Alpha Particle Confinement in Tokamaks

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ABSTRACT

An assessment of diffusive tokamak transport mechanisms of concern for alpha particles indicates that the "stochastic regime" is the only one which appears to pose a real danger for adequate alpha confinement. This fact, in conjunction with the threshold character of that mechanism, allows one to decide whether an alpha born at a given location will be lost or confined, according to a very simple criterion. Implementing this criterion numerically results in a new code for the assessment of alpha confinement, which is orders of magnitude faster than earlier codes used for this purpose.

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Achieving ignition in tokamaks depends critically on these machines confining the alpha particles produced by the D-T fusion reaction long enough that their energy goes into keeping the plasma at fusion temperatures. Tokamaks are now entering a domain of operation where the loss rates of alpha (or alpha-like) fusion products can be measured, and a good theoretical understanding of these loss rates is essential in designing future machines. In this work, we describe progress we have made in achieving the needed theoretical understanding.

The progress is embodied in a new code (RIPLOS) for calculating tokamak alpha loss, which is much faster than both the guiding-center (gc)¹⁻³ and the "mapping-method" (mm)^{4,5} codes which up to now have provided the best numerical tools for assessing the transport, and whose results are in reasonable agreement with the results of the gc codes in the benchmark cases run thus far. For the guiding-center code, a typical ensemble consists of 500 particles, and takes several hours on the Cray, while for RIPLOS, an ensemble of 40,000 particles takes only about 1 minute, implying a boost in the speed per particle between the two codes of order 10^4 - 10^5 . The code's simplicity (and thus speed) is made possible by two conceptual steps. First, an assessment of the existing body of analytic theory of diffusive mechanisms of possible concern for alpha transport indicates that, of all the potential candidates, the only mechanism which really appears to pose a problem for alpha confinement is the "stochastic regime" of ripple transport,⁶ generalized⁷ to apply to the ripple generated by low- n MHD activity, as well as to that from toroidal-field (TF) coils, for which the original theory was developed. This simplification allows one to focus exclusively on the particular *properties of stochastic ripple transport*. Second, *this stochastic mechanism* has a threshold character spatially, and this fact permits us to distill a very simple criterion for deciding whether an alpha born at a given location will be adequately confined or not. The implementation of this criterion results in the RIPLOS code.

We first review the various diffusive mechanisms which have been considered analytically. Table I summarizes the assessment. Each mechanism is described by a radial diffusion coefficient D , which, using the approximate formula $\tau_\alpha \approx a^2/(4D)$ (a is the tokamak minor radius), yields the alpha confinement time τ_α . "Acceptable confinement" of the alphas then holds when $\tau_\alpha \geq \tau_{st}$, with τ_{st} the slowing-down time. For definiteness, we shall evaluate the various D 's at $r = a/2$ for a particular set of INTOR parameters: $R = 5.3\text{m}$, $a = 1.0\text{m}$, $B = 5.5\text{T}$, $n_e(a/2) =$

$1.5 \times 10^{20} \text{ m}^{-3}$, $q_0 = 1.2$, $q(a/2) = 1.3$, $q'(a/2) = 0.4$, $T_i = T_e = T$, with $T(a/2) = 8 \text{ keV}$, $T(0) = 20 \text{ keV}$, and with the number of TF coils $n = 12$. The qualitative results are unchanged using parameters for other machines in the range of TFTR to ITER. For the INTOR parameters, one has $\tau_{ei} \approx 0.15 \text{ sec}$, so that $D \approx a^2$ ($4\tau_{ei} \approx 1.7 \text{ m}^2 \text{ sec}$ or greater is unacceptable).

