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TITLE: INTRODUCTION TO MULTIPLE SCATTERING AND THE PION NUCLEUS OPTICAL POTENTIAL

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ABSTRACT

A brief introduction to t-matrices, multiple scattering and the pion-nucleus is given.

INTRODUCTION TO MULTIPLE SCATTERING AND THE PION NUCLEUS OPTICAL POTENTIAL

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In this first section, which should take about 10 lectures, I hope to lay the groundwork for the several lectures to follow. Professor J. R. Taylor will start the first part of the course and must be excused from some lectures. I hope to give 10 lectures that would be the first part of the course. I hope to give the second part of the course in the next section. I should like to give the third part of the course in the next section.

TWO BODY THEORY

Let us start with the two body problem. We consider a particle of mass m moving in a potential $V(r)$. The wave function $\psi(r)$ satisfies the Schrödinger equation

$$\nabla^2 \psi + k^2 \psi = V(r) \psi$$

where $k^2 = 2mE$. The wave function $\psi(r)$ can be expanded in terms of partial waves

$$\psi(r) = \sum_l A_l j_l(kr) Y_{lm}(\theta, \phi)$$

where j_l is the spherical Bessel function and Y_{lm} is the spherical harmonic. The wave function $\psi(r)$ can also be expanded in terms of plane waves

$$\psi(r) = \int d^3q e^{i\mathbf{q} \cdot \mathbf{r}} \tilde{\psi}(\mathbf{q})$$

where $\tilde{\psi}(\mathbf{q})$ is the Fourier transform of $\psi(r)$. The wave function $\psi(r)$ can also be expanded in terms of scattering states

$$\psi(r) = \sum_l A_l e^{i\mathbf{q} \cdot \mathbf{r}} Y_{lm}(\theta, \phi)$$

where $e^{i\mathbf{q} \cdot \mathbf{r}}$ is the plane wave and Y_{lm} is the spherical harmonic.

$$\psi(r) = \sum_l A_l e^{i\mathbf{q} \cdot \mathbf{r}} Y_{lm}(\theta, \phi) \quad (1)$$

In the two body problem, the wave function $\psi(r)$ can be expanded in terms of plane waves

$$\psi(r) = \sum_l A_l e^{i\mathbf{q} \cdot \mathbf{r}} Y_{lm}(\theta, \phi) \quad (2)$$

We express $\psi(r)$ as

$$\psi(r) = \sum_l A_l e^{i\mathbf{q} \cdot \mathbf{r}} Y_{lm}(\theta, \phi) \quad (3)$$

Formally we can write

$$\psi(r) = e^{i\mathbf{q} \cdot \mathbf{r}} + (E - H_0)^{-1} V(r) \psi(r) \quad (4)$$

where we have implicitly assumed that the plane wave boundary condition is satisfied. The plane wave boundary condition is satisfied if the wave function $\psi(r)$ is a solution of the Schrödinger equation

$$(E - K)e^{i\vec{q} \cdot \vec{r}} = 0 \quad (5)$$

To evaluate this equation we insert "one" in the form

$$1 = \int d\vec{r}' \delta(\vec{r} - \vec{r}') = \frac{1}{(2\pi)^3} \int d\vec{q}' d\vec{r}' e^{i\vec{q}' \cdot (\vec{r} - \vec{r}')} \quad (6)$$

We have thus introduced a complete set of eigenstates of K i.e.

