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BOOTSTRAP CURRENTS IN RADIO-FREQUENCY-DRIVEN TOKAMAK EQUILIBRIA

by

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Fusion Power Program

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ABSTRACT

Interest in the bootstrap current arising from neoclassical transport in tokamaks has increased recently in view of certain experimental observations. In this study, the bootstrap current is calculated for a number of RF current-driven tokamaks. Two-dimensional, self-consistent, steady-state tokamak MHD equilibria are obtained by including both the transport-driven bootstrap current and the externally driven RF current. The self-consistency is accomplished by iterating between the two-dimensional MHD equilibrium calculation and the current calculation (including bootstrap and RF ray-tracing). A useful parameter, $\Gamma = 1 - (\gamma/\gamma^B)$, is defined as a measure of the RF current drive power reduction achieved when bootstrap currents are included in the calculation. Here, $\gamma = \bar{n}_e R_0 I / P_{CD}$ is the current drive efficiency without the benefit of bootstrap current and $\gamma^B = \bar{n}_e R_0 I / P_{CD}^B$ is the higher efficiency obtained when bootstrap effects are included. For INTOR-type reactor parameters with $\bar{n}_e = 0.7 - 1.4 \times 10^{20}/m^3$, $\Gamma = 0.4 - 0.9$ (40%-90% reduction of RF power) is obtained. Calculations for other reactor parameters of interest are also carried out. It is found that for reactor-grade plasmas, the bootstrap current contribution to the toroidal current is, in general, important. An approximate scaling law for Γ , based on the parametric survey performed, is also obtained.

I. INTRODUCTION

Based on engineering and economic considerations, steady-state operation of tokamak reactors seems desirable.[1-3] Many advantages of steady-state operation have been identified, for example, in reducing thermal fatigue, mechanical fatigue, plasma disruption and reactor down time. Due to extensive theoretical and experimental studies, we are gaining confidence that we understand the physics of noninductive current drive, and this encourages us to more seriously weigh the relative costs and benefits of current drive for tokamak reactors.

To achieve steady-state operation, an important issue is how to sustain the required current. Various methods of current drive have been proposed. For example, radio-frequency (RF) current drive [3-5] and neutral beam injection [6] have been studied for this purpose. In particular, it has been shown that, using multiple RF waves, it is possible to generate the current required for steady-state tokamak reactors in the first stability regime.[5] However, the required RF power appears to be a significant fraction of the fusion power. For example, the normalized current drive efficiency for a wide range of cases surveyed in Reference 5 is close to the minimum required value [1] of $\gamma=0.4$. To improve this and make a reactor more attractive, it is desirable to reduce the required RF power for a given tokamak equilibrium. Inspired by the recent experimental observations,[7] the bootstrap current produced by neoclassical transport seems to be a good candidate to accomplish this improvement.

In the work reported here, we extend the previous study [5] to investigate this possibility. For self-consistent steady-state two-dimensional (first-stability regime) equilibria, the bootstrap current and

its contribution to RF current drive efficiency are calculated quantitatively for typical tokamak reactor parameters.

II. MODEL FOR THE CALCULATIONS

It may be helpful to summarize the problem studied here first. Considering an axisymmetric tokamak equilibrium, we want to calculate the bootstrap current associated with the neoclassical transport in this equilibrium. Furthermore, we want to ensure that the total (bootstrap + RF) current needed for maintaining the steady-state equilibrium is self-consistent with the equilibrium. In other words, the objective of the calculation is to establish a steady-state two-dimensional tokamak equilibrium that is maintained by the internally transport-driven bootstrap current and the externally driven RF current which together are consistent with the equilibrium maintained.

The method for calculating self-consistent MHD equilibria in RF current-driven tokamaks, in the absence of an emf and the bootstrap current, has been discussed in detail elsewhere.[4-5] The basic theory for including the bootstrap current in the calculation has also been developed.[8] Here, we summarize the equations of the model used in the present study, as follows. (All units in this paper are SI units except where otherwise noted.)