TABLE I: Alpha Transport Coefficients

Mechanism	Equation	Value(m^2/sec)
symmetric neoclassical	$D_{bn-f} \approx \nu_{st}(q\rho)^2/\epsilon^{1/2}$	0.044
symmetric neoclassical	$D_{bn-s} \approx \nu_{st}(q\rho)^2/\epsilon^{3/2}$	0.46
superbanana (ripple)	$D_{sb-l} \approx \nu_{\perp} r^2/\alpha^5 \delta^{1/2}$	1.8×10^{-4}
superbanana (ripple)	$D_{sb-r} \approx \nu_{\perp} r^2/\alpha^2 \epsilon^{1/2}$	4.3×10^{-3}
banana-drift (ripple)	$D_{bd} \approx \nu_{\perp} \delta^{3/2} n^{9/4} q^{13/4} \rho R/\epsilon^{3/2}$	1.7×10^{-2}
stochastic regime (ripple)	$D_s \approx \epsilon^{1/2}(\pi/\Omega_b)v_B^2\langle J^2 J^2 \rangle$	
MHD ($n = 1 = m$, $\delta = .01$)		39.6
TF ($n = 12$, $\delta = .001$)		4.75
electrostatic turbulence	$D_E \approx (\pi/\Omega_b)v_E^2\langle J^2 J^2 \rangle$	$0.51(0.022) \approx 1.1 \times 10^{-2}$
magnetic turbulence	$D_b \approx (\pi/\Omega_b)(v_{\parallel}\delta)^2\langle J^2 J^2 \rangle$	$2.8(0.022) \approx 6.2 \times 10^{-2}$

Theory exists for diffusive alpha transport due to axisymmetric ("neoclassical"),^{8,9} ripple,^{6,7,10-12} and turbulent^{7,13-15} mechanisms. The first five of these are collisional mechanisms, all linear in either the slowing-down frequency $\nu_{st} \equiv \tau_{ei}^{-1}$ or the perpendicular scattering frequency $\nu_{\perp}$. The last three (below the second double line) are collisionless. D_{bn-f} and D_{bn-s} are the 'fast' and 'slow' variants of the banana transport appropriate to alphas, the former applying when the alpha birth speed v_0 is so much greater than the critical speed v_b that pitch-angle scattering may be neglected [$\epsilon(v_0/v_b)^3 > 1$], and the latter applying in the opposite limit [$\epsilon(v_0/v_b)^3 \leq 1$]. D_{sb-l} and D_{sb-r} are two limits of "superbanana" ripple transport,^{10,12} in which the geometric parameter $\alpha \equiv \epsilon/(nq\delta)$ is small enough that alphas can entrap in the ripple wells over a significant region of the poloidal plane. The quantity $\delta \equiv \delta B/B$ characterizes the magnitude of the field perturbations, due to MHD modes or to TF coils. Thus, in general, the field has the form $B = B_0[1 + \delta \cos(m\theta - n\phi)]$. When δ is below the threshold value δ_c needed for the stochastic regime to be operative, the banana-centers of toroidally trapped alphas can still drift due to the ripple, at the collisional "banana-drift" rate¹¹ D_{bd} . From Table I, one notes that, basically because alphas

are nearly collisionless ($\nu_d \approx 6.5 \text{ sec}^{-1}$, $\nu_- \approx 0.19 \text{ sec}^{-1}$), these collisional mechanisms are weak.

The collisional mechanisms having been eliminated, one is left with the "stochastic regime,"^{6,7} governed by D_s , and the magnetic (D_b) and electrostatic (D_E) collisionless turbulent mechanisms. To account properly for the large gyroradius and perpendicular drifts of alphas, and also (in the case of D_s), for the possibility of low- n , time-dependent ripple from MHD perturbations, the appropriate D for each of these has been generalized⁷ from its form in earlier theories.^{6,13,14} These "orbit-averaging" effects included by the generalization are represented by the factor $\langle \omega^2 J^2 \rangle \equiv \langle J_{l_0}^2(z_g) J_{l_0}^2(z_b) \rangle$ appearing in each of the three D 's: the $J_l(z)$ are Bessel functions, physically corresponding to the fraction of the gyro- and bounce-period that a particle spends locally resonant with the perturbations inducing the transport. For D_s , this factor describes the "variable lingering period" during which a particle takes a radial step in the random-walk process.

