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IMPROVED PLUG VALVE
COMPUTER-AIDED DESIGN OF PLUG ELEMENT

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4,878,652

PLUG VALVE

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4 Claims

1. An improved plug valve including a valve body having an inlet passage and an outlet passage therethrough, a valve chamber between said inlet and outlet passages, a valve plug having substantially the same shape as said valve chamber and being rotatably disposed therein, said valve plug having a flow passage therethrough, said flow passage disposed such that with said valve plug in a first position, said flow passage is in alignment with said inlet and said outlet passages and provides fluid communication therebetween, and with said valve plug in a second position, said flow passage is not in alignment with said inlet and outlet passages and said valve plug interrupts fluid communication therebetween; and means for rotating said valve plug between said first position and said second position; wherein the improvement comprises:

said valve plug and said valve chamber having the shape of a pseudosphere.

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INTRODUCTION

The purpose of this document is to present derivations of equations for the design of a plug valve and to present a computer program which performs the design calculations based on the derivations.

The valve is based on a plug formed from a tractrix of revolution called a pseudosphere. It is of interest to be able to calculate various parameters for the plug for design purposes. For example, the surface area, volume, and center of gravity are important to determine friction and wear of the valve. A computer program in BASIC has been written to perform the design calculations.

The appendix contains a computer program listing and verification of results using approximation methods. A sample run is included along with necessary computer commands to run the program.

VALVE DESCRIPTION

The Improved Plug Valve was granted a patent in July 1989, NO. 4,878,652.

Wear is a fact of life that must be considered in the engineering design of any piece of equipment. In the past, the effects of wear have been minimized by using lubricants and very hard materials resistant to wear. Valves use the later technique for the design of seat and trim. Even by using very hard materials, leaks through the seat remain a serious problem in many applications. The seat and trim are subjected to various wear processes such as abrasion and erosion which, over time, eventually cause the valve to fail. Stem leaks are also a problem in present valve design.

The design of the Improved Plug Valve accepts wear as a fact of life and compensates for it by the novel shape of the plug. See Figure 1. Instead of a spherical element or frustum of a cone, two common shapes currently used to manufacture quarter turn plug valves, the Improved Plug Valve uses a pseudosphere as the plug. A pseudosphere is a shape that looks like a trumpet bell. This unique shape has the property of even wear over its whole surface. As the valve is opened and closed, the seat and trim will always match, thus remaining leak-free through the seat. Since the plug has an even wear feature, it is expected to have a long life and need a minimum of maintenance.

Nevertheless, this valve design incorporates features which make it easy for the valve to be maintained. Both plug and stem can be disassembled and serviced independently.

Using this same pseudosphere concept, a stem seal has been designed which prevents leaks along the stem. Tertiary 'O' ring seals are added at key places as insurance against leaks to the environment and to prevent dust, dirt, and grease in the environment from entering the valve internals.

Figure 1. New Plug Valve

BASIS FOR VALVE DESIGN

A unique shape was selected for this plug valve; it is in the shape of a tractrix of revolution. This shape gives it the unique feature of having even wear on the surface as it rotates against the body. This means that the plug will always be properly aligned and that the plug and body will always fit perfectly together. As such, there will never be any leakage around the body/plug interface because of a mismatch created by wear.

This feature makes this valve particularly reliable and leak free. It will be suitable for any application which requires a valve to open and close a large number of cycles over its lifetime, or where it is in a place where maintenance cannot be performed. Examples of such areas are an unmanned spacecraft on an interplanetary mission, use in the deep ocean, in a severe radiation environment, and in propulsion units.

Tractrix Description and Development

A tractrix is defined as the locus of points, the tangent to such locus of points, from point of tangency to x-axis, is a constant. We denote the length of the tangent, from the curve to the x-axis

as $PQ = a$

$$\frac{dy}{dx} = - \frac{y}{\sqrt{a^2 - y^2}} \quad \text{EQ. 1}$$

Transposing, we have

$$\boxed{dx = - \frac{\sqrt{a^2 - y^2}}{y} dy} \quad \text{EQ. 2}$$

Integrating, using Pierce's integral tables:

$$x = -\sqrt{a^2 - y^2} + a \operatorname{Sech}^{-1} \frac{y}{a} + C$$

The initial conditions are:

$$y = a \text{ @ } x = 0$$

substituting yields $C = 0$

recognizing that

$$\text{Sech}^{-1} \frac{y}{a} = \text{Cosh}^{-1} \frac{a}{y}$$

The final equation for the tractrix is,

$$x = -\sqrt{a^2 - y^2} + a \text{Cosh}^{-1} \frac{a}{y} \quad \text{EQ. 3}$$

We will have need for equations 2 and 3 in subsequent sections.

