $\alpha = 1.5$), amplifier saturation does not change the motion; therefore, the modes are either weakly damped or weakly growing.

These short-wavelength, unstable modes should be easy to eliminate by using an analog processor to extract mode information from position-detector

Fig. 15.

The average plasma trajectory in the horizontal plane obtained from the central position detectors, and the normalized total $l = 2$ field provided by equilibrium windings and feedback.

data and using modal rather than local feedback control. Although the modal calculations show that this nonlinearly saturated instability does not seriously affect the confinement (that is, keeping the plasma off the wall), in practice the oscillatory signals led to increased energy consumption and an increased probability of feedback amplifier tubes arcing. The achievable energy-consumption level determines the size of a feedback-stabilized, high-beta stellarator reactor (see Sec. II).

E. Helical Instability

An interesting effect sometimes observed involved the direct coupling of the feedback $l = 2$ field to the plasma $l = 1$ helical shape. The coupling was produced by a substantial off-axis shift of the plasma. Data are shown in Fig. 22. The solid curve gives the plasma position for the two orthogonal coordinates (at $\pm 45^\circ$) labeled M and J. The M coordinate shows a large displacement, which produced saturation of the position detector. The J coordinate shows the plasma oscillating at the frequency of the natural helical oscillation $\omega_1 \cong 2 \mu s^{-1}$, evidently being driven by the feedback system.

An analysis based on small- δ , sharp-boundary theory leads to the following equation of motion for the helical displacement of the plasma:

$$\frac{\partial^2}{\partial t^2} \delta + \omega_1^2 \delta + \zeta \frac{\partial \delta}{\partial t} = \frac{2\omega_1^2 \xi_x G \sqrt{2}}{B_0 h a (2 - \beta)} \left(\delta + \tau_1 \frac{\partial \delta}{\partial t} \right)_{t-\tau} \quad (21)$$

where the plasma shape is given by

$$r = a[1 + \delta \sin(\theta - hz) + \dots]$$

The symbols are as previously defined except for ζ , which gives the damping of the helical oscillations that occurs without feedback ($1/\zeta \cong 2.5 \mu s$). The condition for instability from Eq. (21) is

$$\xi_x > \frac{\zeta B_0 h a (2 - \beta)}{2 \sqrt{2} \omega_1 G (\omega_1 \tau_1 \cos \omega_1 \tau - \sin \omega_1 \tau)} \cong 0.7 \text{ cm} \quad (22)$$

That is, if the plasma is displaced far enough in the x-direction, the natural helical oscillation will be destabilized by the feedback system and the oscillations will grow until limited by saturation of the feedback amplifiers.

The practical importance of this phenomenon was the increased tendency toward tube-arcing. The feedback system would still respond in a time-average sense to changes in plasma position. Since the feedback system moves the plasma to the axis, the inequality (22) would not in proper operation be satisfied over a long period of time.

HORIZONTAL PLANE

SHOT 5480

$\tau_1 = -3 \mu s$

40 μs

SHOT 5466

$\tau_1 = 3 \mu s$

40 μs

—— POSITION
- - - - CURRENT

Fig. 16.

Plasma behavior with feedback and the velocity gain reversed, and with the correct polarity.

Because of the geometrical arrangement of the position detectors, the instability depends on shifts in the horizontal plane and involves the $\sin(\theta - \text{hz})$ component of the helical displacement. However, without having two detectors one-half wavelength apart, this type of positive feedback can exist regardless of where in a wavelength the position detector is placed, for a shift in some direction.

VII. CONCLUSIONS

The toroidal high-beta stellarator equilibrium is in good quantitative agreement with recent diffuse-profile theory. This theory also carries over to the experimentally important gross mode and provides an analytic expression for the growth rate.

The toroidal equilibrium has an initial dynamical phase associated with the approach of the plasma helical deformations to equilibrium. Of greatest importance is the plasma bumpiness δ_0 , the dynamic behavior of which was determined experimentally from excluded flux measurements. An initial δ_0 transient is present which, in agreement with theory, can be correlated with a directly measured inward force on the plasma occurring at the same time.

The important loss mechanisms in the sector are resistive diffusion and, possibly, particle endloss, with beta decreasing significantly during the time of the experiment.

Feedback stabilization is shown to increase the confinement time to the limit imposed by the sector

Fig. 17.

Observed plasma behavior with low and high feedback gain.

Fig. 18.

A model calculation assuming only the gross displacement mode. The effect of varying the feedback gain is shown.

Fig. 19.

A model calculation assuming longitudinal modes, showing the plasma trajectories at four evenly spaced points around a torus for low and high feedback gain.

configuration, many growth times of the $m = 1$ instability. An important drawback of local feedback control has been identified in that normally stable short longitudinal wavelength modes can be driven unstable by feedback. Local control results in $m = 1$ "turbulence" produced by unstable or weakly damped short wavelength modes. A feedback driven instability involving direct coupling of the feedback $\ell = 2$ field to the plasma helical motion is also possible.

