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Informal Report

MASTER

**Design Study for
Achromatic and Isochronous
180° Bending Systems for PIGMI**

University of California

LOS ALAMOS SCIENTIFIC LABORATORY

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D. A. Swenson

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DESIGN STUDY FOR ACHROMATIC AND ISOCHRONOUS 180° BENDING SYSTEMS FOR PIGMI

by

H. Inoue and D. A. Swenson

ABSTRACT

A Pion Generator for Medical Irradiation (PIGMI) possessing many technical innovations is being developed at the Los Alamos Scientific Laboratory. The heart of the PIGMI facility is a linear accelerator designed to provide a 650-MeV beam of protons at a beam current of 100 μ A. The accelerator will be 120 m long and suitable for installation at major hospital sites. Obviously, a shorter machine may be more convenient to the medical profession. One possibility of reducing the total length of the facility is to bend the linac back on itself at the halfway point. However, the 180° bending system used for this purpose must have the very special properties of being both achromatic and isochronous in order to transport the proton beam without destroying the transverse or longitudinal dimensions of the bunch. This report addresses the design of two such possible systems.

I. DESIGN REQUIREMENTS

It is expected that the 180° bending system would be positioned at about the halfway point in the linac structure corresponding to an energy of 250 MeV. The kinematic character and assumed phase space of the beam at this point are shown in Table I. The bending system would consist of several bending and quadrupole magnets; design parameters for these magnets are given in Table II. The problem is to find an appropriate combination of these magnets to satisfy both the achromatic and the isochronous conditions.

Based on the R-matrix formalism for a beam transport system, the design study* can be readily performed by using the computer program TRANSPORT.¹

* See Appendix for a brief explanation of how the design work for this report was accomplished.

TABLE I
CHARACTERISTICS OF THE PROTON BEAM AT THE ENTRANCE
OF THE BENDING SYSTEM

<u>Characteristic</u>	<u>Value</u>
Central trajectory	
Energy (E)	250.0 MeV
Momentum (p)	729.1 MeV/c
Magnetic rigidity ($B\rho$)	24.32 kG·m
Velocity in units of c (β)	0.6136
Phase space (beam waist)	
Horizontal size (x)	5.0 mm
Horizontal angle (θ)	4.0 mrad
Vertical size (Y)	5.0 mm
Vertical angle (ϕ)	4.0 mrad
Longitudinal length (ℓ)	5.0 mm
Momentum spread ($\Delta p/p$)	0.2 %

TABLE II
SOME DESIGN PARAMETERS FOR THE BENDING MAGNET
AND QUADRUPOLE MAGNET

<u>Parameter</u>	<u>Value</u>
Bending magnet	
Bending radius	1.40 m
Field strength	17.4 kG
Field gradient	0
Pole gap	8.0 cm
Angle of pole face	0°
Quadrupole magnet	
Effective length	20.0 cm
Aperture	6.0 cm

In terms of the R-matrix, the achromatic condition requires that (to first order)

$$R_{16} = 0 \quad (1)$$

and

$$R_{26} = 0 . \quad (2)$$

Equation (1) means that the horizontal position of an outgoing particle at the exit of the system is independent of the momentum of the particle. Equation (2) represents the independence of the horizontal angle of the outgoing particle to its momentum.

The isochronous condition can be realized if the path length of a particle is proportional to the velocity of this particle. In R-matrix formalism, the path length difference ΔL between an off-momentum particle and the central-momentum particle is given by

$$\Delta L = -R_{56}(\Delta p/p) .$$

The fractional momentum spread $\Delta p/p$ can be replaced by $(1-\beta^2)^{-1/2}(\Delta\beta/\beta)$, where $\Delta\beta/\beta$ is the fractional velocity spread. The isochronous condition $\Delta\beta/\beta = \Delta L/L$ gives a constraint for the element R_{56}

$$R_{56} = (1-\beta^2)L . \quad (3)$$

This is the necessary condition for making the system isochronous (to first order) for off-momentum particles incoming on the central orbit. It should be realized that the system must be isochronous for any particles within the given momentum spread. This condition further requires that

$$R_{51} = 0 \quad (4)$$

and

$$R_{52} = 0 . \quad (5)$$

These equations assure that the path length is independent of the horizontal position and the horizontal angle of the incoming particle. In a system satisfying Eq. (3), (4), and (5), any two particles entering the system simultaneously will also appear at the exit simultaneously.

Furthermore, it is desirable that any incoming particles leave the system at the exit from the same position with the same angle as at the entrance, that is, the phase space with regard to the position and the angle should remain

unchanged. This means the system should have a unity R-matrix, except for the finite value for R_{56} given in Eq. (3).

The design can be simplified by taking advantage of a symmetric configuration of magnets with respect to a plane perpendicular to the central ray at the 90° bend point. In order to realize the unit transformation matrix mentioned above for the total system, the first-half of the system needs to have the following properties:

(1) A horizontally diverging beam at the entrance must come to an immediate focusing at the center of symmetry ($R_{12} = 0$). The longer path length of a particle diverging outside in the first-half system can be canceled with the shorter one in the second-half to give a path length (to first order) equal to the central ray in the total system. This assures the same transit time for any particles in the diverging *monochromatic* beam.

