A STUDY OF JET RATES AND MEASUREMENT
OF $\alpha_s$ AT THE $Z^0$ RESONANCE

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ABSTRACT

We present jet rates in hadronic decays of $Z^0$ bosons measured by the SLD experiment at SLAC. The data are analyzed in terms of the JADE and recently proposed Durham algorithms, and are found to be in agreement with similar measurements by the LEP experiments, and also with the predictions of perturbative QCD and fragmentation Monte Carlo models of hadron production. After correction for hadronisation effects the 2, 3 and 4-jet rates are well described by $O(\alpha_s^2)$ perturbative QCD calculations. From fits to the differential 2-jet distribution the strong coupling $\alpha_s(M_Z)$ is measured to be $\alpha_s(M_Z) = 0.119 \pm 0.002$(stat.) $\pm 0.003$(exp.syst.) $\pm 0.014$(theory) (preliminary). The largest contribution to the error arises from the theoretical uncertainty in choosing the QCD renormalisation scale.

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1. INTRODUCTION

The SLAC Linear Collider (SLC) produces electron-positron annihilation events at the $Z^0$ resonance which are recorded by the SLC Large Detector (SLD). The SLC/SLD project enjoyed a successful engineering run in 1991; in addition to commissioning the SLD, studies were made of the properties of hadronic decays of $Z^0$ bosons, resulting in preliminary measurements of the strong coupling $\alpha_s$. The first physics run began in February 1992, during which the SLC performance has continued to improve, routinely achieving $Z^0$ production rates of 10-20 per hour. Up to the end of July a sample of about 8000 $Z^0$s had been accumulated by the SLD; approximately 6000 of these events were used for the analysis presented here.

A major achievement of the 1992 run has been the delivery of an intense beam of longitudinally polarized electrons and the observation of decays of the resulting $Z^0$s. Details of the polarization production and measurement system are contributed to this conference, as well as preliminary measurements of the left-right cross section asymmetry. Detailed studies of the properties of hadronic decays of $Z^0$s produced by polarized and unpolarized electrons are also contributed to this conference.

In this analysis we study the multijet structure of the events and determine the strong coupling $\alpha_s$.

2. THE SLD

The detector is shown schematically in Fig. 1 and a detailed outline of its construction and performance is given in [1]. The micro-vertex detector (VXD) and Cherenkov Ring Imaging Detectors (CRID) were not used in this analysis, but are described in separate contributions to this conference.

Charged particles are tracked in the Central Drift Chamber (CDC) which consists of 10 superlayers, each containing 8 layers of axial or stereo sense wires. Tracking is extended to forward angles (10° from the beam axis) by endcap drift chambers. Momentum measurement is provided by a uniform axial magnetic field of 0.6T.

Particle energies are measured in the Liquid Argon Calorimeter (LAC), which contains both electromagnetic and hadronic sections, and in the Warm Iron Calorimeter which forms the outer layer of the hadronic calorimetry, and also provides tracking for muons. The LAC is segmented into approximately 40,000 projective towers and has a resolution of about 15% for the measured $Z^0$ mass. Luminosity is measured from the rate of small-angle Bhabha events detected in forward silicon-tungsten calorimeters mounted close to the beampipe.

#1 An electron beam polarization of ~ 22% has been achieved to date.
3. TRIGGERING AND DATA SELECTION

Two independent triggers were used for hadronic events: an energy trigger requiring a total LAC energy in excess of 8 GeV, and a charged track trigger requiring at least two well-separated tracks in the CDC. The trigger for Bhabha events required typically 10 GeV in both forward and backward luminosity monitors.

A loose selection of hadronic events was then made by two independent methods: one based on the topology of energy depositions in the LAC, the other on the number and topology of charged tracks measured in the CDC. After statistical subtraction of backgrounds, comparison of the number of hadronic events with the number of luminosity Bhabha events indicated a combined triggering and selection efficiency for hadronic events of better than 90% for the LAC method. 99% of the events identified by the CDC method were also identified by the LAC method. The residual contamination in this overlap sample was estimated to be mainly from $\tau$-pair events, calculated to be at the level of 0.3%.

