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H. Schultheis and R. Schultheis
Department of Physics and Astronomy
University of Maryland, College Park, Maryland 20742 U.S.A.

April 1978

U. of Md. TR #78-087
U. of Md. PP #78-193
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ON FOLDING POTENTIALS AND THE ANTISYMMETRIZATION

H. Schultheis* and R. Schultheis*
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ABSTRACT

Antisymmetrization effects in the $^{16}_0 + ^{16}_0$ heavy-ion potential and their dependence on the nucleon-nucleon interaction have been studied. Counterexamples are given for the recent justification of folding potentials by Fleckner and Mosel in terms of a cancellation of antisymmetrization and polarization effects.
Double-folding heavy-ion interaction potentials are customarily determined from the densities projectile and target would have in the absence of each other rather than from the antisymmetric wave function of the combined system. Such potentials fit heavy-ion elastic scattering data rather well, although the method neglects

(i) the polarization of the densities of projectile and target due to the (Coulomb and nuclear) forces between the nuclei, and

(ii) the inter-nucleus exchange terms in the antisymmetrization of the wave function of the combined system.

Recently Fleckner and Mosel\(^1\) have explained the success of the folding method in terms of a cancellation effect between the two approximations: They find that in the vicinity of the strong absorption radius, \(R_{SA}\), the inclusion of polarization (i) leads to a *decrease* in energy, and the inclusion of the antisymmetrization (ii) leads to an *increase* in energy, which compensates the effect of (i). These results have been obtained in an energy-density calculation with Skyrme forces. Motivated by this finding we have studied the \(^{16}\text{O} + ^{16}\text{O}\) interaction energy for a number of different nucleon-nucleon interactions, and we found *counterexamples* for the cancellation effect stated in Ref. 1.

For \(^{16}\text{O} + ^{16}\text{O}\) we have determined the energy expectation value\(^2\) for the fully antisymmetric state \(A\phi_1\phi_2\) and for the state \(\phi_1\phi_2\) without the antisymmetrization between projectile and target, i.e.,

\[
E^A = \langle A\phi_1\phi_2 | H | A\phi_1\phi_2 \rangle / \langle A\phi_1\phi_2 | A\phi_1\phi_2 \rangle
\] (1)

and

\[
E = \langle \phi_1\phi_2 | H | \phi_1\phi_2 \rangle / \langle \phi_1\phi_2 | \phi_1\phi_2 \rangle.
\] (2)

Here \(\phi_1\) and \(\phi_2\) are taken to be the oscillator shell-model ground states of \(^{16}\text{O}\) with centers \(\vec{R}_1\) and \(\vec{R}_2\), respectively. Both states \(\phi\) are antisymmetric
with respect to the 16 nucleons, and \( A \) denotes the full 32-particle antisymmetrization. The antisymmetrization \( A \) has been performed for the non-orthogonal system of single-nucleon states following Ref. 3 (rather than an expansion into an orthogonal basis).

The Hamiltonian \( H \) of the 32-nucleon system

\[
H = \sum_{i=1}^{A} t_i + \frac{1}{2} \sum_{i \neq j}^{A} v_{ij} + \frac{e^2}{2} \sum_{i \neq j}^{Z} \frac{1}{r_{ij}} - T_{CM},
\]

includes the Coulomb energy and the complete center-of-mass term \( T_{CM} \). For the nucleon-nucleon interaction \( v_{ij} \), we have considered finite range forces \(^4\text{--}^8\) with or without soft-core and the general exchange mixture (spin exchange \( P_{\sigma} \), isospin exchange \( P_{\tau} \), space exchange \( P_X = -P_{\sigma} P_{\tau} \)),

\[
v_{ij} = \sum_{n=1}^{2} \frac{u_n(r_{ij})}{n} [W_n + B_n \sigma_n - H_n \tau_n + M_n X_n].
\]

A Skyrme-type interaction \(^9\) has also been considered.

We have determined the energy shift \( \Delta E = E^A - E \) in the vicinity of the strong absorption radius \( R_{SA} \approx 1.5(A_1^{1/3} + A_2^{1/3}) \). For a separation distance \( R = |\vec{r}_1 - \vec{r}_2| = 7 \) fm and an oscillator constant \( b = (\hbar/m_0)^{1/2} = 1.2 \) fm the following energy shifts \(^{10}\) result from including the antisymmetrization (ii) as in Eqs. (1,2):

- **Kinetic energy** \( T \)
  \[ \Delta T = T^A - T = 4.45 \text{ MeV} \]
- **Coulomb energy** \( E_C \)
  \[ \Delta E_C = E_C^A - E_C = -0.02 \text{ MeV} \]
- **Center-of-mass energy** \( T_{CM} \)
  \[ \Delta T_{CM} = T_{CM}^A - T_{CM} = 0.01 \text{ MeV} \]

Thus the total energy shift \( \Delta E = E^A - E \) is equal to

\[ \Delta E = \Delta T + \Delta V + \Delta E_C - \Delta T_{CM} = 4.42 \text{ MeV} + \Delta V \]

where \( \Delta V = V^A - V \) denotes the change in the expectation value of the nuclear potential energy. As \( \Delta E_C \) and \( \Delta T_{CM} \) almost vanish the effect of the antisymmetrization (ii) on the heavy-ion interaction energy is determined by \( \Delta T \) and \( \Delta V \).
The quantities $\Delta V$ and $\Delta E$ are given in Table I for a number of usual nucleon-nucleon interactions. As the table shows, $\Delta V$ and $\Delta E$ may vary by an order of magnitude depending on the choice of the nucleon-nucleon interaction. Therefore, it is not possible to determine a general effect of the antisymmetrization between projectile and target on the heavy-ion interaction energy.

