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LA-UR--88-3881

DE39 003579

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SUBMITTED TO Proc. SPIE 1045, Modeling
 and Simulation of Laser
 Systems: conference
 to be held in Los Angeles,
 Ca, January, 1989

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Simulation Codes for Modeling Free-Electron Laser Oscillators*

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ABSTRACT

This paper reviews the basic characteristics of free-electron laser (FEL) oscillators and describes a group of codes that have been developed to analyze and design rf-linac-driven FELs. The optical performance of an FEL has been treated using 1-D time-dependent (finite pulse) codes that describe the characteristics of the optical temporal pulse shape and spectrum during the evolution of the oscillator from low-intensity small-signal gain conditions to high-intensity large-signal gain conditions. These codes can include frequency-dependent elements, such as narrow-band filters or grating rhombs. Diffraction effects, transverse optical and electron-beam mode properties, misalignments, as well as aberrations on optical elements are modeled with the 3-D code FELEX. This code has now been extended to include the effects of imperfections in the wiggler magnetic field and light emission at higher optical harmonics. A separate accelerator modeling capability allows the use of a numerically-generated electron pulse in FELEX for 3-D integrated numerical FEL simulations.

1. INTRODUCTION

Free-electron lasers have been operated successfully over a wide range of power levels, wavelengths, and pulse widths, both as oscillators and amplifiers. Furthermore, a variety of related free-electron devices which operate according to the same basic FEL mechanism has been proposed. Any FEL device requires major contributions from three quite different technologies: optics, relativistic electron beams, and wiggler magnets. Numerical simulation methods have shown themselves to be essential for the analysis and design of FELs in which the performance of the device is closely linked, frequently in a nonlinear fashion, to the design and actual performance of components from each of the three crucial technologies. In this paper we will describe two simulation codes that have been developed at Los Alamos and give examples of problems addressed by each one. In somewhat less detail, extensions of these codes to treat various related topics will also be discussed. These discussions will point out the basic phenomenology of FELs: a quantitative analysis of these devices requires realistic numerical models of (perhaps imperfect) components from all three basic technologies. Emphasis here will be on Compton-regime FEL oscillators in which plasma effects do not play a significant role⁽¹⁾; amplifiers can be handled readily by the same simulation codes.

*Work performed under the auspices of the U.S. Department of Energy and supported by the U.S. Army Ballistic Missile Defense Organization.

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Fig. 1 is a schematic of a typical FEL oscillator. A pulse of relativistic electrons from an accelerator, say an rf-linear-accelerator, is magnetically guided onto the axis of an optical resonator. The electron pulse, shown in black, moves with axial velocity βc and interacts with an optical pulse, which moves at c , while both pulses are within a static periodic magnetic field generated by a wiggler magnet. The field of the wiggler is transverse to the propagation direction and induces a transverse wobble motion upon the electrons. This transverse current interacts with the transverse electromagnetic field of the optical pulse and energy can flow from the electrons to the light, hence providing optical gain. The FEL resonance condition is a relation among the optical wavelength, the wiggler field amplitude and wavelength, and the electron energy which ensures that net energy transfer will occur after one transit through the interaction region (wiggler). After passage through the wiggler, the electrons are magnetically guided out of the resonator while the optical pulse is reflected back to the wiggler entrance just in time to interact with the next electron pulse from the accelerator.

2. SMALL-SIGNAL GAIN

As in any laser, there can be no coherent light output unless the gain exceeds the round-trip losses of the optical resonator. We shall define gain to be P_{out}/P_{in} , where P_{out} (P_{in}) is the optical power at the exit (entrance) to the wiggler. Often one distinguishes between the low-gain regime, where the gain is proportional to the cube of the wiggler's length, and the high-gain regime, where the gain grows exponentially. Analytical expressions have been derived for the case of untapered wiggler devices driven by perfect electron beams in the low-gain ^[1] and high-gain ^[2] regimes. The effects of imperfect electron-beams with finite energy spread and emittance must be evaluated numerically ^{[3],[4]}. No analytical formulas are available for the small-signal gain of an arbitrarily tapered wiggler (except for a linear taper in $k_w = 2\pi/\lambda_w$ where λ_w is the local period of the field ^[5], and an optical klystron ^[6]) driven by a perfect electron beam in either the low- or the high-gain regime: these must be evaluated numerically, as must gain degradation effects caused by imperfect electron beams.