$K e^{i\vec{q} \cdot \vec{r}} = (q^2/2m) e^{i\vec{q} \cdot \vec{r}}$ so that

$$\psi_{\vec{q}}(\vec{r}) = e^{i\vec{q} \cdot \vec{r}} + 2m \int \frac{d\vec{q}' e^{i\vec{q}' \cdot \vec{r}}}{q'^2 - q^2} \frac{1}{(2\pi)^3} \int e^{-i\vec{q}' \cdot \vec{r}'} v(\vec{r}') \psi_{\vec{q}}(\vec{r}') d\vec{r}' \quad (7)$$

We define

$$t(\vec{q}', \vec{q}) = \frac{1}{(2\pi)^3} \int e^{-i\vec{q}' \cdot \vec{r}'} v(\vec{r}') \psi_{\vec{q}}(\vec{r}') d\vec{r}' \quad (8)$$

as the t -matrix. Thus

$$\psi_{\vec{q}}(\vec{r}) = e^{i\vec{q} \cdot \vec{r}} + 2m \int \frac{d\vec{q}' e^{i\vec{q}' \cdot \vec{r}}}{q'^2 - q^2 + i\epsilon} t(\vec{q}', \vec{q}) \quad (9)$$

The $i\epsilon$ has been added in the Green's function so that we will have only outgoing waves at infinity.

We may write for a spherically symmetric potential

$$t(\vec{q}', \vec{q}) = \frac{(4\pi)^2}{(2\pi)^3} \sum_{l,m} Y_l^m(\vec{q}) Y_l^{m*}(\vec{q}') \int_0^\infty r^2 dr v(r) j_l(r) j_l(q' r) \\ = \sum_{l,m} t_l(q', q) Y_l^m(\vec{q}') Y_l^{m*}(\vec{q}) \quad (10)$$

$$t_l(q', q) = \frac{2}{\pi} \int_0^\infty r^2 dr v(r) j_l(r) j_l(q' r) \quad (11)$$

Note that

$$t_l(-q', q) = (-1)^l t_l(q', q) \quad (12)$$

To establish the relation between t_l to the scattering amplitude let us examine the second term of Eq. 9 for large q .

$$2m \int \frac{d\vec{q}' e^{i\vec{q}' \cdot \vec{r}}}{q'^2 - q^2 + i\epsilon} t(\vec{q}', \vec{q})$$

$$\begin{aligned}
&= (2m)(4\pi) \sum_{\ell m} i^\ell Y_\ell^m(\hat{q}) Y_\ell^{m*}(\hat{r}) \int_0^\infty \frac{q'^2 dq' j_\ell(q' r) t_\ell(q', q)}{q^2 - q'^2 + i\eta} \\
&= (2m)(2\pi) \sum_{\ell m} i^\ell Y_\ell^m(\hat{q}) Y_\ell^{m*}(\hat{r}) \int_{-\infty}^\infty \frac{q'^2 dq' j_\ell(q' r) t_\ell(q', q)}{q^2 - q'^2 + i\eta} \\
&= (2m)\pi \sum_{\ell m} i^\ell Y_\ell^m(\hat{q}) Y_\ell^{m*}(\hat{r}) \int_{-\infty}^\infty \frac{q'^2 dq' (h_\ell^{(+)}(q' r) + h_\ell^{(-)}(q' r)) t_\ell(q', q)}{q^2 - q'^2 + i\eta} \\
&\xrightarrow{r \rightarrow \infty} (2m)\pi \sum_{\ell m} \frac{Y_\ell^m(\hat{q}) Y_\ell^{m*}(\hat{r})}{r} \int_{-\infty}^\infty \frac{q' dq' (i^{-1} e^{iq' r} + i^{2\ell+1} e^{-iq' r}) t_\ell(q', q)}{q^2 - q'^2 + i\eta}
\end{aligned}$$

Completing the contour on the first piece in the upper half-plane (where $e^{iq'r}$ dies for $q \rightarrow +i$) and the second piece in the lower half plane and using Eq. 12 we find

$$m(2\pi)^2 \frac{e^{iqr}}{r} \sum_{\ell m} Y_\ell^m(\hat{q}) Y_\ell^{m*}(\hat{r}) t_\ell(q, q) \quad .$$

Thus

$$f(\theta) = m(2\pi)^2 t(q', q) ; \quad |\vec{q}'| = |\vec{q}| \quad (13)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad (14)$$

and the t-matrix is identified as the scattering amplitude (with constant factors) for $|\vec{q}'| = |\vec{q}|$ i.e. on shell.

