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POLARIZATION AS AN INSTRUMENT TO EXPLORE THE PHENOMENOLOGY

AND DYNAMICS OF VECTOR SPACES

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It is shown on the example of an extensive set of polarization data of p-p elastic scattering at 6 GeV/c and $t = -0.6 \text{ GeV}^2/c^2$ that the careful choice of polarization formalisms to simplify the relationship between observables and amplitudes which nevertheless maintains a maximal amount of flexibility can enormously simplify the determination of the amplitudes as well as can yield clues to the dynamics underlying a reaction in atomic, nuclear, or high energy physics.

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We generally think of our understanding of microscopic phenomena (i.e. phenomena in atomic, nuclear, intermediate energy, and particle physics) in terms of an outer product of various vector spaces. For example, in particle physics we talk about spin-polarization space, isotopic spin space, strangeness space, charm space, etc., and we then try to establish the structure of particle dynamics in each of these spaces.

The exploration of each of these spaces can be thought of in terms of (generalized) polarization experiments, each of which studies phenomena for a particular combination of the vector projections in the particular space. For example, in early pion physics, the cross sections of pion-nucleon scattering were interpreted in terms of projections of isotopic spin vectors, thus yielding the discovery of the dominance of the $3/2, 3/2$ resonance at low energies.

Such explorations generally have two objectives. In the absence of a dynamical theoretical model, the aim is to give a complete phenomenological description of the reaction matrix in the space; in other words, to determine from the experimental observables, the amplitudes of the reaction matrix. If a theoretical prediction already exists, the aim is to test those predictions for the amplitudes.

In either of these cases, it is essential to incorporate two features of the formalism used for the exploration. First, the formalism should offer the simplest possible relationship between the experimental observables and the amplitudes, so that the latter could be determined through a minimal set of experiments and with a least degree of uncertainty. Second, one must preserve as much flexibility as possible in selecting the formalism, because in testing a particular theoretical model, or in trying to gain a clue to

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and insight into the yet unknown dynamical structure, one formalism may be very much more advantageous than another. Hence, a priori locking the analysis into one particular formalism is highly undesirable.

The aim of this note is to describe in general how such an exploration can be carried out and to demonstrate the advantages of such a procedure on a particular set of experimental data, analyzed previously by the traditional, more ad hoc methods. We want to stress that the importance in this note is therefore not in the specific numbers it generates, but in the procedure used to generate them, since that procedure is thus demonstrated to be not only extremely general, applicable to all varieties of present and future polarization experiments, but also capable of yielding, in a simple and quick way, significant results which in the present example have already been certified as such by the conventional methods.

The discussion will be given in terms of the spin space of particles and hence in terms of the usual polarization experiments in the narrow sense of the word, but the procedure may be equally applicable to the exploration of other spaces also. The illustrative set of data is that of elastic $p-p$ scattering at 6 GeV/c, because it constitutes a non-trivial and complete set of polarization experiments. In order to articulate the procedure and its advantages in a way that is comprehensible to and of interest to non-specialists also, the details are relegated to other, more extensive articles, some already available.

In finding a maximally simple relationship between the amplitudes and the experimental observables, one is directed to the so-called optimal formalisms.⁽¹⁾ In these the matrix connecting the experimental observables and the bilinear combination of amplitudes (bicombs) is as diagonal as possible. Total diagonalization is precluded by the reality requirements for the

observables, but in the optimal formalism the matrix consists of a string of 1×1 , 2×2 , 4×4 , and 8×8 submatrices along the main diagonal, and no submatrix larger than 8×8 ever appears in the matrix regardless of how large the spins of the particles may be.