We note that the sign of $\Delta E$ is in variance with the choice of the nucleon-nucleon force. Since the inclusion of polarization degrees of freedom in $\phi_1$ and $\phi_2$ can lead only to an energy decrease (if any), a negative $\Delta E$ from the antisymmetrization (ii) enhances rather than compensates the polarization effect. Thus the nucleon-nucleon interactions with a negative $\Delta E$ in Table I are counterexamples for the cancellation effect stated in Ref. 1 as a justification for the folding procedure.

As an example, Fig. 1 shows how changes in the force parameter affect the antisymmetrization effect $\Delta E$ at the strong absorption radius ($R = 7.6 \text{ fm}$, $b = 1.8 \text{ fm}$). Here a Gaussian soft-core interaction

$$u_n(r_{ij}) = \frac{S_n}{n!} \exp\left(-\frac{r_{ij}^2}{r_n^2}\right), \quad B_n = H_n = 0, \quad W_n = 1 - M_n \quad (n = A, R)$$

with constant strength parameters $S_n$ is used, and $\Delta E$ is plotted as a function of the range ($r_A$ and $r_R$) and Majorana exchange ($M_A$ and $M_R$) parameters of the attractive (A) and repulsive (R) part of the nucleon-nucleon interaction. The solid part of each curve corresponds to the range of parameter values occurring in the standard Volkov\(^7\) and Brink-Boeker\(^8\) interactions. This indicates the amount by which the force parameter may be changed while still maintaining the desired nucleon-nucleon scattering or binding properties of the force. The figure shows that the sign of $\Delta E$ can be reversed by only moderate changes in the force parameters (even without structural changes in the form factor or exchange mixture of the force). The value of $\Delta E$ is particularly sensitive to the range of the force. Therefore (and because of their peculiar exchange
\[ P_x \delta = \delta, \quad P_\sigma \delta = -P_\tau \delta \] 

Skyrme forces may be misleading.

The counterexamples of this note show that the cancellation effect stated in Ref. 1 is in variance with the particular choice of the nucleon-nucleon force. The existing forces are fitted in the limit of saturation density and in the limit of nucleon-nucleon scattering. They may or may not adequately describe the low-density range at \( R_{SA} \) to which heavy-ion elastic scattering data are sensitive. As at present there are no \textit{a priori} reasons to disregard certain types of nucleon-nucleon forces for the derivation of heavy-ion potentials the double-folding method cannot microscopically be justified in terms of a general cancellation effect between certain polarization and antisymmetrization contributions.

The authors are grateful to the University of Maryland Nuclear Theory Group for their hospitality and helpful discussions. Supported by the Deutsche Forschungsgemeinschaft, the U. S. Department of Energy and the University of Maryland Computer Science Center.
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*On leave from University of Tübingen, West Germany.


10. Similar results have been obtained for $R = 8 \text{ fm}$ and for $b = 1.6 \text{ fm}$. 
Table I: Effect of the antisymmetrization between projectile and target on the expectation values $(1,2)$ of the nuclear potential energy $V$ and the total energy $E$ at $R = 7 \text{ fm}$ for a number of different nucleon-nucleon interactions. The force parameters are defined in Eq. (4). The form factors are Gaussian unless otherwise stated.
<table>
<thead>
<tr>
<th>Force</th>
<th>Ref.</th>
<th>S (MeV)</th>
<th>r (fm)</th>
<th>W</th>
<th>B</th>
<th>H</th>
<th>M</th>
<th>ΔV (MeV)</th>
<th>ΔE (MeV)</th>
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<td>-51.9</td>
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<td>0.183</td>
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<td>1.47</td>
<td>0.3742</td>
<td>0.1158</td>
<td>0.0692</td>
<td>0.4408</td>
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<td>Gillet</td>
<td>[6]</td>
<td>-40.0</td>
<td>1.7</td>
<td>0.35</td>
<td>-0.1</td>
<td>0.4</td>
<td>0.35</td>
<td>-2.34</td>
<td>2.08</td>
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<td>Volkov no. 1</td>
<td>[7]</td>
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<td>1.6</td>
<td>0.4</td>
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<td>0.0</td>
<td>0.6</td>
<td>-2.88</td>
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<td></td>
<td>144.86</td>
<td>0.82</td>
<td>0.4</td>
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<td>0.0</td>
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<tr>
<td>Brink-Boeker B₁</td>
<td>[8]</td>
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<td>1.4</td>
<td>0.5136</td>
<td>0.0</td>
<td>0.0</td>
<td>0.4864</td>
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<tr>
<td>Brink-Boeker B₄</td>
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<td></td>
<td>7228.0</td>
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<td>0.991</td>
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</table>
Fig. 1: Shift in nuclear potential energy $\Delta V$ and total energy $\Delta E$ due to the antisymmetrization between projectile and target for $^{16}O + ^{16}O$ at the strong absorption radius. The abscissa gives the relative change of the parameters $r_A$, $r_R$, $M_A$ and $M_R$ of the nucleon-nucleon interaction in Eq. (5). The zero of the abscissa corresponds to the parameter set of the Brink-Boeker interaction $B_1$. The solid part of each curve corresponds to parameter values occurring in the standard interactions as explained in the text.