Because of the complicated physics of an open-ended sector and some essential differences between

Fig. 20.

Plasma trajectories and feedback currents. Short longitudinal wavelength $m = 1$ motion of the plasma is evident.

Fig. 22.

Plasma position and feedback current vs time showing feedback-driven helical instability.

Fig. 21.

A model calculation giving the plasma trajectories around a torus for high gain and no amplifier saturation. A short longitudinal wavelength $m = 1$ instability produced by feedback is apparent.

a sector and a torus, decisive proof of feedback stabilization is not possible without a toroidal experiment that has a longer confinement time than that possible in a sector. For a torus, modal rather than local feedback control could be implemented in a natural way, since the longitudinal modes are well defined. Such an experiment, although large and technically complex, would be relatively definitive because axial particle and thermal losses and other end effects could be eliminated from consideration. The overall conclusion from this work is that feedback-stabilized, high-beta stellarator physics appears favorable at this stage but that the work should naturally progress to a complete torus experiment.

For a fusion reactor to be possible, the feedback power consumption must be sufficiently small. The scaling of feedback power with toroidal radius is favorable, and therefore, from the present standpoint, a fusion reactor seems possible for a sufficiently large radius. In present-day terms,

however, the necessary feedback power capability is enormous, and the desirability of a feedback-stabilized, high-beta stellarator fusion reactor thus depends on technical development of simple and efficient high-power amplifiers.

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APPENDIX A

$\ell = 0$ EQUILIBRIUM MODEL*

The plasma equilibrium in the presence of an $\ell = 0$ field was calculated by using the long-wavelength approximation, neglecting field-line curvature effects. Thus theta-pinch pressure balance relation $p + B^2/2 = B_e^2/2$ could be used, where the external field is a function of z , $B_e(z) = B_0[1 + b_0 \cos(hz)]$. The relation $\nabla p = \mathbf{j} \times \mathbf{B}$ shows that the pressure is constant in a flux tube (see Fig. A-1). In addition, the temperature was assumed constant in a flux tube because of the good thermal conductivity along field lines.

The number of electrons or ions in a flux tube, averaged over a wavelength L of the $\ell = 0$ field, is given by

$$\frac{\partial N}{\partial \phi} = n \int \pi \frac{\partial R^2}{\partial \phi} \frac{dz}{L}, \quad (\text{A-1})$$

where ϕ is the flux, $n(\phi)$ is the electron or ion density, and the function $R(\phi, z)$ gives the radius of a flux surface. If $\partial N/\partial \phi$ and the entropy per particle s are specified as functions of ϕ , the equilibrium pressure profile can be obtained by solving the equation $p = 2n^\gamma \exp[s(\gamma - 1)]$ and the pressure balance relation for $p(\phi)$; that is, solving the equations

$$p = \frac{2\left(\frac{\partial N}{\partial \phi}\right)^\gamma}{\left(\int \pi \frac{\partial R^2}{\partial \phi} \frac{dz}{L}\right)^\gamma} \exp[s(\gamma - 1)],$$

$$\frac{\partial R^2}{\partial \phi} = \frac{1}{\pi \sqrt{B(z)^2 - 2p}}, \quad (\text{A-2})$$

Fig. A-1.
Geometry of a flux tube.

*A system of units with $\epsilon_0 = \mu_0 = c = 1$ is used.

where γ is the ratio of specific heats.

The evolution of the equilibrium was calculated from the equations describing the evolution of a straight theta-pinch equilibrium.

$$\frac{\partial}{\partial t} \frac{\partial N}{\partial \phi} \bigg|_{\phi} = -\frac{n}{B} \frac{1}{r} \frac{\partial}{\partial r} \left(r \eta \frac{\partial B}{\partial r} \right) - \frac{\eta}{B} \frac{\partial B}{\partial r} \frac{\partial}{\partial r} \left(\frac{n}{B} \right),$$

$$\frac{\partial s}{\partial t} \bigg|_{\phi} = \frac{\eta \left(\frac{\partial B}{\partial r} \right)^2 + \frac{1}{r} \frac{\partial}{\partial r} \left(r \kappa \frac{\partial T}{\partial r} \right)}{p} - \frac{\partial s}{\partial r} \frac{\eta}{B} \frac{\partial B}{\partial r}, \quad (\text{A-3})$$

where $T = T_e = T_i = p/(2n)$. Classical values were used for the resistivity η and the thermal conductivity κ . The thermal force is small and can be neglected. A fall-off of the main field was also assumed, that is, $B_0 \rightarrow B_0 \exp(-t/\tau)$.

With an $\ell = 0$ field, the right-hand sides of Eqs. (A-3) are functions of z and were evaluated at $hz = \pi/2$, which is intermediate between the field maximum point, $hz = 0$, and the field minimum point, $hz = \pi$. The effect of evaluating the right-hand sides at a different location in a wavelength was fairly small.

