PROPERTIES OF ION TEMPERATURE GRADIENT DRIFT INSTABILITIES IN H-MODE PLASMAS

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ABSTRACT

Experimental results from tokamaks such as DIII-D and JET have indicated that the electron density profile in H-mode ("high-confinement") discharges can be nearly flat over most of the plasma, and, in some cases, even inverted (outwardly peaked). These conditions have very interesting implications for pictures of anomalous thermal transport based on the presence of ion temperature gradient (η_i) drift instabilities. The present paper includes (i) a new derivation of the ion temperature gradient threshold for weak density gradient η_i modes when ion transit resonances are taken into account; and (ii) the first derivation of threshold conditions for the onset of η_i -modes when η_i is negative along with the properties of these negative η_i instabilities when the thresholds are exceeded. Possible consequences for confinement in H-mode plasmas are discussed.

MASTER

I. INTRODUCTION

One of the most important problems currently encountered in tokamak research involves understanding the physics responsible for the favorable confinement properties observed in so-called H-mode discharges. For example, design studies for future ignition experiments such as CIT (Compact Ignition Torus) have indicated that acceptable performance levels in such auxiliary-heated divertor plasmas will require maintaining H-mode rather than the more common L-mode-type ("low" mode) energy confinement trends. Although the energy confinement time in H-mode plasmas is typically observed to be about a factor of 2 better than L-mode cases, the thermal transport nevertheless remains very anomalously large. Hence, confinement pictures based on the presence of drift-type microinstabilities such as ion temperature gradient (η_i) modes continue to be plausible.

Previous theoretical and experimental studies have supported the proposition that η_i -type drift instabilities¹⁻³ can account in large part for the observed non-neoclassical features of ion thermal losses in high density ohmic discharges as well as auxiliary-heated L-mode plasmas.⁴⁻⁹ The improved confinement in pellet-fueled high density ohmic plasmas in the ALCATOR C, TFTR, ASDEX, and TEXT tokamaks has been shown to be consistent with results from transport codes which include models for both trapped-electron and η_i -type drift instabilities.^{4-7,9} More direct supporting physical evidence has come from fluctuation measurements carried out on the TEXT tokamak. Results have indicated (i) the appearance of an ion feature in the spectrum of the high density, gas-fueled cases where the η_i -modes are predicted to be present and (ii) the absence of such a feature in the improved confinement pellet-fueled cases where the η_i ($\equiv d \ln T_i / d \ln n_i$) parameter is inferred to fall below the stability threshold (i.e., $\eta_i < \eta_{ic}$).⁹ Although uncertainties in the profile measurements preclude definitive comparisons with specific threshold conditions, the qualitative trends noted in these studies^{8,9} are indeed very suggestive that the η_i -modes could be present.

With regard to H-mode plasmas, experimental results from tokamaks such as DIII-D¹⁰ and JET¹¹ have characteristic electron density profiles which can be nearly flat over most of the discharge and can even be outwardly peaked in some instances. For the weak density gradient (very large η_i) limit, it has been previously pointed out that a relatively weak form of toroidal η_i insta-

bilities could be responsible for the improved but still strongly anomalous thermal losses observed.¹² Onset of instability in these cases is *independent* of the density gradient scale length (L_n) and requires only a sufficiently steep ion temperature gradient.¹²⁻¹⁶ In the present paper attention is focused on situations where η_i is negative. This scenario ($\eta_i < 0$) is particularly relevant to H-mode plasmas because electron density profiles on DIII-D, for example, have been observed to be relatively flat, while the corresponding Z_{eff} profiles appear to be centrally peaked.¹⁷ Moreover, H-mode discharges with outwardly peaked electron density profiles have been measured in both the DIII-D¹⁰ and the JET¹¹ tokamaks. Motivated by these considerations, we have proceeded to derive for the first time the onset conditions for negative η_i modes. The properties of these instabilities are quite different from those of ion temperature gradient modes either with $\eta_i > 0$ or with flat ion density gradients ($\eta_i \rightarrow \infty$). Specifically, since $|\gamma/\omega_r| \ll 1$ (with γ and ω_r being the growth rate and real frequency) for these instabilities, even away from the threshold, it can now be argued that a weak turbulence analysis of the resultant confinement characteristics is reasonable.