The analysis presented here used charged tracks measured in the CDC. A set of cuts was applied to the data to select well-measured tracks and events well-contained within the detector acceptance. Tracks were required to have:

- a fit quality of $\sqrt{2}\chi^2 - \sqrt{2}N_{df} - 1 < 15$, where $N_{df}$ is the number of degrees of freedom for the track fit
- a closest approach to the beam axis within 10 cm, and within 20 cm along the beam axis of the nominal interaction point
- a polar angle, $\theta$, with respect to the beam axis with $|\cos \theta| < 0.80$
- a minimum momentum transverse to this axis of $p_\perp > 150$ MeV/c².

Events were required to have:

- a minimum of five such tracks
- no track with a momentum larger than 100 GeV
- a thrust axis direction, $\theta_T$, within $|\cos \theta_T| < 0.71$
- a minimum charged visible energy, $E_{vis}$, greater than 0.2$M_\pi$, where all tracks were assigned the charged pion mass.

After applying these cuts, distributions of track multiplicity, polar angle, momenta and event $\theta_T$ and $E_{vis}$ were reproduced by Monte Carlo simulations. 674 unpolarized and 3163 polarized events survived these cuts. For this analysis the unpolarized and polarized data samples were combined. The total residual contamination from background sources was estimated to be negligible.²

² Beam-related backgrounds are discussed in [4].
4. STUDY OF JET RATES AND MEASUREMENT OF $\alpha_s$

The measurement of jet production rates provides an intuitive way to determine the strong coupling $\alpha_s$, since, to first order in perturbative QCD, the rate of three jet events is directly proportional to $\alpha_s$. One possible way to define jets in events is the “JADE algorithm”, [11,12] in which pairs of particles are clustered together in an iterative procedure until the cluster-pair masses $M_{ij}$ satisfy

$$y_{ij} = \frac{M_{ij}^2}{E_{vis}^2} > y_{cut} \tag{1}$$

where $E_{vis}$ is the visible energy in the event. Cluster or particle pairs with $y_{ij} \leq y_{cut}$ are thereby recombined into a single cluster. The number of clusters remaining resolved at the end of the process is defined to be the jet multiplicity of the event.

The JADE algorithm has been widely used as a procedure for defining jets in both experimentally measured hadronic events[25–29] and in perturbative QCD calculations at the parton level,[13–17] allowing theory to be compared with experiment after taking account of hadronization effects. There is no fixed value for $y_{cut}$, rather, calculations and measurements are done as a function of $y_{cut}$. For the original JADE algorithm $M_{ij}$ is defined by

$$M_{ij} = 2E_iE_j(1 - \cos \vartheta_{ij}) \tag{2}$$

where $E_i$ and $E_j$ are the energies of the two particles or clusters $i$ and $j$ and $\vartheta_{ij}$ is the angle between them.

More recently a new jet algorithm has been introduced called the “Durham” (D) algorithm [18,19,20] based on a jet resolution criterion related to transverse momenta rather than to invariant masses:

$$M_{ij} = 2\min(E_i^2, E_j^2)(1 - \cos \vartheta_{ij}) \tag{3}$$

The analysis described below has been carried out with both algorithms as well as with the $E$, $E0$ and $p$ schemes[21] which are variations of the JADE algorithm. To second order in QCD perturbation theory, the $E0$ and the original JADE algorithm are equivalent.

