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Particle Deposition in Granular Media

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Prepared by

Chi Tien

Department of Chemical Engineering and Materials Science

Syracuse University

Syracuse, NY 13244

MASTER

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Abstract

Presented in this report is a summary of the studies carried out at Syracuse University under Grant No. DE-FG-02 88 ER 13930, "Particle Deposition in Granular Media" during the period June 1, 1988 to May 31, 1992.

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I. Introduction

The overall objective of the studies conducted under Grant No. DE-FG-02 88 ER 13930, "Particle Deposition in Granular Media," is to obtain a quantitative understanding of aerosol deposition from gas streams flowing through granular media leading to the formulation of a framework for describing the process. Such information is of significant importance to the development of rational design of granular filtration systems for various applications.

The project period of this study extended from June 1, 1988 to May 31, 1992. The following investigations were carried out:

- (1) Transient Behavior of Granular Filtration of Aerosols
- (2) Stochastic Simulation of Aerosol Deposition in Granular Media.

In addition, during the beginning phase of the grant, a significant amount of time was devoted to the completion of studies which were initiated prior to June 1, 1988 under the Contract DE No. DE-AC02 79 ER 10386. Therefore, a brief description of these investigations was also included.

Among the various investigations conducted during the period June 1, 1988 - May 31, 1992, the results of some of these studies have been published in the literature while publication of others is still pending. In preparing this report, we have kept the description of these published results to a minimum in order to conserve space.

II. Transient Behavior of Granular Filtration

Granular filtration, as a fixed-bed process, is inherently non-steady state in nature. As the deposited aerosols within a granular filter increases during the course of filtration, there is a continuing change of the filter medium structure such that both the local permeability and the unit collector efficiency change with time. Information which can be used to predict this change of media structure as a function of the extent of deposition is therefore required for the design of granular systems.

During this project period June 1, 1988 - May 31, 1992, we conducted extensive measurements to gather data which can be used to establish the relationship between the extent of deposition and the local permeability and collection efficiency. The first part of our work was directed at monodispersed aerosols, namely, aerosols of the same size. Once the problem of the mono-dispersed aerosols case was satisfactorily solved, the problem concerning polydispersed aerosols, which is of greater practical significance was then examined.

II.1. Effect of Deposition on Granular Filtration Performance - Monodispersed Aerosols. For monodispersed aerosol filtration in granular beds under the condition of constant throughput, its dynamic behavior may be desolved by the following set of equations (Tien, 1989)

$$u_s \frac{\partial c}{\partial z} + \frac{\partial \sigma}{\partial \theta} = 0 \quad (2.1)$$

$$\frac{\partial \sigma}{\partial \theta} = (u_s)(\lambda) c \quad (2.2)$$

$$-\Delta p = -(\Delta p)_o \int_0^L \frac{(\partial p / \partial z)}{(\partial p / \partial z)_o} dz \quad (2.3)$$

where c and σ denote the particle concentration (vol/vol) and the specific deposit (vol of deposited particle/vol of bed). z and θ are the axial distance and the corrected time defined as $t - \epsilon z/u_s$, where t is the time, u_s , the superficial velocity, and ϵ the bed porosity. λ is the filter coefficient which characterizes the rate of filtration and is proportional to the so-called unit collector (or single collector) efficiency (Tien, 1989). Δp is the pressure drop across a filter of length L and $(\Delta p)_o$ is the value of (Δp) when the filter bed is free of deposited aerosols.

The effect of the media structure change due to the presence of deposited aerosols within the media results in a change in λ and in $(\partial p / \partial z)$. The effect of deposition may therefore be represented as

$$\frac{\lambda}{\lambda_o} \frac{\eta}{\eta_o} = 1 + \alpha_1 \sigma^{\alpha_2} \quad (2.4)$$

$$\frac{(\partial p / \partial z)}{(\partial p / \partial z)_o} = 1 + \beta_1 \sigma^{\beta_2} \quad (2.5)$$

where η denotes the unit- (or single-) collector efficiency, α_1 and α_2 (or β_1 and β_2) are empirical constants.

The difficulty associated in establishing relationships such as Equations (2.4) and (2.5) arises from the fact that the effect of deposition is a local and time-dependent phenomenon. To establish these two relationships, Walata et al. (1986) proposed a method based on collecting data using filter beds of different heights and extrapolation of these results to zero

height. Subsequently, Takahashi et al. (1981) applied the method of Walata et al. in establishing the first correlations between λ/λ_0 [or $(\partial p/\partial z)/(\partial p/\partial z)_0$] with σ .

By applying the procedure of Walata et al., correlations such as Equations (2.4) and (2.5) can be readily established under specified operating conditions. On the other hand, the extrapolation procedure which requires the use of data collected with beds of different height is tedious and time consuming. Accordingly, as the first investigations of this project, the possible simplifications of the method of Walata et al. was explored. As a result of this simplification, collection of a large body of data of λ/λ_0 vs. σ [and $(\partial p/\partial z)/(\partial p/\partial z)_0$ vs. σ] over a wide range of conditions became possible. This, in turn, made it possible to develop correlations which can be used for estimating the change in λ [and $(\partial p/\partial z)$] under different operating conditions.

The main results of our work may be summarized as:

- (a) If the filter bed height is less than ten times of the unit collector height (which is approximately equal to the filter grain diameter) the results (i.e. λ vs. σ) using average values of σ gives a sufficiently accurate representation of the local situation.
- (b) Correlations between the empirical constants of Equations (2.4) and (2.5) and operating variables were established and can be used for predicting the transient behavior of granular filtration of monodispersed aerosols.

The results of this work was published (J. Aerosol Sci., 22, 187-200, 1991). A copy of the reprint is appended.

II.2. Effect of Deposition in Granular Filtration - Polydispersed Aerosols. For most of the practical applications of granular filtration, the gas stream to be treated is likely to have aerosols of different sizes. For polydispersed aerosols, it is obvious that the effect of

deposition cannot be described by the amount of deposition (i.e., σ) alone but also by its composition as well. The problem is further complicated by the fact that the composition of the deposited particle is not a variable which can be easily controlled in experiments.