The remainder of this paper is organized as follows: In Sec. II the threshold conditions and basic characteristics of η_i -modes are derived in a sheared slab geometry with ion transit resonances included for normal ($\eta_i > 0$), flat ($\eta_i \rightarrow \infty$), and inverted ($\eta_i < 0$) density profiles. The features of the negative η_i modes are compared with those of the familiar positive η_i instabilities, and both analytic and numerical results are presented here. The possible role of these instabilities in H-mode plasmas is discussed in Sec. III, and a brief summary of the main conclusions of the paper are given in Sec. IV.

II. ANALYSIS OF LINEAR PROPERTIES OF η_i -INSTABILITIES

The instability threshold conditions for ion temperature gradient drift modes are derived in this section. We consider a sheared slab geometry and construct a theoretical model consisting of the Boltzmann response for electrons and a collisionless gyrokinetic ion response which includes wave-particle transit resonances. This corresponds to the frequency ordering, $k_{\parallel} v_{te} \gg \omega \sim k_{\parallel} v_{ti}$ with $k_{\parallel}$ being the parallel wave number and v_{ti} being the thermal velocity of species j . Long to moderate perpendicular wavelength

modes ($k_{\perp}^2 \rho_i^2 < 1$), which would most strongly influence thermal transport, are considered. In particular, the long wavelength modes [$k_y^2 \rho_i^2 \lesssim (L_T/R)^{1/2}$] are predominantly sound-wave-like in nature and are well extended along the magnetic field line. Basically, the mode structure is "flute-like" without significant 'ballooning' to the outside unfavorable curvature region. Hence, toroidal curvature modifications to the sheared slab results here are relatively minor.^{13,16}

The well-known eigenmode equation¹⁹ is obtained by imposing the quasineutrality condition on the electron ion density responses; i.e.,

$$\frac{d^2}{dx^2} \phi + Q(x) \phi = 0, \quad (1a)$$

with

$$Q(x) = -b_s + \tau \frac{\tau + 1 + (\tau + \frac{1}{\Omega} - \frac{\eta_i}{2\Omega}) Z_i \xi_i + \frac{\eta_i}{\Omega} \xi_i^2 (1 + Z_i \xi_i)}{(\tau + \frac{1}{\Omega} + \frac{\eta_i}{2\Omega}) Z_i \xi_i + \frac{\eta_i}{\Omega} \xi_i^2 (1 + Z_i \xi_i)}, \quad (1b)$$

where $x = (r - r_0)/\rho_s$, $\Omega = \omega/\omega_{ce}$, $\xi_i = \omega/k_{\parallel} v_{ti}$, $\tau = T_e/T_i$, $b_s = k_y^2 \rho_s^2$, and Z_i is the usual plasma dispersion function for ions with argument ξ_i . The second order differential equation, Eq. (1), is valid when $\rho_i^2(k_x^2 + k_y^2) < 1$. In analyzing Eq. (1), the usual procedure is to first consider the fluid ion limit where $|\omega| \gg |k_{\parallel}| v_{ti}$ is satisfied. Using the large-argument (ξ_i) asymptotic form of the Z-function in Eq. (1), we obtain,¹

$$\frac{\partial^2}{\partial x^2} \phi + \left[-b_s + \frac{1 - \Omega}{\Omega + \frac{1 + \eta_i}{\tau}} + \left(\frac{L_n}{L_s} \right)^2 \frac{x^2}{\Omega^2} \right] \phi = 0. \quad (2)$$

This Weber equation then yields the following dispersion relation:

$$(1 + b_s) \Omega^2 + \left(b_s \frac{1 + \eta_i}{\tau} - 1 + i \frac{L_n}{L_s} (2l + 1) \right) \Omega + i \frac{L_n}{L_s} \left(\frac{1 + \eta_i}{\tau} \right) (2l + 1) = 0, \quad (3)$$

where l is the radial eigenmode number. Two different roots of Eq. (3) can be easily identified in the simple limit where $|b_s(1 + \eta_i)/\tau| \ll 1$ and $|(L_n/L_s)(1 + \eta_i)/\tau| \ll 1$; namely,

$$\Omega \approx i \frac{L_n}{L_s} (2l + 1) \left(\frac{1 + \eta_i}{\tau} \right), \quad (4a)$$

and

$$\Omega \approx 1 - i(2l+1) \frac{L_n}{L_s} \left(\frac{1+\eta_i}{\tau} + 1 \right). \quad (4b)$$