(2) A parallel beam at the entrance must be transformed to a parallel beam at the center of symmetry ($R_{21} = 0$). This assures an outcoming parallel beam at the exit of the full system. To assure the same transit time, the path length difference must be canceled within the first-half of the system ($R_{51} = 0$).

(3) An off-momentum particle entering along the central trajectory must travel on a parallel dispersive trajectory at the center ($R_{26} = 0$). This makes the total system achromatic both in position and in angle.

(4) The isochronous condition for half the system must be gained by replacing L with $L/2$ in Eq. (3).

(5) The mode of the vertical motion of particles must be chosen compatible with the restrictions imposed on the horizontal motion. A point-to-point imaging ($R_{34} = 0$) and a parallel-to-parallel transformation ($R_{43} = 0$) could be the best choice for the vertical motion.

The properties mentioned above are summarized in Table III in terms of the constraints for the transformation matrix elements.

For practicality, the separation between the incoming and outgoing beams must be about 2 m or less.

II. DESIGNED SYSTEMS

A flat semicircular bending magnet can achieve isochronous but not achromatic bending. When two identical 90° bending magnets with a quadrupole magnet located

TABLE III
CONSTRAINTS ON THE TRANSFORMATION MATRIX R AT THE SYMMETRY
CENTER OF THE BENDING SYSTEM

<u>Desired Optical Condition</u>	<u>Constraint</u>
Point-to-point imaging	
Horizontal plane	$R_{12} = 0$
Vertical plane	$R_{34} = 0$
Parallel-to-parallel transformation	
Horizontal plane	$R_{21} = 0$
Vertical plane	$R_{43} = 0$
Achromatic condition	
In angle	$R_{26} = 0$
Isochronous condition	
Horizontal displacement	$R_{51} = 0$
Momentum dependence	$R_{56} = -0.0623^a (L/2)$

^a The coefficient is chosen for 250-MeV protons.

between them are used, a system can be made either achromatic or isochronous (depending on the field strength of the quadrupole magnet), but it cannot be made simultaneously achromatic and isochronous. Therefore, it is apparent that three or more separate bending magnets are necessary to build the desired system. Two possible systems are discussed below.

A. System 1

An achromatic and isochronous system consisting of three bending magnets is shown in Fig. 1. This system satisfies all constraints in Table III. Particle trajectories of special interest are shown in Fig. 2. The z-axis coincides with the central ray. The x- and y-axes indicate the deviation from the central ray in horizontal and vertical motions, respectively. A monochromatic beam diverging at the entrance forms an intermediate double focusing image at the symmetry center of the second bending magnet B_2 , and converges again to the exit.

A representative trajectory of $\theta_0 = 4$ mrad and $\phi_0 = 4$ mrad is drawn with solid lines in Fig. 2. An incoming monochromatic beam parallel to the central ray

Fig. 1. An achromatic and isochronous bending system -- System 1.
(The bend radius of each magnet is 1.4 m.)

Fig. 2. Particle trajectories of special interest in System 2.
(The solid line and the dotted line show trajectories of particles of central momentum. The solid line labelled $\delta = 0.2\%$ is that of an off-momentum particle.)

again becomes parallel at the symmetry center and is transported to an outgoing parallel beam at the exit. A representative trajectory of $x_0 = 5$ mm and $y_0 = 5$ mm is drawn with dotted lines. An off-momentum particle incoming on the central ray at the entrance deviates horizontally from the central ray in the first bending magnet and travels on a dispersive trajectory, which becomes parallel to the central ray at the symmetry center, so the particle is again on the central ray at the exit. A typical trajectory for $\Delta p/p = 0.2\%$ is shown as a solid line in the XZ-plane labelled $\delta = 0.2\%$. A relatively large dispersion at the symmetry center is necessary to make the system isochronous. The field strengths of the quadrupole magnets are listed in Table IV. The large negative values of Q_5 and Q_7 , compared with the counterparts of Q_6 and Q_8 , realize the desired large dispersion. The drift lengths between quadrupole magnets are 0.2 m; the other drift lengths are 0.3 m. An additional constraint is applied to keep the beam size in the system small.

For this system the separation between the incoming and outgoing beams is 5.6 m, which makes it unattractive for many practical applications. In order to build a system with smaller separation, it is necessary to employ more magnets involving bends to both directions.