Spring Force Required

A spring is needed in the design of the improved plug valve to provide an axial force to hold the plug against the seat. Internal pipeline pressures are trying to force the plug away from the seat. This must be prevented in order to make the valve leak-tight around the seat. An axial spring force of twice the value of the force due to internal pressure is needed to prevent leaking around the seat.

An unbalanced force is felt inside the slot due to the variation in radius from top to bottom of the plug. The area upon which the pressure acts is given by:

$$\begin{aligned} \text{area} &= 2 y_1 W - 2 y_2 W \\ &= 2W (y_1 - y_2) \end{aligned}$$

where W = slot width.

The force caused by the pipeline pressure is:

$$\text{Force} = \text{Pressure} \times \text{area}$$

$$F = p \cdot 2W (y_1 - y_2)$$

The contact pressure, P_c , between plug and seat is given by:

$$P_c = 2F/Ap$$

The spring force is:

$$F_s = 2F$$

Example using typical dimensions for a 1" valve:

$$p = 100 \text{ psi}$$

$$W = .5 \text{ in.}$$

$$y_1 = 1.5 \text{ in.}$$

$$y_2 = .58 \text{ in.}$$

$$A_p = 6.00 \text{ in.}^2$$

$$F = 100 \times .5 \times 2 \times (1.5 - .58)$$

$$F = 92 \text{ LB}$$

Force on plug due to pipeline pressure.

$$F_s = 2 \times 92$$

$$F_s = 184 \text{ LB}$$

Spring force required.

$$p_c = 2 \times 92 / 6.00$$

$$p_c = 30.66 \text{ psi}$$

Contact pressure.

WEAR ON PLUG SHAPES

Plug valves are made with a frustrum of a cone or a sphere as the rotating element. These shapes wear non-uniformly, and as a result, begin to leak around the element. A tractrix of revolution has a uniform wear characteristic over its entire surface. This feature permits the plug and seat of the valve to fit perfectly even after a long period of use.

In this section a mathematical development proves that the tractrix will indeed provide uniform wear. Similar mathematical arguments are presented for the frustrum of a cone and a sphere showing the non-uniform wear of these shapes.

Tractrix Uniform Wear Property

It is of interest to determine what curve for a body of revolution would produce uniform wear on a set of mating surfaces. See Sketch below. We seek a curve for which the vertical pressure, based on the projected area, is constant. Define parameters as follows:

Let:

W = Work done in one revolution

μ = Coefficient of friction

P = vertical pressure per unit horizontal area, psi

F = Total axial force or weight on body of revolution, lb

A = Horizontal projected area of rubbing surfaces, in.²

Then $P = \frac{F}{A}$ vertical pressure

N = Normal pressure, psi

H = Horizontal pressure, psi

$\overline{RQ}$ = Length of tangent to curve from point R to x-axis

Sketch of Force and Pressure on
a Body of Revolution.

by similar triangles:

$$\frac{\overline{RQ}}{y} = \frac{N}{P}$$

rearranging $\overline{RQ} = \frac{1}{P} (Ny)$

since P is constant by definition, $\overline{RQ}$ is directly proportional to the product, Ny . Wear on the rubbing surfaces is proportional to the work done in one revolution

$$W = fs$$

where

$$f = \mu N \quad s = 2\pi y$$

Then $W = \mu N \cdot 2\pi y = 2\pi\mu \cdot Ny$

It follows that if we make Ny constant, the wear will be constant. If Ny is constant, then length RQ must be constant to be the necessary and sufficient condition for the wear along the surface to be constant. The curve for which the length of the tangent to the curve from a tangent point on the curve to the x-axis is constant is a tractrix.

Frustrum of a Cone

Figure 1. Wear on the Frustrum of a Cone.

Wear is proportional to work done in one revolution.

At the large radius of the cone

$$W_2 = \mu N 2\pi R_2$$

At the small radius of the cone

$$W_1 = \mu N 2\pi R_1$$

The normal force, N , is the same at both the large and small radius since the angle is the same, but, R_2 at the top is larger than R_1 . Therefore, there is more wear near the top than near the bottom of the cone frustum. Eventually this wear will permit a leak path to develop around the plug.

Ball Element

Figure 2. Ball Element

Wear is proportional to the work done in one revolution. Wear at the center region of the sphere is:

$$W_c = \mu N \cdot 2\pi R$$

R = radius of sphere

Wear at another radius, R_1 , is:

$$W_1 = \mu N \cdot 2\pi R_1$$

Assuming N is constant, then the wear at the center region will be greatest of anywhere on the ball. Eventually, this wear will permit a leak path to develop around the ball at the center radius. The design is such that the socket surrounding the ball cannot be tightened to compensate for this wear.

DERIVATION OF EQUATIONS

There are several reasons why it would be useful to know several parameters about the plug. Certainly, the coordinates of the curve are important so the form can be laid out. In addition, surface area, volume, arc length, center of gravity, and the slot design all would be useful information as design and manufacturing data. Therefore, a series of equations have been developed to calculate these parameters for the plug element. This section shows the mathematical development of these equations for a tractrix and the tractrix of revolution as applied to plug design.