For the usual case of $\eta_i > 0$, Eq. (4a) represents the sound wave destabilized by the ion temperature gradient (i.e., the familiar η_i -mode), and Eq. (4b) corresponds to the shear-damped universal electron drift wave. However, when the density profile is inverted such that $\eta_i < 0$ (throughout this paper $L_s, L_T > 0$ is assumed), the situation changes. It is intuitively clear that the η_i -mode should be unstable for negative values of η_i with $|\eta_i| \gg 1$, since the dependence on L_n should be weak.¹² Indeed, in the limit of $L_n \rightarrow \infty$, Eq. (3) becomes,

$$(1+b_s) \left(\frac{\omega}{\omega_{*Ti}} \right)^2 + \left(\frac{b_s}{\tau} + i \frac{L_T}{L_s} (2l+1) \right) \left(\frac{\omega}{\omega_{*Ti}} \right) + i \frac{L_T}{L_s} \left(\frac{2l+1}{\tau} \right) = 0. \quad (5)$$

Specifically, for $b_s/\tau, (L_T/L_s)(2l+1) \ll 1$,

$$\left(\frac{\omega}{\omega_{*Ti}} \right) \approx \pm \left(\frac{2l+1}{\tau} \right)^{1/2} \left(\frac{L_T}{L_s} \right)^{1/2} e^{\pm \frac{\pi}{4}}, \quad (6)$$

so that there exists an unstable mode regardless of the sign for L_n . As the inverted density profile peaks further, the $|\eta_i|$ value decreases. Then, Eqs. (4a) and (4b) become relevant again. Noting that $\Omega = \omega/\omega_{*e} \propto -\omega$, for $L_n < 0$, we see that the root described by Eq. (4a) is stable for $\eta_i < -1$ while another root described by Eq. (4b) is unstable for $\eta_i < -(1+\tau)$. Therefore, for $L_n < 0$, we should identify the root given by Eq. (4b) as an η_i mode based on the physics origin of its thermodynamic driving force, despite the fact that this mode happens to be propagating in the ω_{*e} direction. Of course, Eq. (4) is an approximate solution of Eq. (3). Straightforward inspection of Eq. (3) indicates that marginal stability occurs when $\eta_i = -1$ at $\Omega = 0$, and when $\eta_i = -(1+\tau)$ at $\Omega = 1$. These results imply that the η_i -mode is unstable for $\eta_i > -1$ and $\eta_i < -(1+\tau)$. It is already clear that a threshold at $\eta_i = -1$ should not be accepted since the fluid approximation ($|\omega| \gg |k_{||} v_{ti}|$) breaks down badly at $\Omega = 0$. On the other hand, another threshold at $\eta_i = -(1+\tau)$ which occurs when $\Omega = 1$, does not violate the fluid approximation. When $k_{||}$ is evaluated for typical mode widths given by Eq. (2), $|k_{||} v_{ti}/\omega| \sim |L_n/L_s|^{1/2}$ i.e., the fluid approximation is satisfied if $|L_n| < L_s$.

We next proceed to analyze the kinetic eigenmode, Eq. (1), for the purpose of evaluating the threshold value of η_i more accurately.

a) Normal Density Profile (Positive η_i -Modes): For $0 < L_n/L_s \ll 1$, results from a number of numerical studies^{20,21} have indicated that $1 < \eta_{ic} < 2$. This implies that the fluid description (which leads to $\eta_{ic} = -1$) cannot be used even as the lowest order approximation. Therefore, we must deal with the strong ion Landau damping region where $|\omega| \lesssim |k_{\parallel}|v_{ti}$. If we use the small ξ_i (large x) asymptotic formula for the Z-function, i.e., $Z \cong i\sqrt{\pi}\xi_i e^{-\xi_i^2}$, then Eq. (1b) simplifies to

$$Q_{\infty}(x) = \frac{(\tau - b_s)(\Omega\tau + 1) - (\tau + b_s)\frac{\Omega}{2}}{1 + \frac{\Omega}{2} + \Omega\tau} - \frac{i}{\sqrt{\pi}} \frac{(2\tau)^{1/2}(\tau + 1)}{1 + \frac{\Omega}{2} + \Omega\tau} \left(\frac{L_n}{L_s} \right) x, \quad (7)$$

