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A COMPUTATIONAL COMPACT TORUS EXPERIMENT

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ABSTRACT

We describe a typical 2D magnetohydrodynamic (MHD) calculation of rundown of plasma in a coaxial, magnetized gun and injection of the plasma and reconnection of the embedded magnetic fields to form a compact toroidal plasma.

INTRODUCTION

The two-dimensional (r, z, t) magnetohydrodynamic code, HAM,¹ has been used to model the performance of the magnetized, coaxial plasma gun injecting a plasma ring into a copper flux conserving chamber, as used in the Beta-II experiment.^{2,3} The inclusion of all the experimental hardware in the calculation makes it very lengthy, but it gives a wealth of information and insight into every aspect of the experiment--breakdown, production of the gun plasma, field-line reconnection, and formation of a compact toroid.

We will display these results with some brief interpretations. The calculations will lead to simpler, more fundamental studies to elucidate different pieces of the physical processes involved. The qualitative results of the calculations appear correct, but a quantitative calibration of the numerical against the laboratory experiment remains to be done. Two principal defects are immediately evident in the laboratory: the code has no model of the desorption of gas or the sputtering of impurities from the gun, and these appear to be important in creating energy losses from the plasma and enhancing the resistivity. The impurity input depends upon the current density and the plasma temperature, which sputters material locally or desorbs other gases from the electrodes, and upon the total plasma-wall contact time. Evidently, numerical experiments could be performed to improve future gun designs along this line. The second defect of the simulation is that, being axisymmetric, it is unable to model MHD instabilities. Stability calculations by Dalhed⁴ and laboratory results show that the guide field of 500 gauss (G) used here would lead to rapid-tilting instability after the toroid is formed. Future simulations can avoid this by keeping the guide field below Dalhed's estimate of the maximum, which, in this geometry, is $B_G(\text{kG}) < 1.6 I_0(\text{MA})$. The total toroidal current is I_0 .

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Most of this report simply presents the computational results with rather brief explanations. Some of the more important observations are as follows:

1. The gun produces two plasmas, a very dense one pressed radially outwards on the anode, and a lower density plasma around the cathode which accelerates ahead of the dense plasma to become the compact toroid. The dense plasma never really separates from the gun and falls back into it at the end of the discharge. Laboratory diagnostics are not yet sufficient to determine if this is correct.

2. The field-line reconnection seems to be affected by plasma flow through the gap between the gun and the flux conserver (see Fig. 1) and even more by plasma flow into the expansion region of the flux conserver.

3. The plasma resistivity is anomalous in the reconnection regions, using a model according to Sgro, ⁵ and, although theory would indicate that the reconnection rate should be independent of resistivity, ⁶ this point needs to be confirmed in more elementary simulations.

4. The efficiency of converting the capacitor bank energy into plasma and magnetic energy trapped in the toroid is only a few percent. This could be improved by varying the power input to the gun and exercising better control over the reconnection process. The parameters of the laboratory experiment will be varied to improve this.

5. The current and voltage variations have considerable effects on the final toroid produced, which needs to be understood in terms of some simpler model. The laboratory gun crowbars the applied voltage at the peak current, an effect not described here.

6. The laboratory experiment comes to equilibrium with about double the poloidal flux provided by the gun embedded in the compact torus. This 'flux amplification' is a three-dimensional effect of reconnection of large-amplitude helical instabilities and is qualitatively explained by evaluating the minimum-energy final state with the initial total magnetic helicity provided by the gun currents.

INITIAL CONDITIONS

The geometry and dimensions of the problem are illustrated in Fig. 1. The computational grid, consisting of a 223×46 mesh of one-centimeter squares, is also shown in Fig. 1(c). The external guide field has a value of 500 G and the poloidal flux in the annular gun barrel is -1.94×10^6 G-cm².

FIG. 1. Geometry and computational grid. (a) Sizes of the experiment; (b) actual computer figure; and (c) computational grid.

The gas fill was 3.125 atm cc. of equivalent deuterium (D), initially distributed according to a Gaussian profile centered about the mid-point of the gun

$$p(z) = 5.13 \times 10^{15} \left\{ \exp -0.0028 (z - 50)^2 \right\} . \quad (1)$$

A uniform background density of $1.87 \times 10^{12} \text{ cm}^{-3}$ was assumed in the guide space. The plasma temperature in the grid was initialized at 0.1 eV for both the ions and the electrons, except in the region $22 \text{ cm} \leq z \leq 28 \text{ cm}$ where the temperature was increased to 5 eV to initiate the discharge problem. A Saha ionization model was employed and the plasma resistivity is the sum of Spitzer and electron-neutral resistivity and Sgro's anomalous resistivity. The geometry and external circuit parameters are given in Table 1.