The study which was carried out during the project period consisted of the following phase:

(a) **Experimental determination of the effect of Deposited Particle Size.** In order to delineate the effect of the size of the deposited particle, special measurements were devised as follows. An experimental filter was first loaded with deposited aerosol of a given size (d_{pj}) by passing a monodispersed aerosols of diameter d_{pj} through the filter. After the specific deposit of the filter reached a pre-determined value, dilute aerosols of diameter d_{pi} was then used as the test aerosol and its collection efficiency determined. The following findings were obtained:

(a) The relationship between the increase in the collector efficiency and the extent of deposition may be expressed as

$$\frac{\eta(i/j)}{(\eta)_o} = 1 + \alpha_1(i/j)\sigma^{\alpha_2(i/j)} \quad (2.6)$$

where (i/j) refers to the case in which aerosols of diameter d_{pi} are the aerosols in question and the filter is loaded with deposited aerosols of diameter d_{pj} . In other words, it was found that the power-law expression which was found useful in representing the effect of deposition on collector efficiency for monodispersed aerosols is also applicable to polydispersed aerosols. The constants $\alpha_1(i/j)$ and $\alpha_2(i/j)$, of course, are different from α_1 and α_2 of Equation (2.4). [In fact, α_1 and α_2 of Equation (2.4) may be written as $\alpha_1(i/i)$ and $\alpha_2(i/i)$].

(b) For polydispersed aerosols of N different sizes, the effect of deposition is assumed to be

$$\eta_i/(\eta_i)_o = 1 + \sum_{j=1}^N \alpha_1(i/j) \sigma^{\alpha_2(i/j)} \omega_j \quad (2.7)$$

where the total specific deposit, σ , is

$$\sigma = \sum_{j=1}^N \sigma_j \quad (2.8a)$$

and

$$\omega_i = \sigma_i / \sigma \quad (2.8b)$$

Experiments were carried out which confirm the expression of Equations.

(c) Experimental data suggest that $\frac{\eta(i/j)}{(\eta_i)_o} - 1$ at specified values of σ (i.e. $\sigma =$

10^{-4} and $\sigma = 10^{-3}$) may be correlated directly with d_{pi}/d_{pj} .

This part of study was recently published (J. Aerosol Sci., 23, 525-537, 1992). A copy of the reprint is appended.

III. Simulation of Aerosol Deposition of Granular Media

As a companion study of those described in II, the possible use of computer simulation to analyze the deposition process was undertaken. The potential utility of computer simulation is that one may obtain information in detail about the accumulation of deposited aerosols within filter media, the morphology of the deposit and its evolution. As the publication of the results of this phase of study is still pending, a detailed description of the study is presented below.

Tien *et al.* (1977) and Wang *et al.* (1977) postulated the principles of simulating particle deposition from suspensions flowing past collectors. Their principle was subsequently adopted by a number of investigators; Kanaoka *et al.* (1980), Tsiang *et al.* (1982) and Ramarao and Tien (1988) applied this technique for analyzing aerosol deposition in fibrous media. The work of Kanoaka *et al.* dealt with diffusive aerosols. For diffusive aerosols, particle trajectories are not deterministic. Consequently, the traditional trajectory analysis for determining collection efficiencies is not applicable. Instead Kanoaka *et al.* used the simulation technique for estimating the initial collection efficiencies. This approach was subsequently adopted by Gupta and Peters (1985) in their work of aerosol deposition on isolated spheres.

Another adoption of the simulation method is its use in examining the effect of deposition on collection efficiencies. Both Beizaie *et al.* (1981) and Pendse and Tien (1982) demonstrated the feasibility of such applications. Application of the simulation method to obtain deposition results under conditions of practical interests, however, have not yet been made because of the excessive computational requirement.

In this study, a more efficient algorithm of simulation than those used before was developed in order to study granular aerosol filtration under conditions of practical interest. In conjunction with the simulation studies, an empirical correction factor was introduced in order to account for a variety of effects such as the bouncing-off of impacting particles and the change of the flow field around the collector when the extent of deposition becomes significant.

Principle of Simulation

Simulation of aerosol deposition may be carried out based on the premise proposed by Tien *et al.* (1977) and Wang *et al.* (1977) previously. The procedure of simulation is composed of the following steps:

- (a) Selection of a model for granular media characterization,
- (b) Generation of particle trajectories,
- (c) Determination of adhesion of impacting particles,
- (d) Inventory of deposited particles.

In sum, simulation yields the results of the number of particles deposited as a function of the number of particles flowing past (or through) a collector as well as the positions of the deposited particles. The later information gives the morphology of deposited particles. The former results can be used to determine directly the unit collector efficiency defined as

$$\eta = \frac{dN}{dM} \quad (3.1)$$

where N is the number of the deposited particles and M , the number of particles flowing past the collector.

Selection of Porous Media Model

To begin simulation, one must first select a granular media model. For a reasonably accurate prediction of aerosol deposition rates, it is obvious that an acceptable model must adequately describe the flow field within the medium. At the same time, the model must be sufficiently simple so that its use in simulation is practical.

Among the several models which have been applied to aerosol filtration studies, only the dense cubic packing model has been found to give good results on deposition rates (Gal *et al.*, 1985). The structure of this model and its flow field expressions are, however, too complex to be used in simulation. On the other hand, for the simpler models, neither the Happel nor the constricted-tube model was found to give reasonable results as pointed out by Paretsky *et al.* (1973) and Pense *et al.* (1978).

The failure of Happel's model in predicting correct collection efficiency is attributed to the underestimation of the jetting effects of fluid flow in porous media (Snaddon and Dietz, 1980). To correct this deficiency, Vaidyanathan *et al.* (1989) and Vaidyanathan (1989) modified Happel's model by considering the entry of fluid into a Happel's cell being through a window of a given size (which may be characterized by the angle, 2β over which the window extends) instead of the entire half sphere of the cell (see Fig. 1 for a schematic representation). Furthermore, Vaidyanathan *et al.* assumed that granular media may be represented by an assembly of such cells with the window sizes following a certain distribution function. Although this idea was found useful in improving the accuracy of predicting hydrosol deposition, its use in simulation is nevertheless impractical because of its excessive computational demand.

As an alternative to Vaidyanathan's modification, it was assumed in this study that the windows for all the cells have the same size. The window size was determined by carrying out a trajectory calculations using different window sizes and comparing the calculated collection efficiencies with experiments. It was found that a window of about 20 degrees in size gives reasonably good agreement with experiments. This modification was then used in this simulation study.

Determination of Particle Trajectories

According to the assumption of Tien et al. (1977), aerosol particles enter into a Happel cell one at a time and are randomly distributed over the window. Trajectories of aerosols can then be determined by integrating the equations of particle motion with the knowledge of the initial particle velocities and positions.