TABLE IV
THE FIELD STRENGTHS OF THE QUADRUPOLE MAGNETS
FOR THE SYSTEM 1

<u>Quadrupole Magnet</u>	<u>Field Strength (kG at 3 cm)</u>
Q_1, Q_{16}	-9.63
Q_2, Q_{15}	5.13
Q_3, Q_{14}	-7.04
Q_4, Q_{13}	5.34
Q_5, Q_{12}	-11.17
Q_6, Q_{11}	9.02
Q_7, Q_{10}	-14.10
Q_8, Q_9	3.33

B. System 2

A somewhat more complicated system consisting of eight bending magnets is shown in Fig. 3. Involving bends in both directions, this system is relatively large, but the separation between the incoming and the outgoing beams is only 2 m. All constraints in Table III are satisfied, which is demonstrated in the particle trajectories plotted in Fig. 4. (The meaning of the lines is the same as in Fig. 2 for System 1.) The symmetry center is at the center of the singlet quadrupole Q_8 . In the first-half of the system, optical properties of point-to-point imaging and parallel-to-parallel transformation are again realized for the monochromatic beam. Therefore, any monochromatic beam can be transported to the one with the same phase space in the full system. Off-momentum particles incoming on the central trajectory are dispersed horizontally by the first bending magnet B_1 and brought by the second bending magnet B_2 to an intermediate image near the quadrupole Q_5 . The angular dispersion at this point can yield a large spatial dispersion at the symmetry center and sufficient path length difference to realize the desired isochronous condition in the full system. The field strength of each quadrupole magnet is given in Table V. The drift spaces are 0.3 m, except those between quadrupole magnets, which are 0.2 m.

Fig. 3. An achromatic and isochronous bending system that involves bends in both directions -- System 2. (The bend angle of each magnet is 45° ; the bend radius is 1.4 m. All bending magnets are identical.)

Fig. 4. Particle trajectories in System 2. (The meaning of the lines is the same as for those in Fig. 2.)

TABLE V
THE FIELD STRENGTHS OF THE QUADRUPOLE MAGNETS
FOR THE SYSTEM 2

Quadrupole Magnet	Field Strength (kG at 3 cm)
Q_1, Q_{15}	-6.01
Q_2, Q_{14}	8.77
Q_3, Q_{13}	-5.87
Q_4, Q_{12}	5.35
Q_5, Q_{11}	-2.88
Q_6, Q_{10}	-7.71
Q_7, Q_9	4.73
Q_8	-3.71

The separation of 2 m between the input and output beams will facilitate installation of both halves of the accelerator in the same beam channel, and this system can readily be designed for even smaller separations. Finally, it should be pointed out that all bending magnets are identical, which would be advantageous from the cost aspect.

III. SECOND-ORDER EFFECTS

Second-order calculations were made for both systems for the field strengths given in Tables IV and V. The results are compared here with the corresponding first-order calculations.

For System 1, the difference in the calculated beam size between the first- and second-order calculations is quite small at the exit, but a relatively large difference of about 30% is seen in y size at Q_{15} . This implies some distortion in the vertical angle at the exit. A more detailed second-order calculation is necessary for this system if one is to use it.

For System 2, the difference in calculated beam size is negligibly small over the entire path length of the system. The maximum difference of about 1% is seen near B_0 for x and at the exit for y. Thus, the second-order effect is seen to be less significant for this system.

IV. EFFECT OF THE FRINGING FIELD

To first order, the fringing field of a bending magnet affects only the vertical focusing of the beam if the pole face rotation angle is zero. For this case the transform matrix for the fringing field has only one nonzero, off-diagonal element R_{43}

$$R_{43} = \frac{1}{\rho_0} \tan\left(K_1 \frac{g}{\rho_0}\right) ,$$

where ρ_0 , g , and K_1 are the bending radius of central trajectory, total gap of magnet, and an integral related to the extent of the fringing field, respectively. Typical values of K_1 are given below for four types of fringing field boundaries:

- (1) Linear drop-off of the field, $K_1 = 1/6$
- (2) Clamped "Rogowski" fringing field, $K_1 = 0.4$
- (3) Unclamped "Rogowski" fringing field, $K_1 = 0.7$
- (4) "Square-edged" nonsaturating magnet, $K_1 = 0.45$.

The effect of the fringing field was checked for the proposed systems. The field strengths of the quadrupole magnets necessary to keep the systems achromatic and isochronous were calculated for various types of the fringing field of the bending magnets. Table VI shows the results for some values of K_1 with a gap of 4 cm. Table VII shows the results for the gaps of 6, 8, and 10 cm when the value of K_1 is 0.5. As seen in the tables, the differences between the field strengths are very small. The effect of the fringing field of the bending magnets can be compensated for by choosing the field strengths of the quadrupole magnets. For comparison, Table VII includes the results without the fringing field.

TABLE VI

FIELD STRENGTHS (KG) OF THE QUADRUPOLE MAGNETS FOR VARIOUS
VALUES OF K_1 (THE GAP OF THE BENDING MAGNETS IS 4 cm)

Quadrupole Magnet	K_1				
	<u>0.167</u>	<u>0.4</u>	<u>0.45</u>	<u>0.5^a</u>	<u>0.7</u>
Q_1	-6.00	-6.02	-6.01	-6.01	-5.99
Q_2	8.77	8.76	8.76	8.76	8.76
Q_3	-5.79	-5.85	-5.87	-5.88	-5.93
Q_4	5.32	5.35	5.35	5.36	5.38
Q_5	-2.84	-2.85	-2.86	-2.87	-2.91
Q_6	-7.68	-7.68	-7.68	-7.68	-7.68
Q_7	4.70	4.72	4.72	4.73	4.74
Q_8	-3.62	-3.69	-3.70	-3.72	-3.78

^a The value taken for default of K_1 in the TRANSPORT code.