The $n$-jet rates $R_n(y_{cut})$ reconstructed from the SLD data with the D algorithm are shown in Fig. 2 for the cases $n = 2, 3, 4, \geq 5$. The data were corrected by standard procedures (see eg.[12,22]) for the effects of initial state radiation, detector acceptance and resolution, analysis cuts, unmeasured neutral particles, decays of unstable particles and hadronization. The $n$-jet rates were corrected to the parton level by applying
bin-by-bin correction factors $C(i)$

$$R_{n,\text{corr.}}(i) = C_n(i) \times R_{n,\text{meas.}}(i),$$

where the correction factors for each bin $i$ are calculated by comparing M.C. at the parton level with M.C. after full detector simulation:

$$C_n(i) = \frac{R_{n,\text{parton}}(i)}{R_{n,\text{MC}}(i)}.$$ 

The Monte Carlo events with detector simulation were subjected to the same analysis criteria and cuts as the data. Also shown in Fig. 2 are the predictions of the JETSET 6.3\textsuperscript{[23]} and HERWIG 5.3\textsuperscript{[24]} perturbative QCD and fragmentation Monte Carlo programs, which are seen to be in agreement with the data.

$R_3(y_{\text{cut}})$ and $R_4(y_{\text{cut}})$ have been calculated to next-to-leading and leading order, respectively, in QCD perturbation theory\textsuperscript{[13-17]}. $R_2(y_{\text{cut}})$ is derived by applying the unitarity constraint $R_2 = 1 - R_3 - R_4$. The free parameters in the calculations are the QCD interaction scale $\Lambda_{\overline{M}S}$ and the renormalization scale factor $f = \mu^2/E_{cm}^2$.

In Fig. 2 the data points at any value of $y_{\text{cut}}$ are strongly correlated with those at other $y_{\text{cut}}$ values, as the whole dataset is used in calculating the $R_n$ at each $y_{\text{cut}}$ value. To avoid these correlations when fitting the QCD calculations to the data, it is conventional to fit to the $D_2$ distribution,\textsuperscript{[25,29]} defined as the slope of the $R_2$ distribution:

$$D_2(y_{\text{cut}}) \equiv \frac{R_2(y_{\text{cut}}) - R_2(y_{\text{cut}} - \Delta y_{\text{cut}})}{\Delta y_{\text{cut}}},$$

where each event enters only once. $D_2(y_{\text{cut}})$ for the D-algorithm is shown in Fig. 3. Also shown are two fits of the calculation to $O(\alpha_s^2)$ by Kunszt and Nason\textsuperscript{[17]} (KN). A second calculation, by Kramer and Lampe\textsuperscript{[13-16]} (KL), was fitted to the data and yielded almost identical results (not shown); this is discussed further in Section 5. In the first fit (dashed line) the renormalization scale factor $f$ was fixed to unity and the single parameter $\Lambda_{\overline{M}S}$ was fitted in the range $y_{\text{cut}} \geq 0.04$, yielding the preliminary result $\Lambda_{\overline{M}S} = 477 \pm 41$ MeV. In the second case (solid line) a two-parameter fit to $\Lambda_{\overline{M}S}$ and $f$ was performed in the range $y_{\text{cut}} \geq 0.02$, yielding $\Lambda_{\overline{M}S} = 227 \pm 18$, $f = 1.3 \pm 0.2 \times 10^{-3}$. All errors are statistical only. To $O(\alpha_s^2)$ $R_4$ is only calculated to leading order and $R_5$ does not contribute at all. Therefore the regions of $y_{\text{cut}}$ where the fits were performed were selected by requiring that $R_4$ be less than 1% for $f = 1$ and that $R_5$ be less than 1% for free $f$.\textsuperscript{[25]} These $\Lambda_{\overline{M}S}$ values can be translated into $\alpha_s(M_Z)$ measurements using the renormalization group equation,\textsuperscript{[30]} giving $\alpha_s(M_Z) = 0.125 \pm 0.002$ and $0.120 \pm 0.002$ respectively.
A similar analysis was performed for the E0, E and p schemes. The results are shown in Table 1. As an example, the $n$-jet rates calculated with the fitted values of the parameters $\Lambda_{\overline{MS}}$ and $f$ are shown in Fig. 4, together with the corrected data for the E0 scheme.