1. Assignment of Initial Positions

The initial positions of the succeeding entering particles over the window at the fluid envelope may be given by the spherical polar coordinates (R_b, θ, ϕ). For any position (R_b, θ, ϕ), $0 \leq \theta \leq \beta$ ($\beta = \pi/2$) and $0 \leq \phi \leq 2\pi$, the probability of the particle's center being within an area element over the window is proportional to the volumetric flow rate through the element. Thus, one has

$$\text{probability } (0 \leq \theta \leq \beta) = 1 \quad (3.2)$$

$$\text{probability } (0 \leq \phi \leq 2\pi) = 1 \quad (3.3)$$

$$G_1(\theta) = \text{probability } (\theta_{in} \leq \theta) = \frac{\sin^2\theta}{\sin^2(\pi/9)} \quad (3.4)$$

$$G_2(\phi) = \text{probability } (\phi_{in} \leq \phi) = \phi/2\pi \quad (3.5)$$

From the cumulative probability density functions, $G_1(\theta)$ and $G_2(\phi)$, of random variables θ_{in} and ϕ_{in} , respectively, one can find these random variables by using the two uncorrelated random numbers R_1 and R_2 distributed uniformly over (0, 1) as follows:

$$\theta_{in} = G_1^{-1}(R_1) = \sin^{-1}(\sqrt{R_1} \sin \pi/9) \quad (3.6)$$

$$\phi_{in} = G_2^{-1}(R_2) = 2\pi R_2 \quad (3.7)$$

A random number generator, RAN 1 (Press et al., 1987), was used in this study to generate sets of R_1 and R_2 .

2. Particle Trajectory Equations

Assuming that inertial, viscous drag and gravitational forces are the only forces acting on a particle in motion¹ and furthermore that particle motion in ϕ direction is absent, the equations of particle motion for the coordinate system shown in Fig. 1 are given as

$$N_s \frac{d^2 r^*}{dt^{*2}} + \frac{dr^*}{dt^*} - N_s r^* \left(\frac{d\theta}{dt^*} \right)^2 - v_r^* + N_G \cos \theta = 0 \quad (3.8)$$

$$N_s \frac{d^2 \theta}{dt^{*2}} + \frac{d\theta}{dt^*} + \frac{2N_s}{r^*} \frac{dr^*}{dt^*} \frac{d\theta}{dt^*} - \frac{v_\theta^*}{r^*} - N_G \frac{\sin \theta}{r^*} = 0 \quad (3.9)$$

$$\phi = \phi$$

¹In other words, the mechanisms of particle deposition are inertial impaction, sedimentation and interception.

where r^* , θ , and ϕ are the dimensionless coordinates of the position of the particle center and t^* the dimensionless time. r^* , t^* as well as the other quantities present in Equations 3.8) and (3.9) are defined as:

$$R_c = d/2 \quad (3.10.a)$$

$$r^* = r/R_c \quad (3.10.b)$$

$$t^* = tu/R_c \quad (3.10.c)$$

$$v_\theta^* = v_\theta/u_s \quad (3.10.d)$$

$$v_r^* = v_r/V_s \quad (3.10.e)$$

$$N_{St} = \rho_p d_p^2 u / (9\mu d_s) \quad (3.10.f)$$

$$N_G = (\rho_p - \rho)g d_p \quad (3.10.g)$$

where

t = time

R_c = collector radius

v_r = radial velocity component

v_θ = angular velocity component along θ -direction

The fluid velocity components (dimensionless) v_r^* and v_θ^* can be found from the flow-field expression for modified Happel's model (Vaidyanathan, 1989) if the change in flow field due to particle deposit is negligible. The expressions of v_r^* and v_θ^* are given in

Appendix A.

The initial conditions are

$$r^* = b \quad \theta = \theta_{in} \quad \phi = \phi_{in} \quad (3.11.a)$$

$$v_r^* = -\alpha \cos \theta \quad \text{at} \quad t = 0 \quad (3.11.b)$$

$$v_\theta^* = \alpha \sin \theta \quad (3.11.c)$$

where

α : flow intensification factor defined as (Vaidyanathan, 1989)

$$\alpha = 2/(1 - \cos 2\beta) \quad (3.12)$$

Thus, by specifying its initial condition, one can determine the trajectory of a particle by integrating Equations (3.8) and (3.9). This procedure, however, is impractical because of the very large number of particle trajectories to be determined in a simulation. Instead, only a small number of particle trajectories corresponding to specified positions of entry were determined by integrating Equations (3.8) and (3.9). Particle trajectories with entering positions other than the specified ones may be estimated by an interpolation procedure described below.

3. Estimation of Particle Trajectories by Interpolation. Because of the axisymmetry of the flow field, the trajectory of a particle can be described by the r -coordinate of the particle's position as a function of the corresponding θ -coordinate and its initial θ -coordinate value, or, $r^*(\theta_{in}, \theta)$. Based on this feature, the interpolation method was developed as follows. First, N_p discrete positions of entry at the window of the modified Happel cell (namely, at θ_i for $i = 1, 2, \dots, N_p$ with $\theta_{N_p} = \pi/9$) were specified. For a particle entering at θ_i , the r -coordinate of its trajectory could be expected to be greater than θ_i . Its trajectory,

therefore, may be described by a set of M_p -i discrete values of $r^*(\theta_{in}=\theta_i, \theta_j)$ with $j = i, i+1, \dots, M_p$, $M_p = \pi/2$. This set of r values were then obtained from integrating Equations (2.8) and (2.9) with the initial position being $r^*=b$, and $\theta=\theta_i$. In other words, for a given set of values of θ_i , the trajectory meshes which can be used in the interpolation scheme were created.