The Grad-Shafranov equation for the poloidal flux, $\psi(R,Z)$, in cylindrical coordinates (R,ϕ,Z) is given by

$$R^2 \nabla \cdot \left[\frac{\nabla \psi(R,Z)}{R^2} \right] = -4\pi^2 \left[\mu R^2 \frac{dp}{d\psi} + F \frac{dF}{d\psi} \right], \quad (1)$$

Here, $p(\psi)$ and $F(\psi) = RB_t$ are the plasma pressure and diamagnetism, respectively; μ is the free-space permeability; B_t is the toroidal component of the magnetic field $\vec{B}$; and ϕ is the toroidal angle. The equation for $F(\psi)$ is [8]

$$\frac{dF}{d\psi} + \frac{\mu F}{\langle B^2 \rangle} \frac{dp}{d\psi} = - \frac{\mu}{2\pi} \left\{ \frac{\langle j_{\parallel}^{RF} B \rangle}{\langle B^2 \rangle} + \frac{\langle j_{\parallel}^{NC} B \rangle}{\langle B^2 \rangle} \right\} = - \frac{\mu}{2\pi} \{G(\psi) + H(\psi)\}, \quad (2)$$

where $\langle \rangle$ denotes the average over a flux surface, [4,9]

$$G(\psi) \equiv \langle j_{\parallel}^{RF} B \rangle / \langle B^2 \rangle, \quad H(\psi) \equiv \langle j_{\parallel}^{NC} B \rangle / \langle B^2 \rangle,$$

and $j_{\parallel}^{RF}$ and $j_{\parallel}^{NC}$ are the parallel (to the magnetic field) current density due to RF waves and neoclassical transport, respectively. The plasma boundary is parameterized by

$$R = R_0 + (R_0/A) \cos(t + d \sin t) \quad (3)$$

$$Z = \kappa (R_0/A) \sin t, \quad (4)$$

where R_0 = major radius, A = aspect ratio, d = triangularity, κ = elongation, and $0 \leq t < 2\pi$. The RF current density is calculated on each flux surface according to

$$G(\psi) = \sum_m [j_{\parallel} / (P_{RF} B)]_m \langle P_{RF} \rangle_m,$$

summing over multiple modes with a two-dimensional ray-tracing calculation. The function in brackets is determined by the standard RF current drive theory and is given explicitly, for example, in Reference 5; the quantity $\langle P_{RF} \rangle$ is simply the flux surface averaged RF heating power density.

The neoclassical current density is calculated according to Reference 9. In the absence of an electric field ($E_{\parallel}=0$), using $n_e = n_i Z_i$ and converting to flux coordinates, Eq. (6.122) of Reference 9 can be transformed to give

$$H(\psi) = \frac{RB_0 \epsilon^{1/2}}{\langle B^2 \rangle} \left\{ -K_{13} \frac{dp}{d\psi} + \left(\frac{5}{2} K_{13} - K_{23} \right) n_e \frac{dT_e}{d\psi} + K_{13} L n_i \frac{dT_i}{d\psi} \right\}, \quad (5)$$

where

$$K_{m3} = K_{m3}^{(0)} [1 + a_{m3} v_{*e}^{1/2} + b_{m3} v_{*e}]^{-1} [1 + c_{m3} v_{*e} \epsilon^{3/2}]^{-1} \quad (6a)$$

and

$$L = [1 + v_{*i}^2 \epsilon^3]^{-1} [1 + v_{*e}^2 \epsilon^2]^{-1} \left[\frac{1.17 - 0.35 v_{*i}^{1/2}}{1 + 0.7 v_{*i}^{1/2}} - 2.1 v_{*i}^2 \epsilon^3 \right]. \quad (6b)$$

The coefficients in Eq. (6a) for K_{13} and K_{23} are of order unity and are slowly varying functions of the ion charge state; in the code the values are interpolated from tables provided in Reference 9. These expressions, which were derived in the limit of large aspect ratio ($\epsilon \ll 1$) assuming circular flux surfaces, are adapted to the numerically calculated flux surfaces in our code

by generalizing the definitions of terms. Thus, the major radius of a flux surface is the average of the maximum and minimum distances from the symmetry axis,

$$R = R(\psi) = \frac{R_{\max} + R_{\min}}{2} \Big|_{\psi}; \quad (7a)$$

the magnetic field is the median value on a flux surface

$$B_0 = B_0(\psi) = \frac{B_{\max} + B_{\min}}{2} \Big|_{\psi}; \quad (7b)$$

and the magnetic well depth on a flux surface is

$$\epsilon = \epsilon(\psi) = \frac{B_{\max} - B_{\min}}{B_{\max} + B_{\min}} \Big|_{\psi}. \quad (7c)$$