All of the gain evaluations referred to have been 1-D calculations. For large values of the gain, or for devices in which the transverse size of the electron beam is comparable to, or larger than, the transverse size of the optical beam, the 1-D approximation is inadequate and 3-D effects must be included. At Los Alamos, the 3-D code FELEX has been developed to treat those effects. We discuss FELEX more fully below; here, wish to point out that the small-signal gain can also depend upon the transverse mode of the light ^[7] and, furthermore, a multiple-pass calculation is needed to find a fully self-consistent small-signal gain solution ^[7] because the mode profile of the light can be changed by both the gain process within the wiggler and by the optical components of the resonator.

Finally, all of the above discussion has ignored the pulsed nature of the electron-beam excitation: this is appropriate only if the length of the electron pulse is much larger than the slippage distance. Since electrons travel at a velocity less than the speed of light, any given point on the envelope of the optical pulse will slip over a finite length of the electron pulse and therefore will be driven by a range of different currents. Similarly, any electron will see a range of different optical field amplitudes during a transit through the wiggler. The optical gain will therefore be less than the value obtained by using the maximum current in a cw-calculation. The gain of a short pulse is a function of the cavity length, as discussed in the next section. Pulse effects can be numerically evaluated with the 1-D simulation code FELP, while mode distortion effects are dealt with in FELEX.

3. FELP AND 1-D SIMULATIONS

FELP is a 1-D time-dependent FEL simulation code. Physical phenomena such as the start-up time of an oscillator and its small-signal gain can be modeled as well as optical cavity length detuning (desynchronization) effects. Typical electron pulse lengths from rf-linacs are in the range of a few picoseconds to perhaps thirty picoseconds. These lengths are much shorter than typical wiggler lengths (1-5 m). Thus, as shown in Fig. 1, it is clear that the cavity lengths L must be adjusted such that $2L/c = T_p$, where T_p is the time between successive pulses from the linac. Otherwise, the optical pulse will not overlap with an electron pulse in the wiggler on the next pass, and, therefore will experience no gain and will decay exponentially according to the magnitude of the resonator losses. For reasons discussed below in the example, the laser will have finite output over a small range of cavity lengths about that for exact synchronism.

The mathematical model used in FELP is due originally to Colson and Ride ^[8] and has been frequently discussed in the literature ^{[9] - [11]}. FELP can also be used in the periodic-boundary approximation which was invented by Colson ^[12] to treat optical spectral phenomena for problems in which the electron pulse is many slippage lengths long. In both the finite-pulse and periodic boundary condition modes, the transverse variation of optical and electron beam properties is ignored. The

on-axis variation of the intensity due to the spatial properties of a resonator mode ^[13] is, however, included, and transverse emittance effects can be modeled as an effective energy spread ^[3].

The variation of the steady-state output of an FEL oscillator as a function of the length of the optical cavity near that for exact synchronism is called a cavity-length detuning, or desynchronism, curve. The width of this curve decreases as the electron pulse length decreases. There would be no curve (or one with infinite width) if the source was a continuous electron-beam. Table 1 shows typical parameters for the Los Alamos FEL oscillator, and Fig. 2 shows a calculated and measured desynchronism curve. The experimental curve has been normalized to the same peak height as the theoretical curve, and the location of the peak of the experimental curve has been arbitrarily set at zero detuning, the location of the peak of the calculated curve. Fig. 3 shows steady-state optical pulse shapes for three values of detuning, and Fig. 4 shows the corresponding three optical spectra (time integrated). Negative δl means a cavity length shorter than exact synchronism. The reason that the laser runs for shorter cavity lengths, but shuts off when the cavity is lengthened, is related to the gain mechanism: the trailing edge of the optical pulse is preferentially amplified. This results in a distortion of the pulse shape such that the velocity of the peak of the pulse is actually less than c , as if there were an index of refraction n greater than one: $v = (c/n) < c$. Hence, the synchronism condition is changed slightly to $2L/v = T_e$, and clearly the numerator of the left-hand side must be reduced from L to $L - \delta L$ to maintain equality.

