ANL/PHY-93/1

ARGONNE NATIONAL LABORATORY
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**LASER COOLING OF A STORED ION BEAM -
A FIRST STEP TOWARDS CRYSTALLINE BEAMS**

by

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**A DISSERTATION SUBMITTED TO
THE FACULTY OF THE PHYSICAL SCIENCES
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY**

**THE UNIVERSITY OF CHICAGO
DEPARTMENT OF PHYSICS**

September 1992

This work was carried out while the author held a Research Associate appointment in the Physics Division at Argonne National Laboratory.

Supported in part through the Argonne Physics Division by the U.S. Department of Energy, Office of Basic Energy Sciences.

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CHAPTER 1

Introduction

Section 1.1 Motivation - Ordered Coulomb Systems

In 1985, Schiffer and Kienle suggested that a dense stored beam of fully-stripped heavy ions, cooled to a very low temperature in the co-moving frame, might form an ordered Coulomb system [SCH1]. Such a beam represents a special case of a one-component plasma (OCP). The classical OCP consists of an ion species which is free to move, surrounded by a neutralizing background charge which does not participate in the dynamics [ICH1]. In a storage ring, the confining forces of the magnetic elements in the ring balance the repulsive space-charge force of the ions. The external confinement force is equivalent to the effect of the neutralizing background charge in the OCP. The beam, which may have a very large longitudinal velocity, is quasi-harmonically confined in the transverse directions. The behavior of a one-component plasma may be characterized by the plasma parameter Γ , defined by

$$\Gamma = \frac{(Ze)^2}{KT}$$

where Ze is the ion charge, T is the temperature, K is the Boltzmann constant and a is given by

$$a = \left(\frac{4}{3} \pi \rho \right)^{-\frac{1}{3}}$$

where ρ is the ion number density. Γ is thus just the ratio of interparticle Coulomb potential energy to thermal energy for the OCP. The infinite OCP has been studied theoretically using various numerical techniques [HANS1,BRU1]. Systems with Γ greater than about 5 exhibit correlations characteristic of a liquid and are often referred to as "condensed"; systems with Γ greater than 170 have long-range order and are referred to as "crystalline" [HANS1]. In the high- Γ limit, the infinite OCP has the symmetry of the body-centered cubic (BCC) crystal [BRU1]. We deal here with systems in which quantum effects are negligible, due to the large interparticle spacing. The quantum mechanical analogue to a high- Γ OCP is known in the literature as a Wigner solid, due to Wigner's early work on the behavior of electrons in alkali metals [WIG1].

The classical ordered OCP thus represents a new type of matter. It is an ordered state analogous to a crystalline solid, but with a density many orders of magnitude lower. Typical values of the parameter a for an ordered OCP in a storage ring would be of order ten microns, corresponding to a number density of order 10^8 cm^{-3} . This may be compared to typical crystalline solid densities, which are of order 10^{22} cm^{-3} . In addition to representing the ultimate in phase space density for a storage ring, the "crystalline beam" may be of fundamental interest to the solid state physicist as well. Such a system also offers the potential for atomic spectroscopic measurements which are nearly free from the adverse effects of Doppler broadening. Other uses for a very cold stored ion beam, such as in spectroscopic tests of special relativity, have been suggested [GRI1].

Research with ordered Coulomb systems in storage rings is not as advanced as that in ion traps (Chapter 3). Several "beam cooling" techniques have been developed to provide stored beams with very low momentum spread and high particle density. Typical applications are in the accumulation of antiprotons for high energy physics research and in the production of nearly monoenergetic stored beams for high resolution nuclear physics experiments. The physics of these cooled beams is no longer that of independent particles. Such beams exhibit collective behavior and suffer from dynamical instabilities which are best understood using the methods of plasma physics. However, until quite recently, the prospect for producing a fast stored beam with $\Gamma > 1$ was considered to be remote.

Historically, the first information on the behavior of very cold stored beams came out of the pioneering experiments on the NAP-M storage ring at Novosibirsk [DEM1]. This machine employed electron cooling [BUD1] to produce a low

temperature stored beam of protons. Longitudinal temperatures of about 1 K (measured in the co-moving frame) were reported. Collective motion of the particles in the beam, in the form of longitudinal plasma waves, was observed. Most surprising, however, was the reported suppressed coupling between the transverse and longitudinal motions of the protons [DIK1]. Electron cooling is primarily a method for cooling the longitudinal motion of the beam; it has limited effectiveness in cooling the transverse motions. For dense beams, the much higher transverse temperature should limit the achievable minimum longitudinal temperature, as the transverse energy is coupled to the longitudinal motion by Coulomb collisions in the beam plasma. The NAP-M experimenters, however, achieved longitudinal temperatures that had an anomalous dependence on beam current. This result led to the interpretation that longitudinal ordering of the beam particles caused suppression of the intrabeam Coulomb scattering [DIK1].