TABLE VII
QUADRUPOLE FIELD STRENGTHS (kG) FOR BENDING MAGNET
GAPS OF 6, 8, AND 10 cm (THE VALUE OF K_1 IS 0.5)

Quadrupole Magnet	Gap of the bending magnets (cm)			
	<u>6</u>	<u>8</u>	<u>10</u>	<u>Not Specified</u>
Q_1	-6.00	-6.01	-6.00	-5.91
Q_2	8.77	8.76	8.76	8.79
Q_3	-5.85	-5.88	-5.91	-5.75
Q_4	5.34	5.36	5.37	5.30
Q_5	-2.86	-2.87	-2.89	-2.87
Q_6	-7.68	-7.68	-7.68	-7.68
Q_7	4.72	4.73	4.74	4.69
Q_8	-3.68	-3.72	-3.76	-3.59

REFERENCE

1. Karl L. Brown *et al.*, "A Computer Program for Designing Charged Particle Beam Transport Systems," Stanford Linear Accelerator report SLAC-91 (February 1974).

APPENDIX

TRANSPORT RUNS FOR THE SYSTEMS

(Reformatter Output)

Most of the design work reported in the body of this report was performed on the portable data terminals of the CDC-6600 at LASL. A reformatter, written by D. Swenson, was used to execute the TRANSPORT fitting calculation at the terminals. The figures in this appendix are prints of the outputs of the TRANSPORT runs for the Systems 1 and 2 by this reformatter. These are itemized as:

- (1) Input data and fitting procedure
- (2) Result of fitting
- (3) Particle trajectories of special interest
- (4) Input for second-order calculation
- (5) Comparison of the beam envelopes in the first- and the second-order calculations.

Figures A-1, A-2, and A-3 are for first-order calculations through half of System 1. Figures A-4 and A-5 are for the first- and second-order calculations for the full systems. Figures A-6, A-7, and A-8 are for first-order calculations through half of System 2.

```

CALL,TRANX
TROUT4 - TRANSPORT OUTPUT REFORMATTER WITH OPTION 4
TRANSPORT 78/04/04 17.37.12
TEST
DATAPL= 0
13. 2.  OFFSIG
1.  .5  .00001  .05  .00001  .5  .00001  .72912  BEAM
3.0 0.3 D1
5.01 0.2 -9.63 3.0 Q1
3. 0.2 D2
5.01 0.2 5.13 3.0 Q2
3.0 0.2 D3
5.01 0.2 -7.03 3.0 Q3
3. 0.2 D4
5.01 0.2 5.34 3.0 Q4
3. 0.3 D5
16. 5. 4.0  GAP
2. 0.0 A1
4. 1.09955 17.3725 0.0 B1
2. 0.0 A1
3. 0.3 D6
5.01 0.2 -11.17 3.0 Q5
3. 0.2 D7
5.01 0.2 9.02 3.0 Q6
3. 0.2 D8
5.01 0.2 -14.10 3.0 Q7
3. 0.2 D9
5.01 0.2 3.33 3.0 Q8
3. 0.3 D10
2. 0.0 A2
4. 1.09955 17.3725 0.0 B2
10. -2. 6. 0.0 0.001 R26
10. -5. 6. -3.86 0.001 R56
10. -1. 2. 0.0 0.001 R12
10. -2. 1. 0.0 0.001 R21
10. -3. 4. 0.0 0.001 R34
10. -4. 3. 0.0 0.001 R43
10.2 -3. 3. 3.5 0.001 R33
13. 4. PR1
SENTINEL

*CORRECTIONS*
( 1.6596E+01) 0.000 .002 -.005 -.001 .003 .001
( 1.9598E-02) 0.000 -.000 .000 .000 -.000 .000
TRANSPORT 78/04/04 17.37.17
TEST
DATAPL = 0
*COVARIANCE ( FIT .0 )
0.000
0.000 .029
0.000 -.813 .043
0.000 .415 -.867 .010
0.000 .967 -.896 .572 .001
0.000 -.173 .194 -.155 -.233 .001
0.000 0.000 0.000 0.000 0.000 0.000 0.000
0.000 .174 -.169 .112 .027 -.013 0.000 .000
11= 116