Note that the second term is small for typical values of the turning point x when $L_n/L_s \ll 1$. Indeed, when $L_n/L_s \ll 1$, the first term of Eq. (7) *alone* determines the stability conditions. If we consider Eq. (1) as a time-independent Schrödinger equation problem with the effective potential $V(x) [\equiv -Q(x)]$, then the sign of $Q_{\infty}(x)$ determines the existence of an integrable solution. Numerical solutions indicate that $|\Omega| \ll 1$ is always satisfied at marginal stability. Hence, from the first term of $Q_{\infty}(x)$, we can see that the potential barrier is formed only when

$$\eta_i > \eta_{ic}^{(+)} = \frac{2(\tau - b_s)}{\tau + b_s}, \quad (8)$$

and there exists an integrable solution (i.e., unstable mode). If $0 < \eta_i < \eta_{ic}^{(+)}$, the eigenfunction becomes unbounded in space; i.e., it is stable.

In fact, Eq. (8) is obviously just the small- b_i -limit of the local kinetic result of Kadomtsev and Pogutse.²² ($\eta_i^{KP} = 2/[1 + 2b_i(1 - I_1/I_0)]$, where I_0 , I_1 are modified Bessel functions. It is indeed expected that we can recover the local kinetic result in the $L_n/L_s \rightarrow 0$ limit.

b) Flat Density Profile (Weak-Density-Gradient η_i -Modes): As L_n is increased, the second term of $Q_{\infty}(x)$ in Eq. (7) becomes larger. Furthermore, Eq. (2) indicates that the potential well at the rational surface,

$V_{eff}(0) = -Q(0) \simeq \frac{\tau}{1+\eta_i}$, becomes shallower. The eigenfunction accordingly extends more into the large x region, thereby supporting the significance of the second term of $Q_\infty(x)$. In this case, we can approximately solve the eigenmode equation by using WKB procedures. The appropriate quantization condition is

$$\int_{-x_T}^{x_T} dx \sqrt{Q(x)} = (l + 1/2)\pi, \quad (9)$$

where l is the radial eigenmode number and x_T is the turning point in the complex plane. Noting that x_T is determined by the balance of the first and second terms of Eq. (7), $x_T \sim \frac{L_n}{L_s} \gg 1$. On the other hand, the ion Landau damping point $x_i \sim \Omega \frac{L_n}{L_s}$ never becomes much greater than one at marginal stability. Hence, the integral can be approximated by ignoring the details of $Q(x)$ for $x < x_i$; i.e.,

$$\int_0^{x_T} dx \sqrt{Q_\infty(x)} = (2l + 1) \frac{\pi}{4}. \quad (10)$$

As mentioned earlier, $|\Omega\tau| \ll 1$ when $0 < \frac{L_n}{L_s} \ll 1$. When l is increased, $|\Omega\tau|$ at marginal stability also increases. Nevertheless, $|\Omega\tau + 1| \ll \eta_i$ is always satisfied. In the flat density limit ($L_n \rightarrow \infty$), Eq. (10) yields

$$\eta_{ic} = \frac{3}{2} \sqrt{\frac{\pi}{2}} \left(1 + \frac{T_i}{T_e}\right) \frac{L_n}{L_s} (2l + 1). \quad (11)$$

The finite Larmor radius corrections here should be ignored in order to be consistent with the ordering $|\Omega\tau + 1| \ll \eta_i$ [i.e., $b_s \eta_i$ in Eq. (7) is also ignored in comparison to η_i]. Equation (11) indicates that the threshold condition for η_i instability is independent of L_n . It also applies to negative values of L_n provided $|\eta_i| \gg 1$ is satisfied. For practical applications to tokamak confinement considerations, such as η_i -mode-based transport models, it is desirable to have an expression for η_{ic} which is valid for arbitrary values of L_n/L_s . In fact, if we can ignore the ' $\Omega\tau$ ' term in Eq. (7), then Eq. (10) yields the following expression:

$$(1 + b_i)^3 (\eta_{ic} - \eta_{ic}^{(\gamma)})^3 = \frac{9\pi}{8} \left(1 + \frac{T_i}{T_e}\right)^2 \left(\frac{L_n}{L_s}\right)^2 (2l + 1)^2 (\eta_{ic} + 2). \quad (12)$$

As mentioned earlier, the finite-gyroradius corrections should be ignored for $L_n/L_s \gtrsim 1$, in order to be consistent with the ordering $|\Omega\tau + 1| \ll \eta_i$. This can be done by replacing $(1 + b_i)^3$ with 1, since the details of $\eta_{ic}^{(+)}$ do not affect the answer in the weak density gradient limit ($L_n/L_s \gtrsim 1$).