While the Novosibirsk experiment does not provide conclusive evidence of ordering in a storage ring, the experimental parameters may be consistent with a theoretically predicted transition from uncorrelated motion to longitudinal order [SCH3]. The ordering parameter for the longitudinal motion only has been estimated to have been as high as $\Gamma \sim 10$ [MÖL1]. This experiment will be discussed in greater detail in Chapter 3.

Recently, more modern storage rings, equipped with electron cooling devices, have approached the conditions of the NAP-M experiment [BOS1, STE1]. More promising however, is the advent of laser cooling for fast stored ion beams [JAI1]. The first laser cooling in a storage ring was observed in the Heidelberg Test Storage Ring (TSR) in 1990 [SCHR1]. Using 13 MeV $^7\text{Li}^+$ ions, the experimenters achieved a longitudinal temperature of about 3 K. The beam exhibited collective longitudinal motion, although the transverse temperature was many orders of magnitude higher. At about the same time, laser cooling experiments were begun using the ASTRID storage ring at the University of Århus in Denmark. Using a beam of 100 keV $^7\text{Li}^+$ ions, a longitudinal temperature of 1 mK was achieved [HAN1]. The use of a laser as a sensitive diagnostic probe of beam dynamics was also demonstrated in the ASTRID ring.

It is these first results of laser cooling in ASTRID and the subsequent attempts to understand them and to improve upon them that form the experimental basis of this thesis. The emphasis is on the physics of the strongly coupled beam plasma and the implications for the attainment of crystalline beams. The theoretical study, by means of molecular dynamics simulations, of the ordered

OCP in a storage ring was the motivation for the experimental work and is reviewed in some detail. Some new MD results are presented. A complete overview follows.

Section 1.2 Overview of the Thesis

The thesis is separated into three main parts. The first part deals mostly with theory and background material necessary for an understanding of both the physics of the OCP confined in a storage ring and the experimental attempts to reach the strongly coupled regime. The subject matter is decidedly interdisciplinary, drawing from plasma physics, accelerator physics, atomic and laser physics, and statistical physics. Chapter 2 provides a very basic introduction to storage ring dynamics and the terminology used to describe it. Chapter 3 describes the modeling of the OCP in a storage ring using molecular dynamics. Described also in Chapter 3 are the conventional beam density and temperature limitations in storage rings. The NAP-M experiment is discussed in greater detail. Chapter 4 provides an introduction to the physics of laser cooling and its application to storage rings.

The second part of the thesis concerns laser cooling and diagnostic experiments in the storage ring ASTRID. Chapter 5 describes the experimental setup - the ring and diagnostic systems. The thesis summarizes four distinct periods of experimental endeavor in ASTRID. These are treated chronologically to emphasize the development of the experimental technique and the consequent improvement in characterization of the physical system. Chapter 6 reports the results of the first ASTRID experiments with ${}^7\text{Li}^+$ beam. These experiments achieved the lowest longitudinal temperature ever reported in a cooled ion beam [HAN1]. Resonant laser diagnostics of stored ion beams were introduced with very promising results. Chapter 7 covers a series of experiments involving laser cooling of ${}^{166}\text{Er}^+$ ions. These experiments, which were unsuccessful, have important implications for the application of laser cooling to storage rings. The return to cooling experiments with ${}^7\text{Li}^+$ ions is the subject of Chapter 8. The experimental technique was refined and the interplay between laser cooling and beam plasma dynamics was investigated. Laser acceleration of the ${}^7\text{Li}^+$ ions was also studied. Transverse laser probing of lithium beam [WAN1] is also reported in Chapter 8, as are the results of further studies aimed at characterizing the storage ring performance.

The final section of the thesis concerns the future of crystalline beam experiments. Chapter 9 summarizes the experimental results in the context of their implications for the attainment of an ordered OCP in a storage ring. Chapter 10 outlines the proposed improvements to the experimental program in ASTRID for the next generation of laser cooling experiments.

CHAPTER 2

A Brief Introduction to Storage Rings

The following sections provide an introduction to the physics of storage rings. Only the concepts and nomenclature necessary for the description of the experimental results and the molecular dynamics simulations of ordered beams are treated.

Section 2.1 Basics

A storage ring is a device for containing a circulating beam of charged particles. The particles may be electrons, protons, partially or fully stripped ions, antiprotons or negative ions, to name a few. Today's storage rings are used in a wide range of physics research, ranging in energy from a few eV to almost 1 TeV for the Fermilab Tevatron. The most common uses are for nuclear and high energy physics research, and for storage of electrons to produce synchrotron radiation.