```

Fig. A-1. TRANSPORT run for the 45° + 90° + 45° bending system --
(1) input data and fitting procedure.

```

*OFFSIG      *
*BEAM        * 1.000000      .73 GEV/C
*D1          * 3.0          .300 M
*Q1          * 5.01         .200 M  -9.630 KG  3.000 CM (FX=   -.347 M  , FY=
*D2          * 3.0          .200 M
*Q2          * 5.01         .200 M  5.132 KG  3.000 CM (FX=   .745 M  , FY=
*D2          * 3.0          .200 M
*Q3          * 5.01         .200 M  -7.035 KG  3.000 CM (FX=   -.487 M  , FY=
*D4          * 3.0          .200 M
*Q4          * 5.01         .200 M  5.339 KG  3.000 CM (FX=   .718 M  , FY=
*D5          * 3.0          .300 M
*GAP         * 16.0      5.0  4.000E+00
*A1          * 2.0          0.000 D
*B1          * 4.000      1.10 M  17.372 KG  .00 ( 45.001 D )
*A1          * 2.0          0.000 D
*D6          * 3.0          .300 M
*Q5          * 5.01         .200 M -11.167 KG  3.000 CM (FX=   -.296 M  , FY=
*D7          * 3.0          .200 M
*Q6          * 5.01         .200 M  9.021 KG  3.000 CM (FX=   .440 M  , FY=
*D8          * 3.0          .200 M
*Q7          * 5.01         .200 M -14.100 KG  3.000 CM (FX=   -.228 M  , FY=
*D7          * 3.0          .200 M
*Q8          * 5.01         .200 M  3.326 KG  3.000 CM (FX=   1.131 M  , FY=
*D10         * 3.0          .300 M
*A2          * 2.0          0.000 D
*B2          * 4.000      1.10 M  17.372 KG  .00 ( 45.001 D )
*R26         * 10.0      -2  6  0.000 / .001
              -.000
*R56         * 10.0      -5  6 -3.860 / .001
              -3.860
*R12         * 10.0      -1  2  0.000 / .001
              -.000
*R21         * 10.0      -2  1  0.000 / .001
              .000
*R34         * 10.0      -3  4  0.000 / .001
              .000
*R43         * 10.0      -4  3  0.000 / .001
              -.000
*R33         * 10.2      -3  3  3.500 / .001
              3.498
*PR1         * 1
-2.8892E+10  -1.8550E-11  0.      0.      0.      5.1487E+00
5.8070E-10   -3.4612E-01  0.      0.      0.      -3.5527E-14
0.           0.          3.4976E+00  2.7303E-12  0.      0.
0.           0.          -3.4494E-11  2.8591E-01  0.      0.
2.9890E-10   -1.7821E-01  0.      0.      1.0000E+00 -3.8600E+00
0.           0.          0.      0.      0.      1.0000E+00
*LENGTH*     6.2 M

```

Fig. A-2. TRANSPORT run for the 45° + 90° + 45° bending system --
(2) result of fitting.

Monochromatic Beam						
Z(m)	NAME	Parallel Beam		Diverging Beam		p/p=0.2% Y(cm)
		X(cm)	Y(cm)	X(cm)	Y(cm)	
0.000	BEAM	.5000	.5000	1.0000E-04	1.0000E-04	
.300	D1	.5000	.5000	.1200	.1200	
.500	Q1	.6379	.3737	.2403	.1628	
.700	D2	.9257	.1324	.4115	.1647	
.900	Q2	1.073	.1014	.5181	.1904	
1.100	D3	1.073	.3385	.5554	.2411	
1.300	Q3	1.286	.4974	.7056	.2436	
1.500	D4	1.727	.5679	.9784	.1977	
1.700	Q4	1.900	.7271	1.098	.1792	
2.000	D5	1.745	1.110	1.040	.1911	.0
3.100	B1	.7214	2.540	.5436	.2391	.0820
3.400	D6	.3473	2.946	.3449	.2537	.1244
3.600	Q5	.1835	2.333	.3095	.1887	.1958
3.800	D7	8.8758E-02	.8997	.3677	5.5612E-02	.3174
4.000	Q6	1.9422E-02	.4230	.3340	7.4364E-02	.3539
4.200	D8	.1321	1.743	.2111	.2119	.3023
4.400	Q7	.3142	2.268	.1589	.2556	.3684
4.600	D9	.5890	1.930	.1705	.1998	.5605
4.800	Q8	.8026	1.760	.1664	.1608	.6965
5.100	D10	1.021	1.749	.1371	.1257	.8101
6.199	B2	1.445	1.749	2.9347E-04	3.4976E-04	1.030
TRANSPORT 78/04/04 17.37.18						
TEST						
5.010000	Q1	-9.63000				
5.010000	Q2	5.13226				
5.010000	Q3	-7.03464				
5.010000	Q4	5.33943				
5.010000	Q5	-11.16736				
5.010000	Q6	9.02062				
5.010000	Q7	-14.10000				
5.010000	Q8	3.32649				
STOP						
/						

Fig. A-3. TRANSPORT run for the 45° + 90° + 45° bending system --
(3) particle trajectories of special interest.