TABLE 1. Initial conditions of the experiment.

External circuit	
capacitance, mfd	0.058
inductance, nh	50
resistance, Ω	0
voltage, kV	-35
External field guide	0.5 kG
Density profile in gun, independent of radius	$\rho(z) = 1.7 \times 10^{-8}$ $\exp[-0.0028 (z - 50)^2] \text{ g.cm}^{-3}$ $\equiv 5.2 \times 10^{15} \exp[-(z - 50)^2 / 19^2] \text{ cm}^{-3}$
Minimum plasma density in gun	$6.25 \times 10^{-12} \text{ g/cm}^3$
Plasma temperature in gun	0.1 eV (ions and electrons)
Initial value of rA_0 (= poloidal flux/ 2π) in inner gun electrode	-0.309 [MG.cm ²]

GUN RUNDOWN

The initial density profile is illustrated in Fig. 2(a), and the initial hot spot of $T_e = 5$ eV is around 25 cm. The code does not carry the induction term ($\partial E/\partial t$) in Ampere's law, and so flux has to diffuse into the gun. However, the initial density and temperature allow the fields to diffuse quite rapidly up the gun barrel to ionize and accelerate the main mass of the gas pill. The laboratory discharge displays eight filaments at the gas valves at 50 cm, and future simulations will try to force breakdown nearer this region.

An intermediate view of the rundown is shown in Fig. 2(b), and the separation of fast and slow plasmas is already apparent. The electron temperature is highest at 45 cm, behind the density peak at 60 cm. The current path runs down the outer electrode (anode) and through the plasma in the region of the density peak to drive the magnetic piston, which pushes and heats the plasma. The $1/r$ fall-off of this B field accounts for the radial variation of the plasma velocity. Momentum flow and electron temperature are shown in Figs. 2(d) and (e).

The state of the plasma as it emerges from the gun is shown in Fig. 3.

TOROID FORMATION

The density, flux, and momentum flow as the plasma runs into the flux conserver, where the field lines reconnect, and how a compact toroid is formed, are shown in Figs. 4-9. The first visible reconnection (Fig. 6) takes place around 0.8 μ secs as plasma flows out of the gap between the gun and the flux conserver. The momentum flow just inside the flux conserver has just developed a stagnation point (Fig. 6) and some plasma is beginning to flow back into the gun. At this time, the forward momentum has slowed considerably, to be followed, at $t = 10.1$ μ secs, by expansion into the main chamber. The toroid formation takes place between 11.9 and 15.6 μ secs in a manner similar to that first discussed by Fatschek (see Ref. 6) for toroid reconnection. The toroid bounces gently in the main chamber as the bulk of the plasma accelerates back into the gun.

FIG. 2. Plasma density simulation (a) initial gas density contours in the gun; (b) density at 2.9 μsecs; (c) poloidal current-flows through anode, across the plasma, and back to the cathode at 2.9 μsecs; (d) momentum flow at 2.9 μsecs; (e) electron temperature at 2.9 μsecs. Peak is still back in loss density region. Density and temperature contours vary by factors of 10.0. Peak density is 10^{15} cm^{-3} .

FIG. 3. Final conditions at $t = 3.8 \mu\text{secs}$ of gun rundown. Figures are (a) density, (b) ion temperature, (c) electron temperature, (d) flows velocity, (e) plasma momentum, and (f) current flow. Contour levels differ by factor of 10.0.

FIG. 4. Plasma density contour between 3.8 and 10.1 μ secs. Plasma flow almost halts at 8.0 μ secs before plasma expands into the main chamber.

FIG. 5. Plasma density contours between 11.9 and 19.2 μ secs show complete toroid formation and plasma retreating into the gun.

FIG. 6. Flux contours between 3.8 and 10.1 μ secs. A tiny amount of flux reconnects at 8.0 μ secs around the gun-conserver gap.

FIG. 7. Flux contours between 11.9 and 19.2 μs shows completion of the reconnection process.

FIG. 8. Plasma momentum vectors between 3.8 and 10.1 μ secs. Observe stagnation of the flow at 8.0 μ secs. This stagnation point at 10.1 μ secs is in the reconnection zone.

FIG 9. Plasma momentum between 11.9 and 19.2 μsecs . Observe that the compact toroid is almost stationary.