<table>
<thead>
<tr>
<th>scheme</th>
<th>$(f = 1)$</th>
<th>$(f \text{ fitted})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Lambda_{\overline{MS}}$ (MeV)</td>
<td>$\chi^2/d.o.f$</td>
</tr>
<tr>
<td>D</td>
<td>477 ± 41</td>
<td>7/8</td>
</tr>
<tr>
<td>E0</td>
<td>258 ± 35</td>
<td>14/8</td>
</tr>
<tr>
<td>E</td>
<td>528 ± 50</td>
<td>9/4</td>
</tr>
<tr>
<td>P</td>
<td>326 ± 48</td>
<td>5/8</td>
</tr>
</tbody>
</table>

Table 1 Results of fitting $O(\alpha_s^2)$ QCD calculations to SLD data, for fixed and variable renormalization scales. The errors are statistical only.

5. SYSTEMATIC ERRORS

The systematic uncertainties in this measurement can be divided into two categories. One contains the experimental systematic errors which arise from limited acceptance, efficiency and resolution of the detector, and from biases and imperfection in detector simulation and in event reconstruction programs and due to selection criteria applied to the data for this analysis. The other encompasses the theoretical uncertainties which arise from hadronization and from unknown higher order corrections, in addition to uncertainties in the theoretical calculations themselves. Our study of experimental systematic errors is still in progress. We describe below preliminary results for those systematic effects which we have so far considered.

The selection cuts described in Section 3 were varied. $\alpha_s$ was recalculated at each point. The greatest deviations from the central value were $\Delta\alpha_s(\text{exp.}) = \pm0.003$.

The hadronization process in the Monte Carlo simulation is another source of uncertainty since we have to depend on models describing the transition from partons to hadrons. The difference between $\alpha_s$ obtained with the JETSET 6.3 and HERWIG 5.3 M.C.'s, using two different hadronization models, gives an estimate of the error: $\Delta\alpha_s(\text{had.}) = \pm0.003$, which is the same for all of the jet finding algorithms used.
Another estimate of the error introduced by the shower model is obtained by varying the parameter $Q_0$, which determines the lower cutoff for parton branching in the parton shower. $Q_0$ was varied from 0.5–10 GeV and $\alpha_s$ calculated for each value of $Q_0$. The uncertainties $\Delta \alpha_s(Q_0)$ vary between ±0.002 and ±0.005 for the different recombination schemes.

We also compared two different theoretical calculations of the jet rates, by Kunszt and Nason[17] (KN) and by Kramer and Lampe[14–16] (KL). Both sets were fitted to the corrected data. The difference in $\alpha_s$ for the two methods $\Delta \alpha_s$ (calc.) $< \pm$0.005 is much smaller than the statistical error and can safely be neglected. A much larger source of uncertainty is the choice of the renormalization scale factor $f$. The difference between the results for $\alpha_s$ using $f$ = 1 and for $f$ as a free parameter is an estimate of the error arising from the scale ambiguity. This dependence of $\alpha_s$ on $f$ is shown in Fig. 5. Uncertainties introduced by varying the fit range of $y_{cut}$ were studied and found to be negligible.

The results for $\alpha_s$ for all algorithms, taken to be the average of the two fits, and all sources of uncertainty considered are summarized in Table 2. The last column in this table lists our estimate of the total errors, obtained by adding all errors in quadrature. The total errors are dominated by the theoretical uncertainties due to variation of the renormalization scale.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$\alpha_s(M_Z)$</th>
<th>$\Delta \alpha_s$(stat.)</th>
<th>$\Delta \alpha_s$(exp.)</th>
<th>$\Delta \alpha_s$(had.)</th>
<th>$\Delta \alpha_s(Q_0)$</th>
<th>$\Delta \alpha_s$(scale)</th>
<th>$\Delta \alpha_s$(tot.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0.125</td>
<td>±0.002</td>
<td>±0.003</td>
<td>±0.004</td>
<td>±0.007</td>
<td>±0.010</td>
<td></td>
</tr>
<tr>
<td>E0</td>
<td>0.112</td>
<td>±0.002</td>
<td>±0.003</td>
<td>±0.002</td>
<td>±0.007</td>
<td>±0.009</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>0.119</td>
<td>±0.002</td>
<td>±0.003</td>
<td>±0.003</td>
<td>±0.005</td>
<td>±0.013</td>
<td>±0.015</td>
</tr>
<tr>
<td>P</td>
<td>0.120</td>
<td>±0.002</td>
<td>±0.003</td>
<td>±0.003</td>
<td>±0.005</td>
<td>±0.009</td>
<td>±0.012</td>
</tr>
</tbody>
</table>