The values given for D_E and D_b in Table I use current estimates of the amplitudes of the turbulent fluctuations in the plasma interior $\tilde{b} \equiv \tilde{B}_r/B \approx 10^{-4}$, $e\tilde{\phi}/T \approx 10^{-2}$, and assume a turbulent spectrum characterized by perpendicular wave number $\Delta k_\perp \approx 0.1/\rho_i$, with ρ_i the thermal ion gyroradius. One sees that the orbit-averaging factor (enclosed in angular brackets in Table I) reduces the turbulent mechanisms from the range where they would be of concern ($D \sim 1 \text{ m}^2/\text{sec}$) to a value not of concern ($D \sim 0.01 \text{ m}^2/\text{sec}$).

In Table I, we consider two cases of D_s , one due to the ripple from an MHD mode with $n = 1 = m$ and typical ripple amplitude $\delta \sim 0.01$, and the second due to TF ripple, so that $n = 12$, $m = 0$, and typical ripple halfway out of $\delta \sim 0.001$. For both cases, D_s is quite substantial. The low- n case is especially large, and would represent essentially instantaneous loss of all trapped alphas, if the mechanism were operative. (However, as will be discussed shortly, the mechanism probably is not operative, due to a very large threshold for low- n ripple.)

From the preceding, we conclude that one need only focus on transport from the stochastic mechanism. For the stochastic regime to be operative, a threshold condition on the ripple strength δ must hold, given for circular cross-section tokamaks by

$$\delta > \delta_s \equiv (\epsilon/\pi n q)^{2/3}/(2\rho q'). \quad (1)$$

This mechanism is easily understood. Conservation of energy and magnetic moment implies a constant value of B at the banana tip. The pres-

ence of ripple causes a displacement of the banana tip, both radially and in toroidal angle. When this displacement is comparable to the ripple period, the tip motion becomes stochastic and the banana tip moves vertically (conserving B) in a random walk, until the orbit intersects the wall near the midplane. The successive banana tip positions form an area-preserving map similar to the standard map⁶ and onset of stochastic behavior can be estimated using the Chirikov overlap criterion, giving Eq. (1) without the factor of 2 in the denominator. As in the case of the standard map, the actual stochastic threshold occurs at a value of δ at least a factor of 2 smaller than the Chirikov estimate, and must be determined numerically. Equation (1) represents a reasonable approximation to the true threshold. The large threshold value for small n is simply due to the long ripple period, and consequently large tip displacement required to produce stochastic motion.

The nature of the loss process implies that particles make impact with the wall near the midplane, at a point on the orbit shortly before reaching maximum banana width. The mean distance below the midplane is proportional to the mean tip displacement for a single bounce. Toroidally, the impact is more likely to occur where the field ripple causes an outward drift. The guiding center codes¹⁻³ give results consistent with this. Thus, while RIPLoS presently does not give detailed information about the alpha wall deposition distribution, but only the total loss, it may also be possible to have it provide this more detailed sort of loss information.

For the case of TF ripple, even for $\delta \sim \delta_*$, the stochastic regime diffusion coefficient D_s is large enough to be of concern. For the present sample parameters, for $r = a/2$, one has $\delta_* \simeq 2.2 \times 10^{-3}$ for $n = 12$, so that (cf. Table 1) $D_s(n = 12) \simeq 23 \text{ m}^2/\text{sec}$. Due to the strong sensitivity both of D_s on δ , and of δ on spatial position, D_s grows rapidly ($\tau_\alpha \ll \tau_{s1}$) as one moves radially outward from the boundary curve C in the poloidal plane where $\delta = \delta_*$ (cf. Fig 2). Thus, a simple approximate rule for alpha confinement may be given: *banana-trapped alphas whose turning points are in a region where $\delta < \delta_*$ will be confined, while those with turning points in the region $\delta > \delta_*$ will be lost in a time much less than τ_{s1} .* (If the ripple is due to internal MHD activity, the second portion of this rule must be modified a bit: *alphas with turning points having $\delta > \delta_*$ will be rapidly moved to the boundary of this stochastic region.*)

For low- n ripple, one notes from Table 1 that D_s , if operative, would expel alphas extremely rapidly from the regions of localization of the MHD modes inducing the transport. A combined scenario can be envisioned, with

MHD transporting alphas out from the central birth region, with TF-ripple taking over to transport them to the plasma edge. However, since from Eq. (1), $\delta_s \sim n^{-3/2}$, one has $\delta_s(r = a/2, n = 1) \simeq 0.085$, i.e., low- n ripple has a very large stochastic threshold. (Low- n MHD modes can typically produce ripple of a few percent during a sawtooth collapse, however, not far from this threshold.) Thus, as long as the MHD ripple is below this value, the alpha ripple transport from this source should be small, governed by the collisional result D_{bd} . In the numerical results reported here we shall assume this, and consider only loss due to TF ripple.

