NATURAL FLAVOR CONSERVATION

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Abstract

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ABSTRACT

It is shown that the fiat of Natural Flavor Conservation for an arbitrary U(1)-unbroken gauge theory require that all quarks of a given charge and helicity transform identically under the gauge group.

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I. Introduction

Experimentally, flavor-violating neutral currents in low energy transitions are absent or at least highly suppressed up to α_G order. And it appears as if the Yukawa couplings of Higgs scalars with the fermion fields also conserve flavor.

For strange particles, for instance, interactions of the form $(s\bar{d})(\mu\bar{\mu})$ are ruled out by the $K_L^0 \rightarrow \mu^+ \mu^-$ branching ratio, $(K_L^0 \rightarrow \mu^+ \mu^-)/K_L^0 \rightarrow \text{all} = 10^{-8}$, as well as interactions of the form $(s\bar{d})(s\bar{d})$ which are ruled out by the $K_L^0 - K_S^0$ mass difference, $m(K_L^0) - m(K_S^0) = 3.5 \times 10^{-6} \text{ eV}$, in whose calculation they contribute¹.

For charmed particles, $D^0 - \bar{D}^0$ mixing excludes $|\Delta C| = 1$ neutral currents of order G_F and it is likely that this is true for α_G order, although the best limit² on charm changing neutral currents is $< 2.6\%$ and from the charm decay rates $\lesssim 2\%$.

One could build models that conserve any number of flavors in the neutral currents by adjusting the values of the various parameters in theory. But when one fiddles a priori with these parameters one ruins manifest renormalizability. The best way to insure that gauge models satisfy the experimental constraints on neutral currents is to require that flavor conservation is a requisite for all neutral currents and is assured "naturally". "Natural" flavor conservation (NFC) means that flavor conservation follows from the group structure and representation content of the theory, rather than from the particular values of the parameters (i.e., coupling constants).

This elevation of flavor conservation from the observed absence of flavor changing neutral currents to a general theoretical constraint for all flavors in gauge theories, pressed by a few authors³, was realized in a paper by Glashow

and Weinberg⁴. There they stated the conditions for $SU(2)_L \times U(1)$, (WS)⁵, to naturally conserve all quark flavors in the neutral currents. These conditions for NFC are that:

R1: All quarks of a given charge and helicity must have the same values of T_3 and T^2 .

R2: All quarks of a given charge receive their mass either through the couplings of precisely one neutral Higgs meson or through an $SU(2)$ -invariant mass term, but not by both mechanisms.

While the former condition is necessary and sufficient for NFC, the latter is only sufficient. This point will be discussed more fully later.

In this paper, we would like to consider the general problem of applying NFC to an arbitrary local gauge theory, broken down to a $U(1)$ subgroup (WET). There are two compelling reasons to consider NFC in this context.

Firstly, it is possible that WS may not be the operative gauge group of the weak and electromagnetic interactions (i.e., $SU(2)_L \times SU(2)_R \times U(1)$ ⁶ is not ruled out).

Secondly, generalizing Glashow and Weinberg's work was found necessary to discuss broader questions concerning NFC and calculability recently⁷.

The general formalism for spontaneously broken local gauge theories, necessary for this, will be introduced in Section II. In Section III, we show that if one requires the neutral currents to conserve all fermion flavors up to order α_G , naturally, then all quarks of a given charge and helicity must transform identically under the gauge group. Also, the demand of NFC for the neutral Higgs exchanges yields the condition that all Yukawa couplings of Higgs with fermions are simultaneously diagonalizable, in a natural manner, with the

mass matrix. Although the former condition is virtually indisputable, the latter condition's necessity is questionable. Conclusions and open questions are discussed in Section IV. The appendix is reserved for the determination of the two gauge boson exchange graphs.

II. General formalism.

This section will be devoted to the explication of a general formalism of gauge theories⁸. A few original conventions are contained herewith, oftentimes for the sake of specificity. It is hoped that this formalism will facilitate the following discussion of natural flavor conservation in gauge theories irrespective of the particular group employed to describe the weak and electromagnetic interactions. We would first like to motivate a Lagrangian, $\mathcal{L}$, for these interactions.

The theory will contain a number of elementary spin- $\frac{1}{2}$ fields, $\alpha_p(x)$, and spin-zero fields, $\beta_q(x)$. We require that $\mathcal{L}$ be invariant under a local gauge transformation with respect to, in general, a direct product of simple Lie groups:

$$G = \otimes G_i \quad (1)$$

with generators T^i satisfying:

$$[T^i_\alpha, T^j_\beta] = i C^i_{\alpha\beta\gamma} T^i_\gamma \quad (2)$$

where $C^i_{\alpha\beta\gamma}$ are the structure constants for G^i and where repeated indices are summed over. These generators will induce an equal number of gauge fields $A^i_\mu(x)$. Gauge invariance demands that the fields transform as representations of the group. So $\psi_j(x) [\phi^k(x)]$, which are the $j = 1, 2, \dots, n$ [$k = 1, 2, \dots, m$] multiplets containing the elementary fields $\alpha_p(x)$ [$\beta_q(x)$] and $A^{i\alpha}_\mu(x)$ will transform according to the representations D^j_F [D^k_B] and the regular representation of G respectively.

