A THEORETICAL STUDY OF MICROWAVE HEATING IN ELMO BUMPY TORUS*

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Abstract

Because the plasma in ELMO Bumpy Torus (EBT) is produced and heated by microwaves, and macrostability is dependent on microwave propagation and energy deposition is a crucial element in EBT theory. A central feature of the microwave heating study is the development of a ray-tracing code capable of operating with a wide variety of three-dimensional (3-D) plasma equilibrium configurations. Results are presented for plasma parameters characteristic of EBT-I and projected parameters characteristic of EBT-II. In EBT-I the extraordinary mode is very heavily damped, and the ordinary mode is weakly damped. The ordinary mode may, however, play an important role in the heating of EBT-I through mode conversion. In EBT-II the ordinary mode should be strongly damped. The code is being improved to include finite temperature effects on the ray paths and absorption at second and higher harmonic resonances.

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The EBT is a large aspect ratio ($R_T/a = 10$) toroidal magnetic trap in which microwave heating is used to produce steady-state, high-beta plasmas. The confined plasma consists of a high-beta ($\beta \sim 30\% - 50\%$) annulus, mirror-confined component situated at the midplane of each of the 24 identical mirror sectors and a less energetic circulating ("toroidal core") component. The 24 magnetic coils are arranged to produce a 2:1 mirror ratio on the ring axis of the torus (0.65 T at the midplane, 1.3 T at the coil plane). The plasma is formed and heated by 18-GHz Klystron amplifiers producing 60 kw (cw) which is divided into 24 equal parts in order to provide power separately to each sector of the torus. In addition, 30 kw of microwave power at 10.6 GHz is available for profile heating and lower-off-resonant heating [1-3].

In addition to plasma heating, the microwaves play an essential role by producing the high-beta, hot electron annuli which provide macroscopic stability for the toroidal plasma component. Clearly a knowledge of the heating profiles, the microwave field distribution, and the effect of the microwaves on the particle distribution function is crucial to the understanding of EBT operation and scaling. A number of previous studies have dealt with the theory of electron cyclotron heating in specific devices (see references in [3]). However, these studies have treated the heating due to an arbitrary specified microwave field structure or have made simplifying assumptions such as plane stratification of the plasma of the plasma. In EBT, however, the plasma and cavity have very complicated and nonsymmetrical geometry. Figure 1 is an equatorial section showing the outline of the coil casings in an EBT sector and magnetic field lines for a typical $\beta = 40\%$, 3-D equilibrium. Also shown are the mod B contours for cyclotron resonance ($\Omega_e = \omega$) and second har-

*Research sponsored by the Office of Fusion Energy (ETM), U.S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

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monic resonance ($2\Omega_e = \omega$) as well as the right-hand cutoff for the extraordinary mode, $\omega_{RH} = \Omega_e/2 + (\Omega_e^2/4 + \Omega_{pe}^2)^{1/2} = \omega$. Thus the assumption of plane stratification is inadequate, and more detailed calculations are necessary in order to determine the spatial heating profiles. This entails a long-term research project which is far from complete at this time [4]. In this paper we concentrate on one important aspect of the EBT microwave heating study, the development of a ray-tracing code capable of operating in arbitrary plasma equilibrium configurations.

A ray-tracing computer code, RAYS, has been developed and is capable of operating with a wide variety of plasma equilibrium configurations. It has been written in a modular form so that equilibrium geometries and wave dispersion relations are readily interchangeable. As of now RAYS calculates ray paths, local wave number k , and local electric field polarization using the Appleton-Hartree dispersion relation, as well as fractional power deposition using the finite temperature dispersion relation [4].

