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Appropriate Sizing of Solar Heating Systems

Paul Bendt

SERI

Solar Energy Research Institute

A Division of Midwest Research Institute

1617 Cole Boulevard
Golden, Colorado 80401

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PAUL BENDT

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PREFACE

This report documents work done under task 3473.99 in the Solar Thermal Conversion Branch of the Solar Energy Research Institute.

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SUMMARY

It is generally assumed that a solar heating system should be sized by minimizing its life cycle cost. This study shows, however, that the uncertainty in future economic trends makes the results of such a procedure questionable. The design conditions for minimum cost are extremely broad and all practical systems have a solar fraction within the limited range of 30% to 90%. Thus, by choosing only three collector areas that give systems within this range, one is assured of selecting a nearly optimal system for any realistic economic scenario. Selecting one of these three systems is essentially equivalent to economic optimization, but simpler. Procedures are derived in this paper for determining the sizes of the three systems. The conclusion is that the collector areas should be about $1/8$, $1/5$, and $1/3$ of the building floor area. This rule of thumb eliminates the need to design solar systems individually, allowing the possibility of mass-produced homes with standardized solar heating systems.

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SECTION 1.0

INTRODUCTION

Choosing the collector size of a solar heating system is a more important problem than sizing a conventional heating system. For a conventional system, the primary cost is that of fuel, and the amount of fuel required to meet a given load does not depend on the furnace size. As long as the furnace is large enough to supply the load, its size is unimportant. The standard procedure is to install an oversized system that, much of the time, will be off, burning no fuel and costing nothing.

In contrast, for a solar system the primary cost is the initial capital investment (or the mortgage payment thereon) and this must be paid whether the system is on or off. Hence, it is economically advantageous to have a system that is operating most of the time. Accurate knowledge of the load and the performance of the collectors is therefore necessary. It is then possible to design a system that will supply the desired amount of heat without being excessively large or expensive. The present report addresses this problem of system sizing.

It is generally not practical to build a solar heating system that supplies 100% of the desired heat. A system large enough to handle the most severe weather would not be used at full capacity most of the time. It is cheaper to build a smaller solar system supplemented with a conventionally fueled furnace. A balance must then be found between the incremental expense of a larger solar system and the expense of fuel. This is an economics optimization problem.

The term "optimum" is frequently used and misused. Technically, the "optimum system" is defined as the one design, incorporating both solar and conventional heaters, that supplies the required heat for minimal total cost. It is often assumed or implied that building any other system amounts to wasting money. If, however, the additional cost of a nonoptimal system is small, an individual may choose to pay that difference for other benefits, such as architectural aesthetics, comfort, or security of energy supply. As shown in this report, there is substantial freedom in system design with very small added costs and thus these benefits become an important consideration. Although the "optimum system" is well defined mathematically it is not the only system worth building. It should serve only as a guide to indicate whether or not a particular design is reasonable or completely unreasonable.

The sizing problem is further complicated by economic uncertainties such as future fuel costs or collector maintenance costs. By making different assumptions about these costs, the deduced optimum systems differ tremendously, and thus economic optimization can be little more than an academic exercise.

Despite such uncertainties, architects designing solar heating systems need to be able to select appropriate sizes. This paper presents a rational approach to the task, using economic analysis but also recognizing its problems and its inability to include all considerations. The approach leads to a simple rule of thumb, and after this rule is derived it can be applied without concern for

economics, optimization, or their attendant problems. This paper presents the derivation of the rule. In addition, a nontechnical booklet has been prepared at the Solar Energy Research Institute (Bendt and Soto 1980) to enable anyone to effectively use the rule without necessarily understanding the technical derivation.

The discussion starts by considering classical economic optimization as applied to solar heating systems. An example is used to illustrate the new approach and to derive representative values for important quantities. The next three topics are quantitative calculations of the impact of economic uncertainties, the extra cost of building a nonoptimal system, and the limits of expected variations. With these three considerations, one can then derive a sizing procedure for the sample problem that is simpler than the classical economic optimization. The next two sections generalize the procedure so that it covers different systems and climates, and the procedure is finally expressed as a rule of thumb. The report finally shows that the proposed rule of thumb is a satisfactory sizing procedure for a very wide range of conditions.