That there may be many fermion multiplets (whose representations need not be equivalent to each other) reflects the fact that there may be "generations"

of quarks and leptons. The subject of generations as well as flavors will be discussed in section II.

m multiplets of boson fields are included in the formalism since the gauge group may require more than one Higgs multiplet to break the symmetry sufficiently.

For clarity these "generation" indices will be suppressed in the following treatment, as well as any strong interaction indices (i.e. color) on the fermion fields. Then the fields transform under G in the following way:

$$\psi(x) \rightarrow \psi'(x) = \exp[-i\vec{t} \cdot \vec{\theta}] \psi(x) = U(\theta) \psi(x) \quad (3)$$

$$\phi(x) \rightarrow \phi'(x) = \exp[-i\vec{L} \cdot \vec{\theta}] \phi(x) = U'(\theta) \phi(x)$$

where $\vec{t} \cdot \vec{\theta} = t^\alpha_\alpha \theta^\alpha$ and $t_\alpha(L_\alpha)$ is the matrix representing the abstract generator T_α in the representation D_F (D_B) of G furnished by the fermion (boson) fields:

$$[t_\alpha, t_\beta] = i C_{\alpha\beta\gamma} t_\gamma \quad (4)$$

$$[L_\alpha, L_\beta] = i C_{\alpha\beta\gamma} L_\gamma$$

and $\theta_\alpha = \theta_\alpha(x)$ is an arbitrary group parameter. Under infinitesimal transformations the field variations are:

$$\delta\psi(x) = -i t_\alpha \theta_\alpha(x) \psi(x) \quad (5)$$

$$\delta\phi(x) = -i L_\alpha \theta_\alpha(x) \phi(x)$$

$$\delta A^\alpha_\mu(x) = -\frac{1}{g} \partial_\mu \theta^\alpha + C_{\alpha\beta\gamma} \theta^\beta A^\alpha_\mu$$

(no sum on α) where g^α are arbitrary coupling constants, independent of α within each subgroup, that characterize each simple Lie group in G .

For the present we will also subsume all subgroup indices, i , into the generator indices α, β, γ , etc. Then the most general renormalizable gauge-

invariant Lagrangian is:

$$\mathcal{L} = L_A + L_\phi + L_\psi + L_M \quad (6)$$

where

$$\begin{aligned} L_A &= -\frac{1}{4} F_{\alpha\mu\nu}^2 \\ L_\phi &= \frac{1}{2} (D_\mu \phi)_p (D^\mu \phi)_p - p(\phi) \\ L_\psi &= \bar{\psi} \gamma^\mu D_\mu \psi \\ L_M &= -\bar{\psi} (m_0 + \Gamma_p \phi_p) \psi \end{aligned} \quad (7)$$

In $\mathcal{L}_A$ the gauge covariant curl $F_{\mu\nu}^\alpha$ is:

$$F_{\mu\nu}^\alpha \equiv \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + g_{\alpha\beta\gamma} A_{\beta\mu} A_{\gamma\nu} \quad (\text{no sum on } \alpha) \quad (8)$$

The structure constants, $C_{\alpha\beta\gamma}$, are of course zero when α, β or γ are from different subgroups in G .

In $\mathcal{L}_\phi$, D_μ is the covariant derivative and $p(\phi)$ is a real G -invariant 4th order polynomial in ϕ . That is

$$(D_\mu \phi)_p \equiv \partial_\mu \phi_p - ig_{\alpha\beta\gamma} (L_p)_{\alpha\beta\gamma} \phi_q A_{\alpha\mu} \quad (9)$$

Since $p(\phi)$ is G -invariant, then $\delta p(\phi) = 0$.

Since θ_α are arbitrary infinitesimal parameters, we have the condition of gauge invariance on $p(\phi)$:

$$\frac{\partial p(\phi)}{\partial \phi_p} (L_p)_{\alpha\beta\gamma} \phi_q = 0 \quad (10)$$

In $\mathcal{L}_\psi$, the covariant derivative D_μ is

$$(D_\mu \psi)_n \equiv \partial_\mu \psi_n - ig_\alpha (t_\alpha)_{nm} \psi_m A_{\alpha\mu} \quad (11)$$

Since the left- and right-handed fermion fields may transform differently under G , it is useful to decompose the generators and fields into their chiral components:

$$t_\alpha = L t_\alpha^L + R t_\alpha^R \quad (12)$$

where $L = \frac{1}{2} (1 - \gamma_5)$ and $R = \frac{1}{2} (1 + \gamma_5)$.

t_α^L and t_α^R separately satisfying the commutation relations (4). and

$$\psi = \psi^L + \psi^R \quad (13)$$

where $\psi^L \equiv L\psi$ and $\psi^R \equiv R\psi$,

so that now t_α^L (t_α^R) is the matrix representing the abstract generator T_α in the representation D_F^L (D_F^R) of G furnished by the fields ψ_L (ψ_R).

