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ABSTRACT

This report discusses some of the physics issues anticipated in field-reversed mirrors. The effect of current cancellation due to electrons is described. An estimate is made of the required impurity level to maintain a field-reversed configuration. The SUPERLAYER code is used to simulate the high- $\beta$  2XIIIB results, and favorable comparisons require inclusion of quasilinear RF turbulence. Impact of a quadrupole field on field-line closure and resonant transport is discussed. A simple self-consistent model of ion currents is presented. Conditions for stability of field-reversed configurations to  $E \times B$  driven rotations is determined.

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## INTRODUCTION

The principal advantage of a field-reversed-mirror (FRM) configuration is that there is no external toroidal field. Consequently  $\beta$  is high, and the configuration can be made small and simple, particularly if it can be sustained by neutral beams. The problems are numerous. Can the configuration be generated with ion current; that is, do self-generated electron currents cancel? How is the equilibrium sustained, and is it pulsed? Finally, conventional wisdom suggests that in the absence of any toroidal field [1] the system is unstable if the layer is too long (tearing), if the aspect ratio is too small (MHD), or if the Larmor orbit is too small (MHD). In the face of these is a stable configuration feasible? It is the purpose of this paper to examine these questions and to indicate our level of understanding at this time.

In Sec. 1, we describe the equations governing equilibrium, emphasizing the role played by electrons. The consequences of constraints imposed by current cancellation are discussed. Next, a comparison is made between 2XIB and the two-dimensional code called SUPERLAYER. One defect of the code is the restriction to axisymmetry. We next assess the role quadrupole fields play upon injection (the effective stretching of the source), equilibrium (the opening of field lines), and transport (the degrading of confinement through single-particle resonances). This section concludes with an analytic model of an ion layer. Finally, in Sec. II, we address stability, where we consider the influence of finite Larmor radius (FLR) and large aspect ratio on rotation-driven instabilities.

### 1. EQUILIBRIUM AND TRANSPORT

#### 1.1 Electron Physics

In an FRM configuration generated and sustained by neutral beams, the first point to resolve is the extent to which electrons cancel the ion current. At a field null, electrons by resistive drag will speed up to the ion velocity unless there is some other competitive viscous force allowing a net current, such as generated by slower moving, higher  $Z$  impurities (Ohkawa current [2]). Away from a null, electron current must be due to  $E \times B$  drifts, and so it is bounded by the potentials sustained across flux surfaces. For substantial current cancellation, the  $E \times B$  velocity must be of the order of ion speed, requiring a potential ( $e\phi$ ) drop of the order of the ion temperature ( $T_i$ ). Consequently, on the open field lines where  $T_e \ll T_i$  and  $e\phi \sim T_e$ , electron currents are negligible. On the other hand, on closed field lines there is no such limitation, and we must solve explicitly for the self-consistent potential. In both of these cases the external viscous force that is required to maintain this differential rotation comes from the interaction of the electrons on the open field lines with the walls.

To study these effects due to electrons [3], we consider an axisymmetric hybrid model wherein the ion dynamics are determined from a two-space, three-velocity particle code (SUPERLAYER)[4]. The magnetic and electric fields are given by

$$\mathbf{B} = \frac{1}{r} \nabla \psi \times \hat{\theta}, \quad \mathbf{E} = -\nabla \phi - \frac{1}{rc} \frac{\partial \psi}{\partial t} \hat{\theta}, \quad (1)$$

and the self-generated toroidal field can be made negligible. The electrons are treated as an  $E \times B$  fluid given by

$$n_e = n_i Z_i, \text{ quasi-neutral} \quad (2)$$

and

$$\begin{aligned} E + \frac{v_e}{c} \times B &= \frac{m_e v_{ei}}{n_e} \sum n_i Z_i^2 (v_i - v_e) = n(j - e \sum n_i v_i Z_i (1 - \frac{Z_i}{Z_{eff}})) \\ &= n(j - j^{OHKAWA}), \end{aligned} \quad (3)$$

with

$$v_{ei} = \frac{4}{3} \sqrt{\frac{2\pi}{m_e}} \frac{n_e e^4 \ln \Lambda}{T_e^{3/2}}. \quad (4)$$