The functions K_{13} , K_{23} , and L are sensitive to electron and ion collisionality, which are given in terms of the connection length:

$$\nu_{*e} = Rq \frac{\tau_e^{-1}}{v_e \epsilon^{3/2}} \quad (7d)$$

and

$$\nu_{*i} = Rq \frac{\tau_i^{-1}}{v_i \epsilon^{3/2}} \quad (7e)$$

where $q(\psi)$ is the safety factor, $v_j = (T_j/m_j)^{1/2}$ is the thermal speed of species j , and τ_j is a collision time [9] for species j . In the banana regime, $v_{*e} \ll 1$, $v_{*i} \ll 1$, a typical trapped particle completes a bounce orbit before suffering a detrapping collision and under such circumstances the bootstrap current is maximized. In the plateau and Pfirsch-Schluter regimes, $v_{*e} \gg 1$, $v_{*i} \gg 1$, this neoclassical current becomes small. We find, for example, the values of K_{13} , K_{23} , and L near the magnetic axis ($\epsilon \approx 0.01$) at $v_{*e} = v_{*i} = 0.1$ have $\sim 75\%$ - 80% of their values at $v_{*e} = v_{*i} = 0.001$, whereas they drop to $\sim 5\%$ - 10% of that magnitude when $v_{*e} = v_{*i} = 10.0$. The value of $H(\psi)$, which depends in detail on the pressure and temperature gradients, drops in approximately the same ratio. Hence, our numerical calculation accounts for the spatial variation of $H(\psi)$ by explicitly evaluating collisionality, well depth, gradients, etc., on each flux surface. We expect our results for $H(\psi)$ to be accurate except for extremely noncircular (e.g., bean-shaped) or low aspect ratio tokamaks. (We note that $O(\epsilon)$ corrections to the neoclassical theory have recently been published [10] but have not been carried out explicitly for the bootstrap term.)

The specific functional form used for $p(\psi)$ is

$$p(\psi) = p_0 \tilde{\psi}^\alpha, \quad (8)$$

where

$$\tilde{\psi} \equiv \frac{\psi - \psi_b}{\psi_a - \psi_b}$$

is the normalized poloidal flux; $\psi_b \equiv \psi$ at the boundary, $\psi_a \equiv \psi$ at the magnetic axis, and α is a constant. The plasma density and temperature profiles of the j -th species are described by

$$n_j(\psi) = n_{j0} \bar{\psi}^{\alpha_n} \quad (9)$$

$$T_j(\psi) = T_{j0} \bar{\psi}^{\alpha_T} \quad (10)$$

with $\alpha_n + \alpha_T = \alpha$ in order to be consistent with Eq. (8).

To carry out the calculations based on the model just described, the RF-current computer code [4,5] has been modified to incorporate the bootstrap current into the calculations. The calculation starts by specifying density and temperature profiles and an initial profile for the diamagnetism $F(\psi)$ consistent with a tokamak equilibrium typical of the first regime of stability. The flux surface geometry is computed from Eq. (1), and knowledge of $\psi(R,Z)$ permits a calculation of macroscopic parameters such as total current, I , volume averaged beta, β , and edge safety factor, q_b . The safety factor, $q(\psi)$, is constrained to be monotonic in ψ with a value at the magnetic axis $q_a \approx 1.1$. The function $H(\psi)$ is then explicitly determined for the initial equilibrium, and the code then employs an optimizing routine to infer the RF power spectrum needed to supply the remaining current density, $G(\psi)$, to reproduce this equilibrium (see Figure 1). Near the magnetic axis, where $H(\psi)$ is small, the RF seed current is generally provided by the fast wave, which should penetrate readily to high density, while the slow (lower hybrid) wave fills in the necessary current density near the surface. At this point, the actual function $G(\psi)$ is calculated for the inferred power spectrum, an updated $F(\psi)$ is found from Eq. (2), and the flux function $\psi(R,Z)$ and the computed $H(\psi)$ and

$G(\psi)$ are iterated. When the converged solution is obtained, the total current from the RF waves and neoclassical transport is self-consistent with the MHD equilibrium. A more detailed description of the code can be found elsewhere. [11]

Convergence is achieved when the parameters I , β , and $q(\psi)$ are numerically stable and vary by less than a few percent with successive iterations. Careful selection of candidate wave modes (discrete values of frequency and index of refraction) allows the solution to converge rapidly, typically in fewer than ten iterations. Care must be taken to be sure that $q(\psi)$ is well behaved. The RF power is invested mainly in driving current near the magnetic axis, where the bootstrap contribution is small (see Figure 1); thus the RF power is sensitive to q_a . In this study all converged equilibria have axial safety factors in the range $q_a = 1.12 \pm 0.14$. Running time is on the order of 5-10 minutes on Cray computers for ten rays in the wave power spectrum.