The equations solved to obtain the ray trajectories are the Hamiltonian form of the geometrical optics equations (see for example, references [5,6]).

$$\frac{dr}{dt} = v_g = \frac{\partial \Omega}{\partial k} = - \frac{\partial D / \partial k}{\partial D / \partial \omega} \quad (1)$$

$$\frac{dk}{dt} = - \frac{\partial \Omega}{\partial r} = \frac{\partial D / \partial r}{\partial D / \partial \omega} \quad (2)$$

where the waves satisfy a dispersion relation of the form $D(k, \omega, r, t) = 0$. For time invariant media ($\partial/\partial t = 0$), the arc length along the ray, s , is a convenient parameter and the above equations can be reduced to

$$\frac{dr}{ds} = \frac{1}{|v_g|} \frac{dk}{dt} = -\text{sgn}\left(\frac{\partial D}{\partial \omega}\right) \frac{\partial D / \partial k}{|\partial D / \partial k|} \quad (3)$$

$$\frac{dk}{ds} = \frac{1}{|v_g|} \frac{dk}{dt} = \text{sgn}\left(\frac{\partial D}{\partial \omega}\right) \frac{\partial D / \partial k}{|\partial D / \partial k|} \quad (4)$$

Equations (3) and (4) are the ones integrated in the tracing code. The Appleton-Hartree dispersion relation [5], a high frequency limit ($\omega \gg \Omega_i$) of the cold plasma dispersion relation, takes the form

$$D(k, \omega, r) = 1 - \frac{c^2 k^2}{\omega^2} - \frac{2\alpha(1-\alpha)}{2(1-\alpha) - \beta \sin^2 \theta \pm \Gamma(\theta, r)} \quad (5)$$

$$\text{where } \Gamma(\omega, \theta, r) = [\beta^2 \sin^4 \theta + 4\beta(1-\alpha)^2 \cos^2 \theta]^{1/2}$$

$$\alpha(\omega, r) = \omega_{pe}^2(r)/\omega^2$$

$$\beta(\omega, r) = \Omega_e^2(r)/\omega^2$$

The $+$ ($-$) sign in Eq. (5) corresponds to the ordinary mode (extraordinary mode) of propagation respectively. The group velocity is most conveniently calculated in a local coordinate system with $\hat{z}$ taken along the local magnetic field and $\hat{x}$ taken in the B, k plane. The group velocity is resolved into components along $\hat{k} = k/|k|$ and $\hat{\theta} = k \times B / |k \times B|$.

The right-hand sides of Eqs. (3) and (4) can be obtained analytically by a straightforward but tedious differentiation of Eq. (5). The unit vectors in Eqs. (3) and (4) are expressed in terms of a cartesian coordinate system with origin located at the origin of the model equilibrium. These equations are then numerically integrated to obtain the cartesian components of $\underline{r}$ and $\underline{k}$ as functions of arc length s and the initial conditions of the ray $\underline{r}^0$ and $\underline{k}^0$. RAYS has been run using a model cylindrical mirror equilibrium which gives a 2:1 mirror ratio vacuum field. The density profile was modeled to be constant along field lines and to decrease in moving radially away from the z axis. Figure 2 shows the results of injecting extraordinary mode rays at varying angles into a mirror equilibrium from a point located at the mirror midplane ($z = 0$). The plasma parameters were chosen to be representative of EBT-1; $\omega/2\pi = 18$ GHz, $(\omega_{pe}/\omega)_{MAX} = 0.5$, and $V_e/c = 0.034$. The solid curves are the projection of the ray path onto the equatorial plane. The projection of the local wave vector $\underline{k}$ is indicated at various points along the ray. Also shown in Fig. 2 are the magnetic field lines and equatorial cross sections of the cyclotron resonant surface and right-hand cutoff surface. It can be seen that rays injected from the low magnetic field region near the midplane do not penetrate to the cyclotron resonance surface but are reflected near the right-hand cutoff.