CURRENT-VOLTAGE GUN BEHAVIOR

There are several characteristics of the plasma gun discharge voltage and current which coincide with stages of the plasma evolution described above. At point A ($t = 3.8 \mu\text{secs}$), in Fig. 10, the gun current is at its maximum negative value of -600 kA as the plasma emerges from the gun. The first reconnection is taking place as the current swings through zero and the plasma flow reverses in the region of the gun-conserver gap (Fig. 8).

At point C ($t = 10 \mu\text{secs}$), the current is at its maximum positive value of 240 kA and the plasma has just started to expand into the main chamber of the flux conserver (Fig. 11). Formation of the compact toroid takes place between D and E when the voltage has dropped close to zero (Fig. 12). The reconnection processes evidently clamps the voltage around -1 keV . The effects of the positive current flow are barely seen in the flux conserver until this point in time. In the laboratory experiment, the gun short-circuits near the insulator and no positive current flows in the gun at all. Future simulations will invoke this crowbarring effect.

DETAILS OF RECONNECTION

One-dimensional cross-sections of the experiment at $r = 13 \text{ cm}$ are shown in Figs. 13-17. The main chamber is between 135 and 175 cm. The first density plot shows the difference between the expanding plasma and the denser column destined to retreat into the gun. The reconnection point is around 130 cm, where the density is lowest and the resistivity (Fig. 14) is anomalously high. Fig. 15 displays separately the classical part of the resistivity on the same scale as Fig. 14. The very high resistivity back in the gun bore arises from the minimum plasma density and temperatures allowed in the simulation, $p_{\min} = 5.10^{12} \text{ cm}^{-3}$, $\tau_{\min} = 0.1 \text{ eV}$, and is irrelevant to the reconnection processes in the flux conserver. Reconnection is dominated, evidently, by anomalous resistivity. The magnetic field in Fig. 16 reflects the flux contours of Fig. 7 at this time. The negative B_z (-1.5 kG) in the expander is part of the reversed flux which is reconnecting. The field (4 kG) in the conserver between 110 and 140 cm, and the negative B_z at the gun muzzle, is part of the guns' initial poloidal field. This section through the field profile varies because of the inward radial flow of plasma in the reconnection zone around 140 cm.

A - 3.8 μ secs. End of rundown
 B - 8.0 μ secs. Reconnection starts
 in gun conserver gap

FIG. 10. External circuit parameters: 0 - 8.4 μ secs. These and Figs. 11 and 12 show voltage and current variations during the discharge. The code dumps and restarts many times in a run, so time histories are collected from many files.

FIG. 11. External circuit parameters: $t = 8.5 - 18 \mu\text{sec}$.

FIG. 12. Total gun current falls steadily to zero.

FIG. 13. Plasma density at $r = 13$ cm, $t = 13.6$ μ sec shows compact toroid at $2 \cdot 10^{14}$ and the bulk of the plasma peaking at $4 \cdot 10^{15}$.

FIG. 14. Total resistivity at $r = 13 \text{ cm}$, $t = 13.6 \mu\text{sec}$, is largely anomalous.

FIG. 15. The classical resistivity at $r = 13.0 \text{ cm}$, $t = 13.6 \mu\text{sec}$, is very low in the hot plasma in the flux conserver.

FIG. 16. $B_z(z)$ at $r = 13.0 \text{ cm}$, $t = 13.6 \mu\text{sec}$ shows reversed field as the torus forms.

FIG. 17. Toroidal magnetic field at $r = 13.0 \text{ cm}$, $t = 13.6 \mu\text{sec}$.

An examination of Figs. 4-7 shows that B_z traces along the axis of the laboratory experiment are much less informative of the details of the reconnection and reversed flow. The toroidal field (~300 G) is much smaller than the poloidal field throughout the flux conserver (Fig. 17).

Radial plots at the entrance to the expander, where $z \approx 134.5$ cm, show that the plasma resides in a hollow cylinder of radius close to that of the cathode (9.8 cm) [see Fig. 18]. The J_{θ} profile (Fig. 19) shows the plasma current and the large reverse current induced in the copper walls of the conserver. Characteristically, the current peaked at the outside of the plasma column. The resistivity (Fig. 20) shows large η in the low-density interior plasma, and the insulating coils around the flux conserver prevent plasma currents flowing into the copper wall. The $b_z(r)$ plot (Fig. 21) shows the reversed field and the 500 G external guide field. The toroidal field (Fig. 22) is quite small at this time.