Calculated desynchronism curves are in good agreement with experimental data ^[14]. The modulation shown in Figs. 3 and 4 is due to the sideband instability ^{[9]-[13]}. Sidebands can be eliminated with an intracavity optical filter: a recent extension of FELP to include a grating rhomb to suppress sidebands, and calculations for the Boeing burst-mode experiment, can be found in ^[15].

Table 1: Parameter values for Los Alamos 10 μ m FEL oscillator

Electron beam	
peak current	100 A
E/mc^2	40
$\Delta\gamma/\gamma$	1.5%
FWHM	10ps
Wiggler magnet	
length	100 cm
wavelength	2.73 cm
field amplitude	3000 g
untapered	
Optical resonator	
optical wavelength	10 μ m
Rayleigh range	50 cm
round-trip loss	5%

4. FELEX AND 3-D SIMULATIONS

FELEX^[16] is a 3-D FEL oscillator simulation code which is used to examine transverse optical mode structure effects and optical interactions with finite-transverse-emittance electron-beams. Within the wiggler magnet, 3-D trajectories of individual simulation electrons are followed. The optical field is driven by macroscopic currents constructed from the electron phase-space. The electron dynamics includes beams with arbitrary phase-space distribution. Optical gain and refractive effects are self-consistently calculated as the light interacts with the electrons inside the wiggler. At the exit of the wiggler, the optical field is propagated through the optical resonator and back to the entrance of the wiggler where it interacts with the next electron pulse: the steady-state reached after many passes therefore represents a complete self-consistent FEL oscillator solution. The code can be run in three different modes: single wave-front (no pulse effects, CW solution); multiple wave-fronts with periodic-boundary-conditions ^{[12],[17]}; and multiple wave-fronts with finite optical and electron pulses.

Figure 5 exhibits the equation of motion for the optical electric field which is numerically solved in FELEX. Plane

polarization is assumed for light of carrier wave-vector $k_0 = 2\pi/\lambda_0$. Also shown in Fig. 5 is the form of the magnetic scalar potential χ_m for a curved-pole-face wiggler^[18]. The rectangular coordinate system takes the xz plane to be the plane of the electrons wiggler motion so that a flat-pole-face wiggler would be represented by $k_{\omega y} = 0$ and $k_{\omega x} = k_{\omega}(z)$. The current density j that appears in the source term S is obtained by numerical solution of the electrons' equations of motion which are exhibited in Fig. 6. Here γmc^2 is the total energy of an electron, $\beta_x(\beta_y)$ is the scaled transverse x-velocity v_x/c (y-velocity v_y/c), and b_i^s are the components of an externally-imposed static magnetic field. The quantity a_s which appears in the equation for ψ is defined by $a_s = (e E_0/mc^2 k_0)$ and represents the dimensionless vector potential of the optical field in the same way that a_{ω} , defined in Fig. 5, is the dimensionless vector potential of the wiggler magnetic field. The electron equations are generalizations to 3-D of the usual 1-D equations^[8], and the 1-D optical evolution equation can be deduced from Fig. 5 by projection of the total electric field onto one mode^[13].

An example of a recent application of FELEX is integrated numerical modeling of free-electron laser oscillators^[18]. A numerically-generated electron micropulse from a complete numerical model of the rf-linac/beam transport system is used in FELEX to obtain FEL performance. Fig. 7 shows the current profile of the numerically-generated electron pulse: the mean energy, rms energy width, and transverse emittances vary with position within this pulse profile^[16]. Recent Los Alamos 10 μ m oscillator experiments^[19] have used four different wigglers: a 1-m wiggler approximately linearly tapered in λ_w (W1); wiggler W1 preceded by an 8.75-cm-long prebuncher located about 30 cm upstream of the main wiggler (W2); a 1-m wiggler parabolically-tapered in λ_w (W3); and wiggler W3 with the same prebuncher as in W3^[20]. The prebuncher had a fixed wavelength of 3 cm.