In $\mathcal{L}_M$, the bare mass, m_0 , must be gauge invariant,

$$[t_\alpha, \gamma_0 m_0] = 0 \quad (14)$$

If the Yukawa interaction in $\mathcal{L}_M$ is to be invariant, then we must have:

$$[t_\alpha, \gamma_0 \Gamma_p] = - (L_p)_{pq} \gamma_0 \Gamma_p \quad (15)$$

$\gamma_0 \Gamma_p$ must transform under G according to the representation D_B . The Γ_p matrices include the coupling constants in the Yukawa interaction.

Some of our conventions and normalizations are:

We choose the generators and corresponding gauge fields to be Hermitian

$$T_\alpha^\dagger = T_\alpha, \quad A_{\gamma\mu}^\dagger = A_{\alpha\mu} \quad (16)$$

so the structure constants are real

$$C_{\alpha\beta\gamma}^* = C_{\alpha\beta\gamma} \quad (17)$$

then if $F^{\alpha\mu\nu} F_{\mu\nu}^\alpha$ is G-invariant then the structure constants are totally antisymmetric.

We choose the component fields ϕ_p to be Hermitian

$$\phi_p^\dagger = \phi_p \quad (18)$$

so the L_α are pure imaginary

$$(L_\alpha)^*_{pq} = - (L_\alpha)_{pq} \quad (19)$$

then if $(D_\mu \phi)_p (D^\mu \phi)_p = (D_\mu \phi, D^\mu \phi)$ is G-invariant, then the L_α are antisymmetric

$$(L_\alpha)_{pq} = - (L_\alpha)_{qp} \quad (20)$$

Also $\bar{\psi} \equiv \psi^\dagger \gamma_0$, $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$, $\gamma_5 \equiv i\gamma_1 \gamma_2 \gamma_3$ and the metric is $(+---)$.

Of course the theory contains no explicit mass terms for the gauge fields. These mass terms $\frac{1}{2} m^2 A_\mu^\alpha A^{\mu\alpha}$ would ruin the gauge invariance of our theory, as we can see by the transformation laws for A_μ^α in Eq. (5). Mass terms would also ruin the renormalizability of the theory.

Consider the gauge invariance condition on $p(\phi)$, Eq. (10). If we differentiate this equation once more and evaluate it at the value of ϕ , γ , that minimizes $p(\phi)$. (There may be more than one), then

$$\left(\frac{\partial^2 p(\phi)}{\partial \phi_m \partial \phi_p} \right)_{\phi=\lambda} (L_\alpha)_{pq} \lambda_q = 0 \quad (21)$$

If we expand $p(\phi)$ about its minimum, then,

$$p(\phi) = -\frac{1}{2} M_{pm}^2 (\phi-\lambda)_p (\phi-\lambda)_m + \dots \quad (22)$$

where M_{pm}^2 is the scalar mass matrix:

$$M_{pm}^2 = - \left(\frac{\partial^2 p(\phi)}{\partial \phi_p \partial \phi_m} \right)_{\phi=\lambda} \quad (23)$$

Thus from Eq. (21),

$$(M^2)_{mp} (L_\alpha)_{pq} \lambda_q = 0 \quad (24)$$

for all α , satisfies the Goldstone theorem.⁹

Define S to be the M-dimensional subgroup of G which remains a symmetry of the vacuum, and L_α the generator of S , which is in general some linear combination of the original generators. Then parametrize the fields $\phi(x)$ by:

$$\phi(x) = \exp \left[\sum_\alpha i \xi_\alpha(x) L_\alpha / \lambda \right] (\lambda + \eta(x)) \quad (25)$$

The sum is over those $(N-M)$ L^α which do not annihilate λ . The vector $\eta(x)$ represents as many independent fields as there are dimensions in the part of the n-dimensional representation space orthogonal to all $L^\alpha \lambda$. The number of field parameters $\xi_\alpha(x)$, associated with the broken subgroup of G , is equal to $(N-M)$.

Choose the gauge function such that

$$\theta_\alpha(x) = \xi_\alpha(x) / \lambda \quad (26)$$

that is,

$$\phi \rightarrow \phi' = \exp[-i \xi_\alpha L_\alpha / \lambda] \phi \quad (27)$$

then the G-invariance of $\mathcal{L}$ will yield a Lagrangian dependent only on the n fields and gauge fields A_μ^α , $N-M$ of which are now massive. The gauge field mass matrix is

$$(M^2)^{\alpha\beta} = g^\alpha g^\beta (L^\alpha \lambda, L^\beta \lambda), \quad (\text{no sum over } \alpha, \beta) \quad (28)$$

The $N-M$ Goldstone bosons are absorbed into the longitudinal components of the $N-M$ massive vector bosons:

$$A_\mu^{\prime\alpha} = A_\mu^\alpha - \frac{1}{g^\alpha \lambda} \partial_\mu \xi^\alpha + O(\xi^2) \quad (29)$$

The vector mesons corresponding to the surviving M-dimensional symmetry

subgroup S remain massless. So the number of independent degrees of freedom remains the same. This is the Higgs mechanism.¹⁰ In this gauge $\phi'(x)$ has no components in the subspace spanned by the Goldstone bosons, which is the space spanned by $L^\alpha \lambda$. Thus

$$(L^\alpha \lambda, \phi'(x)) = 0 \quad (30)$$

defines the unitary gauge.