For any particle with entry position other than those specified, for example, with $\theta_i < \theta_{in} < \theta_{i+1}$, the radial position of the particle at θ_j , $j = i + 1, i + 2, \dots, M_p$, were approximated by the following linear interpolation formula

$$r^*(\theta_{in}, \theta_j) = r^*(\theta_i, \theta_j) + \left[\frac{r^*(\theta_{i+1}, \theta_j) - r^*(\theta_i, \theta_j)}{\Delta \theta_i} \right] (\theta - \theta_i) \quad (3.13)$$

where

$$\Delta \theta_i = \theta_{i+1} - \theta_i \quad (3.14)$$

$$r^*(\theta_i, \theta_i) = R_j/R_c = b \quad (3.15)$$

Similarly, the radial position of the particle at any given θ , with $\theta_j < \theta < \theta_{j+1}$, were approximated as

$$r^*(\theta_{in}, \theta) = r^*(\theta_{in}, \theta_j) + \left[\frac{r^*(\theta_{in}, \theta_{j+1}) - r^*(\theta_{in}, \theta_j)}{\Delta \theta_j} \right] (\theta - \theta_j) \quad (3.16)$$

where

$$\Delta \theta_j = \theta_{j+1} - \theta_j \quad j = i + 1, i + 2, \dots, M_p \quad (3.17)$$

To reduce the interpolation error, finer meshes were used in the region near the axis of symmetry, where the curvature of streamlines changes rapidly. Specifically, for the region $\theta \leq 1^\circ$ the increment of entry points, $\Delta\theta_i$, was taken to be 0.1° . For the region $1^\circ < \theta \leq 20^\circ$, $\Delta\theta_i$ was 0.25° ; i.e.

$$\text{for } \theta \leq 1^\circ \quad \Delta\theta_i = 0.1^\circ \quad \Delta\theta_j = 0.1^\circ \quad (3.18.a)$$

$$\text{for } 1^\circ < \theta \leq 20^\circ \quad \Delta\theta_i = 0.25^\circ \quad \Delta\theta_j = 0.25^\circ \quad (3.18.b)$$

$$\text{for } 20^\circ < \theta \leq 90^\circ \quad \Delta\theta_j = 0.25^\circ \quad (3.18.c)$$

For particles entering the region with its θ -coordinate less than 0.1° , their trajectories were determined directly by integrating Equations (3.8) and (3.9) with the initial conditions given by Equation (3.11).

Similar interpolation procedure was used to estimate particle velocities.

Determination of the Outcome of an Approaching Particle

There are three possible outcomes for any particle approaching the collector:

- (1) deposition on the collector surface,
- (2) deposition on a previously deposited particle, or
- (3) escape from collection.

An approaching particle escapes collection if the following conditions are satisfied by its trajectory:

$$r^*(\theta, \phi) > 1 + N_R \quad (3.19.a)$$

and

$$d_i > 2N_R \quad \text{for all } i \quad (3.19.b)$$

where d_i is the distance (nondimensionalized in terms of the radius of a collector) between the center of the particle and the center of i -th deposited particle.

The requirements for a particle to be deposited are: (1) The particle has to make contact with either a collector or any one of the previously deposited particles, and (2) there must be a sufficiently strong attractive force such that once the particle makes contact with the collector or any previously deposited particles, adhesion occurs.

To determine whether or not the first condition is satisfied, the particle's trajectory must be compared with the positions representing the collector surface as well as the surface of all previously deposited particles. In principle, this is accomplished by incrementing the θ -coordinate in steps of $\Delta\theta_i$ along the particle trajectory, checking whether either one of the two relations, Equations (3.19.a) and (3.19.b), is not satisfied. Checking in this manner, however, requires excessive computer time.

To reduce the time requirement, the following scheme was devised:

- (1) The entire upper half region of the modified Happel's cell is discretized along the θ - and ϕ -coordinates, depicted in Fig. 2. The size of a grid in the direction of ϕ was 4 degrees, while the size of a grid in the direction of θ was made exactly the same as that of trajectory meshes as stated before.

Accordingly, the positions of deposited particles are stored in a three-dimensional grid system and identified by their positions and by the order of their deposition. In this way, one can easily identify the likely location of deposition for an approaching particle, thus significantly reducing computer time.

- (2) For a grid with no deposited particles, it is obvious that the possibility of an approaching particle's being deposited inside this grid is nil. Thus, the only

possibility to be checked is that of an approaching particle's being deposited on the collector surface as it passes through the empty grid.

- (3) Furthermore, in each and every grid the highest radial-coordinate position of a deposited particle is monitored and updated as the simulation progresses.

Accordingly, if the radial position of an approaching particle in a grid is much higher than the highest radial position of deposited particles in the grid, particle deposition cannot take place in the grid, and the particle will move to the next grid.

- (4) If deposited particles are present in a grid, and the approaching particle does not make contact with the collector while the particle is within the grid, then the particle is first returned to one of its previous positions, and the distances between the approaching particle and all the previously deposited particles in the grid are calculated to see if particle deposition takes place on a previously deposited particle. This calculation continues while the particle moves forward.

- (5) For an approaching particle located at a position close to the boundary between two adjacent grids in terms of ϕ , all the deposited particles in these two grids were included in the comparison, since the approaching particle may be deposited on either one of the deposited particles in the two adjacent grids.

A more detailed description of the algorithm used in carrying out the checking process can be seen from the flow chart (see Fig. 3).

Criterion for Deposition

Two criteria were considered for determining adhesion of impacting particles:

(1) Criterion of Particle Bounce-off. Dahneke (1972) stated that when a particle collides with a surface, the bounce-off or adhesion of the particles onto the surface depends upon the magnitude of the particle's incident normal velocity being greater or less than the capture limit velocity, V_{esc} defined as

$$V_{esc} = \left[\frac{2\Delta E}{m} \frac{1-e^2}{e^2} \right]^{1/2} \quad (3.20)$$

where ΔE is the particle-surface adhesion energy, m is the particle mass, and e is the coefficient of restitution defined as the ratio of the normal particle velocity at the instant of rebound to the normal particle velocity at the instant of contact.

As pointed out by Tsiang (1982), there are certain uncertainties in applying Equation (3.20) for adhesion criterion. Dahneke (1972) showed that the elastic flattening of a sphere and/or indentation of a surface caused by the molecular attractions between the sphere and the surface molecules may cause a substantial increase in adhesion energy. The adhesion energy of flattened particles may be 20 times that of unflattened particles. Furthermore, when a particle collides with a previously deposited particle, accurate estimation of the particle velocity upon impact (in particular, the velocity normal to the previously deposited particle) is often difficult since particle trajectories were determined without considering the effect of deposition. These two problems make the use of the adhesion criterion questionable as shown in the later results.

(2) Capture Probability. As an alternative to account for the various complications not considered in the simulation such as the change in flow field due to deposition, the bounce off of impacting particles and the possible hydrodynamic interaction effect, the capture probability was introduced in simulation as an empirical correction factor. It is

defined as the probability of an approaching particle to be collected on a deposited particle or a collector when the approaching particle makes contact with the deposited particle or the collector based on the assumption that there is no change of flow field due to particle deposits and no hydrodynamic interaction between the approaching particles and the deposited particle (including the collector).