The basic components of such a device are magnetic elements for directing and containing the beam and a vacuum system for minimizing the interactions of the stored particles with the rest gas. In addition, an injection system for producing and accelerating the charged particles is required. The storage ring may itself be an accelerator, but, strictly speaking, operation in storage ring mode implies fixed energy.

Dipole magnets are used to cause the beam particles to follow a closed path, which normally lies in a horizontal plane (Figure 2-1). Magnetic quadrupoles are employed to confine the beam to a small transverse size. The principle of alternating gradient (AG) focusing [COU1] is used in the confining system. Quadrupole magnets act as focusing lenses in one transverse plane and as defocusing lenses in the other (Figure 2-1). A quadrupole that focuses in the horizontal (vertical) plane is referred to as an F (D) quadrupole. Net focusing

(stability) can be achieved by judicious placement of F and D quadrupoles and field-free regions. The arrangement of magnetic elements in a storage ring is often referred to as the "magnetic lattice" of the machine.

Figure 2-1 (a) A typical storage ring layout, showing dipole and quadrupole magnets (b) A representation of the ideal orbit, illustrating the local coordinate system (c) A cross-section through an F-quadrupole, illustrating magnetic field lines and lines of force.

Section 2.2 Betatron Motion

Particles whose initial positions in the ring differ from the ideal closed orbit will undergo transverse oscillations about the reference orbit. This motion is known as betatron motion. The oscillations are not strictly harmonic because of the alternating gradient configuration. The betatron motion for a plane (the x -plane

is used for illustration) may be described in terms of x , the transverse position of the particle relative to the ideal orbit; s , the longitudinal position of the particle in the machine; and $x' = dx/ds$, the angle the trajectory makes with the reference trajectory. The local coordinate system is shown in Figure 2-2. The linearized equation of motion may be written

$$\frac{d^2x}{ds^2} - K_x(s)x = 0$$

where $K_x(s)$ is the local focusing (or defocusing) coefficient at position s in the ring. An analogous equation can be written for the y -plane. K is determined by the strength of the quadrupole magnets and has the property

$$K_x(s) = -K_y(s)$$

The solution for the betatron motion for a single particle (subscript j) may be written as follows

$$x_j(s) = \sqrt{\epsilon_{jx}\beta_x(s)} \cos(\Psi_x(s) + \delta_j)$$

The function β_x is the envelope function for the transverse motion and is determined by the strengths and placements of the quadrupole magnets. It is periodic in the circumference C of the machine. Ψ_x is the betatron phase advance, given by

$$\Psi_x(s) = \int_0^s \frac{ds}{\beta_x(s)}$$

β_x and Ψ_x characterize the machine; ϵ_{jx} and δ_j are determined by the initial conditions for a given particle. The number of oscillations performed by a particle per round trip in the storage ring is known as the tune, designated Q_x or Q_y for horizontal or vertical motion. The tune is related to the phase advance by

$$2\pi Q_x = \Psi_x(C) = \int_0^C \frac{ds}{\beta_x(s)}$$

The parameter ϵ_x is also determined by the initial conditions of the particle's orbit.. The significance of ϵ_x may be seen by considering a fixed position s_i in the ring. The trajectory of a particle passing s_i may be mapped from turn to turn by plotting a point in the x - x' plane for each passage (Figure 2-2). After many turns, the particle has mapped out an ellipse in the x - x' plane. The maximum transverse displacement of the particle at position s_i is just $\sqrt{\epsilon_x \beta_x(s_i)}$. The area of the ellipse in x - x' space is $\pi \epsilon_x$, independent of the position s_i . The so-called single particle emittance, ϵ_x , is a constant of the motion. The orientation of a given particle's ellipse will change from place to place in the ring, but the area is constant. The invariant may be parametrized as [COU1]

$$\epsilon_x = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

where

$$\alpha = -\frac{1}{2} \frac{d\beta}{ds}$$

and

$$\gamma = \frac{1 + \alpha^2}{\beta}$$

The ellipse is characterized by the three numbers α , β , and ϵ . The ellipse in Figure 2-2 has $\alpha=0$ and is referred to as "upright". At position s_i , each of the many particles in the beam will trace an ellipse geometrically similar to that in Figure 2-2. The emittance of a collection of particles is defined as the area of the ellipse containing a set fraction of the single particle ellipses. Various fractions are in common use, such as 95% or, for a Gaussian emittance distribution, 2σ of the projection of the distribution into the spatial plane. The acceptance or admittance of the machine is the largest emittance a particle may have and still circulate without encountering an aperture.