SECTION 2.0

ECONOMIC OPTIMIZATION OF A SOLAR HEATING SYSTEM

The next five sections of this paper treat a particular installation and derived values that refer to it specifically. The text, however, is generally applicable to all systems, and at the end of the paper the values are generalized to apply to any building, location, collector type, and collector orientation.

Suppose one wishes to size a solar heating system for the collectors and the home described in Table 2-1. As discussed by Beckman, Klein, and Duffie (1977), some components such as the storage tank, collector heat exchanger, and pumps should be sized proportional to collector area. Thus, by sizing the collector, one is implicitly sizing these components as well. Beckman et al. finds that other components, such as the load heat exchanger, should be sized proportional to the house heat loss. Since they give rules of thumb for sizing these other components, this issue is not addressed here. The size of the collector and the components proportional to it is addressed; thus, the one quantity, collector area (A in m^2), completely specifies system size.

The f-chart method (Beckman et al. 1977) was used to analyze the performance of a solar home described in Table 2-1, and the results are plotted as the points in Fig. 2-1. These points were then fit to the functional form $f = a \ln(1 + A/b)$ using nonlinear least squares to find a and b . The resulting equation

$$f = 0.372 \ln(1 + A/6.83) \quad (1)$$

is also plotted in Fig. 2-1 and gives a good fit to the data points.

The present value of the cash stream for installing, operating, and maintaining the heating system is approximately proportional to the initial cost; thus

$$\langle C_s \rangle = P_s (2000 + 200 A) \quad , \quad (2)$$

where $\langle \rangle$ denotes present value and P_s is the proportionality constant obtained from economic analysis. Similarly, the present value of fuel costs is

$$\langle C_f \rangle = P_f [600(1 - f)] \quad . \quad (3)$$

The total life cycle cost (LCC) is

$$\langle C \rangle = \langle C_s \rangle + \langle C_f \rangle = P_s (2000 + 200 A) + 600 P_f [1 - 0.372 \ln(1 + A/6.83)] \quad . \quad (4)$$

The minimum occurs where the derivative with respect to A vanishes:

$$0 = \frac{d\langle C \rangle}{dA} \Big|_{A^*} = 200 P_s + 600 P_f \frac{-0.372}{A^* + 6.83} \quad ,$$

or

$$A^* = 1.12 P - 6.8 \quad , \quad (5)$$

where

$$P = P_f / P_s$$

and the solar fraction at this optimum, f^* , is given by

$$f^* = 0.372 \ln(0.164 P) \quad . \quad (6)$$

Table 2-2 shows A^* , f^* , and relative cost $\frac{\langle C \rangle}{600 P_f}$ for different values of P .

Table 2-1. SPECIFICATIONS FOR SAMPLE SOLAR HEATING SYSTEM

Location	Boulder, Colorado
Collectors	
Type	Single glazed, selective absorber, water
$F_r \alpha$	0.8
$F_r U_L$	7 W/C m ²
Size	1 m ² each
Cost	\$200 each
Storage size	75 L of water/m ² of collector
Load heat exchanger; ϵU_{min}	600 W/C
House loss	300 W/C
Domestic water heating load	55 MJ/day
Cost of pumps, heat exchangers, ducting, etc.	\$2,000
Total heating load	100 GJ/yr
Collector orientation	South
Collector tilt	55 degrees
Payments	
Down payment	10%
Term of mortgage	20 yr at 10% interest
Annual payments	0.1041 $\times$ initial cost
Cost of conventional fuel	\$6/GJ
Fuel inflation rate	12%
General inflation rate	8%

Figure 2-1. Solar Fraction vs. Size for System Described in Table 2-1

Table 2-2. OPTIMUM SOLAR SYSTEM SIZES AND BENEFITS FOR VARIOUS ECONOMIC FUTURES

P	A* (m ²)	f*	<C>/600 P _f	Comment
5	0	0	1.0	no solar system
7	1	0.05	1.47	solar system loses \$
10	4	0.18	1.29	solar system loses \$
15	10	0.33	1.11	solar system loses \$
20	16	0.44	0.98	solar system saves \$
30	27	0.59	0.81	solar system saves \$
50	49	0.78	0.61	solar system saves \$
70	72	0.91	0.48	solar system saves \$
100	105	1.00	0.38	solar system saves \$