Tractrix Curve

The basic equations for the tractrix curve are developed in an earlier paragraph. The tractrix is the locus of points of a curve the length of whose tangent is constant from a point on the curve to an intersection point on the x-axis. When the curve is rotated about the x-axis to form a body of revolution, the resulting figure resembles a trumpet bell. In the field of non-Euclidian geometry, this shape is known as a pseudosphere. It is a surface of constant negative curvature. The coordinates of the tractrix curve can be calculated using equation 3.

Arc Length

The arc length of the tractrix from any two points on the curve is easy to develop.

An element of arc length is

$$ds = - \sqrt{1 - \left(\frac{dx}{dy}\right)^2} dy$$

But, by inverting equation 2, we have

$$\frac{dx}{dy} = - \frac{\sqrt{a^2 - y^2}}{y}$$

Substituting

$$ds = - \sqrt{1 - \left[\left(- \frac{\sqrt{a^2 - y^2}}{y} \right)^2 \right]} dy$$

simplifying

$$ds = - \sqrt{1 + \frac{a^2 - y^2}{y^2}} dy$$

$$ds = - \sqrt{1 + \frac{a^2}{y^2} - \frac{y^2}{y^2}} dy$$

$$ds = - \frac{a}{y} dy$$

Integrating between the limits of y_1 and y_2 yields:

$$s = -a \int_{y_1}^{y_2} \frac{dy}{y}$$

$$= -a \ln y \Big|_{y_1}^{y_2}$$

$$= -a [\ln y_2 - \ln y_1]$$

$$= a [\ln y_1 - \ln y_2]$$

$$\boxed{s = a \ln \frac{y_1}{y_2}}$$

EQ. 4

Volume

The volume of the pseudosphere is found by integration of the tractrix body of revolution.

The volume of an elemental strip of volume of the pseudosphere is

$$d(\text{vol}) = \pi y^2 dx$$

$$\text{vol} = \pi \int y^2 dx$$

substituting Eq. 2

$$\text{vol} = -\int \pi y^2 \frac{\sqrt{a^2 - y^2}}{y} dy$$

$$\text{vol} = \frac{\pi}{2} \int_{y_1}^{y_2} -2 y^2 (a^2 - y^2)^{1/2} dy$$

using formula 132, p. 251 of math handbook (Ref. 3) yields.

$$\text{vol} = \frac{\pi}{2} \left[\frac{(a^2 - y^2)^{3/2}}{3/2} \right] \Big|_{y_1}^{y_2}$$

$$= \frac{\pi}{3} \left[(a^2 - y^2)^{3/2} \right] \Big|_{y_1}^{y_2}$$

$$\text{vol} = \frac{\pi}{3} \left[(a^2 - y_2^2)^{3/2} - (a^2 - y_1^2)^{3/2} \right]$$

$$\text{let } Z1 = \sqrt{a^2 - y_1^2}$$

$$Z2 = \sqrt{a^2 - y_2^2}$$

$$\boxed{\text{vol} = \frac{\pi}{3} [Z2^3 - Z1^3]}$$

EQ. 5

Surface Area

Wear calculations are dependent on the contact surface area. It is therefore important to calculate this parameter.

The equation to calculate the surface area of the pseudosphere is developed as follows:

$$A = \int_{y_1}^{y_2} 2 \pi y dx$$

substituting equation 2 into this expression, yield:

$$\begin{aligned} A &= -2 \pi \int_{y_1}^{y_2} y \frac{(a^2 - y^2)^{1/2}}{y} dy \\ &= -2\pi \int_{y_1}^{y_2} (a^2 - y^2)^{1/2} dy \end{aligned}$$

Using formula 131, p. 251 math handbook (Ref. 3)

$$A = -\pi \left(y \sqrt{a^2 - y^2} + a^2 \sin^{-1} \frac{y}{a} \right) \Big|_{y_1}^{y_2}$$

from the IBM handbook (Ref. 4) we have:

$$\sin^{-1} x = \arctan \frac{x}{\sqrt{1 - x^2}}$$

$$A = -\pi \left(y \sqrt{a^2 - y^2} + a^2 \arctan \frac{y/a}{\sqrt{1 - (y/a)^2}} \right)$$

EQ. 6

$$A = -\pi (y \sqrt{a^2 - y^2} + a^2 \arctan \frac{y}{\sqrt{a^2 - y^2}})$$

$$\text{let } Z = \sqrt{a^2 - y^2}, \quad Z1 = \sqrt{a^2 - y_1^2}, \quad Z2 = \sqrt{a^2 - y_2^2}$$

$$A = -\pi (y Z + a^2 \arctan \frac{y}{Z}) \Big|_{y_1}^{y_2}$$

substituting limits

$$A = \pi (y_1 Z1 + a^2 \arctan \frac{y_1}{Z1} - y_2 Z2 - a^2 \arctan \frac{y_2}{Z2})$$