CALL,TRANSY
 TROUT - TRANSPORT OUTPUT REFORMATTER WITH OPTIONS 1-6

```

CALL,TRANSY
TROUT - TRANSPORT OUTPUT REFORMATTER WITH OPTIONS 1-6
? 4
TRANSPORT 78/04/07 11.28.14
TEST
DATAPI= 0
13. 2. OFFSIG
1. .5 4. .5 4. .5 .2 .72912 BEAM
17. SEC
3. 0.3 D1
5.0 0.2 -9.6300 3.0 Q1
3. 0.2 D2
5.00 0.2 5.1322 3.0 Q2
3. 0.2 D3
5.00 0.2 -7.0346 3.0 Q3
3. 0.2 D4
5.00 0.2 5.3394 3.0 Q4
3. 0.3 D5
16. 5. 4.0 GAP
2. 0.0 A1
4. 1.09955 17.3725 B1
2. 0.0 A1
3. 0.3 D6
5.00 0.2 -11.1674 3.0 Q5
3. 0.2 D7
5.0 0.2 9.0206 3.0 Q6
3. 0.2 D8
5.00 0.2 -14.1000 3.0 Q7
3. 0.2 D9
5.00 0.2 3.3265 3.0 Q8
3. 0.3 D10
2. 0.0 A2
4. 1.09955 17.3725 0.0 B2
13. 4. PR1
4. 1.09955 17.3725 0.0 B2
2. 0.0 A2
3. 0.3 D11
5.00 0.2 3.3265 3.0 Q9
3. 0.2 D12
5.00 0.2 -14.1000 3.0 Q10
3. 0.2 D13
5.00 0.2 9.0206 3.0 Q11
3. 0.2 D14
5.00 0.2 -11.1674 3.0 Q12
3. 0.3 D15
2. 0.0 A3
4. 1.09955 17.3725 0.0 B3
2. 0.0 A3
3. 0.3 D16
5.00 0.2 5.3394 3.0 Q13
3. 0.2 D17
5.00 0.2 -7.0346 3.0 Q14
3. 0.2 D18
5.00 0.2 5.1322 3.0 Q15
3. 0.2 D19
5.00 0.2 -9.6300 3.0 Q16
3. 0.3 D20
13. 4. PR2
SENTINEL

```

Fig. A-4. TRANSPORT run for the $45^\circ + 90^\circ + 45^\circ$ bending system
 System 1 -- (4) input for second-order calculation.

Z	NAME	First Order		Second Order	
		X	Y	X	Y
0.000	BEAM	.5000	.5000	.5000	.5000
.300	D1	.5142	.5142	.5142	.5142
.500	Q1	.6817	.4077	.6817	.4077
.700	D2	1.013	.2113	1.013	.2113
.900	Q2	1.192	.2157	1.192	.2157
1.100	D3	1.208	.4156	1.208	.4156
1.300	Q3	1.467	.5539	1.467	.5539
1.500	D4	1.985	.6014	1.985	.6014
1.700	Q4	2.195	.7489	2.195	.7489
2.000	D5	2.031	1.127	2.031	1.127
3.100	B1	.9071	2.552	.9072	2.552
3.400	D6	.5050	2.957	.5068	2.957
3.600	Q5	.4097	2.341	.4077	2.341
3.800	D7	.4938	.9014	.4917	.9014
4.000	Q6	.4870	.4295	.4900	.4300
4.200	D8	.3917	1.756	.3998	1.756
4.400	Q7	.5095	2.283	.5182	2.283
4.600	D9	.8307	1.940	.8426	1.940
4.800	Q8	1.076	1.767	1.090	1.767
5.100	D10	1.311	1.753	1.326	1.753
6.199	B2	1.774	1.749	1.787	1.750
7.299	B2	1.311	1.753	1.317	1.759
7.599	D11	1.076	1.767	1.080	1.774
7.799	Q9	.8307	1.940	.8333	1.948
7.999	D12	.5095	2.282	.5101	2.293
8.199	Q10	.3917	1.756	.3920	1.765
8.399	D13	.4870	.4295	.4900	.4331
8.599	Q11	.4938	.9014	.4997	.9035
8.799	D14	.4096	2.341	.4176	2.348
8.999	Q12	.5050	2.957	.5175	2.967
9.299	D15	.9070	2.551	.9300	2.561
10.398	B3	2.031	1.127	2.084	1.141
10.698	D16	2.194	.7487	2.248	.7669
10.898	Q13	1.985	.6012	2.032	.6283
11.098	D17	1.467	.5537	1.501	.5974
11.298	Q14	1.208	.4154	1.234	.4709
11.498	D18	1.191	.2156	1.215	.2779
11.698	Q15	1.013	.2114	1.031	.2604
11.898	D19	.6816	.4078	.6912	.4348
12.098	Q16	.5142	.5143	.5184	.5264
12.398	D20	.5000	.5000	.5018	.5002