SECTION 3.0

REASONABLE ECONOMIC SCENARIOS

Table 3-1 presents three different economic scenarios that might be used to compute the LCC of this system. The third scenario assumes a long system life, which is achieved by periodic refurbishment of the system. As can be seen from the resulting values of P , the choices give very different results. For the first case, a solar system is completely impractical; for the second, it is marginal; and for the third, it is overwhelmingly beneficial. Of greater concern to the sizing problem is that the "optimum" size varies from no solar system to a 67 m², \$15,000+ system, which is quite a range. But how is one to choose an economic scenario when this involves predicting the future--items like fuel costs and life expectancy? A scenario like the second is usually chosen, but there is no justification for it except that the result, marginally economic solar systems, seems intuitively correct.

Table 3-1. SPECIFICATIONS OF POSSIBLE ECONOMIC SCENARIOS

	Scenario		
	1	2	3
Fuel inflation rate	10%	11%	12%
Operation and maintenance (O&M)			
inflation rate	10%	8%	8%
Discount rate	12%	10%	8%
Life expectancy	10 yr	20 yr	50 yr
Annual O&M			
(% of system)	1%	1%	1%
Refurbishment	none	none	20% of system every 10 years
Mortgage:			
Down payment	100%	10%	10%
Terms	cash	20 yr @ 10%	20 yr @ 10%
P_s	1.092	1.167	2.419
P_f	9.16	22.1	160.0
P	8.4	19.0	66.0
A^*	3 m ²	14 m ²	67 m ²
f	0.14	0.41	0.86
$\langle C_s \rangle$	\$2,840	\$5,600	\$37,250
$\langle C_f \rangle$	\$4,730	\$7,690	\$13,440
$\langle C \rangle$	\$7,570	\$13,290	\$50,690

SECTION 4.0

PERMISSIBLE VARIATION ABOUT OPTIMUM DESIGN

Once an economic scenario is chosen, the preceeding analysis can be used to obtain the size of the minimum LCC system. It is seldom possible, however, to build a system exactly this size because of architectural constraints and the fact that components are available only in standard sizes. The allowable degree of deviation from the optimal design depends on how much one is willing to lose as a result of building a system that is not optimal. Since the curve of LCC vs. size is quadratic in the vicinity of the minimum, small variations cause negligible losses. A simple criterion that is used here is that the total LCC shall not be more than 5% above the minimum possible. This is actually a very restrictive condition because the uncertainty in the total LCC can be almost an order of magnitude, as can be seen from the bottom line of Table 3-1. Using this criterion, one can calculate a range of nearly optimal system sizes, which is done in Table 4-1 for scenarios 2 and 3. (Scenario 1 is deleted because it is unfavorable for solar energy.) As can be seen, the ranges are quite large. It is easier to express them as ranges in f rather than ranges in size, and the acceptable range in f is about ± 0.15 from the optimal design.

Let f_l^* and f_u^* represent the lower and upper limits of the range of nearly optimal system sizes; then

$$f_l^* = f^* - 0.15 = 0.372 \ln(0.164 P) - 0.15 \quad (7a)$$

and

$$f_u^* = f^* + 0.15 = 0.372 \ln(0.164 P) + 0.15 \quad (7b)$$

The corresponding collector areas are

$$A_l^* = 0.75 P - 6.8 \quad (8a)$$

and

$$A_u^* = 1.68 P - 6.8 \quad (8b)$$

The asterisk indicates that these values depend on the economic scenario; that is, they are functions of P . These bounds are not exactly the same as the 5% excess cost criterion, although they are close. For reasonable economic scenarios, in fact, these bounds are slightly more strict. Throughout this paper, the term "nearly optimal system size" refers to this range.