III. RESULTS OF THE CALCULATIONS

In this report, we basically extend the work of Reference 5, which studied RF current driven tokamak reactors in the first stability regime, by including the self-consistent bootstrap current in the present calculations. For comparison, all of the equilibria in Reference 5 have been recalculated in the present work. Thus, an extensive parameter range has been considered ($3.6 \leq A \leq 8.0$, $1.0 \leq \kappa \leq 2.4$, various density and temperature profiles, etc.). In addition, typical cases of TIBER II (Tokamak Ignition/Burn Experimental Reactor) [12] and JT-60 [13] have also been studied. Here, a note should be made before the results are presented. Ideally, we would like to study exactly the same equilibrium with and without the presence of the bootstrap

current such that the effect of bootstrap current can be singled out. In practice, however, the local bootstrap current density occasionally exceeds the current density of the corresponding equilibrium of Reference 5 over a small region of the plasma. Nevertheless, the excess current density is never more than ten percent. In these exceptional cases (denoted by boxes in the figures), the convergent equilibrium with bootstrap current is a little different from the equilibrium without bootstrap current. For example, I and β may differ by as much as $\sim 10\%$.

For noninductive current drive, an important parameter for reactor design is the normalized current drive efficiency, γ , defined as

$$\gamma = \frac{\bar{n}_e R_0 I}{P_{CD}}, \quad (11)$$

where $\bar{n}_e$ is the volume averaged electron number density in units of 10^{20} m^{-3} , R_0 is the major radius in m, I is the total toroidal current in MA, and P_{CD} is the RF power heating the plasma in MW. As the definition shows, γ is a global quantity characterizing how effectively the RF power is used in driving plasma current in a tokamak.

To quantify the contribution of the bootstrap current to an equilibrium, we define the "figure of merit" for the bootstrap current as

$$\Gamma = \frac{P_{CD} - P_{CD}^B}{P_{CD}} = 1 - \frac{P_{CD}^B}{P_{CD}}, \quad (12)$$

where P_{CD} and P_{CD}^B are the required RF wave powers without and with the bootstrap current, respectively, for the same equilibrium. Namely, Γ is the fraction of savings in RF power due to the presence of bootstrap current. For

nearly identical equilibria (with and without bootstrap current), i.e., approximately the same $\bar{n}_e$, R_0 , I , electron temperature, profiles, etc., as we surveyed in this study, we have

$$\Gamma = 1 - \frac{\gamma}{\gamma^B}, \quad (13)$$

where γ and γ^B are the normalized RF current drive efficiency without and with the bootstrap current, respectively. Whereas γ may depend sensitively on physics and technology (for example, being influenced by how close to the speed of light the current-carrying electrons are traveling), we expect γ^B to have a similar dependence; hence Γ , the fractional power savings, should remain insensitive to RF physics and technology and be a useful figure of merit. Moreover, Γ is a better measure of bootstrap benefits than the ratio of total toroidal bootstrap current, I_B , to total current, I . This is because I_B , the integral of the bootstrap current density, supplies no information regarding the bootstrap current profile. We expect bootstrap current near the magnetic axis to be more valuable than near the low density edge because it is less efficient to generate RF-driven current in the high density interior. Thus, the RF power savings due to the bootstrap effect is a better measure of worth. For all the equilibria presented in this paper, we calculated the efficiency, γ , without the bootstrap contribution; with the bootstrap contribution, we calculated γ^B and thus Γ .

To consolidate our numerical results and summarize with an analytic fit, we study the Γ variation with global parameters. In the limit $I_B \ll I$, we may approximately write

$$\Gamma = 1 - \frac{\gamma}{\gamma_B} = \frac{I_B}{I}. \quad (14)$$

For nearly circular flux surfaces and $A \gg 1$, we have

$$I_B = \int_0^{\psi_b} d\psi \frac{2\pi r}{2\pi R B_p} j_{||}^{NC}. \quad (15)$$