RIPLOS reads numerically generated equilibria, which can have arbitrary cross-sectional shape and pressure and current distributions. Using a randomly generated birth distribution conforming to a chosen analytic form, it applies the simple loss criterion given above, using the generalization of Eq. (1) to an arbitrary equilibrium. In Fig. 1 are shown the radial profiles used to describe (a) the alpha birth distribution, (b) safety factor $q(r)$, (c) $q'(r)$, and (d) the ripple strength δ , and stochastic threshold δ_s , for INTOR. δ was fit using the form

$$\delta = \delta_0 \exp\{[(R - R_0)^2 - bz^2]^{1/2}/w\},$$

with $R_0 = 4.58$ m, $w = 0.63$ m, and $b = 0.25$, which can be made to match the ripple contours and values fairly well in the domain with $\delta > 5 \times 10^{-4}$. Values smaller than this are not relevant.

In Fig. 2 the loss and confinement regions are shown for INTOR parameters, assuming $\delta_a \equiv \delta(r = a, \theta = 0) \simeq 0.01$, and $q_a \equiv q(r = a) \simeq 2.5$. As q_a or δ_a are increased, one sees from Eq. (1) that the position of C moves radially inward and, particularly because of the strong r -dependence of the alpha birth profile, the loss fraction increases rapidly with δ_a and still more rapidly with q_a , providing a simple explanation of the strong q_a -dependence noted earlier by Tani, et al.² The dependence on q_a is of particular concern, because greater q_a is more desirable from the standpoint of MHD stability, and also of remaining below the threshold $\beta_a^{fb} \propto q'$ for direct (i.e., non-diffusive) loss due to fishbones.¹⁶

In Fig. 3 are shown benchmark comparisons of RIPLOS results (curves) with those from the gc code (individual points), for INTOR, and also for TFTR, plotting loss fraction versus q_a . The plots of Figs. 1 and 2 correspond to the INTOR point at $q_a = 2.5$. Each RIPLOS curve consists of ten points, each point coming from an ensemble of 40,000 particles. One notes that the RIPLOS and gc results are in agreement, to within better

than a factor of 2. The strong sensitivity of the results to the q profile, a poorly known quantity, indicates that higher accuracy than this is not worth the expense. The saturation of the loss fraction at large values of q_a is due to all trapped alphas being lost. As already indicated, operation at higher q -values, preferable from an MHD-stability standpoint, is excluded by the requirement of acceptable alpha confinement. Moreover, due to the very strong q_a -dependence of δ_* , one can only gain slightly in the permissible q_a by designing the tokamak with decreased δ_a , while keeping the loss fraction constant.

Summarizing, we have developed a new numerical tool (RIPLOS) for assessing total alpha loss in tokamaks of arbitrary cross section, which is many orders of magnitude faster than previous codes developed for studying this problem, and which yields results in reasonable agreement with those from the earlier codes. The code's simplicity is made possible by the realization that stochastic ripple-transport should be the chief loss mechanism for alphas, and by making use of the threshold character of that mechanism.

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Figures

FIG. 1. INTOR radial profiles for (a)alpha birth distribution, (b)safety factor $q(r)$, (c) $q'(r)$, and (d) ripple strength δ , and stochastic threshold δ_s .

FIG. 2. Position of the loss and confinement regions for INTOR parameters, with $\ell_a \simeq 0.01$, $q_a \simeq 2.5$.

FIG. 3. Loss fraction versus q_a , comparing results from the RIPLOS (curves) and guiding-center (individual points) codes, for INTOR and TFTR.

Fig. 1

Fig. 2

Fig. 3