Since in constructing an optimal formalism the quantization directions of the particles can be left unspecified, we have in fact a multiply infinite set of optimal formalisms, each of them equally optimal. When constraints

other than Lorentz- or rotation invariance apply, the optimal set is tightened by the additional symmetries relating the quantization axes of the various particles to each other. (2)

In the specific reaction involving four particles with spin $1/2$ each, and with the imposition of time reversal invariance, parity conservation, and identical particle constraints, as appropriate for elastic proton-proton scattering, these additional relations allow quantization directions only in the following ways:

- a) All four particles are quantized in the direction normal to the reaction plane (transversity formalism); or
- b) All four particles are quantized in the reaction plane, and the quantization directions are pairwise parallel (or antiparallel).

A special case of this alternative is the helicity formalism. The angle describing the particular quantization direction in the reaction plane will be denoted by β .

If a set of experimental observables is complete enough to determine the amplitudes completely, the set, of course, yields a set of amplitudes in any formalism (though the effort spent in obtaining these amplitudes may vary enormously). These different sets of amplitudes can be easily described in terms of the picturesque result that as β varies, the values of a given amplitude for the various values of β describe an ellipse in the complex amplitude plane. Thus for the reaction in question, determining the amplitudes for the transversity case, determining them for one case in the reaction plane, and constructing the ellipse giving the other cases in the reaction plane constitute a complete determination of the amplitudes in any optimal formalism.

Since the relationship between transversity amplitudes and the amplitudes in one of the cases in the reaction plane can also be given easily, the actual numerical determination needs to be performed only for one particular formalism.

We now turn to the application of this to the 6 GeV/c data for p-p elastic scattering. (3-8) We have chosen $|t| = 0.6 \text{ GeV}^2/c^2$ for the actual analysis since at that value the data appear to be fairly extensive and possess relatively small uncertainties. Table I summarizes the data we used.

The general relationship between observables and bicomps for this reaction is given in Reference 9 for an arbitrary optimal formalism in the case when only Lorentz- or rotation invariance holds. The additional imposition of the three symmetries mentioned earlier yields the results given in Table II for the observables tabulated in Table I. In Table II the results are given both in the transversity formalism and in the helicity formalism. The latter agrees with the results given in Table I of Reference 8.

The numerical determination of the values of the amplitudes from the experimental data in terms of the transversity amplitudes is very easy in this case, and can be carried out with a hand calculator in an hour or less. In contrast, the corresponding procedure in terms of the helicity amplitudes appears forbidding and can probably only be done on an electronic computer. This illustrates one of the advantages of maintaining flexibility in the choice of an optimal formalism, namely that one can then select that particular formalism in which the information contained in the experimental data is most easily accessible.

The results are given in Figure 1. It is noteworthy that in this formalism the determination of the magnitudes of the amplitudes can be carried out completely separately from the determination of the phases, and hence

the uncertainties in the former turn out to be quite small even though the uncertainties in the latter are, for some amplitudes, considerable.

With the help of the easily derivable relationships between transversity amplitudes and helicity amplitudes, one then can determine the helicity amplitudes also. They are given in Figure 2. As expected, in the helicity formalism the determination of magnitudes and that of the phases are coupled and hence the uncertainties are larger. In principle, these uncertainties can be lessened by tracing out the very complicated correlations among the uncertainties in the various amplitudes, but in practice such a task is almost unmanageable.

The numerical results of our analysis are, of course, similar to those carried out previously by Johnson et al.⁽¹⁰⁾. A comparison with that reference highlights the advantages of the optimal approach in several ways: The present paper arrives at the same transversity amplitudes as Reference 10 but we show systematically why those amplitudes should be considered (they are optimal) and what other amplitude systems should be considered "in the running". While Reference 10 considers, of course, only the two traditional sets (helicity and transversity), we point out that these two are, in fact, only examples of the infinitely many optimal frameworks which should all be considered for suitability and as clues toward dynamics.

Once the helicity amplitudes are determined, one can also give the amplitudes in any other formalism with quantization axes in the reaction plane by drawing the ellipses along which the amplitudes move when β is varied. This is also shown in Figure 2. One can see that the ellipses for the various amplitudes are similar and are oriented identically. The only difference among the amplitudes in this respect is that the amplitude b is out of phase by

45° with the other amplitudes as it revolves along the ellipse.