In a gauge theory with an unbroken subgroup S, the generators of S will be those M independent linear combinations of the generators, $C_\alpha^i T_\alpha$, that leave λ invariant:

$$C_\alpha^i T_\alpha \lambda = 0 \quad (31)$$

The gauge field mass matrix, given by Equation 28 is real symmetric.

Thus, we can find a complete set of orthogonal eigenvectors, $u_{\alpha i}$:

$$(u^2)_{\alpha\beta} u_{\beta i} = m_i^2 u_{\alpha i} \quad (\text{no sum on } i) \quad (32)$$

where

$$\sum_\alpha u_{\alpha i} u_{\alpha j} = \delta_{i,j} \quad (33)$$

$$\sum_i u_{\alpha i} u_{\beta i} = \delta_{\alpha,\beta}$$

These u_i can be used to define new canonically normalized vector fields

$$Z_\mu^i = u_\alpha^i A_\mu^\alpha \quad (34)$$

Using (33):

$$A_{\alpha\mu} = \sum_i u_\alpha^i Z_\mu^i \quad (35)$$

Then the gauge meson mass term in $\mathcal{L}'$ can be written as

$$\frac{1}{2} v_{\alpha\beta}^2 A_{\alpha\mu} A_\mu^\beta = \frac{1}{2} \sum_i m_i^2 Z_\mu^i Z^{\mu i} \quad (36)$$

The Z_μ^i fields describe, therefore, particles of definite mass -- they are the physical gauge fields.

In this basis one defines new generators

$$\bar{T}^i \equiv C_\alpha^i T_\alpha \quad (37)$$

These C_α^i are related to the $u_{\alpha i}$ through Eqs. (28) and (31),

$$u_\alpha^i = \frac{N^i C_\alpha^i}{g_\alpha} \quad (38)$$

where N^i is a normalization factor for each u_α^i . From Eq. (33)

$$N^i \equiv \left[\sum_\alpha \frac{C_\alpha^{i2}}{g_\alpha^2} \right]^{-1/2} \quad (39)$$

We call the M unbroken generators of S, Eq.(31), Q^i :

$$Q^i \equiv \bar{T}^i \quad \text{for } i = 1, 2, \dots, M \quad (40)$$

where nonvanishing C_α^i 's are associated with the fields neutral under Q^i .

Also, the M eigenvectors corresponding to zero eigenvalues, $u^2 u^i = 0$, will be relabeled: $p^i \equiv u^i$, where $T_i \in S$. Finally, we call the M massless vector fields $A_\mu^i \equiv Z_\mu^i$ for $i \leq M$.

III. Natural Flavor Conservation

As stated previously, the theory may contain more than one set of fermion fields which, at this point, transform as different representations of G. Consider, for example, one has n sets of fields (multiplets) transforming as n k_n -vector representations of G:

$$\psi^i = \begin{pmatrix} \alpha_1^i \\ \alpha_2^i \\ \vdots \\ \alpha_{n1}^i \end{pmatrix}, \quad i = 1, 2, \dots, n \quad (41)$$

where α_j^i is a fermion field that is the j^{th} component of the i^{th} multiplet. Decomposing ψ^i into left- and right-handed components, according to one has:

$$\psi_L^i = \begin{pmatrix} \alpha_1^i \\ \alpha_2^i \\ \vdots \\ \alpha_{n1}^i \end{pmatrix}_L \quad \text{and} \quad \psi_R^i = \begin{pmatrix} \alpha_1^{i'} \\ \alpha_2^{i'} \\ \vdots \\ \alpha_{n1}^{i'} \end{pmatrix}_R$$

where the primes indicate that the left- and right-handed representations also need not be equivalent (contain the same number and type of field). Every distinct type of field contained in these multiplets is called a "flavor". Flavors having both left and right chirality are considered one flavor.

The existence of flavors, their type and number, for both quarks and leptons is not well understood. Some attempts to explain flavor¹¹ require that these multiplets transform under a continuous group, K, in the "horizontal" direction. That is, gauge theories normally work in the "vertical" or j-space

of a multiplet. The "horizontal" group acts in the i-space of different multiplets.

Although some of these horizontal symmetries may also "explain" mixing angles and mass relations, they contain undesirable features such as Goldstone bosons or superheavy vector bosons.

Another interpretation of flavor requires the multiplets to transform, also in the horizontal direction, under a discrete¹¹ symmetry. This device avoids the problems associated with the continuous theories but is less physically appealing and usually less ambitious.