The resultant form of the algebraic equation, Eq. (12) (with $1 + b_i$ replaced by 1), can be readily solved exactly to yield:

$$\eta_{ic} = \eta_{ic}^{(+)} + \sqrt[3]{b + \sqrt{b^2 - a^3}} + \sqrt[3]{b - \sqrt{b^2 - a^3}}, \quad (13a)$$

where

$$\begin{aligned} \eta_{ic}^{(+)} &= 2(1 - b_i)/(1 + b_i) \\ a &= \frac{3\pi}{8} \left(1 + \frac{T_i}{T_e}\right)^2 \left(\frac{L_n}{L_s}\right)^2 (2l + 1)^2 \\ b &= 3 \left(1 + \frac{1}{2} \eta_{ic}^{(+)}\right) a. \end{aligned}$$

When $0 < L_n/L_s \ll 1$, $b^2 \gg a^3$, and the second term on the right side of Eq. (13) provides the nonlocal correction to $\eta_{ic}^{(+)}$. The third term on the right side of Eq. (13) is negligible in this case. When L_n/L_s becomes large enough so that $b^2 < a^3$, the second and third terms of Eq. (13) become complex. However, since they are complex-conjugates to each other, η_{ic} remains a real value. In this case, the following equivalent formula minimizes the possibility of confusion in choosing the right complex root:

$$\eta_{ic} = \eta_{ic}^{(+)} + \sqrt{\frac{3\pi}{2}} \left(1 + \frac{T_i}{T_e}\right) \frac{L_n}{L_s} (2l + 1) \cos \left[\frac{1}{3} \cos^{-1} \left(\frac{b}{a^{3/2}} \right) \right]. \quad (13b)$$

If $L_n/L_s \gg 1$, the a^3 term dominates the right side of Eq. (13), and we thereby recover Eq. (11).

Before proceeding to the analysis of negative η_i modes, it is appropriate here to compare our threshold result for the flat density gradient limit, Eq. (11), with recent estimates¹³⁻¹⁶ for the onset of flat density gradient toroidal η_i instabilities. In terms of the critical ion temperature gradient scale length (L_{Ti}), Eq. (11) can be rewritten as

$$(\epsilon_{L_{Ti}})_c = \frac{2}{3} \sqrt{\frac{2}{\pi}} \frac{L_s}{R} / \left(1 + \frac{T_i}{T_e}\right). \quad (14)$$

With $\epsilon_{L_T} \equiv L_T/R$, $L_s = Rq/\hat{s}$, and $\hat{s} = rq'/q$ Romanelli has reported that when ion ∇B -drift and curvature drift resonant effects are taken into account, the threshold for toroidal η_i -modes in the weak density gradient limit can be approximated by

$$(\epsilon_{L_T})_c = \frac{3}{4} \left(1 + \frac{T_i}{T_e} \right). \quad (15)$$

The absence of shear (L_s) dependence in Eq. (15) is due to the fact that the ion transit motion is neglected. Comparing the sheared-slab and toroidal results one can see that for values of L_s/R around 1.4, Eqs. (14) and (15) are in agreement. If $T_i = T_e$, then Eqs. (14) and (15) yield $(\epsilon_{L_T})_c \simeq 0.375$. This value, in turn, is consistent with more recent toroidal studies^{15,16} which have indicated the flat gradient threshold to be $(\epsilon_{L_T})_c \simeq 0.35$ under similar conditions. Hence, in the limit of weak density gradients, the present sheared-slab result Eq. (14) and the approximate toroidal kinetic results^{15,16} provide similar estimates for the critical L_T .