In this limit, the integrand may be expressed in terms of Eq. (5). However, $\epsilon = \epsilon(\psi)$, $B_p = B_p(\psi)$, and K_{13} , K_{23} and L are functions of v_{*e} and v_{*i} , which are functions of ψ , so Eq. (15) cannot be integrated analytically. To proceed to a reasonable fit by global parameters, we carried out the following manipulations: (i) Replaced ϵ and B_p by their values at the boundary, i.e., A^{-1} and B_{pb} , respectively; (ii) Replaced K_{13} , K_{23} and L by their values in the limits of $A \gg 1$ and $v_{*} \ll 1$; (iii) Put in the explicit forms for $p(\psi)$, $T_e(\psi)$ and $T_i(\psi)$; (iv) Made use of $n_e = Z_{eff} n_i$, $\mu I = 2\pi a B_{pb}$, and the Shafranov definition of peak poloidal beta, $\beta_{I0} = 2\mu p_0 / (\mu I / 2\pi a S)^2$, where p_0 is the peak pressure, $a = R_0/A$, and S is the shape factor, $S = \oint dl / 2\pi a \psi_b$. After these manipulations, we obtain an approximate expression for Γ , given by

$$\Gamma = f C A^{-1/2} \beta_{I0}, \quad (16)$$

where

$$C = C_1 - C_2 \frac{\alpha_T}{\alpha} - C_3 \frac{\alpha_T}{\alpha} \frac{n_{eo}}{Z_{eff}} \frac{T_{i0}}{p_0}. \quad (17)$$

The coefficients, C_1 , C_2 , and C_3 are slowly decreasing functions of Z_{eff} which we approximate by their values at $Z_{eff}=1.5$: $C_1 = 2.1$, $C_2 = 1.3$, and $C_3 = 1.2$; and f varies slowly as a complicated function of geometry, profiles, etc. Although Eq. (16) was derived in an unrealistic limit ($A \gg 1$, circular plasma, $I_B \ll I$, etc.), the functional form well describes our numerical results, as we shall see.

The first set of calculations modeled JT-60 [13] with lower hybrid current drive (LHCD), assuming a range of central temperatures, T_{e0} . In all cases $n_{e0} = 0.13 \times 10^{20} \text{ m}^{-3}$, $I = 1.5 \text{ MA}$, $Z_{eff} = 1.5$, and peaked T_e and broad n_e profiles are assumed. As T_{e0} increases, the collisionality drops, and ν^* enters the banana regime. At the lowest temperature ($\sim 3 \text{ keV}$) we find waves at a frequency $f = 2.3 \text{ GHz}$ penetrate and create a centrally peaked current profile. At $T_{e0} \sim 7 \text{ keV}$, the waves do not penetrate to the magnetic axis; either anomalous current density diffusion must occur, or higher frequency (e.g., 3.7 GHz) is needed.

We calculated the LHCD power required to achieve an MHD equilibrium both without and with the bootstrap contribution. Our calculations show that, for the postulated profiles, bootstrap contributions to LHCD on JT-60 are negligible ($I' < 0.01$) with the present value of $T_{e0} = 6 \text{ keV}$. However, if T_{e0} can be raised further, and if the density could become more peaked, perhaps $I' > 0.10$ can be achieved.

Next, we turned our attention to INTOR. We assumed that the fast wave can provide the central seed current density at reactor conditions, and LHCD is only used in the low density region. We studied the benchmark INTOR [14] ($R_0 = 4.9 \text{ m}$, $A = 4.2$, $\kappa = 1.6$, $T_{e0} = 35 \text{ keV}$, $I = 8.0 \text{ MA}$, $B = 5.5 \text{ T}$) with three different densities: $\bar{n}_e = 0.7, 1.0$, and $1.4 \times 10^{20} \text{ m}^{-3}$. Figure 1 shows the converged equilibrium at $\bar{n}_e = 1.0 \times 10^{20} \text{ m}^{-3}$, which has $\beta = 0.053$, $q_a = 1.1$,

$q_b = 2.4$, and $\beta_{I0} = 3.9$. The variation of collisionality (ν^*) and profile gradient terms in $H(\psi)$ with flux surface is shown in Figure 2 for the same equilibrium. Note that $H(\psi)$ is small near the axis and near the boundary. It is noted that the power savings in this case ($\bar{n}_e = 1.0 \times 10^{20} \text{m}^{-3}$), due to the presence of the bootstrap current, is very significant. The equilibrium with bootstrap current requires only $P_{CD}^B = 31 \text{ MW}$, whereas without the bootstrap current we found the necessary RF power is $P_{CD} = 93 \text{ MW}$. Thus, γ^B is three times larger than γ .