EQ. 7

Center of Gravity

The center of gravity of a pseudosphere is derived as follows:

$$\bar{x} = \frac{\int_{y_1}^{y_2} x d(vol)}{vol}$$

$$\bar{x} = \frac{\int \pi y^2 x dx}{vol}$$

substituting eq. 2 and 3

$$X1 = \frac{\bar{x} \text{ vol}}{\pi} = \int_{y_1}^{y_2} y \left(\sqrt{a^2 - y^2} + a \cosh^{-1} \frac{a}{y} \right) \left(-\frac{\sqrt{a^2 - y^2}}{y} \right) dy$$

simplifying

$$X1 = \int y (a^2 - y^2) dy \quad - \quad a \int y \sqrt{a^2 - y^2} \cosh^{-1} a/y dy$$

1st intergal
2nd intergal

Evaluation of the first integral:

$$\begin{aligned}
 &= -\frac{1}{2} \int -2 y (a^2 - y^2) dy \\
 &= -\frac{1}{2} \left[\frac{(a^2 - y^2)^2}{2} \right] \Big|_{y_1}^{y_2} \\
 &= -\frac{1}{4} \left[a^2 - y_2^2 - (a^2 - y_1^2) \right] \\
 &= -\frac{1}{4} \left[(a^2 - y_2^2) - (a^2 - y_1^2) \right]
 \end{aligned}$$

Evaluation of the second integral:

$$\frac{-a}{-2} \int -2 y \sqrt{a^2 - y^2} \cosh^{-1} \frac{a}{y} dy$$

rearrange

$$\frac{a}{2} \int \underbrace{\left(\cosh^{-1} \frac{a}{y} \right)}_u \underbrace{(-2 y \sqrt{a^2 - y^2} dy)}_{dy}$$

This can be integrated by parts

$$\int u dv = uv - \int v du.$$

A definition for v and du is needed.

$$v = \int dv = \int -2 y (a^2 - y^2)^{1/2} dy$$

$$v = \frac{(a^2 - y^2)^{3/2}}{3/2}$$

$$u = \cosh^{-1} a/y$$

$$du = \frac{d(a/y)}{\sqrt{a^2/y^2 - 1}}$$

$$= \frac{-a/y^2 dy}{\frac{\sqrt{a^2 - y^2}}{y^2}}$$

$$du = \frac{-a dy}{y\sqrt{a^2 - y^2}}$$

substituting

$$\frac{a}{2} \left[\frac{2}{3} (\cosh^{-1} \frac{a}{y}) (a^2 - y^2)^{3/2} + \frac{2}{3} a \int \frac{\sqrt{a^2 - y^2} (a^2 - y^2) dy}{\sqrt{a^2 - y^2} y} \right]$$

integrate by parts once again.

$$\int \underbrace{(a^2 - y^2)}_u \underbrace{\frac{dy}{y}}_{dv}$$

$$v = \int dv = \int \frac{dy}{y}$$

$$v = \ln y$$

$$u = a^2 - y^2$$

$$du = -2 y \, dy$$

$$uv = \int v \, du.$$

$$= (a^2 - y^2) \, du \, y - \int -2 y \ln y \, dy$$

from formula no. 317, p. 263 math handbook (Ref. 3)

$$= (a^2 - y^2) \ln y + 2 y^2 \left(\frac{\ln y}{2} - \frac{1}{4} \right)$$

$$= (a^2 - y^2) \ln y + y^2 (\ln y - .5)$$

adding all terms together gives;

$$x1 = \frac{-(a^2 - y^2)^2}{4} + \frac{a}{3} (a^2 - y^2)^{3/2} \cosh^{-1} \frac{a}{y} +$$

$$\frac{a^2}{3} (a^2 - y^2) \ln y + \frac{a^2 y^2}{3} (\ln y - .5)$$

$$\text{let } Z = \sqrt{a^2 - y^2}$$

$$\text{Then } Z^2 = a^2 - y^2$$

$$Z^3 = (a^2 - y^2)^{3/2}$$

$$x1 = \frac{-Z^4}{4} + \frac{a}{3} Z^3 \cosh^{-1} a/y + \frac{a^2}{3} Z^2 \ln y +$$

$$\frac{a^2 y^2}{3} (\ln y - .5)$$

from IBM handbook

$$\cosh^{-1} \frac{a}{y} = \ln \left(\frac{a}{y} + \sqrt{\left(\frac{a}{y} \right)^2 - 1} \right)$$

$$= \ln \left(\frac{a}{y} + \sqrt{a^2 - y^2} / y \right)$$

$$\bar{x} = \frac{\pi}{vol} \left[-\frac{Z^4}{4} + \frac{a}{3} Z^3 \ln \left(\frac{a}{y} + \frac{Z}{y} \right) + \frac{a^2}{3} Z^2 \ln y + \frac{a^2 y^2}{3} (\ln y - .5) \right] \Big|_{y_1}^{y_2}$$