The optical resonator for the 3-D finite pulse FELEX simulations was configured to be identical with that used experimentally. Two copper mirrors were used, each with about 98.5% reflectivity. Outcoupling was provided by a 1mm-diameter hole in one mirror. The hole, plus various apertures within the resonator, provided an additional loss about 0.8%, so that the total round-trip loss was about 3.8%.

Figs. 8–10 show typical results, in this case for wiggler W2. Fig. 8 shows the evolution of the extraction efficiency vs pass number until a steady-state value of about 2.25% is reached. Fig. 9 shows the extraction efficiency within the electron pulse—note that only the central part of the electron beam contributes. Hence, one would expect that the optical pulse would be somewhat shorter than the electron pulse: this is shown in Fig. 10 which is a plot of the amplitude of the optical field vs longitudinal position z and transverse position x .

The extraction efficiency results of these calculations are shown compared with experimental data in Table 2. The agreement in most cases is reasonable in view of the following qualifications: all calculations were done at exact synchronism because at the present time FELEX cannot handle desynchronism; the measurements are maxima along a detuning curve. All of the calculations used the same numerically-generated electron pulse while the experiments may have been done under various accelerator settings. No sideband suppression was used experimentally^[19]; the finite-pulse calculations could not be done with enough resolution to resolve sidebands (see Fig. 3a and b) due to computer limitations. Finally, the data for W3 were taken hurriedly and may not represent the true experimental optimum. Nevertheless, these calculations represent the most sophisticated numerical simulations of FEL oscillator performance yet done, and the experiments have attained the highest extraction efficiency yet reported for the short-wavelength FEL oscillator.

Table 2: Calculated and measured extraction efficiency

Wiggler Type	calculated η_c (%)	measured η_M (%)
W1	1.93	1.7
W2	2.25	3.1
W3	2.17	1.4
W4	3.8	4.4

5. FURTHER EXTENSIONS OF FELEX

Another recent extension of FELEX was to model phenomena expected in the Boeing burst-mode ring resonator experiment^[21]. This work describes a method of altering the resonator to compensate for the optical mode distortions produced by a large value of saturated gain. Also modeled is the performance of a grating rhomb in the resonator. Other

recent extensions of FELEX include the calculation of harmonics in 3-D [22],[23], the inclusion of aberrations on optical elements of a resonator [24], and the simulation of refractive effects in an FEL oscillator [25]. Here we briefly discuss one other extension, namely, the modeling of the effects of wiggler field errors (due to imperfect magnetic material or improper wiggler assembly) upon FEL performance[26]. Fig. 11 shows a contour plot of the small-signal gain of the Los Alamos 1-m untapered wiggler driven by a 300 A beam with a 2π mm-mr emittance. On the vertical axis is rms field error in percent; on the horizontal axis is displacement of the electron-beam centroid at the entrance to the wiggler. Maximum displacement on the horizontal axis is $200\text{ }\mu\text{m}$; the direction of the displacement is almost along the negative y-axis (the yz plane is the plane of betatron motion). The figure shows that 0.5% field errors would reduce the gain from about 2.3 to about 1.1 if the electrons are injected exactly on the geometrical axis of the wiggler. However, if the electrons are injected off axis by $200\text{ }\mu\text{m}$, the full gain is almost perfectly recovered. Field errors cause the electrons to wander off-axis[26] and hence out of the optical beam, thus lowering the gain. Fig. 11 shows that much of this degradation can be overcome if the electrons are injected at the proper off-axis location initially.

6. SUMMARY

Two FEL simulation codes have been developed at Los Alamos to model FEL behavior: the 1-D code FELP is used to study the interaction of finite optical and electron micropulses and includes the effects of slippage, oscillator start-up and evolution from spontaneous emission, arbitrarily-tapered wigglers, electron energy spread sideband generation, and wavelength-dependent optical filters and grating rhombs. The 3-D code FELEX treats FEL problems with transverse variations of the optical and electron beams: it has been used to study the effects of optical mode distortion—due to gain and refractive effects of the electron-beam as well as apertures and optical elements of the resonator—upon FEL performance. FELEX has been extended to use numerically-generated electron-beams for integrated modeling studies, to calculate coherent spontaneous emission of harmonics in 3-D, to include models of wiggler field errors and optical aberrations, and to study effects of misalignments. Future extensions will include simulations of MOPA systems. These two codes provide a powerful capability for modeling the physics and realistic design constraints of free-electron lasers.

