ON FOLDING POTENTIALS AND THE ANTISYMMETRIZATION

H. Schultheis and R. Schultheis
Department of Physics and Astronomy
University of Maryland, College Park, Maryland 20742 U.S.A.

April 1978

U. of Md. TR #78-087
U. of Md. PP #78-193

UNIVERSITY OF MARYLAND
DEPARTMENT OF PHYSICS AND ASTRONOMY
COLLEGE PARK, MARYLAND

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ORO 5126-38
U. of Md. TR #78-087
U. of Md. PP #78-193

ON FOLDING POTENTIALS AND THE ANTISYMMETRIZATION

H. Schultheis* and R. Schultheis*
Department of Physics and Astronomy
University of Maryland, College Park, Maryland 20742 U.S.A.

April 1978

ABSTRACT

Antisymmetrization effects in the $^{16}\text{O} + ^{16}\text{O}$ heavy-ion potential and their dependence on the nucleon-nucleon interaction have been studied. Counter-examples are given for the recent justification of folding potentials by Fleckner and Mosel in terms of a cancellation of antisymmetrization and polarization effects.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Double-folding heavy-ion interaction potentials are customarily determined from the densities projectile and target would have in the absence of each other rather than from the antisymmetric wave function of the combined system. Such potentials fit heavy-ion elastic scattering data rather well, although the method neglects

- (i) the polarization of the densities of projectile and target due to the (Coulomb and nuclear) forces between the nuclei, and
- (ii) the inter-nucleus exchange terms in the antisymmetrization of the wave function of the combined system.

Recently Fleckner and Mosel¹ have explained the success of the folding method in terms of a cancellation effect between the two approximations: They find that in the vicinity of the strong absorption radius, R_{SA} , the inclusion of polarization (i) leads to a *decrease* in energy, and the inclusion of the antisymmetrization (ii) leads to an *increase* in energy, which compensates the effect of (i). These results have been obtained in an energy-density calculation with Skyrme forces. Motivated by this finding we have studied the $^{16}_O + ^{16}_O$ interaction energy for a number of different nucleon-nucleon interactions, and we found counterexamples for the cancellation effect stated in Ref. 1.

For $^{16}_O + ^{16}_O$ we have determined the energy expectation value² for the fully antisymmetric state $A\phi_1\phi_2$ and for the state $\phi_1\phi_2$ without the antisymmetrization between projectile and target, i.e.,

$$E^A = \langle A\phi_1\phi_2 | H | A\phi_1\phi_2 \rangle / \langle A\phi_1\phi_2 | A\phi_1\phi_2 \rangle \quad (1)$$

and

$$E = \langle \phi_1\phi_2 | H | \phi_1\phi_2 \rangle / \langle \phi_1\phi_2 | \phi_1\phi_2 \rangle. \quad (2)$$

Here ϕ_1 and ϕ_2 are taken to be the oscillator shell-model ground states of $^{16}_O$ with centers $\vec{R}_1$ and $\vec{R}_2$, respectively. Both states ϕ are antisymmetric

with respect to the 16 nucleons, and A denotes the full 32-particle antisymmetrization. The antisymmetrization A has been performed for the non-orthogonal system of single-nucleon states following Ref. 3 (rather than an expansion into an orthogonal basis).

The Hamiltonian H of the 32-nucleon system

$$H = \sum_{i=1}^A t_i + \frac{1}{2} \sum_{i \neq j}^A v_{ij} + \frac{e^2}{2} \sum_{i \neq j}^Z \frac{1}{r_{ij}} - T_{CM}, \quad (3)$$

includes the Coulomb energy and the complete center-of-mass term T_{CM} . For the nucleon-nucleon interaction v_{ij} we have considered finite range forces⁴⁻⁸ with or without soft-core and the general exchange mixture (spin exchange P_σ , isospin exchange P_τ , space exchange $P_x = -P_\sigma P_\tau$),

$$v_{ij} = \sum_{n=1}^2 u_n(r_{ij}) [W_n + B_n P_\sigma - H_n P_\tau + M_n P_x]. \quad (4)$$

A Skyrme-type interaction⁹ has also been considered.