c) Inverted Density Profile (Negative η_i -Modes): As discussed in the beginning of this section the threshold for inverted density profile is

$$\eta_{ic}^{(-)} = - \left(1 + \frac{T_e}{T_i} \right). \quad (16)$$

This equation has been derived from the dispersion relation, Eq. (15), and agrees very well with the numerical results obtained from the WKB and shooting code analyses of the kinetic eigenmode equation, Eq. (1). Figure 1 illustrates the excellent agreement between Eq. (16) and the numerical results for various values of T_e/T_i . Finite Larmor radius corrections to Eq. (16) are negligible for the long wavelength modes considered in this paper ($b_i \lesssim 0.25$). A particularly interesting new feature for the inverted density profile cases is that $\gamma \ll |\omega_r|$ is satisfied for η_i modes over a wide range of negative η_i values. This is demonstrated in Fig. 2 where the complex Ω values obtained from the numerical analysis are plotted as functions of η_i for fixed values of $L_n/L_s = -1$, $b_i = 0.1$, and $\tau = 1$. This differs considerably from the positive η_i case where $\gamma \ll |\omega_r|$ is satisfied only near the threshold, $\eta_{ic}^{(+)}$. The comparison between negative and positive η_i cases is made in Figs. 3a and 3b where η_i is varied for fixed $L_T/L_S = 0.05$, $b_i = 0.1$, and $\tau = 1$. With

regard to the weak density gradient limit, Fig. 3c shows that, as expected, the complex eigenfrequencies are almost symmetric with respect to the sign of L_n , when $|\eta_i| \gg 1$ is satisfied.

The threshold conditions for various values of profile scale lengths (L_T , L_n , and L_s) are numerically calculated and compared to the analytic formulas, Eqs. (8), (11), and (16). As illustrated in Fig. 4a, a very good agreement is found with the exception of cases where $L_n/L_s \simeq -1$. This is not surprising because neither the fluid approximation used for the negative η_i analysis nor the approximate kinetic approach employed in the flat density profile calculations is justified in this limit. Note that the change in mode character is rather abrupt as a function of L_n/L_s in this region. This transition region is shown in detail in Fig. 4b. Additional numerical results for normal and flat density profiles and their comparisons with the theoretical formulas, Eqs. (8) and (13), are presented in Figs. 5, 6, and 7. These include both poloidal and radial wave number dependences as well as the temperature ratio dependencies. The implications of these results for tokamak confinement physics are discussed in the next section.

III. ROLE OF η_i -INSTABILITIES IN H-MODE PLASMAS

In this section, we discuss the results presented in Sec. II and their relevance to tokamak experiments. We begin with the positive η_i cases for which Eq. (13) is applicable. Specifically, for a sharply peaked density profile and low magnetic shear, $L_n/L_s \ll 1$, so that $\eta_{ic} \cong 2(1 - b_i)/(1 + b_i)$. This indicates the shorter wavelength modes can be easily excited. This property is also illustrated in Fig. 5. However, for the associated larger values of b_i (i.e., $b_i \sim 1$) $\rho_i^2 k_z^2$ also tends to be large. Hence, an integral equation formulation is necessary to analyze the stability of these modes in detail.^{23,24}

When L_n/L_s becomes non-negligible either due to flatter density profiles or to stronger magnetic shear, we must consider the full expression for η_{ic} given by Eq. (13). In this case, η_i modes become more difficult to excite if T_i/T_e is raised. This dependence is rather strong and may suggest a reconciliation of high T_i and the relatively benign fluctuations and transport observed in the PLT neutral-beam-heated plasmas.²⁵ This possible link between a τ -dependent η_i -threshold and the PLT result was first suggested by S. Guo *et*