THE COMPACT TOROID

At 13.6 μ secs, as the reconnection is taking place, the poloidal current flow in the expander is frozen into the reconnected flux surfaces (see Fig. 23). Unfortunately for the toroidal field, the reversal of the current flow on the outer field lines produces the bi-directional current flows of Fig. 24. This produces the curious, and probably unstable, $\delta_p(r)$ profile in Fig. 25.

The remaining sequence (Figs. 26-28) shows density, B_z and $J_{\theta}(r)$ along the lines $z = 150$ cm through the center of the expander. The final plot at this time (Fig. 29) shows the relative density in the expander and the plasma flowing through the gun conserver gap at $r = 24$ cm.

Isometric plots of density and flux at 15.9 μ secs are given in Figs. 30 and 31, and they illustrate the relative amounts of each extracted from the gun to form the toroid, which is not easily understood from the contour plots.

Characteristics of the final toroid are summarized in Table II. Equivalent laboratory data are not yet available. The current density is still high enough to cause the resistivity to be quite anomalous.

FIG. 18. Density of entrance to flux expander at $z = 134 \text{ cm}$, $t = 13.6 \mu\text{sec}$. Plasma lies in a hollow cylinder.

FIG. 19. Toroidal current at $z = 134 \text{ cm}$, $t = 13.6 \mu\text{sec}$ shows image current in flux conserver well at 17.0 cm .

FIG. 20. Resistivity at $z \approx 134 \text{ cm}$, $t = 13.6 \mu\text{sec}$ shows plasma and insulating zones.

FIG. 21. $B_z(r)$ at $z = 134 \text{ cm}$, $t = 13.6 \mu\text{sec}$ shows reversed field configuration with 2.5 kG on axis.

FIG. 22. Toroidal magnetic field at $z = 134 \text{ cm}$, $t = 13.6 \text{ } \mu\text{sec}$ is only 0.3 kG compared with poloidal fields of 2-3 kG.

FIG. 23. Poloidal current flow at 13.6 μ sec in compact torus shows positive and negative flow which diminish the total toroidal magnetic field.

FIG. 24. Final current pattern in compact torus at 19 μ sec. Laboratory gun shoots out the positive current flow.

FIG. 25. Toroidal magnetic field profile through the center of the toroid shows positive and negative B.

FIG. 26. Final density profile in toroid shows ring-like plasma.

FIG. 27. The B_z profile in toroid.

FIG. 28. Toroidal current profile in toroid and image current in conserver wall. The peak is close to the density peak.

FIG. 29. Density at $r = 24 \text{ cm}$, $t = 19 \text{ } \mu\text{sec}$ shows compact toroid and plasma flowing through gun-conserver gap.

FIG. 30. Geometric plot of density at $t = 15.9 \mu\text{secs}$ shows toroid and gun plasmas.

FIG. 31. Geometric plot of flux at $t = 15.9 \mu\text{secs}$ shows magnetic trench in the gun and well formed by the compact torus and pinned by the flux conserver.

TABLE 11. Characteristics of the compact toroid.

Time	= 19.2 μ secs
Toroid thermal energy	0.602 kJ
Toroid kinetic energy	0.0252 kJ
Toroid poloidal field energy	0.369 kJ
Toroid toroidal field energy	0.0067 kJ
Total energy in toroid	1.00 kJ
Bank energy supplied	35.00 kJ
I-Ring current	-75.6 kA
Density average	$3.89 \times 10^{14} \text{ cm}^{-3}$
Total particles	3.12×10^{19}
Mean electron temperature	39.0 eV
Mean ion temperature	41.7 eV
Axial velocity at 0-point	-1.25 cm/ μ sec
Field on axis	-1.05 kG
Reversed flux at axis	$-17.2 \times 10^2 \text{ kg cm}^2$
Ring volume	80.1 l or L
Toroidal field at 0-point	-706.0 G

FINAL REMARKS

This report has described a typical simulation of the Beta-II gun experiments. The time and field and density scales agree semi-quantitatively. The main differences are in the temperature, which is dominated by impurities, and in the flux, which is enhanced considerably by three-dimensional instabilities. Formulae have been developed, based on conservation of helicity, to explain the flux enhancement, and experimental efforts are underway to eliminate the impurities. Another difference is that the Beta-II experiment still indicates a high density in the entrance to the flux conserver after reconnection is complete. The computations also achieve reconnection somewhat faster than the experiment.

Despite these differences, further calculations are underway to simulate high energy, lower-density discharges for neutral injection, and for possible demonstration reactor designs.

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