We have determined the energy shift $\Delta E = E^A - E$ in the vicinity of the strong absorption radius $R_{SA} \approx 1.5(A_1^{1/3} + A_2^{1/3})$. For a separation distance $R = |\vec{R}_1 - \vec{R}_2| = 7$ fm and an oscillator constant $b = (\hbar/m\omega)^{1/2} = 1.8$ fm the following energy shifts¹⁰ result from including the antisymmetrization (ii) as in Eqs. (1,2):

Kinetic energy T	$\Delta T = T^A - T = 4.45$ MeV
Coulomb energy E_C	$\Delta E_C = E_C^A - E_C = -0.02$ MeV
Center-of-mass energy T_{CM}	$\Delta T_{CM} = T_{CM}^A - T_{CM} = 0.01$ MeV.

Thus the total energy shift $\Delta E = E^A - E$ is equal to

$$\Delta E = \Delta T + \Delta V + \Delta E_C - \Delta T_{CM} = 4.42 \text{ MeV} + \Delta V$$

where $\Delta V = V^A - V$ denotes the change in the expectation value of the nuclear potential energy. As ΔE_C and ΔT_{CM} almost vanish the effect of the antisymmetrization (ii) on the heavy-ion interaction energy is determined by ΔT and ΔV .

The quantities ΔV and ΔE are given in Table I for a number of usual nucleon-nucleon interactions. As the table shows, ΔV and ΔE may vary by an order of magnitude depending on the choice of the nucleon-nucleon interaction.

Therefore, it is not possible to determine a general effect of the antisymmetrization between projectile and target on the heavy-ion interaction energy.

We note that the sign of ΔE is in variance with the choice of the nucleon-nucleon force. Since the inclusion of polarization degrees of freedom in ϕ_1 and ϕ_2 can lead only to an energy *decrease* (if any), a negative ΔE from the antisymmetrization (ii) *enhances* rather than compensates the polarization effect. Thus the nucleon-nucleon interactions with a negative ΔE in Table I are counterexamples for the cancellation effect stated in Ref. 1 as a justification for the folding procedure.

As an example, Fig. 1 shows how changes in the force parameter affect the antisymmetrization effect ΔE at the strong absorption radius ($R = 7.6$ fm, $b = 1.8$ fm). Here a Gaussian soft-core interaction

$$u_n(r_{ij}) = S_n \exp(-r_{ij}^2/r_n^2), \quad B_n = H_n = 0, \quad W_n = 1 - M_n \quad (n = A, R) \quad (5)$$

with constant strength parameters S_n is used, and ΔE is plotted as a function of the range (r_A and r_R) and Majorana exchange (M_A and M_R) parameters of the attractive (A) and repulsive (R) part of the nucleon-nucleon interaction. The solid part of each curve corresponds to the range of parameter values occurring in the standard Volkov⁷ and Brink-Boeker⁸ interactions. This indicates the amount by which the force parameter may be changed while still maintaining the desired nucleon-nucleon scattering or binding properties of the force. The figure shows that the sign of ΔE can be reversed by only moderate changes in the force parameters (even without structural changes in the form factor or exchange mixture of the force). The value of ΔE is particularly sensitive to the range of the force. Therefore (and because of their peculiar exchange

terms $P_x \delta = \delta$, $P_\sigma \delta = -P_\tau \delta$) Skyrme forces may be misleading.

The counterexamples of this note show that the cancellation effect stated in Ref. 1 is in variance with the particular choice of the nucleon-nucleon force. The existing forces are fitted in the limit of saturation density and in the limit of nucleon-nucleon scattering. They may or may not adequately describe the low-density range at R_{SA} to which heavy-ion elastic scattering data are sensitive. As at present there are no *a priori* reasons to disregard certain types of nucleon-nucleon forces for the derivation of heavy-ion potentials the double-folding method cannot microscopically be justified in terms of a general cancellation effect between certain polarization and antisymmetrization contributions.

