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TO FACILITATE TOKAMAK REACTOR STUDIES

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AN APPLICATION OF TOROIDAL MULTIPOLES TO FACILITATE TOKAMAK REACTOR STUDIES

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ABSTRACT

A method of calculating the vacuum magnetic flux for plasma equilibrium over a range of aspect ratio A and major radius R is presented. Toroidal multipoles are used to fit the vacuum flux for a set of reference equilibria and the fitted moments are then used to parametrize the flux over the design space in A and R . An example is given in which the equilibrium flux for a D-shaped tokamak plasma is predicted and compared with the actual equilibrium code vacuum flux.

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I. INTRODUCTION

Parameter studies for the design of tokamak reactors cover a wide range of values of the major radius R and the aspect ratio A . At any particular (A,R) combination, the plasma equilibrium configuration can be obtained by specifying the plasma outer boundary and other variables, and running one of the equilibrium codes to obtain the plasma parameters and the required vacuum magnetic flux ψ . The flux is then matched by designing an appropriate set of coils. This process is repeated for each (A,R) of interest.

G. V. Sheffield has proposed [1] replacing the second step by fitting the ψ on the boundary with a set of toroidal multipoles, determined independently of a particular coil configuration.

This paper presents a method of providing the plasma equilibrium parameters and the magnetic field by a set of semiempirical equations which can be easily evaluated for any (A,R) in the design space, and gives an example of applying the method to predicting the plasma equilibrium for a D-shaped plasma.

II. METHOD

The general approach consisted of calculating plasma equilibria at several values of aspect ratio A for a single value of the major radius R .

The value of the poloidal flux ψ on the plasma boundary was fit in a least-squares sense to the equation:

$$\psi = \psi_0 + B_V \pi r^2 + 2\pi r(A_{\phi 4} + A_{\phi 6} + A_{\phi 8} + A_{\phi 10}) \quad (1)$$

where $A_{\phi 4}$, $A_{\phi 6}$, and $A_{\phi 8}$ are the vector potential for a toroidal quadrupole, hexapole, and octupole [1], and $A_{\phi 10}$ is the toroidal decapole extension provided in Appendix A. B_V is a uniform vertical field, and ψ_0 is a constant. The number of multipoles required to fit ψ depends on the desired accuracy of the fit.

To evaluate Eq. (1), the location of the null must be provided for each moment. A measure (D) of the goodness-of-fit was provided for each fit by the sum of squares of the differences between the equilibrium code ψ values and the ψ values predicted by the moments.

A set of multipoles was obtained for each value of A. These values were then used to calculate the coefficients for a parabolic function of A:

$$(A-k)^2 = 4p(f(A)-h). \quad (2)$$

Scaling is linear with R for the position of the null R_0 , and the field components scale inversely with R to the (2/3) power.

III. APPLICATION

To cover the design space ($A = 2.5$ to 4.5 , $R = 2.5$ to 4.5 m) [2], plasma equilibria were calculated for a D-shaped plasma at three values of A, for a plasma major radius of $R = 3.6$ m.

The determination of the best way to fit the vacuum fields with the toroidal moments is somewhat arbitrary. An initial value of the goodness-of-fit parameter D was obtained by using the toroidal multipoles up to the octupole order and setting the position of the null for each multipole equal to the plasma magnetic axis from the corresponding equilibrium code run. As an improvement, several alternative approaches were tried to find a smaller value of D : allowing the position of the null common to all multipoles to vary; using one higher term in the multipole expansion; and allowing the position of the null for each multipole to vary independently.

Because the third method led to results which were dependent on the exact details of the minimization process (i.e., there were very many local minima of D) precluding a simple scaling procedure, a combination of the first two approaches was the chosen solution. Allowing the common null position R_0 to vary along the midline showed a strong local minimum for D near the outer edge of the plasma, and another minimum, possibly also local, at a position $R_0 \gg R$.

The radial position of the local minimum of D near the plasma outer edge was chosen as the value of R_0 to use for two reasons: 1) the values obtained for the moments and their associated vertical magnetic fields were convenient for design estimates (at $R_0 \gg R$, the multipole field components bore little relationship to the typical field values), and 2) by using the decapole, the moment values predicted by a quadratic fit to A predicted flux values to the necessary accuracy to justify using this scaling method.

The scaling functions obtained for the moments from the toroidal multipoles are shown in Fig. 1. The quantities plotted are the position of the null

common to all the multipoles R_0 ; B_y , the vertical field (Tesla); and $B_4 - B_{10}$ (Tesla) where

$$B_i = \frac{\mu_0 M_i}{R_0^3}. \quad (3)$$

An example of the use of this method is shown in Figs. 2 and 3. Figure 2 is the flux pattern from an equilibrium code run for a plasma with $A = 4.0$, $R = 3.6\text{m}$. For comparison, the moment values for this case were obtained from Fig. 1, and the flux calculated and plotted in Fig. 3. The average deviation of ψ calculated using the coefficients from Fig. 1 in Eq. (1), from the ψ values obtained by running the equilibrium code (neglecting the additive constant ψ_0), is 1.2% of the increment in flux between the inner and outer edge of the plasma, $\Delta\psi$. The largest deviation for this case is 1.8% of $\Delta\psi$.