*et al.*²⁶ In Ref. 26 the favorable T_i/T_e dependence of η_{ic} was predicted only for the relatively short wavelength modes where $b_i \gtrsim \tau/2$. Hence, the differential equation used²⁶ is only marginally valid. On the other hand, our present result [Eq. (13)] indicates that the favorable T_i/T_e dependence persists even for long wavelength modes ($b_i \ll 1$) when values of L_n/L_s are finite. Figure 6 illustrates the strong T_i/T_e dependence for $L_n/L_s = 1$ and $L_n/L_s = 0.1$. This favorable dependence of the stability threshold on T_i/T_e should persist even in the hot ion mode of TFTR operation. In the TFTR cases, however, the non-Maxwellian ion beam contribution can be significant.²⁷ Hence, Eq. (13), which was derived assuming a Maxwellian distribution for the ions should only be regarded as being indicative of a quantitative physics trend for these situations. Examination of Eq. (13) also indicates that the higher radial eigenmodes should have higher instability thresholds. The difference in the threshold values is small when $(1 + T_i/T_e)L_n/L_s \ll 1$. However, when $(1 + T_i/T_e)L_n/L_s$ becomes non-negligible, the difference becomes larger and indicates that these higher radial eigenmodes are harder to excite linearly. These facts are illustrated in Fig. 7. The linear stability properties of such higher radial eigenmodes suggest that they can become significant²⁸ in cases where the shear is relatively weak and the density profile is rather peaked. As mentioned in Sec. I, recent measurements in DIII-D H-mode plasmas have indicated that the characteristic electron density profiles can be nearly flat and sometimes even inverted (outwardly peaked).^{10,17} Under these circumstances either Eq. (11) for the flat density profile cases or Eq. (16) for the negative η_i situation should be applicable. However, it is difficult to derive a formula which covers both types of conditions. This is because Eq. (16) has been derived from the fluid eigenmode equation where $|\omega| \gg |k_{||}|v_{ti}$ is assumed, while the asymptotic behavior in the region where $|\omega| \ll |k_{||}|v_{ti}$ plays a crucial role in the derivation of Eq. (11). It is worthwhile to remark that in contrast to Eq. (11), Eq. (16) predicts a higher threshold ($|\eta_{ic}^{(-)}|$) for larger T_e/T_i when the density profile is inverted. Also, to the dominant order shown in Eq. (11), the threshold is independent of poloidal and radial wavelengths. This fact has been confirmed from numerical studies using a WKB code and a shooting code. We also note that for flat density profiles the poloidal wave number dependence obtained from our numerical studies is weak.

Rough estimates based on typical scale lengths of profiles obtained from

DIII-D H-mode discharges^{16,29} suggest values of η_i which exceed the thresholds given by Eq. (11) and Eq. (16). For example, using the $T_i(r)$ and $n_i(r)$ profiles for $0.4 \leq r/a \leq 0.5$ from Ref. 29, we obtain $L_{Ti}/R \cong 0.19$, $L_{Ni}/R \cong -1.18$ which yield $\eta_i = -6.21$. As mentioned in the previous section, the negative η_i modes which are likely to be excited in various H-mode discharges are characterized by $\gamma \ll |\omega_R|$. This suggests that a weak turbulence theory of these instabilities is needed to gain further insight into their influence on ion thermal transport. For the weak density gradient limit, the $T_i(r)$ profiles should stay reasonably close to the marginal stability condition in order to be consistent with the observed confinement properties.⁴ This again suggests that a weak-turbulence-type nonlinear analysis of the associated transport coefficient is appropriate. Investigations in this direction for the negative η_i situations are in progress, and the results will soon be presented in another paper. An encouraging feature of this analysis is that it is possible to treat the 'test' mode using fluid equations even for values of η_i near η_{ic} . This makes feasible a nonlinear nonlocal analysis, where the actual radial structure of the eigenmodes is properly taken into account. With regard to other weak-turbulence-type studies, we note that a local kinetic analysis of positive η_i -modes near threshold has recently been presented by Mattor and Diamond.³⁰ Regarding the radial mode structure, the crucial difference between positive η_i mode and negative η_i mode is that the kinetic analysis is essential for the former case while the fluid approximation is valid for the latter. This contrast is demonstrated in Fig. 8.

A preliminary iterative evaluation of the fluctuation spectrum for marginally unstable negative η_i -modes has indicated that the weak turbulence form of the ion diffusivity is considerably smaller than the familiar mixing length estimate.

IV. SUMMARY

Motivated by experimental results for the profile characteristics associated with H-mode plasmas, we have carried out a systematic analytical and numerical study of the linear properties of η_i -instabilities in a sheared slab geometry. Ion transit resonance effects, which can strongly influence the stability thresholds, have been taken into account in our investigation of normal ($\eta_i > 0$), flat ($\eta_i \rightarrow \infty$), and inverted ($\eta_i < 0$) ion density profile sce-

narios. The corresponding analytic threshold criteria given, respectively, by Eqs. (8), (11), and (16) are well supported by numerical results from WKB and Shooting Code programs. With regard to the weak-density-gradient η_i -modes, Eq. (11) is a new result for the ion temperature gradient threshold when ion transit resonances are included. As noted in the discussions following Eqs. (14) and (15), the present criterion from our sheared-slab analysis compares reasonably well with estimates for the critical L_{Ti} from recent investigations of weak density gradient *toroidal* η_i -modes.