There are a number of mechanisms by which the extraordinary mode could eventually be absorbed by the plasma: 1) by Budden tunneling through the right-hand cutoff to the resonance layer, 2) by linear mode conversion to a warm plasma mode and subsequent absorption by Landau damping, and 3) by a complicated process involving multiple reflections, depolarization into the ordinary mode, reversion to extraordinary mode on the high field side of the cyclotron resonant zone, and absorption at the resonant layer. The theory of these processes is in too primitive a state to include them in RAYS at this time. However, we have examined the fate of an extraordinary mode ray originating near the mirror throat which, for example, might arise from depolarization of an ordinary mode ray on reflection. Fig. 2 also shows ray paths and wave vectors for extraordinary mode rays injected at varying angles in the equatorial plane from a point near the mirror coil. On this side, the cyclotron resonance zone is accessible. It can be seen that the rays approach the cyclotron resonance layer with $\underline{V}_g$ nearly perpendicular to $\underline{B}$, and $\underline{k}$ nearly parallel to $\underline{B}$. This is characteristic of parallel stratified media in cold plasma model. The circles on the rays indicate the points at which 1%, 50%, and 99% of the wave power has been absorbed. For the EBT-1 parameters used in the calculation, the waves are completely damped before reaching the resonant layer. The ray paths show a tendency to focus the energy toward the center of the plasma. However, this can hardly be considered significant due to the early damping of the wave energy.

The situation is much less complicated for the ordinary mode. As long as the microwave frequency is higher than the plasma frequency, the entire plasma is accessible to the ordinary mode. For densities and temperatures characteristic of EBT-1, the ordinary mode is only weakly damped (typically <1% per pass through the cyclotron resonance). However, the ordinary mode may play an important role in the heating of EBT-1 since conversion of ordinary mode energy in the high field region can be an important source of extraordinary mode which subsequently propagates to the cyclotron resonance. For a plasma having parameters characteristic of the projected values for

EBT-II, the ordinary mode is heavily damped. Figure 3 shows the result of injecting ordinary mode rays at varying angles into a plasma with $\omega/2\pi = 120$ GHz, $(\omega_{pe}/\omega)_{MAX} = 0.5$ and $V_e/c = 0.09$. The rays are virtually straight with slight curvature due to density gradients. The circles indicate the point at which 1%, 10%, . . . of the wave energy has been absorbed. In this case, up to 60% of the energy is absorbed in a single pass through the resonant zone.

A number of tasks remain in this area of ray tracing theory, and a great deal of additional coding will be required before detailed calculation of heating profiles can be carried out with RAYS. For example, work is currently underway for including the effects of finite temperature on the ray paths and for calculating the energy deposition using Poynting's theorem with the finite temperature conductivity. The formalism makes use of the full finite temperature dispersion matrix and permits inclusion of second and higher cyclotron harmonic resonances. Provision for reflection, partial absorption, and partial depolarization of rays at conducting walls needs to be included in the code. This may be particularly important in understanding complicated absorption processes of the extraordinary mode in EBT-II.

REFERENCES

- [1] R. A. Dandl et al., "Research Program for Plasma Confinement and Heating in ELMO Bumpy Torus Devices," ORNL/TM 4941, June 1975.
- [2] R. A. Dandl et al., in Proceedings of the Fifth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo, Japan 1974 (International Atomic Energy Agency, Vienna, Austria, 1975), Vol. II, p. 141.
- [3] R. A. Dandl, H. O. Eason, and H. Ikegami, "Toroidal Plasma Heating at the Electron-Cyclotron Frequency," to be published in Nucl. Fusion.
- [4] D. B. Batchelor, "Status of the Theoretical Study of Microwave Heating in EBT," ORNL/TM-6320.
- [5] T. H. Stix, The Theory of Plasma Waves, McGraw-Hill, New York, NY (1962).
- [6] I. B. Bernstein, "Geometric Optics in Space and Time-varying Plasmas," Phys. Fluids 18, 320 (1975).

2nd Harmonic Resonance

1st Harmonic Resonance layer

right hand. cutoff

2nd Harmonic Resonance

1

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Figure 1 Assembly of an FRT-I Sector

EXTRORDINARY MODE PROPAGATION

DOTTED VECTORS-WAVE VECTORS

EPI-1 PFD-8.5 OFED-8.702 VFD-0.834 FWD-18 GHz

Fig. 2 Extraordinary Mode Propagation in a Plasma

ORDINARY MODE PROPAGATION

57

Fig 3 Ordinary Mode Propagation in a Plasma

EBT-11 REF-0.5 CFE-0.207 VIE-09 FMU-128 SHZ