Figure 2-2 Positions in the x - x' plane recorded by an observer at a fixed longitudinal position in a storage ring. Twenty consecutive turns are plotted. The ellipse has $\beta = 3$ m, $\alpha = 0$, and $\varepsilon = 5.3$ mm-mrad. The tune is 2.809.

Section 2.3 Dispersion and Chromaticity

The treatment of transverse motion has ignored the velocity dependence of the magnetic guiding force. This may be included, to first order, by introducing the dispersion function D_x , defined by

$$\Delta x(s) = D_x(s) \frac{\delta p}{p}$$

where Δx is the horizontal displacement from the ideal orbit for a particle with momentum deviation δp . D_x is also periodic in the circumference of the machine. The betatron oscillations of an off-momentum particle are centered on the dispersed orbit, so the dispersion contributes to the physical beam size. For a beam whose particles have no correlation between emittance and momentum deviation, the beam size may be written

$$(a_x(s))^2 = \beta_x(s)\epsilon_x + \left(D_x(s) \frac{\delta p}{p} \right)^2$$

where a_x is the horizontal beam size. Most storage rings are built in a horizontal plane; a vertical dispersion may be analogously defined if necessary.

The quadrupole magnets also produce velocity-dependent forces. The tune is thus a function of the particle's momentum. This dependence is quantified to first order as the chromaticity ξ defined by

$$\Delta Q = \xi \frac{\delta p}{p}$$

The chromaticity may be corrected by adding sextupole magnets (magnets with a field proportional to x^2) to regions of non-zero dispersion. The detailed determination of the chromaticity and dispersion functions will not be considered here. The terms are introduced for their utility in discussing the beam behavior.

Section 2.4 Nonlinear Resonances

In the design of a storage ring, the effects of imperfections in the guide and focusing fields on the transverse motion must be considered. Random field errors and nonlinear multipole components act as perturbations to the quasi-harmonic betatron motion. In addition to distorting the lattice parameters from their design values, these imperfections may drive nonlinear resonances, which can lead to beam instability and loss [WIL1]. The physical situation to be avoided is that in which particles follow orbit paths which expose them to perturbations in phase with their betatron motion. For example, a machine with a tune which is an integer is inherently unstable, as the orbit closes on itself.

A more general resonance condition, which anticipates the possibility of coupling between the horizontal and vertical betatron motion, may be written

$$nQ_x + mQ_y = p$$

where n , m , and p are integers. Each combination of n , m , and p describes a so-called resonance line in Q_x - Q_y space. The order of the resonance is $|n|+|m|$.

Generally speaking, the lower the order, the faster is the growth time of the instability. The tunes must be chosen to avoid operation in the vicinity of a resonance line. For storage rings with long storage life-times, resonances of up to about 20th order must be considered. The fractional tunes must be controlled with a precision of about .001.

Section 2.5 Longitudinal Motion

With a single exception, the beams to be discussed in the experimental section of the thesis are "unbunched", that is, the ions are uniformly distributed around the circumference of the storage ring. (The term "bunched" stems from the spatial organization of ions into groups by the radiofrequency electric fields used for particle acceleration.) In the absence of particle interactions, the longitudinal motion of the ions is just steady circulation at constant momentum. For a particle with the ideal velocity v_0 the revolution frequency is just

$$f_0 = \frac{v_0}{C}$$

In the general, relativistic case, particles with slight momentum deviation have an error in revolution frequency given by

$$\frac{\delta f}{f} = \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} \right) \frac{\delta p}{p}$$

where γ is the relativistic factor for the beam and γ_t is a characteristic of the machine, the "transition energy". The expression above arises due to the velocity dependence of the Lorentz force and due to the nonlinear relation between p and v in the relativistic case. Below transition energy, revolution frequency increases with momentum; above transition, higher momentum particles have lower revolution frequencies. The ASTRID ring operates well below transition, where relativistic effects are negligible.

In dense beams with low momentum spread, the interparticle interactions and the electromagnetic interaction of the beam with the storage ring vacuum chamber may lead to longitudinal collective behavior of the beam. An example is the observation of longitudinal charge-density waves in the Novosibirsk

experiments [DEM1]. Sustained coherent phenomena in a storage ring are referred to as instabilities. Instabilities may lead to deterioration of beam quality or to particle loss from the machine. The transverse resonant phenomena described above are a particular class of instability. Specific destabilizing mechanisms relevant to the present considerations will be introduced in later sections.