Table 4-1. RANGE OF NEARLY OPTIMAL SYSTEM SIZES

	Scenario 2			Scenario 3		
	min	opt	max	min	opt	max
A (Size)	5 m ²	14 m ²	27 m ²	40 m ²	64 m ²	94 m ²
f	0.21	0.42	0.59	0.71	0.86	0.98
Δf	0.21		0.17	0.15		0.12

SECTION 5.0

LIMITING RANGE FOR OPTIMUM SYSTEM

5.1 MINIMUM

For an optimally sized solar system, the marginal cost of solar energy equals the cost of auxiliary energy. For such a system to be economical, the average cost of solar energy must be less than the cost of auxiliary energy:

$$\text{average cost of solar energy} < \text{cost of auxiliary energy} = \text{marginal cost of solar energy};$$

that is,

$$\frac{\langle C_s \rangle}{f} < \frac{d\langle C_s \rangle}{df} = \frac{d\langle C_s \rangle}{dA} \left(\frac{df}{dA} \right)^{-1},$$

or

$$\frac{2000 + 200 A}{0.372 \ln(1 + A/6.83)} < 200 \frac{A + 6.83}{0.372} \quad (9)$$

Solving the equality in Eq. 9 for the area gives $A = 14 \text{ m}^2$, and the inequality is satisfied for $A > 14 \text{ m}^2$. This condition corresponds to $f \geq 0.42$, and it is never desirable to build a system smaller than this. If a solar system is economical with $f < 0.42$, then building an $f = 0.42$ system is even more so; and if an $f = 0.42$ system is not economical, then neither is any smaller system. With different costs and locations, this minimum varies from $f = 0.3$ to 0.5 , but in no situation is an active system with $f \leq 0.3$ acceptable.

5.2 MAXIMUM

At the other extreme, it is not practical to build a system with $f > 0.9$ unless either an annual-cycle storage scheme is used or the indoor temperature is allowed to drop during long storms. The second alternative is essentially equivalent to designing an $f = 0.9$ system in which auxiliary energy is simply not supplied when the solar storage cannot maintain the thermostat temperature.

5.3 ACCEPTABLE RANGE

When these results are combined, one can see that for a solar heating system to be economical under any reasonable economic scenario, it should deliver between 0.3 to 0.9 of the total load. This range, $0.3 \leq f \leq 0.9$, shall be called the range of "acceptable" system sizes, meaning that systems outside this range are not acceptable by any reasonable economic criteria.

SECTION 6.0

SMALL/LARGE SIZING PROCEDURE

The results from the last two sections show that: (1) an assumed economic scenario gives a nearly optimal range of ± 0.15 about a central value, a range 0.3 wide; and (2) the acceptable range of sizes corresponds to $0.3 \leq f \leq 0.9$, a range 0.6 wide. Since the width of the nearly optimal range is a significant portion of the width of the acceptable range, calculating the precise mathematical "optimum" becomes an academic exercise. For example, if a cost/performance analysis was performed for three systems with $f = 0.4, 0.6$, and 0.8 , one of these systems would be within ± 0.1 of the optimum given by any scenario and thus well within the nearly optimal range. The entire process of economic analysis and optimization thus reduces to choosing one of these three sizes.

Since there are now only four distinct choices (three sizes plus the option of no solar system), it is quite easy to give the consumer a performance and cash flow analysis for each and allow him or her to choose the system that seems most appropriate. At first it may appear that this requires the consumer to make a value-based decision that was not needed in the classical economic optimization approach, but this is not so. In the classical procedure, the consumer must specify such quantities as the economic life and discount rate. The specification of economic life is a statement of how far into the future one is willing to accept the return on an investment made now; and the choice of discount rate indicates how much one is willing to pay now to receive a dollar at a future time. Such considerations involve value judgments. Most consumers are not economists and do not understand the implications of a particular choice of economic life and discount rate. However, if the consumer is offered a choice of four cash flow sequences, the options are easier to understand, the value judgments explicit, and the decision easier to make.

SECTION 7.0

EXTENSION TO OTHER CLIMATES

Thus far the discussion has focused on a particular sample installation in a particular climate. This section generalizes these results to different climates, and the next section generalizes to different installations.

In Sec. 5.0 a minimum system was derived for which the average and marginal costs of delivered energy are equal. An upper practical limit of $f = 0.9$ was set, but in many climates an $f = 0.9$ system cannot be justified economically. Another upper limit can be obtained by choosing the economic scenario that is most favorable for a solar system and still justifiable. This limit corresponds to large P , and a justifiable limit on P is perhaps $P = 50$. Thus the upper limit becomes the lesser of $f = 0.9$ or $P = 50$. Table 7-1 shows the systems corresponding to these lower and upper limits in three different climates.