Since we are considering WET's, we can assume that the effective gauge group is of the form $G = G' \times U(1)$ with G' not necessarily simple — the electric charge is left unbroken. From Eq. (40) the electric charge is written as:

$$Q = C_\alpha T_\alpha \quad (42)$$

where T_α are the Hermitian electrically neutral generators of G. The complete set of orthonormal eigenvectors, $u_{\beta 1}$, of the neutral vector boson mass matrix, μ_N^2 satisfy Eqs. (32-39), where M is now 1-dimensional and $m_1^2 = 0$ corresponds to the massless photon. Now from (7),

$$\mathcal{L}_\psi = -\bar{\psi} \gamma^\mu D_\mu \psi \quad (43)$$

where ψ is a column vector representing the set of all quark fields: to wit, all our flavor multiplets are put into ψ . D_μ will be block diagonal in this space; each block acting between the same flavor multiplet:

$$(D_\mu \psi)_n \equiv \partial_\mu \psi_n - ig_\alpha (t_\alpha)_{nm} \psi_m A_{\alpha\mu} \quad (44)$$

Since we are concerned here with the current term, the relevant part of

the Lagrangian, $\mathcal{L}_c$, will be:

$$\mathcal{L}_c = ig_\alpha \bar{\psi} \gamma^\mu t_\alpha A_{\mu} \psi \quad (45)$$

So the interaction of the gauge fields with the fermion fields is described by the matrices $g_\alpha t_\alpha A_{\mu}$. In the physical vector boson basis we show that it is just $N_1 \bar{t}^1 Z_\mu^1$. Using Eqs. (33-38) we have:

$$\sum_1 N_1 \bar{t}^1 Z_\mu^1 = \sum_\alpha g_\alpha t_\alpha A_\mu^\alpha \quad (46)$$

Since the mass matrix conserves charge, the gauge mesons corresponding to different charges will decouple and so μ will be block diagonal.

For the sake of simplicity, we assume there is no mixing among the charged gauge mesons. Define then, for the charged interactions

$$T^a = T^a(a,b) \equiv T_a^a + i T_b^a \quad (47)$$

where T^a is the generator from the a th subgroup made up of the a th and b th generators corresponding to the charged gauge field $W_\mu^a \equiv Z_\mu^a$.

Then if we split $\mathcal{L}_c$ into chiral components, then from Eqs. (43) and (45), the couplings of the neutral IVB's, Z^1 , with the fermion neutral current will be:

$$\mathcal{L}_{Nc}^1 = iZ_\mu^1 \{ N_L^1 \bar{\psi}_L \gamma^\mu t_L^1 \psi_L + \text{h.c.} \} \quad (48)$$

and

$$\mathcal{L}_{cc} = iW_\mu^\alpha \{ N_L^\alpha \bar{\psi}_L \gamma^\mu t_L^\alpha \psi_L + \text{h.c.} \} \quad (49)$$

Now, in analogy with Glashow and Weinberg⁴ we impose the condition, that the neutral currents naturally conserve all fermion flavors. In lowest order this requires flavor conservation in all graphs like Figure 1.

Since each quark flavor corresponds to a different eigenvalue of the quark mass matrix, then we require $\bar{t}_1^{L(R)} = C^1 t_1^{L(R)}$ be diagonal in the basis in which the quark mass matrix is diagonal. In the physical quark basis, $\bar{t}_1^L$ and $\bar{t}_1^R$ will become

$$\bar{t}_1^{L(R)} = U_{L(R)} \bar{t}_1^{L(R)} U_{L(R)}^\dagger \quad (50)$$

where $U_{L(R)}$ diagonalize the mass matrix. Since the first condition requires the neutral currents conserve flavors in this basis, then $\bar{t}_1^{L(R)}$ must be diagonal. For $\bar{t}_1^{L(R)}$ to be diagonal for all arbitrary $U_{L(R)}$, they must be functions of the matrix Q . Thus the necessary and sufficient condition for neutral currents to conserve all quark flavors in lowest order is:

$$\bar{t}_1^{L(R)} = f_1^{L(R)}(Q) \quad (51)$$

or

$$\sum_\alpha C_\alpha^1 t_\alpha^{L(R)} = f_1^{L(R)}(Q) \quad (52)$$

This condition is not very useful in its present form. We must relate equation (51) to a statement about the original neutral generators. This can be done easily if one notes that the $\bar{t}_1$ are just an orthogonal set of neutral generators corresponding to mass eigenstates of the neutral gauge bosons and so equation (37) is invertible. So we may write (51) as:

$$t_\alpha^{L(R)} = f^{L(R)}(Q) \quad (53)$$

This is, then, the generalization of reference [4] to an arbitrary gauge group. Condition I is equivalent to requiring that all quarks of a given charge and helicity must have the same value of t_α for all α , where t_α is a neutral generator of G .

Assuming now that Eq. (53) is satisfied, we look at the one loop diagrams of order αG_F which could produce unwanted flavor nonconserving interactions. The diagrams which could prove to be fatal in this order are exhibited in Figure 2.

Consider first the diagrams I with $\alpha = \beta$. They induce an effective Fermi interaction of the form (see appendix):

$$H_I = \alpha G_F \{ 15 L_\mu L^\mu - K_\mu K^\mu \} \quad (54)$$

where K_μ and L_μ are the effective neutral currents:

$$K_\mu = \bar{q} \gamma_\mu Z_L L_q - \bar{q} \gamma_\mu Z_R R_q \quad (55)$$

and

$$L_\mu = \bar{q} \gamma_\mu V_L L_q - \bar{q} \gamma_\mu V_R R_q$$

where

$$Z_L = 4 T_L^{\alpha\alpha\dagger} - T_L^{\alpha\dagger} T_L^\alpha \quad (56)$$

$$Z_R = 4 T_R^{\alpha\dagger} T_R^\alpha - T_R^{\alpha\alpha\dagger}$$

and

$$V_L = T_L^{\alpha\alpha\dagger}, \quad V_R = T_R^{\alpha\dagger} T_R^\alpha \quad (57)$$