Consequently, with Ampere's law, neglecting displacement currents, we obtain

$$\frac{\partial \psi}{\partial t} + v_e \cdot \nabla \psi - \frac{r^2 c^2}{4\pi} \nabla^2 \cdot \frac{1}{r^2} \nabla \psi = n c r j^{OHKAWA} = a r c n j, \quad (5)$$

where the second equality holds when there is only one species having a non-zero velocity. Note the bootstrap current [5]  $v_e \cdot \nabla \psi / n r c$ . There are several simple points that can be obtained from Eq. (5). First, at the null (0-point), if the electron velocity is finite, steady state is not possible in the absence of the seed current. It should be emphasized that any process that breaks the relation  $P_{ei}/n_e = n j$  leads to a seed current. Further, if field reversal is achieved, it can only be lost in a resistive diffusion time.

We estimate the seed current necessary for reversal by examining the steady-state solution of Eq. (5) in the paraxial limit ( $B \ll B_0$ ). For an ion source localized near the position of the null  $r_0$ , write the ion source  $n v_r = (r - r_0) S$  and then integrate over the plasma volume to obtain

$$1 = \frac{\alpha}{a_i} \int_{r_0}^{r_1} dx \exp \left( \frac{S \tau_{skin}}{2 n r_0} ((r - r_0)^2 - (x - r_0)^2) \right), \quad (6)$$

where for  $\beta \sim 1$  we use  $n m_i v_\theta^2 = B_0^2 / 4\pi$ ,

$$l_{skin} = \frac{4\pi r_0^2}{n c^2} = \left( \frac{r_0}{a_i} \right)^2 \tau_{drag}, \quad a_i = v_\theta / \omega_{ci},$$

where  $\omega_{ci}$  is the gyrofrequency at the vacuum field,  $v_\theta$  is the ion thermal velocity, and  $r_1$  is the last closed flux radius where  $B$  is assumed to have its vacuum value. The term multiplying the integral is just the ratio of the self field, due to the Ohkawa current, to the vacuum field.

To apply this result to the slow buildup in the present 2XIIIB experiments, we argue as follows. The injection current must be sufficient to drive the field close to zero while the field lines are open and  $T_e$  is low; in this regime,  $S \tau_{drag} \approx n$  or for the trapped current

$$\frac{I_{\text{trap}} \tau_{\text{drag}}}{L} \propto \frac{r_0^2}{a_i^2} \quad (7)$$

We now compare this with the condition that a well developed field-reversed steady state exists for the same  $T_e$ , noting that this should be pessimistic because  $T_e$  should climb on closed lines. For fixed  $\alpha$ , Eq. (6) may be thought of as giving  $S_{\text{skin}}/n \propto I_{\text{trap}} \tau_{\text{drag}}/L$  as a function of  $r_0/a_i$ . The combination of this result and Eq. (7) forms a minimum in the required trapped current; for  $\alpha = 0.2$  this lies at  $r \approx 2r_0 \approx 4a_i$ . Thus, the required radius is about twice that of 2XIIB, which is roughly four times the volume. Folding in the increased trapping efficiency, we conclude that to achieve field reversal in the experimental regime, a threefold increase in injection current, or an equivalent increase in the energy containment on the open lines, is required. This increase in lifetime to  $\tau_E \approx \tau_{\text{drag}}$  would require better suppression of rf turbulence at high  $B$ . These basic electron physics equations, Eqs. (1-5), coupled with the constraint

$$\oint \frac{dl}{B} \nabla \cdot \mathbf{j} = 0$$

are being coupled with the ion pusher, SUPERLAYER, in a new hybrid simulation code.