Section 2.6 Liouville's Theorem and Beam Cooling

For a many-particle dynamical system, it is impossible to know the initial position and momentum of each particle individually. A statistical description of the system behavior is necessary. For an N -particle system, the state of the system may be specified by the location of a point in the $6N$ -dimensional phase-space of $3N$ generalized coordinates q_i and $3N$ conjugate momenta p_i . A phase space density distribution ρ may be defined [SØR1] such that

$$\rho(q, p, t) dq dp, \text{ where } dq dp = \prod_i dq_i dp_i$$

gives the probability for finding the system in volume $dq dp$ about (q, p) at time t . The Liouville theorem states that the phase-space density ρ is a constant along a trajectory, provided that the forces on the system are derivable from potentials [LIC1]. Note that Coulomb interactions between particles are conservative forces and so do not alter the $6N$ -dimensional phase-space density ρ .

The Liouville theorem in the $6N$ -dimensional phase-space defined above is of little practical value in dealing with storage ring dynamics, where N may be 10^{12} or more. It is, however, possible to derive an analogous theorem in a 6-dimensional phase space [LIC1] when the interactions of the particles can be ignored. The 6-dimensional phase-space density $f(q_1, q_2, q_3, p_1, p_2, p_3, t)$ is defined to be the number of particles in the elemental volume about $(q_1, q_2, q_3, p_1, p_2, p_3)$ at time t . The Liouville theorem (for Hamiltonian forces only) is simply

$$\frac{df}{dt} = 0$$

The 6-dimensional Liouville theorem may be generalized to allow the inclusion of intrabeam interactions such as collisions. The Liouville theorem then leads to the Boltzmann equation with a collision term [LIC1]

$$\frac{\partial f}{\partial t} + \left(\frac{\partial f}{\partial p_i} \dot{p}_i + \frac{\partial f}{\partial q_i} \dot{q}_i \right) = \frac{\partial f}{\partial t} \Big|_{\text{collision}}$$

The six-dimensional phase space density is not conserved.

The transverse planes (x-x') and (y-y') defined above are related to the 6 dimensional phase space. A volume element in 6-dimensional phase-space may be written

$$dV = dx dp_x dy dp_y dz dp_z$$

Concentrating on the x coordinates, we may write

$$dx dp_x = dx d(mc\beta_x \gamma) = mc\beta \gamma dx dx'$$

where β and γ are the relativistic factors. Similar equations may be written for y and z. Thus, Liouville's theorem implies that, for a storage ring at constant energy, the product of the three emittances $\epsilon_x \epsilon_y \epsilon_z$ is conserved, provided that intrabeam interactions can be neglected. Under the condition of no coupling forces between planes, each individual emittance is conserved (i.e. when the Hamiltonian is separable). The terms "phase-space area" and "emittance" are often used interchangeably, but care must be taken in the application of the Liouville theorem to emittance.

The methods of beam cooling are devised to reduce the 6-dimensional phase space volume occupied by a group of particles by introducing external forces to act selectively on particles with large emittance. Electron cooling [BUD1] uses a low momentum-spread electron beam, overlapped with the stored ions in a section of the storage ring. The ions transfer momentum to the colder electrons by Coulomb interactions, thereby reducing the longitudinal momentum spread of the ion beam. Electron cooling is not very effective for cooling the transverse motion, as the transverse temperature of the electron beam is much higher than the longitudinal. Typical electron cooling systems in use today can reduce the momentum spread of a few-MeV proton beam from 10^{-3} to less than 10^{-5} in less than a second.

Higher energy storage rings, such as the antiproton accumulation rings at CERN (3.5 GeV) and at Fermilab (8 GeV), employ stochastic cooling [VAN1] for

reducing beam emittances. The antiprotons, which are created by collisions of a proton beam with a target, occupy a large phase space volume. This volume must be reduced to allow the antiprotons to be injected into a high energy storage ring for use in colliding beam experiments. A stochastic cooling system is essentially a high-bandwidth feedback system which senses errors in particle transverse positions or longitudinal momenta and compensates them by applying correcting "kicks" (via electric fields) to the particles. Cooling times range from seconds to minutes, depending on the bandwidth of the system and the number of particles to be cooled. The cooling limits for existing stochastic cooling systems are not low enough to reach the regime of $\Gamma > 1$, but stochastic cooling may be useful in conjunction with electron or laser cooling to enhance the transverse cooling. Laser cooling will be discussed in detail in Chapter 4.