Again, we demand that $Z_{L(R)}$ and $V_{L(R)}$ be diagonal in the basis in which the quark masses are diagonal. this will imply that $Z_{L(R)}$ and $V_{L(R)}$ be functions of Q . Equivalently,

$$T_{L(R)}^\alpha T_{L(R)}^{\alpha\dagger} = f_{L(R)}(Q) \quad \text{and} \quad T_{L(R)}^{\alpha\dagger} T_{L(R)}^\alpha = g_{L(R)}(Q) \quad (58)$$

From the definition of T^α , Eq. (47), one has:

$$T^{\alpha\alpha\dagger} = T_a^{\alpha 2} + T_b^{\alpha 2} - i[T_a^\alpha, T_b^\alpha]$$

and

$$T^{\alpha\dagger} T^\alpha = T_a^{\alpha 2} + T_b^{\alpha 2} + i[T_a^\alpha, T_b^\alpha] \quad (59)$$

Then from (2)

$$T^{\alpha\alpha\dagger} = T_a^{\alpha 2} + T_b^{\alpha 2} + C_{abc} T_c^\alpha \quad (60)$$

and

$$T^{\alpha\dagger} T^\alpha = T_a^{\alpha 2} + T_b^{\alpha 2} - C_{abc} T_c^\alpha$$

where C_{abc} are the structure constants for G . The last terms in Eq. (60) will in general be a linear combination of neutral generators. Since Condition I requires all of them to be functions of Q , then (58) is equivalent to

$$T_{a(b)}^\alpha = f_{a(b)}(Q) \quad (61)$$

for all α . This is a necessary condition for NFC. Now we would like to make sure that Diagrams II and III yield no new restrictions on quark representations.

Consider Diagrams II with $\alpha = \beta$.

By inspection one can see that these diagrams yield a contribution proportional to $[T_{L(R)}^\alpha, T_{L(R)}^{\alpha\dagger}]$. From (60) this is equal to $2C_{abc} T_c^\alpha$. Since this is just a linear combination of neutral generators, Eq. (53) will ensure flavor conservation and so these diagrams yield no new requirement.

The first three Diagrams of III, with $\alpha = \beta$, produce a change in the neutral-current coupling (as in Ref. 4):

$$\Delta_i = C_{abc} T_c^\alpha \quad (62)$$

Again, (53) ensures NFC. Similarly with the second three diagrams. Thus Eq. (61) defines the necessary condition for flavor conservation in order αG . Only necessary, remember, because we have not yet looked at graphs with $\alpha \neq \beta$.

However, we need not look at the graphs with $\alpha \neq \beta$ since the generators from Eqs. (53) and (61) simply exhaust the set of generators. Thus since any

other conditions that would arise in the case $\alpha \neq \beta$ would be of the form:

$$F(T^\alpha) = f(Q) \quad (63)$$

then no new constraint could be made on the representations since all T^α are functions of Q from the case $\alpha = \beta$. Thus condition (61) is also a sufficient condition.

Now, since all quantum numbers (weak and electromagnetic) are functions of the generators themselves, we may combine (53) and (61) to obtain the final result:

R1': The conservation of fermion flavors in the gauge-boson-mediated neutral current up to αG_F order is equivalent to requiring that all quarks of a given charge and helicity transform identically under G (possess the same quantum numbers).

The remaining source of flavor-changing neutral weak interactions is the second-order exchange of neutral Higgs bosons in Figure 3.

If one believes that these Higgs exchanges are strong enough to consider, then one must require that the coupling of each neutral Higgs boson be such as to naturally conserve all quark flavors.

The interaction of the Higgs scalars ϕ^α with the quark fields will be of the form in Eq. (7). The Lagrangian will also possibly contain a G -invariant bare mass term.

After spontaneous symmetry breaking of G down to $U(1)$, we obtain

$$M = \sum_Q \bar{q}_{OL}^Q M(Q) q_{OR}^Q + \text{h.c.} \quad (64)$$

where

$$M(Q) = \sum_\alpha \Gamma_Q^\alpha \lambda_Q^\alpha + M_Q^0$$

and where $q_{OL(R)}^Q$ is the left- (right-) handed projection of a vector containing all bare quark fields of a given charge Q . $\lambda_Q^\alpha = \langle \phi^\alpha \rangle_Q$ is the vacuum expectation value of a neutral Higgs scalar that contributes to the mass matrix for quarks of charge Q (α runs over all such fields). Γ_Q^α is the associated coupling matrix.

Of course, in general one may have $\lambda_Q^\alpha = \lambda_{Q'}^\alpha$, if there is γ coupling, or $\Gamma_Q^\alpha = \Gamma_{Q'}^\alpha$, if the Q and Q' quarks are both contained in the same left-handed multiplet and the same right-handed multiplet.

Our last condition then requires that each Γ_Q^α and M_Q^0 be diagonal in this basis in which $M(Q)$ is diagonal. If the Lagrangian respects no further symmetry, then "naturalness" in NFC for the Higgs couplings will require that R2 hold.