Table 2 Summary of results for $\alpha_s$ and the errors contributing to the measurement uncertainty. The values for $\alpha_s$ are the average of the results from the two fits.

These results agree within experimental errors with previous measurements from SLC and LEP[31] as well as with our own measurement of $\alpha_s$ from energy-energy correlations.[32]
6. SUMMARY AND DISCUSSION

We have presented an analysis of jet rates from a data sample of about 6000 hadronic $Z^0$s recorded by the SLD. We have determined the value of the strong coupling, $\alpha_s(M_{Z^0})$, using four different jet finding algorithms (E0, p, E and D). These measurements were compared with analytic calculations in complete second order perturbative QCD. The QCD parameter $\Lambda_{\overline{MS}}$ and thus $\alpha_s(M_{Z^0})$, was then determined in fits of the QCD calculations to the corrected data distributions. The average of the four results is thus

$$\alpha_s(M_Z) = 0.119 \pm 0.002(\text{stat.}) \pm 0.003(\text{exp.syst.}) \pm 0.014(\text{theory}).$$

Experimental uncertainties due to the modelling of the detector response lead to relative uncertainties of 3% in $\alpha_s(M_{Z^0})$. The statistical errors are less than 2% in all cases. The theoretical error quoted above is the sum of $\Delta\alpha_s(\text{had.})$, $\Delta\alpha_s(Q_0)$ and $\Delta\alpha_s(\text{scale})$ added in quadrature, for the E scheme, which yields the largest uncertainties. We find that the largest error in this measurement is the theoretical error from varying the renormalization scale $f$. Our result is in good agreement with results from the LEP experiments.

Acknowledgements

The SLD Collaboration is indebted to all the SLAC staff whose efforts resulted in the successful operation of the SLC which produced the events used in this paper. We wish to thank S. Bethke, G. Kramer and Z. Kunszt for helpful comments relating to this analysis.
References

[4] SLD Collab., 'Measurement of the left-right cross section asymmetry in Z boson production at $E_{cm} = 91.4$ GeV', paper submitted to this conference and talk presented by P. Rowson.
[32] SLD Collab., 'A study of energy-energy correlations and measurement of $\alpha$, at the $Z^0$ resonance', paper submitted to this conference.
Figure 1  Quadrant drawing of SLD.
Figure 2  Relative production rates of n-jet events defined in the D-scheme, as a function of the jet resolution parameter $y_{cut}$. The data, corrected for detector effects and hadronization, are compared with model calculations from JETSET 6.3 and HERWIG 5.3.
Figure 3  Measured distribution of $D_2(y_{cut})$ defined in the D-scheme, corrected for detector and hadronization effects, compared with the corresponding analytic $\mathcal{O}(\alpha_s^2)$ QCD calculations of Z. Kunszt and B. Nason. The QCD parameters are the fit results of $\Lambda_{\overline{MS}}$ with $f = 1$ and of $\Lambda_{\overline{MS}}$ and $f$ in the regions of $y_{cut}$ indicated by the arrows.
Figure 4  Relative production rates of \( n \)-jet events defined in the E0-scheme, as a function of the jet resolution parameter \( y_{\text{cut}} \), corrected to the parton level, compared with the prediction of the QCD calculations with the values of \( \Lambda_{\overline{MS}} \) and \( f \) determined from the fit to \( D_2 \). The fit regions are the same as in Fig. 3.
Figure 5  Dependence of $\alpha_s$ on the renormalization scale factor $f$. The error bar indicates the size of the statistical error at each point.