Section 2.7 Schottky Diagnostics of Particle Beams

A particle beam circulating in a storage ring is not a continuous current but an accumulation of a finite number of discrete charges. The discrete nature of the charge can be exploited for diagnostic purposes. Fluctuations in the macroscopic quantities which characterize the beam, such as the average current, the average transverse displacement, average dipole moment, etc., can be detected by suitably designed pickups [CHA1]. From these measurements, much information about the microscopic dynamics can be inferred. Signals due to fluctuations in these macroscopic quantities are generally (and unfortunately) referred to as "Schottky noise". Far from being noise, these signals provide the most widely useful, nondestructive diagnostic information available in modern storage rings. The Schottky signal is also the input to the feedback loop in stochastic cooling systems.

A pickup located at a fixed azimuth θ_p (the variable $\theta = 2\pi s/C$) in a storage ring will detect each particle passage as a delta-like current pulse. A single particle j has a current [CHA1]

$$I_j(t) = q\omega_j \sum_{m=-\infty}^{+\infty} \delta(\theta_j(t) - \theta_p - 2\pi m)$$

Using the Fourier series representation of the periodic delta function and summing over all particles in the ring, the total current may be written

$$I^P(t) = q \sum_{j=1}^N \frac{\omega_j}{2\pi} \sum_{m=-\infty}^{\infty} e^{-im(\theta_j(t) - \theta_s)}$$

A particle circulating steadily in the ring without perturbation has

$$\theta_j(t) = \omega_j t + \theta_j^0$$

where θ_j^0 is the initial longitudinal phase for particle j . We deal here with unbunched beams, so the θ_j^0 are randomly distributed between 0 and 2π . The total current measured by the pickup is thus

$$I^P(t) = q \sum_{j=1}^N \frac{\omega_j}{2\pi} \sum_{m=-\infty}^{\infty} e^{-im(\omega_j t + \theta_j^0 - \theta_s)}$$

The frequency-space representation of the current consists of bands centered at the revolution harmonics $m\omega_0$. The bands have frequency width $m\Delta\omega$ for beam revolution frequency spread $\Delta\omega$. For uncorrelated phases, only the DC ($m=0$) band contributes to the macroscopic current. For revolution frequencies evenly distributed about ω_0 , we have

$$\langle I^P(t) \rangle = qf_0 N$$

which is just the current due to N charges with revolution frequency f_0 .

In practical applications the Schottky signal is amplified and analyzed by a spectrum analyzer. The quantity of interest is thus the power spectral density of the current fluctuations. It can be shown that the shape of the power spectral density gives the distribution of revolution frequencies - and therefore of momenta - in the beam [CHA1]. The average current fluctuation power in a single revolution band is, for uncorrelated particles

$$\langle P_I \rangle_m = \frac{2q^2 \omega_0^2 N}{(2\pi)^2}$$

independent of m . The total power in a single band is thus proportional to the number of stored particles. The shape of the power spectral density will be distorted by correlations or collective interactions in the beam, that is, when the longitudinal phases are not randomly distributed. The analogous fluctuation power for a single band contains a term proportional to N^2 . The Schottky pickup is thus extremely sensitive to collective behavior and coherent beam instabilities.

A Schottky pickup may also be constructed to be sensitive to the transverse motion of the ions. Specifically, the transverse Schottky pickup measures the fluctuations in the dipole moment current of the beam in either the horizontal or vertical plane [CHA1]. The mathematics of the transverse signal will not be developed here. The frequency domain representation of the signals consists of so-called betatron sidebands at frequencies $(m \pm Q_f)f_0$, where Q_f is the fractional part of the tune. The transverse Schottky signals can be used to determine tunes, emittances, and chromaticities, and to diagnose transverse coherent instabilities.

CHAPTER 3

Crystalline Beams

As mentioned in Chapter 1, the NAP-M experimental results may be explained by postulating a spatial ordering of the proton beam. The idea that a cold, heavy-ion beam in a storage ring might be subject to the phase transitions of the classical OCP [SCH1] led a number of authors to study the physics of the OCP in a storage ring geometry. Rahman and Schiffer were the first to apply the molecular dynamics technique to a cold, confined, one-component plasma [RAH1]. It was their discovery of the layered cylindrical shell structure (Section 3.2) that first caught the imagination of many scientists, both theorists and experimentalists. The chronology of the development of the physics of ordered OCP's is summarized in [SCH3]. The field of crystalline beams has advanced to the point today that one group has proposed construction of a storage ring which would be dedicated to crystalline beam studies [BIS1].

This chapter reviews the results of the static and dynamic molecular dynamics calculations that form the basis for the current understanding of the cylindrically-confined OCP. Results of some analytical calculations are also discussed. The physics of the spherically confined OCP, which is relevant to storage of particles in ion traps, is briefly covered. The physical mechanisms which, in a storage ring, are known to hinder the effort to attain a cold, dense beam are reviewed. Finally, the experiment in the NAP-M storage ring in Novosibirsk is discussed in greater detail.