## 1.2 SUPERLAYER Simulation

In addition to the electron physics, which is presently being worked into SUPERLAYER, we have developed a Monte Carlo package to model the drift-cyclotron loss-cone (DCLC) rf activity on open lines and charge exchange off the beams. The former generates transport (i.e., diffusion coefficients) in agreement with the quasilinear model of DCLC. Since the electron physics package is not complete, only energy transfer to electrons has been considered; thus, at present we are only able to simulate the approach to  $B = 0$ . Typical runs contain between 10 000 and 60 000 superparticles. We select the electron temperature (and the rf level, which determines the ion energy) to agree with the experiment, and then determine the remaining equilibrium quantities as a function of injection current. The DCLC turbulence not only modifies the energy distribution, it also causes appreciable radial diffusion. In addition results are sensitive to the stream current imposed. At present, this current is an external parameter. Agreement between simulation and experiment [6] is reasonably good at two field strengths for peak densities,  $\Delta B/B$  and radii. However, the plasma length is too short in the simulation by a factor  $\sim 2$ . This discrepancy is believed to be due to the neglect of a quadrupole field in the simulation, which now brings us the question of what we neglect by assuming axisymmetry.

## 1.3 Quadrupole Fields

An obvious effect of the quadrupole field is the tipping of the field lines away from the axis, effectively lengthening the source. To evaluate this effect, we have superimposed an external quadrupole field in SUPERLAYER and have observed stretching of ion-ring equilibria by the right magnitude. However, the introduction of the quadrupole field raises more subtle issues. First is the question of whether field lines

can then close if the field is reversed. The second issue is how badly do single-particle resonances for axis-encircling particles degrade confinement.

First consider the question of the opening of field lines [7]. To show the scaling, we superimpose externally applied toroidal and quadrupole fields on the field-reversed configuration. As a model, we again assume that  $B_z \gg B_r$  almost everywhere and use the paraxial approximation (except over a small region where field lines turn). In this case, the field-reversed configuration can be written

$$r^2 - r_0^2 = \pm r_0^2 \frac{\eta}{\eta_0}, \quad (8a)$$

$$B_z^2 = B_v^2 \tanh^2 \eta, \quad \eta \geq 0, \quad (8b)$$

$$B_T = \hat{\theta} \frac{r}{R} B_T, \quad (8c)$$

and

$$B_q = B_q \frac{r}{R} (\hat{r} \cos 2\theta - \hat{\theta} \sin 2\theta). \quad (8d)$$

Note that  $\eta_0$  is the value of  $\eta$  on the last closed-field line and is related to the vacuum flux by  $\exp(\psi/\psi_0) = \cosh^2 \eta$ . Also, Eq. (8a) is the equation of a field line ( $\eta$  constant) in the absence of  $B_q$  and  $B_T$  that we now treat as a perturbation. Now the equations of motion of a field line due to this perturbation can be written

$$d\psi / \oint \frac{dl}{B} B_q \cdot \nabla \psi = d\theta / \oint \frac{dl}{B} (B_q + B_T) \cdot \nabla \theta; \quad (9)$$

thus

$$\eta = \eta(z=0) \frac{\sin 2\theta(z=0) - \frac{B_T}{B_q}}{\sin 2\theta - \frac{B_T}{B_q}}. \quad (10)$$

Consequently, field lines are confined if  $|B_T/B_q| > 1$ . This analysis ignores diamagnetic effects on the quadrupole field. To ascertain whether plasma currents can shield out the quadrupole field is an unanswered question and awaits the development of a self-consistent three-dimensional equilibrium code.

Another aspect of asymmetry is how it affects radial transport. Specifically, axis-encircling particles can satisfy resonance conditions, in which case perturbations due to quadrupole fields can enhance particle losses. Luckhardt and Fleishman <sup>8</sup> have suggested that such is the cause of the enhanced electron losses in the RECE-BERTA ring experiment. An important observation is that the sudden losses occur at fixed values at  $\delta = \Delta B/B$  (independent of  $B$ ) and that the losses between these fixed values are proportional to the collision frequency but enhanced by a factor proportional to  $B_q$ . These features suggest that, near the fixed values of  $\delta$ , a sizable fraction of the plasma intercepts the loss region, whereas at other values there is "neoclassical" like transport with an effective banana width  $\propto B_q^{1/2}$ .