Fig. A-5. TRANSPORT run for the $45^\circ + 90^\circ + 45^\circ$ bending system (System 1) -- (5) comparison of the beam envelopes in the first- and the second-order calculations.

```

CALL,TRANXX
TROU4 - TRANSPORT OUTPUT REFORMATTER WITH OPTION 4
TRANSPORT 78/04/06 12.36.29
TEST
DATAPI= 0
13. 2. OFFSIG
1. .5 4. .5 4. .5 .2 .72912 BEAM
3. 0.3 D1
5.01 0.2 -6.0 3.0 Q1
3. 0.2 D2
5.01 0.2 8.77 3.0 Q2
3. 0.3 D3
20. 180. LEFT
16. 5. 4.0 GAP
2. 0.0 A1
4. 1.09955 17.372 0.0 B1
2. 0.0 A1
20. -180. RIGHT
3. 0.3 D4
5.01 0.2 -587 3.0 Q3
3. 0.2 D5
5.01 0.2 5.35 3.0 Q4
3. 0.3 D6
2. 0.0 A2
4. 1.09955 17.572 0.0 B2
2. 0.0 A2
3. 0.3 D7
5.01 0.2 -2.88 3.0 Q5
3. 0.3 D8
2. 0.0 A3
4. 1.09955 17.372 0.0 B3
2. 0.0 A3
3. 0.3 D9
5.01 0.2 -7.70 3.0 Q6
3. 0.2 D10
5.01 0.2 4.73 3.0 Q7
3. 0.3 D11
2. 0.0 A4
4. 1.09955 17.372 0.0 B4
2. 0.0 A4
3. 0.32 D12
5.01 0.1 -3.71 3.0 Q8
10. -1. 2. 0.0 0.001 R12
10. -2. 1. 0.0 0.001 R21
10. -3. 4. 0.0 0.001 R34
10. -4. 3. 0.0 0.001 R43
10. -2. 6. 0.0 0.001 R26
10. -5. 6. -5.74 0.001 R56
10.2 -1. 1. 2.50 0.001 R11
10.2 -5. 3. 2.00 0.001 R33
13. 4. PR3
SENTINEL

*CORRECTIONS*
( 1.8139E+01) 0.000 .001 -.002 .004 -.003 -.005
( 2.1160E+02) 0.000 -0.000 0.000 .000 .000 0.000
TRANSPORT 78/04/06 12.36.35
TEST
DATAPI= 0
*COVARIANCE ( FIT .0 )
0.000
0.000 .032
0.000 -.990 -.929 .003
0.000 -.998 -.880 .986 .023
0.000 0.000 0.000 0.000 0.000 0.000
0.000 -.993 -.864 .975 .997 0.000 .003
0.000 .997 .867 -.981 -.999 0.000 .010
J1= J47

```

Fig. A-6. TRANSPORT run for System 2 -- (1) input data and fitting procedure.

```

*OFFSIG *
*BEAM * 1.000000 .73 GEV/C
*D1 * 3.0 .300 M
*Q1 * 5.01 .200 M -6.010 KG 3.000 CM (FX = -.575 M , FY=
*D2 * 3.0 .200 M
*Q2 * 5.01 .200 M 8.771 KG 3.000 CM (FX= .451 M , FY= -
*D2 * 3.0 .300 M
*LEFT * 20.0 180.00 D
*GAP * 16.0 5.0 4.000E+00
*A1 * 2.0 0.000 D
*B1 * 4.000 1.10 M 17.372 KG .00 ( 45.000 D )
*A1 * 2.0 0.000 D
*RIGHT * 20.0 -180.00 D
*D4 * 3.0 .300 M
*Q3 * 5.01 .200 M -5.872 KG 3.000 CM (FX= -.589 M , FY=
*D5 * 3.0 .200 M
*Q4 * 5.01 .200 M 5.354 KG 3.00 CM (FX= .716 M , FY= -
*D6 * 3.0 .300 M
*A2 * 2.0 0.000 D
*B2 * 4.000 1.10 M 17.372 KB .00 ( 45.000 D )
*A2 * 2.0 0.000 D
*D7 * 3.0 .300 M
*Q5 * 5.01 .200 M -2.883 KG 3.000 CM (FX= -1.232 M , FY= 1
*D8 * 3.0 .300 M
*A3 * 2.0 0.000 D
*B3 * 4.000 1.10 M 17.372 KG .00 ( 45.000 D )
*A3 * 2.0 0.000 D
*D9 * 3.0 .300 M
*Q6 * 5.01 .200 M -7.705 KG 3.000 CM (FX= -.442 M , FY=
*D10 * 3.0 .200 M
*Q7 * 5.01 .200 M 4.731 KG 3.000 CM (FX= .806 M , FY= -
*D11 * 3.0 .300 M
*A4 * 2.0 0.000 D
*B4 * 4.000 1.10 M 17.372 KG .00 ( 45.000 D )
*A4 * 2.0 0.000 D
*D12 * 3.0 .320 M
*Q8 * 5.01 .100 M -3.714 KG 3.000 CM (FX= =1.948 M , FY= 1
*R12 * 10.0 -1 2 0.000 / .001
          -.000
*R21 * 10.0 -2 1 0.000 / .001
          .000
*R34 * 10.0 -3 4 0.000 / .001
          .000
*R43 * 10.0 -4 3 0.000 / .001
          -.000
*R26 * 10.0 -2 6 0.000 / .001
          -.000
*R56 * 10.0 -5 6 -5.740 / .001
          -5.740
*R11 * 10.2 -1 1 2.500 / .001
          2.384
*R33 * 10.2 -3 3 2.000 / .001
          2.000
*PR3 * 1
2.3843E+00 -8.5099E-12 1.3115E-11 -3.2208E-10 0. 4.7358E+00
5.1369E+10 4.1941E-01 1.6207E-09 -9.3750E-10 0. -5.9401E-10
4.4706E-09 -2.7017E-10 2.0000E+00 2.9813E-11 0. -1.1070E-08
1.9320E-09 -2.7503E-12 -6.5377E-10 4.9999E-01 0. -1.7879E-09
3.8499E-10 1.9863E-01 1.1251E-09 -9.9745E-10 1.0000E+00 -5.7400E+00
0. 0. 0. 0. 0. 1.0000E+00
(LENGTH* 9.2 M