Results for the pressure distributions at the initial time and $t = 40 \mu s$ are shown in Figs. A-2 and A-3. The $1/e$ radius, β on axis and excluded flux versus time are shown in Figs. 7 and 8. The initial particle and entropy distributions correspond to a constant-temperature Gaussian pressure profile with the $\ell = 0$ field turned off.

When the $\ell = 0$ field is switched on, the magnetic field pressure at the field minimum point decreases and the plasma bulges. The bulging becomes more extreme and more localized as $\beta_0 \rightarrow 1$. The criterion that allows neglect of curvature terms, $\partial B_z / \partial r \gg \partial B_r / \partial z$, translates roughly into $\Delta z / a \gg 2/\beta$, where Δz is the distance over which the bulging occurs and a is the plasma characteristic radius. Because ha is small ($ha \cong 0.1$), this rough criterion usually is satisfied easily except for β_0 very near 1. The initial configuration shown in Figs. A-2 and A-3 corresponds to $\beta_0 = 0.95$ and is marginal. Curvature effects might be important near the bulge. For lower β_0 , for example $\beta_0 = 0.9$, the bulging is much less localized and curvature effects are probably small.

Fig. A-2.

Calculated radial β profiles at the field maximum ($hz = 0$) and minimum ($hz = \pi$) locations at two times.

Fig. A-3.

Calculated shapes of a family of flux surfaces at two times.

The maximum average beta in the presence of an $\ell = 0$ field is $\beta_{\max} = (1 - b_0)^2$. Small- δ theory allows all β values up to 1; nevertheless, it probably applies at least qualitatively if the limit $\beta \rightarrow 1$ is reinterpreted as $\beta \rightarrow \beta_{\max}$, because the plasma $\ell = 0$ response becomes large as $\beta \rightarrow \beta_{\max}$, just as it does in the small- δ theory for $\beta \rightarrow 1$.

APPENDIX B

$l = 0$ DYNAMICAL MODEL*

A simple dynamical model was used to describe the plasma motion in the presence of an $l = 0$ field. The plasma is assumed incompressible and the sharp-boundary approximation is used. For a long-wavelength $l = 0$ field, the equation of motion for the axial plasma velocity $v_z(z, t)$ is

$$\rho \frac{\partial v_z}{\partial t} = -\frac{1}{2} \frac{\partial}{\partial z} (B_{out}^2 - B_{in}^2), \quad (B-1)$$

where ρ is the constant plasma density, B_{out} the external magnetic field,

$$B_{out} = B_0 [1 + b_0 \cos(hz)], \quad (B-2)$$

and B_{in} the magnetic field inside the plasma obtained from the plasma radius $R(z, t)$ using flux conservation,

*A system of units with $\epsilon_0 = \mu_0 = c = 1$ is used.

$$B_{in} = B_0 \sqrt{1 - b_0 \frac{a^2}{R^2}}. \quad (B-3)$$

A term $-\rho v_z/\tau$ is added to the right-hand side of Eq. (B-1) to provide damping. The final equation, which relates R to v_z , is obtained from the incompressibility condition $\nabla \cdot \mathbf{v} = 0$, which gives

$$\frac{\partial R}{\partial t} = -\frac{R}{2} \frac{\partial v_z}{\partial z} - \frac{\partial R}{\partial z} v_z. \quad (B-4)$$

For small b_0 these equations reduce to the correct dynamical equation for δ_0 (Ref. 8) for the incompressible case $\gamma \rightarrow \infty$. With damping, the dynamical solution approaches in time the equilibrium configuration calculated from the model described in Appendix A for a sharp-boundary profile and $\gamma \rightarrow \infty$.

APPENDIX C

FEEDBACK WITH LONGITUDINAL MODES

In a torus the plasma $m = 1$ displacement, $\xi(z)$, can be written

$$\xi(z) = \sum [a_n \cos(k_n z) + b_n \sin(k_n z)], \quad (C-1)$$

where $k_n = n/R$. The dynamical equation for the coefficients a_n and b_n was assumed to be

$$\ddot{a}_n = -\omega_n^2 a_n + f_n, \quad (C-2)$$

where

$$\omega_n^2 = (2 - \beta) v_A^2 k_n^2 - \gamma^2,$$

with γ the $m = 1$ instability growth rate for $n = 0$. The feedback force, f_n , was defined as the Fourier component of the force as a function of z ,

$$f_n = \int F_{1,2}(z) \cos(k_n z) \frac{dz}{\pi R}, \quad (C-3)$$

with the cosine replaced by sine for the b_n equation. The $F_{1,2}$ force is given by Eq. (2) in terms of the feedback $l = 2$ field.

The number of helical periods around the torus is hR . It is reasonable to assume that the feedback force is given by Eq. (C-3) only for $n \ll hR$. In the calculations the maximum n was taken as $hR/2$.

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