Substituting limits

$$\begin{aligned} \bar{x} = \frac{\pi}{vol} & \left[-\frac{Z_2^4}{4} + \frac{a}{3} Z_2^3 \ln \left(\frac{a}{y_2} + \frac{Z_2}{y_2} \right) + \frac{a^2}{3} Z_2^2 \ln y_2 + \frac{a^2 y_2^2}{3} (\ln y_2 - .5) + \right. \\ & \left. + \frac{Z_1^4}{4} - \frac{a}{3} Z_1^3 \ln \left(\frac{a}{y_1} + \frac{Z_1}{y_1} \right) - \frac{a^2}{3} Z_1^2 \ln y_1 + \frac{a^2 y_1^2}{3} (\ln y_1 - .5) \right] \text{ EQ. 8} \end{aligned}$$

This equation yields the center of gravity location relative to the y axis as shown in the diagram.

COMPUTER PROGRAM DESIGN

The previous mathematical developments represent substantial effort if the calculations were to be done by hand. A computer program could quickly and accurately make these calculations. A permanent record would result. Thus, it was decided to write a computer-aided design program to perform the calculations and print them in a readily understandable format.

Plug Sizing

The principle design input parameter is the pipe line diameter. The maximum diameter of the plug is selected to be 1-1/2 times the pipeline diameter, and the constant, a , for the tractrix is selected to be 3 times the pipeline diameter.

Slot Design

To spread the area of the hole through the plug over the plug surface, an oval shaped hole is selected. The cross-sectional area of the pipe is maintained as the same area through the valve. The width of the oval made half the line diameter while the oval length is adjusted to provide the same area as the pipeline. The following formulas are used to define the dimensions of the oval.

Width of slot

D = pipeline diameter

W = width of slot.

$$W = \frac{1}{2} D$$

Pipeline cross-sectional area, A :

$$A = \frac{\pi}{4} D^2$$

Slot Length,

$$= \frac{(A - \frac{\pi}{4} W^2)}{W}$$

Sketch of slot shape

$$L.O.A. = \ell + W$$

An edge distance of W is selected. Plug length becomes:

$$L = \quad + 3W$$

Under certain circumstances the edge distance may be made larger or smaller. For example, a high pressure valve may require thicker material for the edge distance to help withstand the stress imposed by the higher pressure.

The area ratio for this plug design is based on the fact that the hole goes clear through the plug. Thus the area is removed from the surface twice. The ratio of hole area to total surface area is:

$$AR = 2 A1/AS$$

Parameters

Based on the value of y_1 , the value of x_1 can be found by the basic formula for the tractrix. The value of x_2 is then x_1 plus the overall length of the plug.

Finding the value of y_2 that corresponds to x_2 is more of a problem since the equation contains transcendental terms: it cannot be solved explicitly for y . The Newton-Raphson method is selected to iterate the basic equations to determine the value of y_2 that corresponds to x_2 . This establishes all the values needed to determine the various parameters.

The mid-section of the program calculates the arc length, surface area, volume, and center of gravity of the plug.

These parameters are printed out in a format which labels the parameter and gives its units.

Lastly, the coordinates of the tractrix are calculated and printed out in 0.1 in. increments of y .

With all this information in hand, the design of the plug is complete.

EXAMPLES OF PLUG DESIGN

Calculations were made using the computer program to design a plug for a valve for a pipeline which has a diameter of 1 inch. Two examples are given. The first is for an edge distance of one-half the slot width and the second is for an edge distance equal to the slot width. Computer printouts and diagrams are shown in the accompanying figures and tables.

Diameter	=	1 in.
Projected Area	=	2.438107 in.
Volume	=	3.53032 cu. in.
Arc Length	=	2.244661 in.
Surface Area	=	9.611787 sq. in.
Center of Gravity	=	3.286293 in.
Overall Length	=	2.178101 in.
Slot Length	=	1.678101 in.
Slot Width	=	.5 in.
Slot Area	=	.7854 sq. in.
Area Ratio	=	.1634243

X, in.	Y, in.
2.459814	1
2.759643	.9
3.098528	.8
3.48642	.7
3.937908	.5999999
4.475627	.4999999
4.637916	.473208

Diameter = 1 in.
 Projected Area = 6.006905 in.
 Volume = 8.332142 cu. in.
 Arc Length = 2.84372 in.
 Surface Area = 16.15875 sq. in.
 Center of Gravity = 2.290572 in.
 Overall Length = 2.678101 in.
 Slot Length = 1.678101 in.
 Slot Width = .5 in.
 Slot Area = .7854 sq. in.
 Area Ratio = 9.721048E-02