The inspection of Figure 2 suggests that the dynamics is described in a particularly simple way in a formalism in which the quantization axes are in the reaction plane but normal to the helicity quantization axes. In this third formalism (which might be called in-plane transversity formalism) all five amplitudes are either pure real or pure imaginary within the experimental errors. We do not venture a deeper interpretation of this result, and in fact even its validity may be questioned on the basis of this single set of data with not inconsiderable uncertainties. Yet the observation in itself can serve as an example of how dynamical insight may be obtained from completely utilizing the flexibility that resides in the choice of optimal polarization formalisms.

That in our example the transversity amplitudes⁽¹¹⁾ functioned so well is due to the fact that the available experimental data interface very well with the transversity labels. In general, this indicates that in designing a set of polarization experiments and in choosing an appropriate formalism to analyze those experiments, attention to coordination can save a large amount of experimental effort and expense and a large amount of computer time. In view of the growing importance of polarization experiments in all branches of microscopic physics, the relevance of this point may be considerable.

In summary, we want to list the principal results of this paper.

- 1) We have shown on a well established example that the systematic use of the optimal formalism and the freedom it provides can methodically lead to a simple and quick phenomenological analysis of data.
- 2) The utilization of the additional freedom in the optimal polarization

data can be used to acquire clues toward the nature of the dynamics of the reaction.

3) The utilization of the optimal formalism can systematically lead to a procedure of analysis in which a subset of the amplitude parameters are determined by a subset of experiments, as it is the case for the magnitudes of the amplitudes in the example we present. This is of considerable significance in the motivation for performing polarization experiments since it is thus possible to obtain unambiguous and accurate partial information about the amplitudes even in the absence of a complete set of polarization experiments.

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References

1. G.R. Goldstein and M.J. Moravcsik, *Ann. of Phys. (N.Y.)* 98, 128 (1976), and G.R. Goldstein and M.J. Moravcsik, *Ann. of Phys. (N.Y.)* 126, 176 (1980).
2. G.R. Goldstein and M.J. Moravcsik, "Symmetry Constraints in Optimal Polarization Formalisms with an Application to p-p Scattering", (to be published).
3. R.C. Fernow *et al.*, *Phys. Lett.* 52B, 243 (1974); see p. 246 of the text.
4. I.P. Auer *et al.*, *Phys. Rev. Lett.* 37, 1727 (1976).
5. T.A. Mulera, *AIP Conf. Proc.* 35, 83 (1976); see pp. 88 and 90.
6. I.P. Auer *et al.*, *Phys. Lett.* 70B, 475 (1977); see figure and footnote on p. 478.
7. D. Miller *et al.*, *Phys. Rev.* D16, 2016 (1977).
8. A. Beretvas *et al.*, *Phys. Rev.* D20, 21 (1979); see Table IV.
9. M.J. Moravcsik, *Phys. Rev.* D22, 135 (1980).
10. P.W. Johnson, R.C. Miller and G.H. Thomas, *Phys. Rev.* D15, 1895 (1977).
11. The use of transverse amplitudes for p-p scattering has been advocated for some time by A. Erisch, see for example his "Spin Effects in p-p Scattering", *Rencontre des Morand* (J. Tran Thanh Van, Ed.), 1974, pp. 103-112. While the main motivation was that experiments in which particles are transversely polarized are technologically easier, there was also some support in terms of a simple model of the collision of two balls spinning around axes perpendicular to the collision plane. Our procedure gives some additional support for the use of this set of amplitudes by pointing out that they are optimal.

Table I. Experimental data on p-p elastic scattering at 6 GeV/c and $|t| = 0.6 \text{ GeV}^2/c^2$ used in the present analysis.