Note that we may obtain an equation for t itself by multiplying Eq. 9 by $1/(2\pi)^3 e^{-iq \cdot r} V(r)$ and integrating

$$t(q', q) = V(q', q) + 2m \int \frac{d\vec{q}'' V(q', q'') t(q'', q)}{q'^2 - q''^2 + i\eta} \quad (15)$$

Also note that the energy in the Green's function need not be related to either momentum and we obtain

$$t(q', q, E) = V(q', q) + 2m \int \frac{d\vec{q}'' V(q', q'') t(q'', q, E)}{2mE - q''^2 + i\eta} \quad , \quad (16)$$

an equation for the fully-off-shell t-matrix.

Now that I have written out several of the equations in detail I will begin using operator notation. To write out the full equation one must insert appropriate complete sets of states as was done above.

We define an operator $t(E)$ such that

$$t(\vec{q}, \vec{q}, E) = \langle \vec{q} | t(E) | \vec{q} \rangle \quad (17)$$

We also use

$$G \equiv \frac{1}{E - K - V} \quad , \quad G_0 \equiv \frac{1}{E - K} \quad (18)$$

Useful algebraic identities are:

$$G = G_0 + G_0 V G \quad (19a)$$

$$= G_0 + G V G_0 \quad (19b)$$

In this notation Eq. 16 may be written as (suppressing the variable E):

$$t = V + V G_0 t \quad (20)$$

Using Eqs. 19 one can show that

$$t = V + V G V \quad (21)$$

and

$$t = V + t G_0 V \quad (22)$$

which are useful alternative forms of Eq. 20.

THE MULTIPLE SCATTERING EQUATIONS

We wish to solve the $(A+1)$ body Schrödinger equation given as

$$(K + \sum_{i=1}^A V_i + H_N - E)\psi = 0 \quad (23)$$

where K is the pion kinetic energy, V_i is the pion interaction potential with the i^{th} nucleon, ψ is the total $(A+1)$ -body wave function, and H_N is the nuclear Hamiltonian with eigenstates given by:

$$H_N \phi_n(\vec{r}_1 \cdots \vec{r}_A) = E_n \phi_n(\vec{r}_1 \cdots \vec{r}_A) \quad (24)$$

We will use the notations:

$$G = \frac{1}{E - K - V - H_N} \quad , \quad G_0 = \frac{1}{E - K - H_N} \quad , \quad G_N = \frac{1}{E - K} \quad (25)$$

Introducing the boundary condition as before

$$\psi = \phi_0 e^{i\vec{q} \cdot \vec{r}} + G_0 \sum_{i=1}^A V_i \psi \quad (26)$$

We may identify, as before, the pion-nuclear-ground-state

t-matrix as

$$\begin{aligned} T(\vec{q}, \vec{q}) &= \frac{1}{(2\pi)^3} \int e^{-i\vec{q} \cdot \vec{r}} \phi_0 \sum_{i=1}^A v_i \psi d\vec{r} d\vec{r}_1 \cdots d\vec{r}_A \quad (27) \\ &= \sum T_i(\vec{q}, \vec{q}) \end{aligned}$$

Multiplying Eq. 26 by v_i we have

$$T_i = v_i + v_i G_0 \sum_{j=1}^A T_j \quad (28)$$

To eliminate the pion-nucleon potential we define τ_i such that

$$\begin{aligned} \tau_i &= v_i + v_i G_0 \tau_i \\ &= v_i + \tau_i G_0 v_i \\ &= (1 + \tau_i G_0) v_i \quad (29) \end{aligned}$$

Thus

$$v_i = (1 + \tau_i G_0)^{-1} \tau_i \quad (30)$$

Substituting Eq. 30 in Eq. 28 we get

$$T_i = \tau_i + \tau_i G_0 \sum_{j \neq i} T_j \quad (31)$$

which are the desired multiple scattering equations. We may get the Watson multiple scattering series by iterating these equations

$$T = \tau_i + \sum_{j \neq i} \tau_i G_0 \tau_j + \sum_{\substack{i \neq j \\ j \neq k}} \tau_i G_0 \tau_j G_0 \tau_k + \cdots \quad (32)$$

These two forms of the multiple scattering equations² are the basis of most of the work that has been done on pion nucleus scattering.