X, in.	Y, in.
1.352798	1.5
1.534001	1.4
1.732551	1.3
1.950853	1.2
2.191957	1.1
2.459815	.9999999
2.759644	.8999999
3.098529	.7999998
3.486421	.6999998
3.937908	.5999998
4.030899	.5813271

CONCLUSIONS

Based on the foregoing presentation, the following conclusions are presented:

1. The tractrix is the curve that will give uniform wear for the plug of a valve.
2. Other shapes used for valves, such as the frustum of a cone and a sphere, will wear unevenly and eventually leak around the plug.
3. A series of mathematical developments has provided a means to evaluate the arc length, volume, surface area, and center of gravity of a tractrix of revolution.

An oval shaped hole was selected for the fluid passage through the plug. The cross-sectional area of the oval is identical to the pipe cross-sectional area.

4. A computer program has been written to perform the design calculations for the plug. The program uses the equations developed in this report. Satisfactory verification of the equations and the computer program has been obtained using approximate methods (See Appendix B).

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4. IBM BASIC Handbook, Appendix E.

APPENDIX A
COMPUTER PROGRAM LISTING

COMPUTER PROGRAM LISTINGS

General

Computer programs were written to make the design calculations using the equations derived in the main body of this report. These programs are written in BASIC which was available on the IBM-PC. Two programs were written: one for a low pressure valve and one for a high pressure valve. A listing for each program is included in this appendix. A sample run for each program is also included.

Program Description

Plug size and the slot through the plug have been made proportional to the pipeline diameter. The necessary input is the size of the pipeline. Plug length, volume, arc length, surface area, projected area, center of gravity, and slot dimensions are calculated by the program and printed. This is followed by a listing of the coordinates of the tractrix curve. These parameters completely characterize the design of the plug.

The two programs differ only in statement No. 60. This is the statement which determines the length of the plug. The edge distance is different for the two valve designs depending on whether it is for a high pressure or a low pressure application.

Running a Program

On the IBM-PC, one can access BASIC by typing

```
DOS> BASICA
```

A list of functions will appear at the bottom of the screen. Function key F3 is equal to the LOAD command. The file name given to one of the programs is VALVE1.

```
F3 VALVE1
```

If a hard copy of the printout is desired, the printer has an on-off toggle switch. The command for this is:

```
CTRL = SHIFT PRT SC
```

Function F2 is equal to the RUN command.

```
F2
```

The program will request the pipeline diameter. Input this value in units of inches.

```
INPUT LINE DIAMETER  
? 1
```

The program will run and print out the results. When complete, type

```
SYSTEM
```

to return to DOS.

```

5 REM FILENAME: VALVE1
10 PRINT "      IMPROVED PLUG VALVE"
15 PRINT "      DESIGN CALCULATIONS"
17 PRINT "      HIGH PRESSURE"
20 PRINT
25 PRINT
30 PRINT "INPUT LINE DIAMETER"
31 INPUT D
35 PRINT
40 PRINT
45 PI = 3.14159
50 W = .5*D
55 A1 = .7854*D^2
57 L1 = (A1 - .25*PI*W^2)/W
60 L = L1 + 3*W
65 Y1 = 1.5*D
70 A = Y1^2
75 Z1 = SQR(A^2 - Y1^2)
80 X1 = -Z1 + A*LOG(A/Y1 + Z1/Y1)
85 X2 = L + X1
100 Y2 = EXP(-(L + A*LOG(Y1))/A)
105 Y3 = Y2
110 Z2 = SQR(A^2 - Y2^2)
115 Y2 = Y2 - (X2 - A*LOG(A/Y2 + Z2/Y2) + Z2)/(Z2/Y2)
120 IF Y2 - Y3 < .0001 THEN 200
150 GOTO 105
200 Z2 = SQR(A^2 - Y2^2)
205 V = (PI/3)*(Z2^3 - Z1^3)
210 S = A*LOG(Y1/Y2)
215 PA = PI*(Y1^2 - Y2^2)
220 AS = PI*(Y1*Z1 + A^2*ATN(Y1/Z1) - Y2*Z2 - A^2*ATN(Y2/Z2))
230 V1 = PI/V
240 C = (.3333*A^2*Z2^2)*LOG(Y2) - (.3333*A^2*Z1^2)*LOG(Y1) + (.3333*A^2*Y2^2)*(
LOG(Y2) - .5) - (.3333*A^2*Y1^2)*(LOG(Y1) - .5) - .25*Z2^4 + .25*Z1^4 + .3333*A*
(Z2^3)*(LOG(A/Y2 + Z2/Y2)) - .3333*A*(Z1^3)*(LOG(A/Y1 + Z1/Y1))
250 CB = C*V1
255 PRINT
260 PRINT "DIAMETER = ";D;" IN."
262 PRINT "LENGTH = ";L;" IN."
265 PRINT "VOLUME = ";V;" CU IN."
270 PRINT "ARC LENGTH = ";S;" IN."
275 PRINT "SURFACE AREA = ";AS;" SQ IN."
280 PRINT "PROJECTED AREA = ";PA;" SQ IN."
285 PRINT "CENTER OF GRAVITY = ";CB;" IN."
290 PRINT
295 PRINT "SLOT LENGTH = ";L1 + W;" IN."
300 PRINT "SLOT AREA = ";A1;" SQ IN."
305 PRINT "AREA RATIO = ";2*A1/AS
310 PRINT
315 PRINT "X, IN.          Y, IN."
320 PRINT
325 FOR Y = Y1 TO Y2 STEP -.1
330 Z = SQR(A^2 - Y^2)
335 X = A*LOG(A/Y + Z/Y) - Z
340 PRINT X,Y
345 NEXT Y
350 PRINT X2,Y2
355 END