One can intuit the capture probability to be a function of a number of variables including d_p , V_{∞} , N_{st} , and N_R as well as of the shape of particle clusters or dendrites. Also, it may be a function of the material constants of the impacting particle and the collector. Our objective, however, is pragmatic and is limited to determining an overall value such that the simulation results agree with experimental data. The justification and success of this empirical approach, of course, is the ability of defining such a parameter with consistency and generality. The incorporation of the capture probability in simulation was carried as follows: First, the value of the capture probability is selected arbitrarily. Next, the trajectory of an oncoming particle is monitored until the particle makes contact with either the collector or a previously deposited particle. If the particle makes contact with a collector, a uniform random deviate between 0 and 1 is generated. If the value of the random deviate is less than or equal to the assumed value of the capture probability, the particle is assumed to be deposited. On the basis of best agreement between experiments and prediction, a capture probability is determined empirically.

Simulation Results

Two types of information; namely, the number of particles deposited, N , versus the number of particles flowing past the collector, M , and the positions of deposited particles may be obtained for the simulation. The second type of information gives the deposited

morphology and its evolution with time. The unit collector efficiency can be found in the results of N vs. M according to Equation (3.1).

The extent of particle deposition may be described by the specific deposit, σ , defined as the volume of deposited particles per unit medium value. In terms of N , σ may be written as

$$\sigma = N(1-e)(d_p/d_g)^3 \quad (3.21)$$

where d_p and d_g are the diameter of the aerosol particle and packing grain, respectively.

The transient behavior of aerosol deposition in granular media is characterized by the changes in collection efficiency as the media becomes clogged. Quantitative relationship between η and σ is required for the rational design of granular filtration systems. A body of experimental data of η vs. σ have been collected in recent years (Walata *et al.*, 1986; Takahashi *et al.*, 1986; Jung and Tien, 1991; Jung, 1991). In the following, we compare our simulation results with the more recent data of Jung and Tien (1991). The conditions which were used in the experimental work are listed in Table 1 and the simulation was made using a super computer (IBM 3090-600E at Cornell National Supercomputer Facility, Ithaca, NY).

Simulation Without Considering the Effects of Flow Field Change. Hydrodynamic Interaction and Bounce-off. As an initial attempt to compare simulation with experimental results, the simulation runs were made with the assumption that the flow-field change due to the presence of particle deposits, the hydrodynamic interaction between an approaching particle and previously deposited particles, and the bounce-off of particles upon impact with a previously deposited particle are absent (i.e., capture probability is 1.0). The conditions

used in the simulation runs were the same as those used in experiments of Jung and Tien (1991) and are given in Table 1.

A typical comparison of the simulation and experimental results is shown in Fig. 4. Although both sets of results indicate an increase in η with the increase in σ , they are not in good agreement. The degree of disagreement between them becomes pronounced as the extent of deposition increases. The collection efficiencies predicted by simulations are always greater than those obtained by experiments as specific deposit increases. Similar conclusions were also obtained by Tsiang *et al.* (1982) in their study of aerosol deposition in model fibrous filters.

Simulation Results Considering the Bounce-off of Impacting Particles. It can be expected that the difference between simulation and experiments shown in Fig. 4 may be reduced by including the effect of bounce-off of impacting particles in simulation. The bounce-off of impacting particles can be determined by comparing the incident particle velocity (in normal direction) with the capture limit velocity V_{esc} defined by Equation (3.20).

To determine V_{esc} , it was assumed that the adhesion energy, ΔE , is ten times that given by the classical Bradley-Hamaker theory, as Tsiang assumed previously (Tsiang, 1980). According to the classical Bradley-Hamaker theory, the adhesion energy between two adhering particles of diameter d_1 and d_2 is

$$\Delta E = \frac{Hd_1d_2}{12\delta_o(d_1 + d_2)} \quad (3.22)$$

$$= \frac{Hd_1}{24\delta_o} \quad \text{if } d_1 = d_2 \quad (3.23)$$

where H is the Hamaker constant and δ_o is the separation distance between the particles. The value of δ_o is not known exactly but is often estimated to be about $4 \text{ \AA}$ (Tien, 1989). By substituting Equation (3.23) into Equation (3.21), the capture-limit velocity are given as

$$V_{esc} = \frac{1}{d_1} \left[\frac{H(1-e^2)}{2\pi\delta_o\rho_p e^2} \right]^{1/2} \quad (3.24)$$

The capture limit velocity can, therefore, be calculated if one knows the values of H , e , and δ_o . The coefficient of restitution, e , is given as (Zener, 1941)

$$e = e_o + \exp(-1.7\Lambda) - 1 \quad (3.25)$$

where e_o is the thick-body value of e (i.e., no flexural work), which is estimated to be 0.95, and Λ is the inelasticity parameter defined as (Tien, 1989)

$$\Lambda = \frac{2}{3\pi^{2/5}} \left(\frac{d_p}{d_g} \right)^2 \left(\frac{1}{1+d_p/d_g} \right)^{1/10} \left(\frac{V_{ni}}{V_c} \right)^{1/5} \left(\frac{\pi_p}{\pi_c} \right)^{3/5} \left(\frac{k_c}{k_p+k_c} \right)^{2/5} \quad (3.26)$$

where V_{ni} is the normal incident velocity of a particle and V_c is defined as $1/(k_p k_c)$. d_p and d_g are the particle and the collector diameters, and k_p and k_c are defined for the particle and collector materials, respectively, by $k_i = (1-\bar{\nu}_i^2)/Y_i$, where $\bar{\nu}_i$ and Y_i are Poisson's ratio and Young's modulus for material i . If it is assumed that Equation (3.26) can be applied to the case where two particles of the same sizes are colliding, then one has (Ramarao and Tien, 1988)

$$\Lambda = \frac{(2)^{1/2}}{3\pi^{2/5}} \left(\frac{V_m}{V_c} \right)^{1/5} \quad (3.27)$$

The relevant physical constants for polyvinyltoluene latex particles are found to be (Tsiang, 1980)

$$\rho_p = 1.027 \text{ g/cm}$$

$$d_p = 1.1, 1.02 \text{ } \mu\text{m}$$

$$\bar{v}_p = 0.33$$

$$Y_p = 0.32 \times 10^{11} \text{ dyne/cm}$$

$$H = 1.0 \times 10^{-12} \text{ erg}$$

$$\delta_o = 4 \text{ } \text{\AA}$$

The calculated values of capture limit velocities obtained by using these physical constants and the relevant equations are

$$V_{esc} = 30.51 \text{ cm/sec for } 1.1 \text{ } \mu\text{m latex particles}$$

$$V_{esc} = 15.75 \text{ cm/sec for } 2.02 \text{ } \mu\text{m latex particles}$$

The simulation results including the bounce-off effect are also shown in Fig. 4. It is clear that by including the bounce-off effect, agreement between simulation and predictions improves, but the difference nevertheless exists. The results also show that the effect of bounce-off is significant only at high values of σ .