Section 3.1 Molecular Dynamics

Molecular Dynamics is a numerical technique for simulating the Newtonian dynamics of many-particle systems. In the case of the OCP in storage ring geometry, the motion is determined by the interparticle Coulomb force and the

external confining force. It is convenient to work in the frame moving relative to the laboratory with the average ion velocity. The first model considered by Rahman and Schiffer was a static (i.e. steady confinement force) model with 2-dimensional harmonic confinement [RAH1]. The Hamiltonian may be written

$$H = \sum_{i=1}^N \frac{1}{2} m (v_{xi}^2 + v_{yi}^2 + v_{zi}^2) + \sum_{i=1}^N \frac{1}{2} K (x_i^2 + y_i^2) + \frac{1}{2} \sum_{i \neq j} \frac{(Ze)^2}{r_{ij}}$$

where x and y are the transverse displacements from the central orbit, K is the force constant for the external confining field, and r_{ij} is the interparticle separation. This model Hamiltonian is the first approximation to the real situation in AG storage rings. The focusing force has cylindrical symmetry; the velocity dependence of the focusing force is neglected; and the curved trajectory through the dipole magnets is not considered. To make the connection with storage ring terminology, we can define the "betatron" frequency

$$\omega_\beta = \sqrt{\frac{K}{m}}$$

The MD calculation is performed on a system of N particles in a cubic volume of side L . The number of particles is limited by computer size and speed, but may be up to a few thousand. To simulate a length of stored beam, especially the axial component of the Coulomb force due to adjacent volumes, the volume is periodically repeated (Figure 3-1). The summation over two-particle Coulomb energies is performed using the Ewald technique [EWA1], a method which speeds convergence by defining the order of summation to reflect the physical importance of the terms.

The input to the MD calculation is the particle number N , the temperature T , the cell dimension L , the strength of the focusing force K , and the length of the calculational time step. Dimensionless units are used in the calculation [RAH1], but these will not be introduced here. The calculation proceeds step-wise in time. The forces are calculated at each time step, the system is allowed to evolve, the forces are calculated for the new particle positions, and so forth. The temperature is controlled by rescaling the velocities if the temperature goes above or below the

Figure 3-1 Cell geometry for the molecular dynamics calculations using cylindrical confinement.

prescribed limits. The size of the time step is determined by the physical situation being studied. Typically a time step is a small fraction ($<.01$) of the betatron period. The calculations require the use of a supercomputer, due to the demands of calculating the sum of two-particle Coulomb energies for 10^3 particles.

Section 3.2 Results of Static MD Simulations

The MD simulations for static cylindrical confinement revealed, for large Γ , the surprising arrangement of the beam particles into concentric cylindrical shells (Figure 3-2) [RAH1]. On the surface of a shell, the particles were found to exhibit hexagonal order, similar to that in the infinite 2-dimensional Coulomb solid. The three dimensional correlation functions of the particles are approximately consistent with BCC order [RAH2]. The number of shells was found to depend on the linear charge density of the beam. The linear density may be expressed as the dimensionless parameter λ , defined as the number of particles per length a , where a is the spacing defined in the introduction. The dependence of the configuration of cold static systems on the parameter λ has been studied in great detail [HAS1, HAB2]. In the following, consideration is limited to large Γ , above the crystallization threshold.

Figure 3-2 The equilibrium configuration of a very cold ($\Gamma=20000$) beam in static cylindrical confinement. The plot is a cross section through the beam. The λ parameter (see text) is 26. Reproduced from [SCH3].

For λ less than 0.709, the particles remain on the axis (i.e. the central orbit) of the storage ring. This configuration is known as a "string"; this may be the first configuration to become observable in real storage rings (Section 3.3). The dynamics of the string have been studied analytically by Habs [HAB3]. As λ increases, the particles move off of the axis, forming first a zig-zag pattern and then a helix. For λ greater than about 3.1, the configuration begins to have two layers - a single shell and a string. Two shells are well defined at $\lambda = 5.7$. The number of shells has been empirically parametrized as [SCH3]

$$N_s = 0.8\sqrt{\lambda}$$

The various configurations resulting from different values of λ are depicted graphically in Figure 3-3.

Figure 3-3 Configurations of crystallized ($\Gamma > 10000$) systems for different values of the parameter λ . For multishell systems, the number of particles per shell is proportional to the shell radius. The average density is constant. Lengths are in units of a . Reproduced from [SCH3].