Without the bootstrap contribution, P_{CD} increases as $\bar{n}_e$ increases, but with it we find P_{CD}^B actually decreases; see Figure 3. Figure 4 shows the density dependence of the figure of merit, Γ , for bootstrap current for the benchmark INTOR parameters. The linear increase of Γ with poloidal beta, suggested by Eq. (16), is evident. Note that, at full density and temperature, ~ 80-90% of the INTOR current would be supplied by the bootstrap effect; only ~ 10 MW of noninductive current drive is required.

The INTOR calculations assumed similar density and temperature profiles ($\alpha_n = \alpha_T = 0.7$). However, as Eq. (5) shows, the profiles have a strong influence on the bootstrap current. To investigate the effect of various density, temperature and pressure profiles, we first kept α constant and varied α_T/α . Namely, the equilibria have the same pressure profile but different density and temperature profiles. Then we kept α_T/α constant and varied α to see the effect of pressure profile. The figure of merit for bootstrap current, Γ , is plotted in Figures 5 and 6, respectively. It is found that, as shown in Figure 5, a peaked density profile is more effective in driving the bootstrap current than a peaked temperature profile when the pressure profile is unchanged. The upper abscissa in the figure includes the profile parameters in C from Eq. (17) and shows Γ increasing monotonically

with CB_{I0} . Figure 6 shows, as expected, that the more peaked pressure profile produces more bootstrap current when α_T/α is fixed; for the four cases plotted the function C is nearly constant, so I increases with B_{I0} .

To see the Z_{eff} dependence, three values of Z_{eff} were studied and the results, with I normalized to C , are plotted in Figure 7. The figure shows that our replacement of C_1 , C_2 , and C_3 by constants does not accurately reflect the variation with Z_{eff} . In order to approximately duplicate the calculated Z_{eff} variation, we must multiply Eq. (17) by the factor $[1+0.329(1.5-Z_{eff})]$.

Next, we studied the effect of equilibrium geometry on the bootstrap current by varying the major radius, R_0 , the aspect ratio, A , and the elongation factor, κ . Table I shows, for equilibrium parameters held constant, that I has a negligible change as the major radius is varied by a factor of two from the size of experimental tokamaks (e.g., TIBER II, $R_0=3.0m$) up to commercial reactor dimensions (e.g., STARFIRE [15], $R_0=7.0m$). The aspect ratio and elongation series included the seven equilibria of Reference 5 ($3.6 \leq A \leq 8.0$, $1.0 \leq \kappa \leq 2.0$), and the TIBER II equilibrium ($A = 3.6$, $\kappa = 2.4$). Again we found Eq. (16) is a reasonably good characterization of the results.

For the extensive series of MHD equilibria surveyed in this study, most of the data are well fitted by the form of Eq. (16), even though this equation is strictly valid only for low B_{I0} . In Figure 8 we plot I versus $(CA^{-1/2} B_{I0})$, including all the equilibria surveyed with $Z_{eff}=1.5$. Except for the solid points, all of these equilibria have the same broad pressure ($\alpha=1.4$) which might occur in reactors, and all of these points lie on the straight line,

$$I = 0.26 CA^{-1/2} B_{I0}, \quad (18)$$

with no indicated fall-off as Γ approaches unity, its highest, physically meaningful value. The solid points are for the more peaked pressure profiles ($\alpha = 2.0, 2.5, 3.0$), from Figure 6, indicating that the slope of the straight line would vary with α , becoming less as α increases. When $Z_{\text{eff}} \neq 1.5$, we should multiply the right-hand side of Eq. (18) by $[1 + 0.329 (1.5 - Z_{\text{eff}})]$. A variety of definitions of poloidal beta abound in the literature, but our analysis shows that the Shafranov value [16] used here provides a good fit to our numerical results for Γ .

For the equilibria represented by the square points in Figure 8, the bootstrap current density exceeded the total current density of the initial target equilibrium in some places, so that the bootstrap-driven equilibria are slightly different from the target equilibria. Moreover, in order to achieve equilibria for the largest values of β_{I0} , it was, in fact, necessary to use the more peaked profiles shown as solid points. If the pressure profile were kept constant as in Eq. (18), there would appear to be a "beta limit" beyond which the bootstrap current would be too large to be consistent with equilibrium. This limit, if it exists, is hard to define because there are a number of parameters (such as pressure profile) that can be varied to keep the bootstrap current low. Nevertheless, our results show a distinct tendency of the bootstrap current density to increase with poloidal beta, relative to the required current density and to exceed it by several times for very high beta, second-stability regime equilibria (not shown in this paper).