The effect of deposition is generally manifested in the increase of η with the increase in σ as shown in Fig. 4. However, with the bounce-off consideration, this monotonic

increase in η with σ may no longer be valid. An example of this is shown in Fig. 5 which gives the unit collector efficiency vs. σ for the deposition of $1.1 \mu\text{m}$ aerosol particles in media composed of glass spheres of diameter of $5.25 \mu\text{m}$ at a superficial velocity of 43.55 cm s^{-1} . With inclusion of the bounce-off effect, the simulation results show a decrease of η with σ when the specific deposit exceeds 6×10^{-5} .

Determination of Capture Probability. As shown in Figs. 4 and 5, simulation results fail to agree with experiments regardless of whether the bounce-off effect is included or not. Furthermore, the capture limit velocity which is used as the criterion for adhesion cannot be determined without uncertainty. Accordingly, a more pragmatic and direct alternative to account for the various complications not included in the simulation would be the use of an empirically determined correction factor which we term the capture probability. Such an approach, of course, is arbitrary. However, if a consistent value of this correction factor can be found, the practical purpose of developing the simulation method can then be realized.

The determination of the capture probability can be illustrated by the example shown in Fig. 6 which shows gas η vs. σ under the conditions similar to those of Fig. 4. The simulation results are those obtained with the capture probability, γ equal to 1.0, 0.2 and 0.1, respectively. It is obvious that η decreases as the capture probability decreases. For this case, the experimental data are bracketed by the simulation results corresponding to $\gamma = 0.1$ and $\gamma = 0.2$. By successive trial and error, it was found that the best agreement was obtained with $\gamma = 0.15$. Similar determinations under different simulation conditions were also made and the results are summarized in Table 2. The agreement between the simulation results incorporating the use of γ and experiments are shown in Figs. 7 and 8. It is clear that with the use of this correction factor, good prediction of η vs. σ indeed can be made using the simulation method.

It should be noted that while γ is an empirical factor and varies with a number of variables as shown by the results given in Table 2, the degree of change of γ is, however, relatively small (ranging from 0.15 to 0.23). In the absence of its precise values, an average value of γ based on those given in Table 2 may be used in the simulation.

Deposit Morphology. From the simulation results, one can also obtain information concerning the morphology of particle deposits, which may vary with the extent of deposition and variables such as particle size, gas velocity, and collector size. A quantitative characterization of the deposit morphology and its evolution can be obtained by determining the size and size distribution of the particle clusters and their locations based on the average results of a number of simulations corresponding to the same set of conditions. An example is presented below.

(a) Size of Particle Deposits

The sizes of particle clusters formed in particle deposition may be classified by grouping the cluster in the manner shown in Table 3. The number of particles belonging to each group of clusters at a given specific deposit can be counted in each simulation run. Therefore, from the ensemble average of these runs, one can determine the fractional contribution to deposition by particles in each group corresponding to a given specific deposit. An example of this representation is shown in Fig. 9. It is clear that as the extent of deposition becomes significant, particle deposition takes place mainly on the previously deposited particles, leading to the further growth of the existing particle clusters.

The effect of the Stokes number, N_{St} , on the morphology of particle deposition is shown in Figs. 10. Fig. 10 indicates that a greater N_{St} leads to fewer but larger particle clusters. The effect of the interception parameter, N_R , shown in Fig. 11 which indicates that

clusters. The effect of the interception parameter, N_R , shown in Fig. 11 which indicates that for the same number of deposited particles, a smaller N_R gives to a larger number of smaller dendrites. This behavior may be due to the fact that the probability of an incoming particle's making contact with any previously deposited particles is proportional to the size of the particles.

(b) Positions of Particle Deposits

The morphology of particle deposits can also be analyzed in terms of their positions of deposition. To determine the deposition positions, the front half of the collector surface may be divided into a number of strips in the manner explained in Table 4. For example, a deposited particle with its θ -coordinate falling within a given strip is considered to be deposited in the strip regardless of the value of its radial coordinate and azimuth angle. By counting the positions of deposition from simulation results, one can readily obtain the spatial distribution of particle deposition.

Concerning the effect of particle inertia (i.e., N_{St}) on the position of particle deposition, Fig. 12 shows that at larger N_{St} , particle deposition takes place mainly in the more immediate neighborhood of the stagnation point than when the N_{St} is small, a behavior which is not unexpected.

Conclusions

A simulation algorithm for the study of aerosol deposition in granular media was developed based on the use of a modified Happel model for characterizing the structure of the media. With the use of the algorithm and a super computer, particle deposition under conditions of practical extent can be simulated with reasonable computer time.

The concept of the capture probability was introduced to account for the various complications such as the change of the flow field due to deposition, and the bounce-off of impacting particles which are not considered in the simulation. The value of the capture probability was determined empirically and its value was found to be insensitive to the operating condition. With the use of this correction factor, accurate prediction of the change in the unit collector efficiency with the extent of deposition can be by simulation.

The simulation results make it possible to analyze the particle-deposit morphology in terms of the size and position of particle deposits. It was found that the two parameters, N_{st} and N_r tend to give rise to few but large particle clusters, while larger N_{st} and N_r tend to result in many but small particle clusters. Therefore, one may expect that smaller particles are more likely to form large particle clusters than larger particles at the same specific deposit and gas velocity. Furthermore, particle clusters composed of small particles are expected to be more densely packed clusters made up of large particles. This means that the particle clusters made up of smaller particles are likely to be less permeable and to more effectively reduce the available pore space through which gas flows. As a result, it is expected that pressure drop across the filter bed as well as collection efficiency increases more significantly as filtration progresses. This qualitative trend has been found experimentally.

IV. Studies Initiated Prior to June, 1988

Two investigations initiated prior to June 1, 1980 were completed after the beginning of the Grant DE-FE-02 88 ER 13930. They are:

(1) **Experimental Determination of the Initial Collection Efficiency of Granular Filters.** As discussed in Section II, the transient behavior of granular filtration is described by the extent of its departure from the initial behavior of granular filtration. For this reason, it was important to collect a body of experimental data on the initial collection efficiency and compare them with other results available in the literature. Furthermore, it was thought useful to establish the possible effect due to the types of particle counter used in determining collector efficiency. The bulk experimental work was completed under Contract DE-ACO2-79-10386, but some additional measurements were made after the beginning of the grant. The work was since published (Aerosol Sci. Technol., 11, 168-182, 1989).