This condition is sufficient to insure NFC in the Higgs sector. If we consider the problem with more than one Higgs scalar contributing to the Yukawa couplings for each charge, then we must find a "natural" way to insure flavor conservation — that is, through a further horizontal symmetry of X , K , above and beyond the gauge group employed. We assign the fermion multiplets and Higgs scalars to representations of this additional symmetry in such a way as to guarantee flavor conservation through Higgs exchange. This problem has been called the search for "naturalness groups"¹².

Why introduce more scalars into the theory? One reason is to generate non-trivial calculable mixing angles in the charged currents via mass matrix diagonalization. There are those willing to posit the existence of more Higgs fields and another group symmetry, in order to obtain meaningful mixing angles in the theory.

The naturality group, K , then must serve a dual purpose. It must insure NFC and also limit the Higgs-quark couplings in such a way as to allow calculable mixing angles. The mixing angles obtained in this way are left unaltered by renormalization. In WS it has been shown that calculability necessitates more than two Higgs scalars.

A naturality group can be found for any gauge theory. For instance, one can always find a K -symmetry such that all Higgs couplings are diagonal, yielding a diagonal M . Then trivially all Γ^Q 's will be simultaneously diagonalizable with each other and $M(Q)$, and all mixing angles will be trivial.

Now the last condition states that there will be NFC if the diagonalization of the quark mass matrix is simultaneous with the diagonalization of the Yukawa couplings of the quarks with the neutral Higgs scalars. In general, then, the second condition for NFC is:

$R2'$: The conservation of fermion flavors in the second order Higgs exchange is equivalent to requiring that all quarks of a given charge receive their mass either

- (a) through a G -invariant mass term or
- (b) through the couplings of neutral Higgs mesons; where these couplings are simultaneously diagonalizable in a natural manner with the mass matrix, but not by both mechanisms.

The case $R2'$ (a) is trivial since it is often excluded by the gauge group and representation and the naturality group and representation. More importantly, it is not possible to generate calculable mixing angles in this case.

"Diagonalizable in a natural manner" means that the couplings derive this property from a group symmetry or representation.

IV. Discussion

Condition $R1'$ is quite powerful in limiting the representation content of a WET. Some of the manifestations of this power are:

- (1) Any gauge model consisting of "generations" of quarks will satisfy $R1'$. These include the "standard" $SU(2) \times U(1)$ and $SU(2)_L \times SU(2)_R \times U(1)$ models, as well as the Grand Unified Models based on these - $SU(5)$ and $SO(10)$.
- (2) If every type of quark in a gauge theory are contained in a single multiplet, then there must be "generations" of quarks.
- (3) There must be, in any gauge model, the same number of quarks of every charge contained in a multiplet.
- (4) As in Ref. 13 we can conclude that all irreducible quark representations of G must be either identical or else be completely non-overlapping as far as quark charges are concerned.
- (5) An interesting consequence of $R1'$ is that requiring the neutral currents naturally conserve flavor up to αG_F order results in the absence of flavor-changing currents to all orders. This follows from the condition that $T_\alpha = f(Q)$ for all α and from understanding that all higher order graphs be functions of the T_α . Symbolically,

$$\text{graphs} \sim g(T_\alpha) = g(f(Q)) .$$

So any order graph will be a function of Q , and thus be simultaneously diagonalizable with the mass matrix.

- (6) If we wish to exclude $|\Delta Q| \geq 2$ charged currents from our enquiry¹³, then the gauge group must take the form $G = [SU(2)]^N \times [U(1)]^M$. Then it is apparent that since two sets

of gauge fields from different SU(2) subgroups of G coupled to identical quark multiplets will just represent a redundancy, new phenomenology in the charged currents will take place only when the two sets coupling to non-overlapping multiplets.

In this case R1 takes the form:

All quarks of a given charge and helicity must have the same value of Y_α and T_α^2 for all α and β , where Y_α are the neutral generators of G and where T_3 corresponds to the β^{th} SU(2) subgroup of G.

It has been shown that it is not possible to generate meaningful mixing angles in an arbitrary WET if one applies R2'. Since this consequence is undesirable, we may ignore the condition for flavor conservation from Higgs exchange, and suppress flavor-violating effects by making the Higgs mesons very heavy. Although it is not clear whether it is possible to choose arbitrarily large Higgs masses in gauge theories¹⁴.

The preceding analysis can be trivially extended to Grand Unified theories, where the unbroken subgroup will then possibly include color SU(3).

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APPENDIX

Consider first the two box diagrams in I in Figure 2. The intermediate vector bosons have, in general, a nonzero integral charge (which must of course be the same for α and β). The relevant left and right gauge couplings appear in Figure 4.