Table 7-1. ACCEPTABLE SYSTEM SIZE RANGES IN DIFFERENT CLIMATES

	Albuquerque	Boulder	Seattle
$A_L = \text{min size (m}^2\text{)}$	11	14	19
$A_U = \text{max size (m}^2\text{)}$	39	49	41
$f_L = \text{min solar fraction}$	0.51	0.42	0.34
$f_U = \text{max solar fraction}$	0.90	0.78	0.53

As can be seen, the range of the solar fraction varies substantially with climate, but the range of collector areas does not. This range of areas will be called the range of "reasonable" system sizes. The variables do not have asterisks, which indicates that they do not depend on a particular economic scenario. The total range is from 11 to 47 m^2 and a complete appreciation of this range can be obtained by examining systems of 15, 25, and 40 m^2 ; these three sizes appropriately cover the range in each location.

It may at first be puzzling that the acceptable range of system sizes is approximately the same in sunny climates like Albuquerque and cloudy climates like Seattle, but this can be easily explained. In a sunny climate, an additional square meter of collector area receives a large amount of energy. Hence, it is advantageous to increase collector size until the curve of f vs. A_c levels off at around $f = 0.8$. In contrast, in a cloudy climate the marginal utility of additional collector area is small because the energy incident upon it is small. It is therefore preferable to stay in the steep region of the f curve, around $f \approx 0.4$. In an intermediate climate, such as Boulder, an intermediate value, such as $f = 0.6$, is expected. If an $f = 0.6$ system is sized for Boulder and then built in Seattle, it will deliver less energy, about $f = 0.4$, because there is less sun in Seattle. Conversely, if the same system is installed in Albuquerque, where the climate is favorable for solar

systems, the system will perform better, around $f = 0.8$. Thus the desired system size is, to first order, independent of climate.

This discussion should not be interpreted as saying that a solar system is equally practical in all locations. A system in Albuquerque will deliver more heat and save more fuel than the same system in Seattle, but the range of reasonable system sizes is similar in both climates.

Similarly, collector orientation will make a substantial difference in the amount of energy supplied by a solar heating system and an effort should be made to mount collectors at the optimum angle: due south with a tilt equal to the latitude plus twelve degrees. However, if it is not possible to optimally orient the collector, the system must still be sized. An examination of systems with orientations as far as 60° from south and tilts from 15° to 90° shows that performance may drop by a factor of two but the range of reasonable system sizes does not change. This is not surprising in view of the climate independence discussed above. A poorly oriented collector is formally equivalent to an optimally oriented collector in a less sunny climate.

Precisely the same arguments apply to collector type. One will find that, depending on location and cost, different collector designs are most economical, but the reasonable range of system size does not change. Thus, different collector types should be compared assuming a fixed system size.

SECTION 8.0

RULE OF THUMB

For the particular house studied, the previous section showed that the reasonable system size was between 15 and 40 m², independent of climate, orientation, and other factors. To extrapolate to different houses, consider first the situation of two identical, neighboring houses described by Table 2-1 and each heated by solar energy. If these houses are considered together as one system, this system is described as having twice the heat loss rate, twice the collector area, and twice as much energy delivered by solar energy. Hence the solar fraction and the economic feasibility are the same as for the single house. Thus, if the collector area is scaled proportionally to the heat loss rate, all the economic considerations remain the same. The house described in this study has a loss rate of 300 W/C, so the range of practical system sizes is about 0.05 m²/(W/C heat loss) to 0.13 m²/(W/C). These values scale the range of sizes so that they apply to any building size. As indicated in Sec. 6.0, it is most useful to examine three system sizes for a cost/performance comparison, so a system of intermediate size should be included. The recommended sizes are given, in metric and English units in Table 8-1.

Table 8-1. RECOMMENDED COLLECTOR AREAS

Units	Small	Medium	Large
m ² per W/C of building heat loss	0.05	0.08	0.13
ft ² per Btuh/F of building heat loss	0.3	0.45	0.75

Assuming a well-insulated house with a heat loss rate of about 0.4 Btuh/F per square foot of floor area, or 2.3 W/C m², we can now derive a very useful rule of thumb: the collector sizes that should be considered are about 1/8, 1/5, and 1/3 the floor area of the house. From here, there are many methods available to estimate performance of these systems and to predict cash flow. The consumer can then compare the options to make a decision about how large a solar heating system to install.

This rule of thumb allows designers of tract homes to plan homes with each of these system sizes and to build identical houses in different locations without designing each solar heating system individually.