```


RUN

IMPROVED PLUG VALVE
DESIGN CALCULATIONS
HIGH PRESSURE

INPUT LINE DIAMETER

? 1

DIAMETER = 1 IN.
LENGTH = 2.678101 IN.
VOLUME = 8.332142 CU IN.
ARC LENGTH = 2.84372 IN.
SURFACE AREA = 16.15875 SQ IN.
PROJECTED AREA = 6.006905 SQ IN.
CENTER OF GRAVITY = 2.290572 IN.

SLOT LENGTH = 1.678101 IN.
SLOT AREA = .7854 SQ IN.
AREA RATIO = 9.721048E-02

X, IN.	Y, IN.
--------	--------

1.352798	1.5
1.534001	1.4
1.732551	1.3
1.950853	1.2
2.191957	1.1
2.459815	.9999999
2.759644	.8999999
3.098529	.7999998
3.486421	.6999998
3.937908	.5999998
4.030899	.5813271

0

```

5 REM FILENAME: VALVE2
10 PRINT "      IMPROVED PLUG VALVE"
15 PRINT "      DESIGN CALCULATIONS"
17 PRINT "      LOW PRESSURE"
20 PRINT
25 PRINT
30 PRINT "INPUT LINE DIAMETER"
31 INPUT D
35 PRINT
40 PRINT
45 PI = 3.14159
50 W = .5*D
55 A1 = .7854*D^2
57 L1 = (A1 - .25*PI*W^2)/W
60 L = L1 + 1.5*W
65 Y1 = D
70 A = Y1^2
75 Z1 = SQR(A^2 - Y1^2)
80 X1 = -Z1 + A*LOG(A/Y1 + Z1/Y1)
85 X2 = L + X1
100 Y2 = EXP(-(L + A*LOG(Y1))/A)
105 Y3 = Y2
110 Z2 = SQR(A^2 - Y2^2)
115 Y2 = Y2 - (X2 - A*LOG(A/Y2 + Z2/Y2) + Z2)/(Z2/Y2)
120 IF Y2 - Y3 <= .0001 THEN 200
150 GOTO 105
200 Z2 = SQR(A^2 - Y2^2)
205 V = (PI/3)*(Z2^3 - Z1^3)
210 S = A*LOG(Y1/Y2)
215 PA = PI*(Y1^2 - Y2^2)
220 AS = PI*(Y1*Z1 + A^2*ATN(Y1/Z1) - Y2*Z2 - A^2*ATN(Y2/Z2))
230 V1 = PI/V
240 C = (.3333*A^2*Z2^2)*LOG(Y2) - (.3333*A^2*Z1^2)*LOG(Y1) + (.3333*A^2*Y2^2)*(
LOG(Y2) - .5) - (.3333*A^2*Y1^2)*(LOG(Y1) - .5) - .25*Z2^4 + .25*Z1^4 + .3333*A*
(Z2^3)*(LOG(A/Y2 + Z2/Y2)) - .3333*A*(Z1^3)*(LOG(A/Y1 + Z1/Y1))
250 CG = C*V1
255 PRINT
260 PRINT "DIAMETER = ";D;" IN."
262 PRINT "LENGTH = ";L;" IN."
265 PRINT "VOLUME = ";V;" CU IN."
270 PRINT "ARC LENGTH = ";S;" IN."
275 PRINT "SURFACE AREA = ";AS;" SQ IN."
280 PRINT "PROJECTED AREA = ";PA;" SQ IN."
285 PRINT "CENTER OF GRAVITY = ";CG;" IN."
290 PRINT
295 PRINT "SLOT LENGTH = ";L1 + W;" IN."
300 PRINT "SLOT AREA = ";A1;" SQ IN."
305 PRINT "AREA RATIO = ";2*A1/AS
310 PRINT
315 PRINT "X, IN.          Y, IN."
320 PRINT
325 FOR Y = Y1 TO Y2 STEP -.1
330 Z = SQR(A^2 - Y^2)
335 X = A*LOG(A/Y + Z/Y) - Z
340 PRINT X,Y
345 NEXT Y
350 PRINT X2,Y2
355 END