Assume that particles in this general three-dimensional system pos-

sess three good invariants, then the resonance condition associated with these invariants is

$$m\Omega_z + n\Omega_r + L\Omega_\theta = 0 \quad (11)$$

If particles could maintain this resonance relation, they would simply drift out of the confined region under the influence of a symmetry-breaking field. The fact that the Hamiltonian of the system is a nonlinear function of the invariants (actions) bounds the regions of resonance. The quadrupole field couples degrees of freedom if  $L = 2$  and  $m$  is odd. Then the invariants oscillate about resonant values slowly, compared with their associated frequencies. The amplitudes  $\Delta I$  and island frequency  $\Omega_I \propto B^{1/2}$ . If this swing intercepts a radial limiter, plasma loss results.

To evaluate these effects, we expand about circular orbits and write the Hamiltonian in action-angle variables, viz,

$$H_0 = \Omega_r(I_\theta) r + \Omega_z(I_\theta) z + \omega_{ci}^2(I_\theta) R^2(I_\theta) \frac{m_i}{2} \quad (12)$$

The procedure then is to cast the perturbed Hamiltonian due to  $B_q$  in action angle variables, which have a nearly constant phase when Eq. (11) is satisfied. Then, following Chirikov [9], we make a further canonical transformation to obtain a new Hamiltonian dependent on this slow phase and action and hence deduce the oscillation amplitudes of the original actions:

$$\begin{bmatrix} I_r \\ I_\theta \\ I_z \end{bmatrix} = \begin{bmatrix} -n \\ 2 \\ -m \end{bmatrix} \times B_q^{1/2} M, \quad (13)$$

where  $M$  is a completely determined but complicated decreasing function of  $m$  and  $n$ .

To apply this calculation to REGE-BERTA, we choose the flux to satisfy

$$\psi = \frac{B_0 r^2}{2} \left[ 1 - \delta \exp \left( -\frac{r^2}{L_r^2} - \frac{z^2}{L_z^2} \right) \right], \quad (14)$$

with  $L_z = 11$  cm and  $L_r = 16$  cm, and thus the current-density peaks at  $r = 9$  cm.

We have calculated resonance positions and widths in  $I_r, I_z$  space for 450-keV electrons at various values of  $\delta$ . We identify a "well populated" region corresponding to particles that do not hit the limiter and turn axially at  $Z_t < 10$  cm. We find that, as  $\delta$  is varied, resonances move slowly in the axial tail ( $Z_t > 11$  cm) but rapidly in the well-populated region; e.g., the  $m = 3, n = 0$  resonance sweeps through the latter region as  $\delta$  changes from 0.62 to 0.56, corresponding to a time short compared with the ring-decay time ( $10^{-5}$  s) but slow compared with the island frequency ( $10$  s) for  $B(r = 10 \text{ cm})/B = 0.02$ . Hence, particles lying in  $I$ -space within  $\Delta I$  of the radial limiter are lost during a sweep, while the increase in diffusion coefficient for the remaining particles is neoclassical (banana regime)

$$D \sim (\Delta I)^2_{\nu} \sim B_{\nu} \nu. \quad (15)$$

In this model, the most notable resonances were  $\delta = 0.83$  ( $m = n = 1$ ),  $\delta = 0.55$  ( $m = 3, n = 0$ ) and  $\delta = 0.14$  ( $m = 3, n = 1$ ). In RECE-BERTA, only two resonances have been identified, which we speculate to be the latter two. As for the first resonance, a more exact treatment might predict its onset to lie at  $\delta > 1$  and thus not in present data. There are other weaker resonances intercepting the limiter that have also not been observed. It should be emphasized that our model field is crude, assumes fixed scale lengths, and ignores mirroring in the vacuum field. The weakness in the Hamiltonian model is the inadequate treatment of particles with large excursions. It should be noted that resonances hang up in these regions.