(2) **Analysis of Filter Candle Performance.** This work was made in order to examine the possible use of granular media for filter candles. The work was published (Powder Tech., 58, 243-258, 1989).

Copies of the reprints of both articles are appended.

Notations

b	dimensionless diameter of Happel's cell ($= R_p/R_c$)
c	aerosol concentration
d_p	aerosol diameter
d_{pi}	diameter of aerosol of the i -th type
d_g	filter grain diameter
e	coefficient of restitution
e_o	thick body of value of e
$G_1(\theta), G_2(\phi)$	cumulative probability functions
g	gravitational acceleration
H	Hamaker constant
k_p, k_c	defined as $(1-\bar{v}_i^2)/Y_i$ $i = p, c$, with p and c denoting particle and collector, respectively
L	filter height
M	number of particles approaching a collector
m	mass of an aerosol particle
N	number of particle collected or the number of particle type in polydispersed aerosols
N_G	gravitational parameter, defined by Equation (3.10.g)
N_R	interception parameter, defined as d_p/d_g
N_{St}	Stokes number, defined by Equation (3.10.f)
R_1, R_2	random numbers uniformly distributed over (0,1)
R_b	radius of Happel's cell
R_c	collector radius

r	radial distance
r^*	dimensionless r defined as r/R_c
t	time
u_s	superficial velocity
v_r, v_θ	fluid velocity components along the r - and θ -direction
v_r^*, v_θ^*	dimensionless values of v_r and v_θ , at v_r/u_s and u_θ/u_s
V_{esc}	escape velocity defined by Equation (3.29)
V_c	defined as $1/(k_p k_c)$
V_{ni}	normal incident velocity
w_i	defined as σ_i/σ
Y_i	Young's modulus for material i
z	axial distance

Greek Letters

α	defined by Equation (3.12)
α_1, α_2	empirical constant of Equation (2.4)
$\alpha_1(i/j), \alpha_2(i/j)$	empirical constants of Equation (2.6)
β_1, β_2	empirical constant of Equation (2.5)
γ	adhesion probability
ΔE	particle-collector interaction energy
ΔP	pressure drop
δ_o	separation distance
ϵ	porosity
η	collection efficiency

η_0	initial value of η
η_i	collector efficiency of particle of size i
$(\eta_i)_0$	initial value of η_i
$\eta(i/j)$	collection efficiency of particle of size i in a filter loaded with deposited particles of size j
θ	corrected time defined as $t - z\epsilon/u_s$ or angular coordinate
Λ	inelastic parameter given by Equation (3.26)
λ	filter coefficient
λ_0	initial value of λ
μ	viscosity of fluid
ν_i	Poisson's ratio of material i
ρ, ρ_p	density of fluid and density of particle
σ	total specific deposition
σ_i	specific deposit of particle of size i

Table 1

Conditions Used for Simulation

$d_p(\mu\text{m})$	$d_g(\mu\text{m})$	$u_s(\text{cm/sec})$	N_{st}	N_R	N_G
1.1	262	11.3	3.924×10^{-3}	4.198×10^{-3}	3.95×10^{-4}
1.1	262	15.8	5.491×10^{-3}	4.198×10^{-3}	1.53×10^{-4}
1.1	262	20.8	7.223×10^{-3}	4.198×10^{-3}	2.14×10^{-4}
1.1	525	11.3	1.958×10^{-3}	2.095×10^{-3}	3.95×10^{-4}
1.1	525	20.8	3.605×10^{-3}	2.095×10^{-3}	2.14×10^{-4}
1.1	525	31.8	5.504×10^{-3}	2.095×10^{-3}	1.41×10^{-4}
1.1	525	43.6	7.547×10^{-3}	2.095×10^{-3}	1.03×10^{-4}
2.02	525	11.3	6.219×10^{-3}	3.850×10^{-3}	6.81×10^{-4}

Table 2

Numerical Values of $\bar{\gamma}$

$d_p(\mu\text{m})$	$d_p(\text{cm})$	$u_s(\text{cm/s})$	$\bar{\gamma}$
1.1	262	11.1	0.2
1.1	262	15.81	0.21
1.1	262	20.8	0.23
1.1	525	11.3	0.15
1.1	525	20.8	0.15
1.1	525	31.76	0.18
1.1	525	43.55	0.2
2.02	525	11.3	0.14

Table 3

Classification of Particle Clusters According to Size

Cluster Type	Number of Particles Present in the Cluster
1	1
2	2
3	3
4	4
5	5
6	6 - 10
7	11 - 20
8	21 - 40
9	41 - 60
10	61 - 100
11	101 - 200
12	201 - 400
13	401 - 600
14	601 - 800
15	801 - 1000
16	1001 - 1200
17	1201 - 1400
18	1401 - 1600
19	1601 - 1800
20	1801 - 2000
21	2001 - 2200
22	2201 - 2400

Table 4

**Classification of the Position of Particle Deposits in Terms
of Their Angular Positions in θ -Coordinate**

i-th Strip	Angle from the Axis of Symmetry (Degree)
1	0 - 3
2	3 - 6
3	6 - 9
4	9 - 12
5	12 - 15
6	15 - 18
7	18 - 21
8	21 - 24
9	24 - 27
10	27 - 30

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Fig 2

$XDEP(Num, kk, j)$ = the center position of the Num-th deposited particle in grid (kk, j) in terms of x.

$YDEP(Num, kk, j)$ = the center position of the Num-th deposited particle in grid (kk, j) in terms of y.

$ZDEP(Num, kk, j)$ = the center position of the Num-th deposited particle in grid (kk, j) in terms of z.

$Icount(kk, j)$ = the number of deposited particles in grid (kk, j).

$Rmax(kk, j)$ = the highest position of the deposited particle in grid (kk, j) in terms of r, R_h , plus $3N_R$
(i.e., $Rmax(kk, j) = R_h + 3 N_R$)

IDNUM = the identification numbers of dendrites

IDEN(Num, kk, j) = the identification number of the dendrite where Num-th deposited particle in grid (kk, j) was deposited.

Ndep = total number of deposited particles.

Fig. 4

Fig 5

Fig 8.