SECTION 9.0

VALIDATION

In any validation, there are two critical questions. First, against what is this method to be compared? And second, what is to be the measure of comparison? Validation of this rule of thumb is complicated by the fact that it gives the purchaser the option of making choices based on intangible benefits. Therefore we must also ask: how is the choice between system sizes to be made? There are many possible procedures one can invent for validation.

Let us take a "worst case" that will test the rule of thumb most severely. First, assume the purchaser making the choice of sizes ignores all the intangibles and bases the decision only on life-cycle cost economics. Clearly, for this choice, the most severe comparison is against the LCC optimum, since by definition no method can do better when LCC is the only criterion for judgment. A simple and reliable measure of comparison is the excess cost of not having the option of selecting the "optimum" size. This excess cost is positive by definition, but a scale for judging the results is given in Sec. 4. A 5% excess cost is considered acceptable because it is small compared to the uncertainty in the LCC, which is typically a range of an order of magnitude because of uncertainties in future economics.

Having picked a procedure for validation, one must next choose the cases to be compared. We have selected more than 32,000 cases covering a wide range of systems, houses, locations, and economics. These include not only typical cases likely to be encountered in practice, but also a large number of extreme cases for which the method is likely to break down. The cases comprise every possible combination of values of the principal quantities shown in the first column of Table 9-1. It should be noted, however, that the scaling by house size (Sec. 8) is rigorous and hence only one size of house needs to be treated in the validation. Including different heat loss rates, however, violates the assumptions of scaling and one does not expect the method to be good in such a case.

In spite of the fact that the validation procedure and cases compared are strongly biased against the rule of thumb, this method compares very favorably with the results of an optimal LCC analysis. The average excess cost is 0.725%, much lower than the 5% which would have been judged acceptable. A histogram of excess cost for the 32,256 test cases is shown in Fig. 9-1. In approximately half of the cases considered the optimal solar system is none at all; these cases are therefore grouped separately. For these cases, the rule of thumb also will dictate no solar system and, hence, will carry no excess cost. Approximately 90% of the cases have an excess cost less than 1%, and the tail extending beyond 4% excess cost (approximately 4% of the cases) represents the "wild cases" composed of combinations of extreme values of the design parameters. It is also useful to have a breakdown of the histogram for particular values of the parameters; this breakdown is given in Table 9-1.

The rule of thumb is a simple, easy-to-use method that can include intangible benefits and avoids economic uncertainties. At the same time, this validation shows that the rule of thumb is also a very good method when judged by classical economic considerations.

Table 9-1. BREAKDOWN OF VALIDATION RESULTS BY PARAMETER

	Average Excess Cost (%)	No Solar System	Number of Cases										Total
			Excess Cost (%)										
			<0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	4.0-4.5	>4.5	
Location													
Albuquerque, N. Mex.	0.628	1404	1581	470	170	101	45	43	29	24	18	147	4,032
Atlanta, Ga.	0.438	2115	1153	317	107	76	59	17	34	25	16	113	4,032
Boulder, Colo.	0.859	1682	1459	363	99	80	36	59	21	25	9	199	4,032
Lincoln, Nebr.	0.756	1826	1436	305	114	48	49	6	20	21	22	185	4,032
Los Angeles, Calif.	1.637	1955	1140	223	90	97	58	33	33	33	18	352	4,032
Madison, Wis.	0.846	1852	1439	235	85	61	45	32	29	6	22	226	4,032
Medford, Oreg.	0.351	1968	1387	321	113	66	34	28	21	14	25	55	4,032
New York, N.Y.	0.131	2637	1120	135	23	22	13	35	15	0	5	26	4,032
Collector ^a													
1	0.351	2474	1600	193	75	61	30	30	13	18	14	100	4,608
2	0.176	3050	1203	170	58	37	24	18	9	7	3	29	4,608
3	1.409	1595	1732	451	135	106	53	51	24	19	27	415	4,608
4	0.635	2082	1615	357	111	73	67	51	32	34	16	170	4,608
5	0.559	2503	1292	331	116	87	44	29	37	25	18	126	4,608
6	0.955	1692	1785	449	156	93	68	35	39	13	28	250	4,608
7	0.916	2043	1488	419	150	94	53	39	48	32	29	213	4,608
Heat Loss													
200 W/C													
[1.5 (W/C)/m ²]	0.639	5839	2993	817	238	165	101	78	67	48	44	362	10,752
300 W/C													
[2.3 (W/C)/m ²]	0.610	5134	3671	807	306	143	121	64	61	52	32	361	10,752
400 W/C													
[4.0 (W/C)/m ²]	0.859	4466	4051	745	257	243	117	111	74	48	59	580	10,752
Hot Water													
Not included	0.816	8429	4688	1038	401	267	187	112	134	80	78	714	16,128
90 MJ/day	0.635	7010	6027	1332	400	284	152	141	68	68	57	589	16,128