IV. CONCLUSIONS

In this work, we have studied the bootstrap current in tokamak reactors. The bootstrap current contribution has been shown, quantitatively, to be

important; it can provide a significant fraction (up to 80% or more) of total equilibrium plasma current, depending on the parameter regime of operation.

The figure of merit for the bootstrap current, Γ , has been defined to measure quantitatively the bootstrap contribution to the global current drive efficiency. This is an important parameter, especially for reactor design studies, since it reflects the actual power savings in the external current drive power. Based on our survey in the tokamak reactor relevant regime, a convenient simple analytic fit, Eq. (18), which relates Γ to global parameters, has been obtained; this expression may serve as a convenient formula for reactor studies.

The emphasis of this work has been placed on identifying the bootstrap current contribution and its effect on the improvement of possible tokamak reactor designs. The results from this study are very encouraging. For example, from Table I we observe that a reactor the size of STARFIRE might require an exceedingly large RF power input (415 MW) to achieve continuous operation without bootstrap currents, whereas the bootstrap effect would reduce the power fivefold, to a manageable 81 MW. As our calculations have shown, it is possible to affect the bootstrap contribution by varying the various plasma profiles and geometries. This is a concern for reactor design. Thus, Figure 5 shows a drop in Γ as the density profile becomes broad and flat. In order to benefit substantially from bootstrap current generation, it may be desirable to provide peaked densities by central fueling of the plasma. We must also bear in mind the need to provide a seed current near the magnetic axis. While the self-consistent equilibria computed in our study are driven by low-frequency RF waves, we can imagine that the Γ values are also approximately correct for other drivers (ECRH, neutral beams) since, at least

for small bootstrap contributions, I is roughly proportional to the noninductive current replaced by the bootstrap current [Eq. (14)].

Using multiple RF waves, including bootstrap current, we have shown the possibility of maintaining a steady-state tokamak reactor with a relatively low current drive power. The conventional RF current drive efficiency, γ , can be improved up to a factor of ~ 10 . Whereas a previous study [5] of tokamak reactors with self-consistent RF-driven equilibria reported rather dismal power multiplication, $Q = (\text{fusion power})/P_{CD} = 4-6$, our present calculations show that Q is likely to be enhanced by a factor of two to ten, under favorable conditions, by the bootstrap effect. Thus, an efficient tokamak reactor with the advantages of steady-state operation may be more readily achieved, although the experimental demonstration of bootstrap current in tokamaks has not yet been fully achieved. Redoubled experimental effort in this area is highly desired.

Finally, it may be noted that requiring the bootstrap current to be less than the total current for an equilibrium may represent a type of "beta limit." Accurate results at very high poloidal beta will require more study, with careful attention paid to current density and safety factor profiles. Such issues become prominent if equilibrium calculations are pursued into the second regime of stability.

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Figure 1 INTOR with RF seed current and bootstrap current at $\bar{T}_e = 20$ keV and moderate density, $\bar{n}_e = 1.0 \times 10^{20} \text{ m}^{-3}$. Converged equilibrium has $\beta = 0.053$, $I = 8.0$ MA, and requires total RF power of only $P_{CD} = 31$ MW; hence $\beta^B = 1.3$. (a) Contributions to $G(\psi)$ summed over all the launched RF waves (the dashed lines), $G(\psi)$ summed over all the launched RF waves (the chain-dashed line), $H(\psi)$ due to the bootstrap current (the chain-dotted line), the final $\{G(\psi) + H(\psi)\}$ (the dotted line), and the initial or target $\{G(\psi) + H(\psi)\}$ (the solid line). Magnetic axis is at $\tilde{\psi} = 1$ and plasma edge is at $\tilde{\psi} = 0$. (b) Toroidal current density profile along midplane ($Z = 0$ plane). (c) Launched RF wave trajectories. (d) Safety factor profile along midplane.