THE τ_i OPERATOR

We would like to have an expression for pion nucleus scattering in terms of pion-nucleon t-matrices, but we do not have that since G_0 still contains the nuclear Hamiltonian. The quantity τ_i may be interpreted as the pion-nucleon t-matrix in a nuclear medium. The evaluation of this quantity has a long history and some of the later lectures will be on this subject but I will mention some of the things which have been done.

a) Ignore the nuclear Hamiltonian. This is referred to as the

impulse approximation and gives $\tau_1(E) \approx t_1(E)$, the free pion-nucleon t-matrix.

b) Use closure. Inserting a complete set of nuclear states in the basic equation for τ_1 we have

$$\begin{aligned}\tau_1 &= V_1 + V_1 G_0 \tau_1 \\ &= V_1 + \sum_n \frac{V_1 |n\rangle \langle n| \tau_1}{E - K - E_n}.\end{aligned}\quad (33)$$

Assuming that there exists some effective excitation energy $\bar{E}$ and removing the denominator from the sum we recover the free pion-nucleon t-matrix at a different energy i.e.

$$\tau_1(E) \approx t_1(E - \bar{E}) \quad (34)$$

There are several prescriptions in existence for how to choose $\bar{E}$.

c) Include true pion absorption.³ Using the form

$$\tau_1 = V_1 + V_1 G V_1 \quad (35)$$

the complete spectrum of pion states must be included. Since in the $T = \frac{1}{2}$, $J = \frac{1}{2}$ channel the nucleon pole, corresponding to true pion absorption, is in the sum, the use of the τ_1 in various optical model terms allows this effect to be included.

d) Discuss modification of the τ_1 in nuclear matter. Since this will be discussed in detail in later lectures I will not say more.

FIRST ORDER OPTICAL POTENTIAL⁴

We may project out the scattering to the ground state by taking the expectation values of Eqs. 31 on nuclear variables only

$$\langle 0 | T_1 | 0 \rangle = \langle 0 | \tau_1 | 0 \rangle + \sum_{j \neq i} \frac{\langle 0 | \tau_1 | n \rangle \langle n | \tau_j | 0 \rangle}{E - K - E_n + i\eta}$$

If we require the nucleus to remain in the ground state in intermediate scatterings (the coherence approximation) we obtain:

$$\bar{T}_1 = \tau_1 + \tau_1 g \sum_{j \neq i} \bar{T}_j \quad (36)$$

Note that since we have assumed that everything that leaves the incident channel never comes back we expect the result to be that the resulting calculation is too absorptive. Since the wave function is antisymmetrized the τ_i are all identical and so must be the $\bar{T}_i$.

$$\bar{T} = A\bar{T} + A\bar{T} g \frac{A-1}{A} T \quad (37)$$

$$\text{Defining } \tilde{T} = \frac{A-1}{A} T \quad (38)$$

$$\tilde{T} = (A-1)\tilde{T} + (A-1)\tilde{T}g\tilde{T} \quad (39)$$

and we may identify the first order optical potential as

$$U_0 = (A-1)\tilde{T} \quad (40)$$

If we use a single particle model wave function for the nucleus we have:

$$\tilde{T} = \int \phi^*(\vec{r}) e^{-i(\vec{q}'-\vec{q})\cdot\vec{r}} \tau(\vec{q}',\vec{q},\vec{p}_i,\vec{p}_f) \phi(\vec{r}) d\vec{r} \quad (41)$$