Observable	Value	Reference
P	0.092 ± 0.019	7
C_{NN}	0.060 ± 0.025	7
D_{NN}	0.85 ± 0.03	5
K_{NN}	0.13 ± 0.03	3
C_{LL}	-0.010 ± 0.012	6
C_{SS}	-0.10 ± 0.02	4
C_{LS}	0.00 ± 0.01	6
K_{SS}	0.53 ± 0.20	8
H_{SNS}	0.33 ± 0.24	8
σ	1.00	By Definition

Table II. The observables listed in Table I in terms of transversity and helicity amplitudes. The definitions of the amplitudes are: (1-2, 9)

$$\alpha \equiv (+, +; +, +) \quad \beta \equiv (-, -; -, -) \quad \gamma \equiv (+, +; -, -) \quad \delta \equiv (+, -; +, -) \quad \epsilon \equiv (+, -; -, +)$$

$$a \equiv (+, +; +, +) \quad b \equiv (+, +; +, -) \quad c \equiv (+, +; -, -) \quad d \equiv (+, -; +, -) \quad e \equiv (+, -; -, +)$$

and in $(\lambda, \ell; \Lambda, L)$ the indices λ, ℓ, Λ , and L denote the spin projection of C, A, D, and B ($A + B + C + D$) respectively, along the quantization axes of the particular formalism.

Helicity	Obs.	Transversity
$-2 \text{ Im } \{(a + d + e - c)b^*\}$	P	$ \alpha ^2 - \beta ^2$
$+2 \text{ Re } (ad^* + 2 b ^2 - ec^*)$	C_{NN}	$ \alpha ^2 + \beta ^2 + 2[- \gamma ^2 + \delta ^2 - \epsilon ^2]$
$+2 \text{ Re } (ac^* + 2 b ^2 - ed^*)$	D_{NN}	$ \alpha ^2 + \beta ^2 + 2[\gamma ^2 - \delta ^2 - \epsilon ^2]$
$-2 \text{ Re } (ae^* - 2 b ^2 - cd^*)$	K_{NN}	$ \alpha ^2 + \beta ^2 + 2[- \gamma ^2 - \delta ^2 + \epsilon ^2]$
$-\text{Re } \{ a ^2 - c ^2 - e ^2 + d ^2\}$	C_{LL}	$2\text{Re } (\alpha + \beta)\delta^* + 4\text{Re } \epsilon\gamma^*$
$+2 \text{ Re } (ad^* - ec^*)$	C_{SS}	$-2\text{Re } (\alpha + \beta)\delta^* + 4\text{Re } \epsilon\gamma^*$
$-2 \text{ Re } \{(a + d + e - c)b^*\}$	C_{LS}	$2\text{Im } (\alpha - \beta)\delta^*$
$-2 \text{ Re } (ae^* - cd^*)$	K_{SS}	$2\text{Re } (\alpha + \beta)\epsilon^* - 4\text{Re } \gamma\delta^*$
$2 \text{ Im } \{(-a + c + d + e)b^*\}$	H_{SNS}	$2\text{Re } (\alpha - \beta)\epsilon^*$
$2(a ^2 + c ^2 - d ^2 - e ^2 + 4 b ^2)$	$\sigma \equiv 1$	$ \alpha ^2 + \beta ^2 + 2[\gamma ^2 + \delta ^2 + \epsilon ^2]$

Figure Captions

Fig. 1. The transversity amplitudes determined from the data listed in Table I. The amplitudes are normalized to give unity for the cross section. The arcs represent uncertainties in the phases. The uncertainties on the magnitudes are very small in this formalism. The dashed lines represent a second solution which is equally acceptable but is dropped hereafter for simplicity.

Fig. 2. The helicity amplitudes determined from the transversity amplitudes given in Fig. 1. The shaded areas represent uncertainties. The ellipses give the loci of the amplitudes as the orientation angle β of the quantization axis is allowed to vary from 0° to 180° while keeping the quantization axis in the reaction plane.

Figure 2

Transversity Amplitudes

$$p_{\text{LAB}} = 6 \text{ GeV}/c$$

$$t = -0.6 \text{ GeV}^2/c^2$$

Figure 1