Fig. 1. Free-electron laser oscillator schematic.

Fig. 2. Calculated and measured desynchronism curve for the Los Alamos 10 μm FEL uniform wiggler oscillator.

Fig. 3a. Calculated optical pulse shape at $\delta L = 0$.

Fig. 3b. Calculated optical pulse shape at $\delta L = -20\mu\text{m}$.

Fig. 3c. Calculated optical pulse shape at $\delta L = -40\mu\text{m}$.

Fig. 4a. Calculated optical spectrum at $\delta L = 0$.

Fig. 4b. Calculated optical spectrum at $\delta L = -20\mu\text{m}$.

Fig. 4c. Calculated optical spectrum at $\delta L = -40\mu\text{m}$.

Fig. 5. Optical field equation solved in FELEX.

Fig. 6. Electron equations of motion solved in FELEX.

Fig. 7. Numerically-generated electron pulse for integrated numerical modeling.

Fig. 8. Extraction efficiency vs pass number from FELEX.

Fig. 9. Steady-state extraction efficiency vs position within electron pulse.

Fig. 10. Steady-state optical field amplitude vs x and z .

Fig. 11. Compensation of wiggler field errors by initial electron-beam displacement.

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Fig. 1

Fig. 2

Fig. 3a

Fig. 3b

Fig. 3c

Fig. 4a

Fig. 4b

Fig. 4c

$$\mathbf{E} = E_0(\vec{r}) \exp[i\theta_0(\vec{r})]$$

$$2ik_0 \frac{\partial \mathbf{E}}{\partial z} + \frac{\partial^2 \mathbf{E}}{\partial z^2} + \frac{\partial^2 \mathbf{E}}{\partial y^2} = S(\vec{r})$$

$$S(\vec{r}) = \left(-\frac{4\pi k_0}{c} J \frac{a_0(\vec{r})}{\gamma} F_\mu e^{-i(\phi)} \right)$$

where,

$$F_\mu = J_0(\mu) - J_1(\mu) \quad , \quad \mu = \frac{k_0 a_0^2(\vec{r})}{8\gamma^2 k_\omega} \quad , \quad a_0(\vec{r}) = (eB_0(\vec{r})) / (mc^2 k_\omega)$$

$$\chi_m = -\frac{D(z)}{k_{\omega y}} \cosh(k_{\omega x} x) \sinh(k_{\omega y} y) \sin \left[\int k_\omega(z) dz \right]$$

$$\vec{p}_\omega = -\nabla \chi_m \quad , \quad k_\omega^2 = k_{\omega x}^2 + k_{\omega y}^2$$

Fig. 5

$$\frac{d\gamma}{d(ct)} = -\frac{e}{mc^2} \frac{a_0(\vec{r})}{2\gamma} F_\mu E_0(\vec{r}) \sin[\psi + \theta_0(\vec{r})]$$

$$\frac{d\psi}{d(ct)} = k_\omega(z) - \frac{k_0}{2\gamma^2} \left[1 + \frac{a_0(\vec{r})^2}{2} + (\gamma\beta_x)^2 + (\gamma\beta_y)^2 - a_0 a_0 \cos[\psi + \theta_0] \right]$$

$$\frac{d(\gamma\beta_x)}{d(ct)} = -\frac{e a_0^2 k_{\omega x}^2}{2\gamma} x - \frac{e}{mc^2} (\beta_y B_z^* - \beta_z B_y^*)$$

$$\frac{d(\gamma\beta_y)}{d(ct)} = -\frac{e a_0^2 k_{\omega y}^2}{2\gamma} y - \frac{e}{mc^2} (\beta_z B_x^* - \beta_x B_z^*)$$

$$\frac{dx}{d(ct)} = \beta_x \quad , \quad \frac{dy}{d(ct)} = \beta_y$$

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11