These effects have also been investigated with the exact orbit, single-particle code ORBXYZ. To date comparisons have been limited, but for the principal resonances studied,  $\delta$  is as predicted and the excursions scale with analytic theory.

Such resonance should degrade confinement in the very high  $\beta$ -regime in 2XIIIB. However, if the radius is doubled for significant bootstrapping as required for field reversal, the number of axis-encircling particles should be small, the frequencies should be disparate, and this resonant diffusion should disappear.

#### 1.4 Analytical Model of a Field Reversing Ion Layer

To help gain insight into the physics of field-reversed states, we investigated an analytically solvable case. The model is a Z-independent, axially symmetric entity in which monoenergetic ions of fixed canonical angular momentum  $f \sim \delta(E - E_0) \delta(L - L_0)$  move in the self-consistent fields generated by their motion in the external field  $B_0$ . For the chosen ion-distribution function the ionic azimuthal current is  $j_\theta = v_\theta/r$ . Use of this relation, together with that required by conservation of canonical angular momentum, leads immediately to a differential equation of Bessel form for  $y = v_\theta/v$ , the dimensionless azimuthal velocity of the ions:

$$\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \left( \frac{1}{x} + \frac{1}{x^2} \right) y = 0, \quad (16)$$

where  $x = K(r/a_i)$ , a dimensionless radius,  $a_0$  is the ion gyroradius in the (uniform) vacuum field  $B_0$ , and  $K$  is a constant (later determined) that is a measure of the dimensionless radius of the entity  $K \approx a/a_i$ .

The general solution to this equation is

$$y = AI_2(2x^{1/2}) + BK_2(2x^{1/2}) \quad (17)$$

Boundary conditions to be imposed are (1)  $y = -1$  at  $r = a$  and  $r = s$  (inner radius of entity) and (2)  $a_i = a_0$  at  $r = a$  (i.e., ion gyroradius at outer boundary equals vacuum-field gyroradius). With the above, a closed solution is obtained wherein  $A$  and  $B$  are given as functions of  $s/a$  and  $a/a_i$ , from which follow self-consistent fields and orbits.



Some conclusions from the analysis:

- Field reversal is only possible for  $a/a_0 > 2.0$ .
- Large, thick layers produce higher field reversal:

$$\frac{B(s)}{B_0} \approx - (a/s)^{1/2}, \quad a \gg a_0; \quad a \gg s. \quad (18)$$

- Depending on the sign of  $y$  at  $r = s$ , either field reversal or field enhancement may be achieved.

## 2. STABILITY

To ascertain the stability of an FRM we must include FLR and a large aspect ratio. To examine these effects we relax the length constraint and use the paraxial approximation. We also consider a rigid-rotor equilibrium so that see Eq. (8)

$$p_i = \frac{B_0^2}{8\pi} \frac{1}{\cosh^2 \eta} \quad (19)$$

$$\rho = \rho(0) \frac{1}{\cosh^2 \eta} \quad (20)$$

and the rotation frequency

$$\omega = \omega_{*} + \omega_{\text{ExB}} = \frac{2V_{\text{Alf}}^2}{\omega_{ci} r_0^2} \frac{\eta_0}{1 + \frac{T_e}{T_i}} (1 + K) \quad (21)$$

where

$$V_{\text{Alf}}^2 = \frac{B_0^2}{4\eta\rho(0)} \quad (22)$$

and  $K = T_e/T_i$  for electrostatically confined electrons and comes from the  $E \times B$  drift. Also, the mean particle frequency is given by

$$\omega_2 = \omega_{\text{ExB}} + \langle \omega_{\text{VB}} \rangle = \frac{2V_{\text{Alf}}^2}{\omega_{ci} r_0^2} \frac{\eta_0}{1 + \frac{T_e}{T_i}} \left( K - \frac{1}{2\sinh^2 \eta} \right) \quad (23)$$

We assume field-line lengths scale with  $\eta$ . How we resolve the singularity at  $\eta = 0$  will be described presently.