```

RUN

IMPROVED PLUG VALVE
DESIGN CALCULATIONS
LOW PRESSURE

INPUT LINE DIAMETER

? 1

DIAMETER = 1 IN.
LENGTH = 1.928101 IN.
VOLUME = 2.532831 CU IN.
ARC LENGTH = 2.044514 IN.
SURFACE AREA = 7.52448 SQ IN.
PROJECTED AREA = 2.734933 SQ IN.
CENTER OF GRAVITY = 1.558176 IN.

SLOT LENGTH = 1.678101 IN.
SLOT AREA = .7854 SQ IN.
AREA RATIO = .2087586

X, IN.	Y, IN.
.901865	1
1.087314	.9
1.300569	.8
1.548167	.7
1.839763	.5999999
2.190383	.4999999
2.625272	.3999999
2.829966	.359782

0

APPENDIX B
VERIFICATION OF COMPUTER PROGRAM OUTPUT

COMPUTER PROGRAM VERIFICATION

It is necessary to check the results obtained with the computer program that calculates parameters for the tractrix of revolution. This verification has two purposes; to check the derivation of the equations and to check and verify the numbers obtained from the computer program.

The method used is to take well established equations and use these to approximate the area, volume, etc., of the tractrix of revolution and compare the results with the computer results.

Volume Verification

The volume of the tractrix of revolution for example number 1 is, vol = 2.5328 cu. in. The tractrix of revolution can be approximated by two frustums of a cone.

Sketch B-1. Two frustums of a cone approximate the tractrix of revolution.

It is well known that the volume of a frustum of a cone is given by the following formula:

$$VOL = \frac{1}{3} \pi R_2^2 L \left[1 + \frac{R_1}{R_2} + \left(\frac{R_1}{R_2} \right)^2 \right]$$

substituting values,

$$\begin{aligned} VOL &= \frac{1}{3} \pi (0.6)^2 \cdot .9217 \left[1 + \frac{1.0}{.6} + \left(\frac{1.0}{.6} \right)^2 \right] + \\ &\quad \frac{1}{3} \pi (.359)^2 \cdot 1.0064 \left[1 + \frac{.6}{.359} + \left(\frac{.6}{.359} \right)^2 \right] \\ &= 1.8916 + .7422 \end{aligned}$$

$$VOL = 2.6338 \text{ Cu. In.} \quad \text{Vol of Cone}$$

$$VOL = 2.5328 \text{ Cu. In.} \quad \text{Vol of tractrix of revolution}$$

The tractrix of revolution is contained within the frustum of a cone. Because of the concave sides, the volumes of the tractrix is smaller than the circumscribed cone volume.

It can be seen that these values are what was expected: the cone volume is slightly larger than the tractrix. The values agree within 0.1 cu. in.

Surface Area

Using a similar procedure to the above method, the surface area of the tractrix of revolution can be approximated by the frustum of a cone. In this case, it can be expected that the area of the cone will be slightly larger than the tractrix of revolution due to the slightly smaller radius of the tractrix.

$$\text{let } S = \sqrt{(R_1 + R_2)^2 + L^2}$$

Then the frustum of a cone has a surface area of:

$$A = \pi (R_1 + R_2) S$$

Using values from Sketch B-1

$$\begin{aligned} A &= \pi (1.0 + 0.6) \sqrt{(1 - .6)^2 + (.9217)^2} + \\ &\quad \pi (0.6 + .359) \sqrt{(.6 - .359)^2 + (1.0064)^2} \\ &= 5.0504 + 3.1177 \end{aligned}$$

$$A = 8.1681 \text{ sq. in.} \quad \text{cone area}$$

$$A = 7.5244 \text{ sq. in.} \quad \text{tractrix area}$$

It is found that indeed the frustum of a cone has a larger surface area than the tractrix revolution as expected. The difference is found to be 0.64 sq. in. Values and trends are as expected for this area verification.

Arc Length

The arc length for the side of the tractrix is calculated to be $s = 2.2446$ inches. A first approximation for the side is a straight line joining the two end points. The Pythagorean Theorem yields:

$$s = \sqrt{(1 - .473)^2 + 2.178^2}$$

Arc length of cone

Arc length of tractrix

$$s_c = 2.2408 \text{ in.}$$

$$s_t = 2.2446 \text{ in.}$$

Thus, we see these values are very comparable, differing only in the third decimal place.

Conclusions

The derivation of the equations for volume, surface area, and arc length have been checked using the frustum of a cone as an approximation to the tractrix of revolution. Numerical values agree and trends are as expected. Derivations of equations and computer program results are verified as giving correct values.

END

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