Figure 2 Collisionality and gradient contributions to $\langle j_{\parallel} B \rangle = \langle B^2 \rangle H(\psi)$ for INTOR illustration of Figure 1: $n_{e0} = 1.74 \times 10^{20} \text{ m}^{-3}$, $T_{e0} = 35 \text{ keV}$, $T_{i0} = 18 \text{ keV}$, $\alpha_n = \alpha_T = 0.7$. The lower curves are $-\sqrt{\epsilon} K_{13} dp/d\psi$, $\sqrt{\epsilon} (\frac{5}{2} K_{13} - K_{23}) n_e dT_e/d\psi$, and $\sqrt{\epsilon} K_{13} L dT_i/d\psi$.

Figure 3 Current drive power for INTOR at $\bar{T}_e = 20$ keV without bootstrap current (upper curve) and with bootstrap current (lower curve), for different densities. Box indicates that $\langle j_{\parallel}^{NC} \rangle$ exceeds locally the target equilibrium value of $\langle j_{\parallel} \rangle$. Plasma has $R_0 = 4.9\text{m}$, $A = 4.2$, $\kappa = 1.6$, $d = 0.5$, $B_0 = 5.5\text{T}$, $\alpha = 1.4$, $\alpha_T/\alpha = 0.5$, $T_{e0} = 35$ keV, $T_{i0} = 18$ keV, and $Z_{eff} = 1.5$. All equilibria have $q_a = 1.1$, $q_b = 2.4$, and $I = 8$ MA.

Figure 4 Figure of merit for bootstrap current, Γ , for INTOR as density and poloidal beta increase. Plasma parameters are given in Figure 3.

Figure 5 Density profile dependence of the figure of merit for bootstrap current, Γ . The power requirement of the RF current drive decreases as density profile becomes peaked when pressure profile ($\alpha = 1.4$) is kept constant. Equilibrium parameters for these cases are $A = 6$, $\kappa = 1.6$, $Z_{eff} = 1.5$, $R_0 = 5.7$ m with $q_a \approx 1.0$, $q_b \approx 2.0$, and $\beta_t = 0.04$.

Figure 6 Pressure profile dependence of the figure of merit for bootstrap current, r , when α_T/α is kept constant ($\alpha_T/\alpha=0.5$). Equilibrium parameters for these cases are $A = 6$, $\kappa = 1.6$, $R_0 = 5.7$ m, $Z_{eff} = 1.5$ with $q_a = 1.0$, $q_b = 2.0-2.9$, and $\beta_t = 0.04$.

Figure 7 Effective ion charge (Z_{eff}) dependence of the figure of merit for bootstrap current, r . Equilibrium parameters are $A = 6$, $\kappa = 1.6$, $R_0 = 5.7$ m, $\alpha = 1.4$, $\alpha_T/\alpha=0.214$ with $q_a = 1.0$, $q_b = 1.9$, and $\beta_t = 0.4$.

Figure 8 Figure of merit for bootstrap current, r , as a function of the composite global parameter, $CA^{1/2} \beta_{I0}$, for all of the equilibria calculated with $Z_{eff} = 1.5$. Solid symbols represent narrower pressure profiles than $\alpha = 1.4$ (respectively, $\alpha = 2.0, 2.5, 3.0$ with increasing $CA^{-1/2} \beta_{I0}$).

Table I. Equilibrium Parameters for Major Radius (R_0) Dependence Study

Bootstrap	R_0	B_0	n_{e0}	$\bar{n}_e$	q_a	q_b	β	β_{I0}	I (MA)	$P_{CD}^{(B)}$ (MW)	$\gamma^{(B)}$	Γ
No	3.0	8.3	5.9	2.8	1.0	2.0	0.0585	3.70	7.4	152	0.41	
Yes	3.0	8.3	5.9	2.8	1.1	2.0	0.0584	3.68	7.4	29	2.2	0.81
No	5.0	7.2	4.5	2.1	1.0	2.0	0.0584	3.66	11.	255	0.45	
Yes	5.0	7.2	4.5	2.1	1.1	2.0	0.0585	3.66	11.	52	2.2	0.79
No	7.0	6.6	3.8	1.8	1.0	2.0	0.0582	3.63	14.	415	0.42	
Yes	7.0	6.6	3.8	1.8	1.1	2.0	0.0582	3.70	14.	81	2.1	0.80

2 Note: All cases have $A=4.5$, $\kappa=1.6$, $d=0.5$, $T_{e0} = 24$ keV, $T_{i0} = 15$ keV, $Z_{eff} = 1.5$, $\alpha_T/\alpha = .214$, and $\alpha = 1.4$; the selected magnetic field (and density) values result in the same neutron wall load, 3.5 MW/m^2 , for all reactors.

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