IV. CONCLUSION

The fields from the toroidal multipole can be used to fit the value of ψ on the plasma boundary obtained by exercising an equilibrium code. The fitted multipole moments can in turn be used to calculate simple scaling equations which are used to predict ψ values over a wide range of (A, R) without the necessity of running the equilibrium code. Comparison to equilibrium code results for a particular case shows accuracy of better than 2% from inclusion of multipoles through the decapole. Improved accuracy can be obtained, if desired, by increasing the number of multipoles included in the fit.

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APPENDIX A

The behavior of the toroidal decapole at the null ($r = R_0$, $z = 0$) requires that the following conditions be satisfied:

$$B_z = 0 \quad (\text{A.1})$$

$$\frac{\partial B_z}{\partial r} = 0 \quad (\text{A.2})$$

$$\frac{\partial^2 B_z}{\partial r^2} = 0 \quad (\text{A.3})$$

$$\frac{\partial^3 B_z}{\partial r^3} = 0 \quad (\text{A.4})$$

The simplest combination of dipoles on the Z-axis oriented along the Z-axis, plus a uniform magnetic field B_0 , that will meet these requirements is given by a pair of dipoles above and below the midplane ($r = 0$, $z = \pm Z_0$), each of strength M_{10} , a second pair of dipoles at ($r = 0$, $z = \pm Z_1$), each of strength fM_{10} , and the uniform field. The field equations for this combination take the form:

$$A_\phi = \frac{\mu_0 M_{10}}{4\pi} r \left\{ \frac{1}{D_{00}^3} + \frac{1}{D_{01}^3} + \frac{f}{D_{10}^3} + \frac{f}{D_{11}^3} \right\} + \frac{B_0 r}{2} \quad (\text{A.5})$$

$$\psi = 2\pi r A_{\phi} \quad (A.6)$$

$$B_r = \frac{3\mu_0 M_{10}}{4\pi} r \left\{ \frac{z-Z_0}{D_{ou}^5} + \frac{z+Z_0}{D_{ol}^5} + \frac{f(z-Z_1)}{D_{iu}^5} + \frac{f(z+Z_1)}{D_{il}^5} \right\} \quad (A.7)$$

$$B_z = \frac{\mu_0 M_{10}}{4\pi} \left\{ \frac{[2(z-Z_0)^2 - r^2]}{D_{ou}^5} + \frac{[2(z+Z_0)^2 - r^2]}{D_{ol}^5} \right. \quad (A.8)$$

$$\left. + \frac{f[2(z-Z_1)^2 - r^2]}{D_{iu}^5} + \frac{f[2(z+Z_1)^2 - r^2]}{D_{il}^5} \right\} + B_0$$

where:

$$a^2 = \frac{3-\sqrt{7}}{4}, \quad Z_0 = aR_0 \quad (A.9)$$

$$a^2 g^2 = \frac{3+\sqrt{7}}{4}, \quad Z_0 = gZ_1 = agR_0 \quad (A.10)$$

$$f = \frac{\sqrt{7}-2}{\sqrt{7}+2} \left(\frac{7+\sqrt{7}}{7-\sqrt{7}} \right)^{7/2} \quad (\text{A.11})$$

$$D_0^2 = Z_0^2 + R_0^2 = (a^2 + 1)R_0^2 = \left(\frac{7-\sqrt{7}}{4} \right) R_0^2 \quad (\text{A.12})$$

$$D_1^2 = Z_1^2 + R_0^2 = (a^2 g^2 + 1)R_0^2 = \left(\frac{7+\sqrt{7}}{4} \right) R_0^2 \quad (\text{A.13})$$

$$D_{0u}^2 = (z-Z_0)^2 + r^2, \quad D_{0l}^2 = (z+Z_0)^2 + r^2 \quad (\text{A.14})$$

$$D_{1u}^2 = (z-Z_1)^2 + r^2, \quad D_{1l}^2 = (z+Z_1)^2 + r^2 \quad (\text{A.15})$$

$$B_0 = \frac{\mu_0 M_{10}}{2\pi} \left[\frac{(2Z_0^2 - R_0^2)}{D_0^5} + \frac{f(2Z_1^2 - R_0^2)}{D_1^5} \right]. \quad (\text{A.16})$$

The separatrix pattern for the resulting decapole is shown in Fig. 4.

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- [1] SHEFFIELD, G.V., Nucl. Fusion 19 (1983) 1039.
- [2] TFCX Tokamak Fusion Core Experiment, Preconceptual Design Specification Rev. 0 (SHEFFIELD, G.V., Ed.), Plasma Physics Laboratory, Princeton University, Princeton, NJ (unpublished).

FIGURE CAPTIONS

Fig. 1. Scaling functions for R_0 , B_v , B_4 , B_6 , B_8 , and B_{10} as a function of aspect ratio A .

Fig. 2. Contours of constant vacuum magnetic flux ψ from equilibrium code run. $R = 3.6$ m, $A = 4.0$.

Fig. 3. Contours of constant vacuum magnetic flux ψ calculated by evaluating Eq. (1) with coefficients from Fig. 1. $R = 3.6$ m, $A = 4.0$.

Fig. 4. Separatrix flux pattern for toroidal decapole.

Fig. 1

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Fig. 2

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Fig. 3

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Fig. 4