The perturbed equations for this configuration ( $E_{\parallel} = 0$ ) have been derived previously [10] and will be applied here. For the perturbed radial displacement, we have the equation

$$\frac{1}{r} \frac{d}{dr} \eta r^3 \rho T \frac{d\xi}{dr} = (m^2 - 1) \eta \rho T \xi - r \eta \frac{d\rho}{dr} (\omega^2 + k^2 V_{\text{Alf}}^2) \xi \quad (24)$$

with

$$\rho T = \rho(\eta) (\omega - m\omega_1) (\omega - m\omega_2) - k^2 B^2 / 4\pi. \quad (25)$$

We consider rotation-driven modes which are flute-like. Consequently, we average over a line ( $k^2 = 0$ ) to obtain

$$\frac{d}{d\eta} \left\{ \frac{(\eta + \eta_0)^2}{(\eta^2 + \eta_0^2)} \eta_0 \right\} \rho T \frac{d\xi}{d\eta} = \frac{m^2 - 1}{4} \left\{ \frac{\eta_0}{\eta} \right\} \xi \rho T$$

$$+ \omega^2 \rho \left\{ \frac{\eta_0(\eta + \eta_0)}{\eta^2} \right\} \xi \tanh \eta, \quad (26)$$

when the upper (lower) quantities pertain to the open (closed) field lines. Note that because  $\xi$  is only a function of  $\eta$  through the flute average, the boundary condition or regularity at  $r = 0$  is lost. Now to close the problem we need a constraint at the null. Both solutions are well behaved;  $E_\theta = 0$  does not determine the proper solution at the null. We then have as  $\eta$  approaches zero, the two solutions

$$\xi_1 = \eta^2$$

$$\xi_2 = 1 + \frac{m^2 - 1}{16 \eta_0^2} \eta^2 \ln \eta^2. \quad (27)$$

The next point is to obtain the appropriate linear combination. Recalling Eq. (23), we see that breakdown in the model occurs in the VB drift, which diverges at  $\eta = 0$ , and that an appropriate particle treatment would lead to a finite term. Consequently, we model this effect by writing

$$\omega_2 = 2 \frac{v_{\text{Alf}}^2}{\omega_{ci}^2 r_0^2} \frac{\eta_0}{1 + \frac{T_e}{T_i}} \left( K - \frac{1}{2 \sinh^2(\eta^2 + v^2)} \right), \quad v^2 = \frac{a_i}{r_0 \eta_0}. \quad (28)$$

To obtain this scaling, we have made use of the linear expansion  $B = 2B_0(r - r_0) \eta_0 / r_0$ . Now, near the null the differential equation, Eq. (26), takes the form

$$\frac{d}{d\eta} \frac{\eta}{\eta^2 + v^2} \frac{d\xi}{d\eta} = \frac{m^2 - 1}{4 \eta_0^2} \frac{\eta}{\eta^2 + v^2}.$$

solving by iteration we generate the solution ( $\eta \gg v$ )

$$\xi = 1 + \frac{m^2 - 1}{16 \eta_0^2} \eta^2 (\ln \eta^2 - 1 + \ln v^2), \quad (29)$$

which then determines the linear combination. Note that the other solution diverges logarithmically and must be discarded. It must be remarked that this result is only qualitative. A rigorous treatment requires the solution of the Vlasov equation in the vicinity of the null, where the magnetic field is linear. Our results show that as  $v^2 \rightarrow 0$  (small Larmor radius) the system is most stable ( $K \approx 1.2$ ). Recall that in this limit the solution at the null is  $\xi_1$ . On the other hand, as  $v^2 \rightarrow 1/\eta_0(a_i/r_0 = 1)$  the threshold drops to  $K \approx 0$  ( $E = 0$ ). In this limit the solution at the null is predominantly  $\xi_2$ . These results do not agree with the Los Alamos Scientific Laboratory experiment [11], whose stability threshold is lower yet. It should be emphasized that there is a linear combination at the null with marginal stability at zero rotation; however, the solution is inconsistent with our boundary-layer analysis.

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