where $\vec{p}_i$ is a momentum operator on the ϕ to the right and $\vec{p}_f$ operates on the ϕ to the left. This momentum dependence comes about because the $\tau(t)$ is defined in the π -nucleon center-of-mass and we are now in the π -nucleus center-of-mass. Since $\vec{p}_i$ and $\vec{p}_f$ are of the order of Fermi momenta and there is a factor of $\sim u/m$ reducing them compared with $\vec{q}$ and $\vec{q}'$ it might be thought that they can be ignored. If we do, we have the "factorization approximation".

$$U_0 = (A-1)\tau(\vec{q}',\vec{q})S(\vec{q}'-\vec{q}) ; S(\vec{q}'-\vec{q}) = \int e^{i(\vec{q}'-\vec{q})\cdot\vec{r}} |\phi|^2 d\vec{r} \quad (42)$$

It has recently been found that it is important to correct for this approximation. I will discuss briefly this so-called "angle transformation" in a later lecture.

THE KISSLINGER POTENTIAL⁵

For heavy nuclei the form factor is strongly peaked about $\vec{q}'=\vec{q}$. If the t-matrix is slowly varying it is a reasonable approximation to evaluate it at that point. This is common practice in the nucleon-nucleus optical model. Kisslinger pointed out that, because of the strong p-wave nature of the π -nucleon interaction, the form factor does not dominate the t-matrix, especially for light nuclei. Thus he proposed a potential based on a minimum momentum dependence.

$$U_0 = (A-1)(\lambda_0 + \lambda_1 \vec{q} \cdot \vec{q}') S(\vec{q}'-\vec{q}) \quad (43)$$

This leads to a non-local potential (of a derivative type) in coordinate space.

$$U\psi = (A-1)[\lambda_0 \rho + \lambda_1 \nabla \cdot \rho \nabla] \psi \quad (44)$$

The idea is certainly correct but we now know that another refinement is necessary. The t-matrix used in Eq. 43 continues to become stronger the farther it is taken off-shell. This is equivalent to the statement that the pion-nucleon interaction is zero range. We know that both the pion and the nucleon are not point particles so some factor must be introduced to provide this finite

range. The technique usually used for doing this is the one of separable potentials leading to separable t-matrices. This will be the subject of a later lecture.

It is perhaps worth mentioning in passing that there exists another, similar, potential called the Laplacian potential. To obtain this one notes that, on-shell,

$$\begin{aligned}\vec{q} \cdot \vec{q}' &= -\frac{1}{2} (\vec{q} - \vec{q}')^2 + q^2 \\ &= \omega^2 - \mu^2 - \frac{1}{2} (\vec{q}' - \vec{q})^2\end{aligned}\quad (45)$$

and the t-matrix

$$\lambda_0 + \lambda_1 (\omega^2 - \mu^2) - \frac{1}{2} \lambda_1 (\vec{q}' - \vec{q})^2 \quad (46)$$

is used. Since it involves only differences of momenta it is local, reducing to a Laplacian operator on the density, hence the name.

HIGHER ORDER OPTICAL POTENTIALS

The corrections to the first order optical potential take two forms. One of these is first found in second order scattering the other in third order.

The first effect arises from correlations. If we use a simple single particle product wave function for the nucleus then the intermediate states in the second term of Eq. 32 are automatically required to be in the ground state since if the first τ excites a single particle state the second cannot deexcite it because it operates on a different single particle coordinate. Introducing a correlation operator gives a correction to the first order optical potential which depends strongly on the correlation function and the pion-nucleon range. This effect has been shown⁶ to be approximately equivalent to the pionic Lorentz-Lorenz effect.⁷

The second correction is found to start in the third term and occurs because even a single particle wave function can be excited in the intermediate state when $i=k$.

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