```

Fig. A-7. TRANSPORT run for System 2 -- (2) result of fitting.

Monochromatic Beam						
Z(m)	NAME	Diverging Beam		Parallel Beam		$\Delta p/p$ 0.2%
		X(cm)	Y(cm)	X(cm)	Y(cm)	X(cm)
0.000	BEAM	1.0000E-05	1.0000E-05	.5000	.5000	1.0000E-05
.300	D1	.1200	.1200	.5000	.5000	1.0440E-05
.500	Q1	.2248	.1764	.5847	.4199	1.2973E-05
.700	D2	.3601	.2062	.7586	.2610	1.7641E-05
.900	Q2	.4016	.2900	.7437	.1614	1.7946E-05
1.200	D3	.3183	.5068	.4401	7.5247E-02	1.1866E-05
1.200	LEFT	.3183	.5068	.4401	7.5247E-02	1.1866E-05
2.300	B1	4.9839E-02	1.313	.6909	.2390	8.2009E-02
2.300	RIGHT	4.9839E-02	1.313	.6909	.2390	8.2009E-02
2.600	D4	.1570	1.541	.9723	.3262	.1244
2.800	Q3	.2583	1.443	1.331	.3301	.1748
3.000	D5	.3948	1.102	1.880	.2797	.2500
3.200	Q4	.4682	.9085	2.133	.2688	.2858
3.500	D6	.4782	.8250	2.050	.3110	.2777
4.599	B2	.3713	.5514	1.178	.4727	8.7525E-02
4.899	D7	.3059	.4794	.8087	.5197	2.7238E-03
5.099	Q5	.2857	.3952	.6209	.5097	6.4705E-02
5.399	D8	.2895	.2180	.4193	.4331	.1628
6.499	B3	.2174	.4264	.3690	.1618	.5210
6.799	D9	.1763	.6048	.5751	8.8731E-02	.6081
6.999	Q6	.1854	.5923	.8423	2.5312E-02	.8034
7.199	D10	.2318	.4491	1.276	4.8344E-02	1.150
7.399	Q7	.2468	.3592	1.524	.1316	1.335
7.699	D11	.2212	.2984	1.609	.2764	1.360
8.798	B4	7.1977E-02	8.2208E-02	1.418	.8131	1.127
9.118	D12	1.6919E-02	1.9831E-02	1.223	.9747	.9714
9.218	Q8	5.3021E-05	2.0000E-05	1.192	1.000	.9472

TRANSPORT 78/04/06 11.29.56

TEST

5.010000 Q1 -6.01000
5.010000 Q2 8.77064
5.010000 Q3 -5.87232
5.010000 Q4 5.35398
5.010000 Q5 -2.88341
5.010000 Q6 -7.70461
5.010000 Q7 4.73085
5.010000 Q8 -3.71423

STOP

/

(4) Envelope of the PIGM Beam

Z(m)	NAME	X(cm)	Y(cm)
0.000	BEAM	.5000	.5000
.300	D1	.5142	.5142
.500	Q1	.6264	.4554
.700	D2	.8397	.3350
.900	Q2	.8453	.3319
1.200	D3	.5431	.5124
1.200	LEFT	.5431	.5124
2.300	B1	.6975	1.334
2.300	RIGHT	.6975	1.334
2.600	D4	.9927	1.575
2.800	Q3	1.367	1.480
3.000	D5	1.937	1.136
3.200	Q4	2.202	.9475
3.500	D6	2.124	.8845
4.599	B2	1.238	.7263
4.899	D7	.8646	.7071
5.099	Q5	.6866	.6450
5.399	D8	.5349	.4848
6.499	B3	.6744	.4560
6.799	D9	.8554	.6113
6.999	Q6	1.183	.5928
7.199	D10	1.733	.4517
7.399	Q7	2.041	.3826
7.699	D11	2.118	.4067
8.798	B4	1.813	.8173
9.118	D12	1.562	.9749
9.218	Q8	1.523	1.006

Fig. A-8. TRANSPORT run for System 2 -- (3) particle trajectories of special interest (compilation of three runs).