If we first ignore the generators, then, in the limit that the external momenta are small compared with the masses in the theory, the box diagrams will be (in the 't Hooft gauge):¹⁵

$$B(x,y) = i \left(\frac{g}{4\pi} \right)^4 \int d^4k \, k^2 (k^2 - m_W^2)^{-2} (k^2 - m_x^2)^{-1} (k^2 - m_y^2)^{-1} \times [\bar{u} \gamma_\mu \gamma_\lambda \gamma_\nu u] [\bar{u} \gamma^\mu \gamma^\lambda \gamma^\nu u] \quad (A1)$$

where m_x and m_y are the masses of the intermediate fermions. The integral $I(x,y)$ in (A1) is

$$I(x,y) = \frac{-i\pi^2}{M_W^2} J(x,y) \quad (A2)$$

$$\text{where } x(y) \equiv \frac{M_W^2}{M_x^2} \text{ and } J(x,y) = \frac{K(x)-K(y)}{x-y}$$

$$\text{and } K(x) = (1-x)^{-1} + \frac{(x^2 \ln x)}{(1-x)^2} . \text{ For small } x \text{ and } y, J = O(1).$$

Then we find

$$B(x,y) = \left(\frac{g^2}{16\pi M_W} \right)^2 J(x,y) [\bar{u} \gamma_\mu \gamma_\lambda \gamma_\nu u] [\bar{u} \gamma^\mu \gamma^\lambda \gamma^\nu u] \quad (A3)$$

Breaking up $B(x,y)$ into chiral components and reinstating the generators we then find, for the two box diagrams,

$$B_1(x,y) = \left(\frac{g^2}{16\pi M_W} \right)^2 J(x,y) [\bar{u} \gamma_\mu \gamma_\lambda \gamma_\nu (T_L^{\alpha\beta} T_L^{\beta\alpha} + T_R^{\alpha\beta} T_R^{\beta\alpha}) u] \times [\bar{u} \gamma^\mu \gamma^\lambda \gamma^\nu (T_L^{\alpha\beta} T_L^{\beta\alpha} + T_R^{\alpha\beta} T_R^{\beta\alpha}) u] \quad (A4)$$

$$B_2(x,y) = B_1(T^\alpha \leftrightarrow T^{\alpha\dagger}) \quad (A5)$$

The Dirac matrices can be simplified by using:

$$\gamma^\mu \gamma^\lambda \gamma^\nu = g^{\mu\lambda} \gamma^\nu + g^{\lambda\nu} \gamma^\mu - g^{\mu\nu} \gamma^\lambda - i\epsilon^{\mu\lambda\nu\sigma} \gamma_5 \gamma_\sigma \quad (A6)$$

$$\epsilon_{\mu\lambda\nu\sigma} \epsilon^{\mu\lambda\nu\rho} = 6\delta_\sigma^\rho \quad (A7)$$

$$\gamma_5 L = -L, \quad \gamma_5 R = R \quad (A8)$$

Then it is easy to show that

$$B_1(x,y) = A(x,y) \sum_{i,j=L}^R (1 + 3\delta_{i,j}) [\bar{u} T_i^{\alpha\beta} T_i^{\beta\alpha} X_i u] [\bar{u} T_j^{\alpha\beta} T_j^{\beta\alpha} X_j u] \quad (A9)$$

where

$$A(x,y) = g^4 (8 M_W)^{-2} J(x,y) \quad (A10)$$

and

$$X_{L(R)} = L(R) \quad (A11)$$

Now consider the two crossed diagrams in I. The value of the integral will be the same, thus

$$C_1(x,y) = -\frac{1}{4} A(x,y) [\bar{u} \gamma_\mu \gamma_\lambda \gamma_\nu (T_L^{\alpha\beta} T_L^{\beta\alpha} + T_R^{\alpha\beta} T_R^{\beta\alpha}) u] \quad (A12)$$

$$\times [\bar{u} \gamma_\nu \gamma_\lambda \gamma_\mu (T_L^{\beta\alpha} T_L^{\alpha\beta} + T_R^{\beta\alpha} T_R^{\alpha\beta}) u]$$

$$C_2(x,y) = C_1(T^\alpha \leftrightarrow T^{\alpha\dagger}) \quad (A13)$$

The overall minus sign is due to the direction of fermion flow in the diagrams. Using Equations (A6) - (A8) we find:

$$C_1(x,y) = -A(x,y) \sum_{i,j=L}^R (4 - 3_{ij}) [\bar{u} T_i^{\alpha\beta\dagger} \gamma_\sigma X_i u] [\bar{u} T_j^{\beta\alpha\dagger} \gamma^\sigma X_j u] \quad (A14)$$

Considering $\alpha=\beta$ in these diagrams, we find the sum of B_1 , B_2 , C_1 and C_2 can be written as Equations (54) - (57).

Note that this result does not equal reference (4)'s result in the appropriate limits. Since the conclusions they drew from computing these graphs are obviously not dependent on numerical factors, their remarks remain valid.

FIGURE CAPTIONS

- Figure 1. Lowest order neutral current graphs.
The straight lines are fundamental fermions.
- Figure 2. Graphs of order αG_F that could produce flavor violation.
- Figure 3. Lowest order Higgs exchange.
- Figure 4. Feynman rules for gauge couplings, $X = L, R$.

Figure 1

Figure 3

I.

II.

$$G_X(\psi\psi W) = \left(\frac{ig}{\sqrt{2}} \right) T_X^\alpha \gamma_\lambda X$$

Figure 4

III.

Figure 2