Table 9-1. BREAKDOWN OF VALIDATION RESULTS BY PARAMETER (Concluded)

	Average Excess Cost (%)	No Solar System	Number of Cases										Total
			Excess Cost (%)										
			<0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	4.0-4.5	>4.5	
Fuel Cost													
\$4/GJ	0.272	6031	1280	304	98	87	50	25	33	27	18	111	8,064
\$6/GJ	0.525	4154	2569	622	177	122	65	42	45	32	26	210	8,064
\$8/GJ	0.836	2983	3272	702	272	147	106	86	45	45	51	355	8,064
\$10/GJ	1.271	2271	3594	742	254	195	118	100	79	44	40	627	8,064
Solar Fixed Costs													
\$1000	1.249	2762	2995	728	329	24	148	100	103	69	67	552	8,064
\$2000	0.722	3628	2852	629	211	153	87	72	48	34	29	321	8,064
\$3000	0.513	4274	2576	549	144	109	59	45	26	28	20	234	8,064
\$4000	0.418	4775	2292	464	117	78	45	36	25	17	19	196	8,064
Economic Scenario													
P = 10	0.007	5347	15	3	3	2	1	1	1	1	0	2	5,376
P = 20	0.237	4260	677	154	54	56	36	18	20	20	12	69	5,376
P = 30	0.512	2717	1757	406	117	90	50	29	33	27	18	132	5,376
P = 40	0.794	1626	2477	532	197	116	77	53	35	33	25	205	5,376
P = 50	1.154	951	2874	597	185	142	84	73	61	32	25	352	5,376
P = 60	1.652	538	2915	678	245	145	91	79	52	35	55	543	5,376
TOTAL	0.725	15,439	10,715	2370	801	551	239	253	202	148	135	1303	32,256

^aCollector design:

Collector	F_{r,U_L}	$F_{r,\tau\alpha}$	$\$/m^2$
1	8	0.6	200
2	8	0.6	300
3	4	0.7	200
4	4	0.7	300
5	4	0.7	400
6	2	0.8	300
7	2	0.8	400

Figure 9-1. Histogram of Validation Results

SECTION 10.0

REFERENCES

Beckman, W.; Klein, S.; Duffie, J. 1977. Solar Heating Design by the f-Chart Method. New York: John Wiley & Sons.

Bendt, P.; Soto, R. 1980. Homeowner's Solar Sizing Workbook: A Simplified Method for Sizing Active Solar Space Heating Systems. SP-722-342. Golden, CO: Solar Energy Research Institute.

NOMENCLATURE

A	collector area in m^2 , representing system size
A^*	optimal collector area for a given economic scenario
A_l^*, A_u^*	lower and upper limits of the nearly optimal range of collector areas for a given economic scenario
A_l, A_u	lower and upper limits on the reasonable range of collector areas, independent of economics
$\langle C \rangle$	present value of total energy costs
$\langle C_f \rangle$	present value of auxiliary energy costs
$\langle C_s \rangle$	present value of solar system costs
f	solar fraction corresponding to a particular system size
f^*	solar fraction corresponding to A^*
f_l^*	solar fraction corresponding to A_l^*
f_u^*	solar fraction corresponding to A_u^*
f_l	solar fraction corresponding to A_l
f_u	solar fraction corresponding to A_u
$F_r \tau \alpha$	collector optical efficiency
$F_r U_L$	collector heat loss rate
$\ln$	natural logarithm function, base e
P_s	cash-flow-to-present-value conversion factor for solar system
P_f	cash-flow-to-present-value conversion factor for auxiliary fuel
P	ratio of P_f/P_s , a factor representing an economic scenario

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