Equation (16) and the associated numerical results presented in Sec. II represent the first derivation of threshold conditions for negative η_i -modes along with the properties of these instabilities when the thresholds are exceeded. Of particular significance is the fact that the growth rate is much smaller than the real frequency, even far away from the threshold for these negative η_i -modes. This trend tends to hold true even when toroidal modifications such as trapped particle dynamics are introduced. For example, in earlier work on trapped-particle drift modes by Tang *et al.*³¹ it was shown that the negative η_i form of trapped-ion modes was clearly weaker than its positive η_i counterpart. Moreover, the property $|\gamma/\omega_R| \ll 1$, derived in the present work, is also characteristic of the negative η_i trapped-ion instabilities.

In general, the feature, $|\gamma/\omega_R| \ll 1$, helps to justify the application of the standard weak turbulence procedure³² for calculating the transport associated with the presence of negative η_i modes. The primary physics elements of this nonlinear analysis and their implications for H-mode confinement have been qualitatively discussed in Sec. III. In addition, it should be noted that the stabilizing influence of finite beta effects²⁴ on the shorter wavelength η_i modes can significantly influence the nonlinear results. Specific results for transport along with quantitative details of the analysis will soon be presented elsewhere. The major intent of the present paper has been to establish clearly the linear properties of η_i -modes with special attention focused on the negative ion density gradient scenario common to many steady-state H-mode plasmas.^{11,17}

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Figures

FIG. 1. Plot of $\eta_{ic}^{(-)}$ versus T_e/T_i for $b_i = 0.1$, $L_n/L_s = -0.1$. The solid line and dots correspond, respectively, to the analytic Eq. (16) and to the numerical results.

FIG. 2. Plot of numerically computed Ω versus η_i for $L_n/L_s = -1$, $b_i = 0.1$, and $\tau = 1$. The solid and dashed lines correspond, respectively, to $Im(-\Omega)$ and $Re(-\Omega)$. Note that $\Omega = \omega/\omega_{se} \propto -\omega$, since $L_n < 0$.

FIG. 3. Plot of numerically computed Ω versus η_i for $L_T/L_S = 0.05$, $b_i = 0.1$, and $\tau = 1$. The black and white dots correspond, respectively, to the imaginary and to the real part of the eigenvalues. Figure 3a deals with negative η_i -modes, Fig. 3b deals with the familiar positive η_i -modes, and Fig. 3c deals with the weak density gradient limit of both classes of η_i -instabilities.

FIG. 4. Stability diagram in L_s/L_n versus L_s/L_T space, for $b_i = 0.1$, $\tau = 1$. Dots correspond to numerical results. The lines a, b, and c correspond to analytic results obtained from Eq. (8), Eq. (11), and Eq. (16). Figure 4b is a blowup of Fig. 4a for the weak density gradient regime.

FIG. 5. Plot of η_{ic} versus b_i for $L_n/L_s = 0.1$, $\tau = 1$. The dots and the line correspond, respectively, to numerical and analytical Eq. (8) results.

FIG. 6. Plot of η_{ic} versus T_i/T_e for $b_i = 0.1$. The lower and upper curves correspond, respectively, to the numerical results for $L_n/L_s = 0.1$ and $L_n/L_s = 1$.

FIG. 7. Plot of η_{ic} versus L_n/L_s for $b_i = 0.1$, $\tau = 1$. The black dots are numerical results for the fundamental ($l = 0$) modes. The solid lines correspond to the analytical results (Eq. 13), for $l = 0, 1$, and 2.

FIG. 8. Plot of radial eigenmode structures for $L_T/L_s = 0.05$, $b_i = 0.1$, $\tau = 1$ with the solid and dashed lines denoting the real and imaginary parts of $\varphi(x)$. Ion Landau damping points x_i where $|\omega| = |k_{||}|v_{ti}$ are indicated. Figure 8a corresponds to $\eta_i = 4$ and shows that since $\varphi(x)$ extends considerably beyond x_i , a kinetic analysis is necessary. Figure 8b

corresponds to $\eta_i \approx -4$ and shows that since $\varphi(x)$ is localized within x_i , the fluid approach for such negative η_i cases is justified.

FIG. 1

FIG. 2

FIG. 3(a)

FIG. 3(b)

FIG. 3(c)

FIG. 4(a)

FIG. 4(b)

FIG. 5

FIG. 6

FIG. 7

FIG. 8(a)

FIG. 8(b)