The authors are grateful to the University of Maryland Nuclear Theory Group for their hospitality and helpful discussions. Supported by the Deutsche Forschungsgemeinschaft, the U. S. Department of Energy and the University of Maryland Computer Science Center.

REFERENCES

*On leave from University of Tübingen, West Germany.

1. J. Fleckner and U. Mosel, Nucl. Phys. A277, 170 (1977).
2. H. Schultheis and R. Schultheis, Proc. Symp. on Heavy-Ion Elastic Scattering, Rochester, New York, October 24-26, 1977 (University of Rochester, 1978), p. 519; Bull. Am. Phys. Soc. 22, 1005 (1977).
3. D. M. Brink, Int. School of Physics "Enrico Fermi," Course XXXVI (Academic Press, New York-London, 1966), p. 247.
4. R. A. Ferrell and W. M. Visscher, Phys. Rev. 102, 450 (1956).
5. E. W. Schmid and K. Wildermuth, Nucl. Phys. 26, 463 (1961).
6. V. Gillet, Nucl. Phys. 51, 410 (1964).
7. A. B. Volkov, Nucl. Phys. 74, 33 (1965).
8. D. M. Brink and E. Boeker, Nucl. Phys. A91, 1 (1967).
9. C. Bargholtz, Nucl. Phys. A243, 449 (1975).
10. Similar results have been obtained for $R = 8$ fm and for $b = 1.6$ fm.

TABLE CAPTION

Table I: Effect of the antisymmetrization between projectile and target on the expectation values (1,2) of the nuclear potential energy V and the total energy E at $R = 7$ fm for a number of different nucleon-nucleon interactions. The force parameters are defined in Eq. (4). The form factors are Gaussian unless otherwise stated.

TABLE I

Force	[Ref.]	S (MeV)	r(fm)	W	B	H	M	ΔV (MeV)	ΔE (MeV)
Ferrell-Visscher	[4]	-51.9	1.732	0.317	0.0	0.183	0.5	-3.43	0.99
Schmid-Wildermuth	[5]	-72.98	1.47	0.3742	0.1158	0.0692	0.4408	-3.55	0.87
Gillet	[6]	-40.0	1.7	0.35	-0.1	0.4	0.35	-2.34	2.08
Volkov	no. 1 [7]	-83.34	1.6	0.4	0.0	0.0	0.6	-2.88	1.54
		144.86	0.82	0.4	0.0	0.0	0.6		
Brink-Boeker	B_1 [8]	-140.6	1.4	0.5136	0.0	0.0	0.4864	-7.68	-3.26
		389.5	0.7	1.529	0.0	0.0	-0.529		
Brink-Boeker	B_4 [8]	-1307.9	0.8	0.5231	0.0	0.0	0.4769	-12.98	-8.56
		7228.0	0.4	0.991	0.0	0.0	0.009		
Brink-Boeker	C_1 [8]	-117.4	1.4	0.6185	0.0	0.0	0.3815	-9.62	-5.20
		271.0	0.7	2.635	0.0	0.0	-1.635		
Brink-Boeker	C_4 [8]	-1042.0	0.8	0.8765	0.0	0.0	0.1235	-33.08	-28.66
		5444.0	0.4	2.795	0.0	0.0	-1.795		
Bargholtz	[9]	Skyrme						-0.21	4.21

FIGURE CAPTION

Fig. 1: Shift in nuclear potential energy ΔV and total energy ΔE due to the antisymmetrization between projectile and target for $^{16}_0 + ^{16}_0$ at the strong absorption radius. The abscissa gives the relative change of the parameters r_A , r_R , M_A and M_R of the nucleon-nucleon interaction in Eq. (5). The zero of the abscissa corresponds to the parameter set of the Brink-Boeker interaction B_1 . The solid part of each curve corresponds to parameter values occurring in the standard interactions as explained in the text.

Fig. 1