Fig. 8

Fig. 9

% Contribution to Each Type Cluster to
Total Number of Deposited Particles

(Number of deposited particles = 500)

Fig

Fig 12

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Appendix A. Expression for v_r and v_θ

A general solution for the axisymmetric creeping-flow equations in spherical coordinates is given by Happel and Brenner⁶(1985):

$$\psi(r, \theta) = \sum_{n=2}^{\infty} [A_n r^n + B_n r^{n+1} + C_n r^{n+2} + D_n r^{n+3}] I_n(\mu) \quad (A-1)$$

where

$$I_n(\mu) = \frac{P_{n-2}(\mu) - P_n(\mu)}{2n-1}$$

and where ψ is stream function and $P_n(\mu) = P_n(\cos \theta)$ is the Legendre polynomial of degree n .

The flow field in the modified Happel's model are solved by determining the constants A_n, B_n, C_n , and D_n . The detailed procedure can be seen in Vaidyanathan's dissertation (1989).

The velocity fields, v_r and v_θ , are, therefore, obtained by the following relationships:

$$v_r = \frac{-1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} \quad (A-2)$$

$$v_\theta = \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r} \quad (A-3)$$

In this study, the maximum value of n was chosen as 50 and the values of the coefficients are listed in Table (A-1).

Table A-1

Constants That Determine the Expression for v_r and v_θ

$\beta = 20$ degrees, $\epsilon = 0.34$

For $n = 0, 1, 2, \dots, 50$

A_n	B_n	C_n	D_n
0.28160877114D+03	0.80743418643D+02	-.40040644616D+02	-.32231154516D+03
0.41540557612D+02	0.50849812096D+02	-.15143309124D+02	-.77247060584D+02
0.16126481968D+02	0.34354890477D+02	-.69480506143D+01	-.43533321830D+02
0.68347164122D+01	0.20349818526D+02	-.32101117538D+01	-.23974423184D+02
0.21593765369D+01	0.79454126983D+01	-.10957145122D+01	-.90090747230D+01
-.31696614766D+00	-.15238183501D+01	0.15689793047D+00	0.16838865673D+01
-.13393838129D+01	-.67368474898D+01	0.74114445716D+00	0.73350868456D+01
-.13595597047D+01	-.75033365903D+01	0.78775994369D+00	0.80751363512D+01
-.83414319704D+00	-.49977585181D+01	0.50089167046D+00	0.53310100446D+01
-.19219604799D+00	-.12197884420D+01	0.11978658696D+00	0.12921979030D+01
0.26874393844D+00	0.19093099652D+01	-.16743910571D+00	-.20106147980D+01
0.43674279598D+00	0.32880368201D+01	-.27824153572D+00	-.34465380803D+01
0.35910760256D+00	0.28701864581D+01	-.23246861979D+00	-.29968254409D+01
0.16397822219D+00	0.13846778444D+01	-.10766345614D+00	-.14409926104D+01
-.22234272864D-01	-.20715508705D+00	0.14458362358D-01	0.21493099756D+00
-.12606000320D+00	-.11986927719D+01	0.84004359548D-01	0.12407484156D+01
-.13687844623D+00	-.13650583595D+01	0.92044511730D-01	0.14098922940D+01
-.86849389423D-01	-.90689535277D+00	0.58856579783D-01	0.93488816241D+00
-.20758720603D-01	-.22525180938D+00	0.14198439804D-01	0.23181209018D+00
0.27627278678D-01	0.31762115617D+00	-.18886811973D-01	-.32636162288D+00
0.45292381579D-01	0.54101398198D+00	-.31164517397D-01	-.55514184616D+00
0.37168763843D-01	0.46153588134D+00	-.25704289242D-01	-.47300035594D+00
0.16937961960D-01	0.21817584326D+00	-.11770625582D-01	-.22334317964D+00
-.21625721394D-02	-.29472892328D-01	0.14965623611D-02	0.30138902106D-01
-.12674582389D-01	-.17631572353D+00	0.88565964106D-02	0.18013370951D+00
-.13704129338D-01	-.19707341734D+00	0.96113614417D-02	0.20116618523D+00
-.86528935247D-02	-.12852493288D+00	0.60888220031D-02	0.13108900440D+00
-.20718856261D-02	-.31666846932D-01	0.14639805349D-02	0.32274752023D-01
0.26944973949D-02	0.42788618940D-01	-.19034866760D-02	-.43579629659D-01
0.44097282931D-02	0.72043386332D-01	-.31248019969D-02	-.73328312628D-01
0.36041845635D-02	0.60592970782D-01	-.25603283791D-02	-.61636826967D-01
0.16374515230D-02	0.28294725309D-01	-.11660786595D-02	-.28766098173D-01
-.20323929364D-03	-.36505090611D-02	0.14440344089D-03	0.37093449138D-01
-.12076992835D-02	-.22090896685D-01	0.86253489877D-03	0.22436061070D-01
-.13020895173D-02	-.24432491398D-01	0.93180777753D-03	0.24802773138D-01
-.81971212250D-03	-.15770227307D-01	0.58768499762D-03	0.16002254432D-01
-.19635710999D-03	-.38645410282D-02	0.14110679519D-03	0.39197913430D-01
0.25231788788D-03	0.51095778392D-02	-.18129940143D-03	-.51805963256D-01
0.41241588626D-03	0.85424848844D-02	-.29686877954D-03	-.86580319911D-01
0.33626162551D-03	0.71251274023D-02	-.24240767547D-03	-.72189813523D-01
0.15248535208D-03	0.33028280804D-02	-.11008886825D-03	-.33452245642D-01
-.18623612786D-04	-.41534779215D-03	0.13426350451D-04	0.42054505449D-01
-.11151932205D-03	-.25272407041D-02	0.80656631684D-04	0.25581033944D-01
-.12001957491D-03	-.27767917597D-02	0.86913159655D-04	0.28098981750D-01
-.75415724076D-04	-.17807330983D-02	0.54676631088D-04	0.18014721913D-01
-.18067981638D-04	-.43475542314D-03	0.13119091969D-04	0.43970431281D-01
0.23047919880D-04	0.56732887257D-03	-.16733729586D-04	-.57364306287D-01
0.37639021742D-04	0.94404259534D-03	-.27359633082D-04	-.95432198400D-01
0.30640041400D-04	0.78311589411D-03	-.22293826676D-04	-.79146210884D-01
0.13877139315D-04	0.36124030977D-03	-.10107114701D-04	-.36501022420D-01

Appendix B. Reprints of Publications

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