The MD method has also been applied to static systems in spherical confinement, i.e. the confining force is $\vec{F} = -K\vec{r}$ in spherical coordinates. An interesting application of these calculations is the determination of the minimum energy configuration of few-particle systems. Several such systems are shown in Figure 3-4 [RAF1]. In addition to the MD method, the simple systems ($N=2, 3, 4$) can be determined analytically. A Monte Carlo method has also been employed for $N \leq 10$ [RAF1]. For large numbers of particles, the stable configuration consists of concentric spherical shells [RAH1].

Figure 3-4 *Stable configurations of few-particle Coulomb crystals in spherical confinement. Reproduced from [RAF1].*

The case of isotropic confinement is not merely of theoretical interest. Recent developments in the technology of trapping, storing, and cooling of ions have brought the ordered OCP to within reach of the experimentalist. Laser cooling [HÄN1,WIN1] in ion traps permits attainment of extremely low temperatures - in the sub-millikelvin range (see [CHU1] for a review.) Ions cooled to this temperature regime in Penning or Paul traps have been shown to exhibit order [DIE1,WIN2,GIL1], condensing into regular geometric forms in the case of few particles, and into layered shells for the case of many particles. The ion trap potentials are not spherically symmetric, but the MD method allows for simulations in asymmetric potentials as well. Recently, ordered ion systems have

also been studied in a quadrupole storage ring [BIR1], an ion trap having ring geometry not unlike the case of static cylindrical confinement described above. The predictions of the numerical models for these systems [DUB1,BLU1,RAF1,SCH2] have been beautifully realized in experiment.

Section 3.3 Results of Dynamic MD Simulations

Section 3.3.1 Alternating Focusing

As described in Chapter 2, a real storage ring does not have a static, cylindrically symmetric confining force. The obvious first correction to the static MD model is thus to try to simulate the AG focusing of the quadrupole magnets in the ring. Rahman and Schiffer first treated the AG effect approximately by turning the cylindrically symmetric focusing potential on and off at regular time intervals [RAH2]. The asymmetric fields of the quadrupole magnets are not reproduced in this first approximation, but the intent was to see what the response of the cold system was to transverse perturbation.

Starting with an ordered system ($\Gamma=1800$) in static confinement, Rahman and Schiffer found that the introduction of this alternating focusing led to coherent radial oscillation of the ordered beam, accompanied by a slow transfer of the oscillatory energy into the thermal motion of the other degrees of freedom [RAH2]. This radial oscillatory mode is referred to as the monopole mode. In the absence of cooling, the beam reaches the melting point ($\Gamma\sim 180$) in a time equivalent to a few round trips in the storage ring [SCH4]. The relevant concern for maintaining order is thus the strength of the cooling force in the machine.

A further refinement of the model was to modulate the focusing in the x and y transverse directions independently, i.e. only one plane has focusing switched on at any time. The result of this is to introduce a quadrupole shape oscillation [SCH4] in the beam. Again the oscillatory energy couples slowly into the thermal energy of the other degrees of freedom. The nature of the two types of oscillatory motion is depicted graphically in Figure 3-5.

Figure 3-5 Oscillatory modes of a crystalline system subjected to alternating focusing. Left: monopole oscillations driven by cylindrically symmetric, alternating focusing; the top figure shows the excursions in radius, the bottom figure shows the energy transfer to the longitudinal motion. Right: quadrupole oscillations driven by alternating x and y focusing; the top figure shows the out-of-phase excursions in x and y beam sizes, the bottom figure is again the energy transfer to the longitudinal motion. The unit of time is the betatron period. The full time scale is thus about 8 turns in a typical low energy storage ring. The unit of energy is .01 times the interparticle Coulomb energy $(Ze)^2/a$.

The frequencies of the quadrupole and monopole oscillatory modes are of interest. As depicted in Figure 3-6, the monopole frequency of the beam approaches $\sqrt{2}\omega_\beta$ as the beam temperature is decreased. The quadrupole frequency approaches ω_β . The monopole period for the cold beam can be calculated analytically. Consider an infinitely long arrangement of charges, with linear density λ particles/m, confined by a cylindrically symmetric harmonic force of strength K . At zero temperature, the equilibrium radius is determined by the balance between the force due to the beam's electric field and the confining force. The electric field at the surface is

$$E(r_0) = \frac{Ze\lambda}{2\pi\epsilon_0 r_0}$$

The force balance gives

$$r_0 = \left(\frac{(Ze)^2 \lambda}{2\pi\epsilon_0 K} \right)^{\frac{1}{2}}$$

If we now perturb the radius of the cylinder to $r_0 + \delta r$, the total force at the surface is

$$F = -Kr_0 \left(1 + \frac{\delta r}{r_0}\right) + \frac{Ze\lambda}{2\pi\epsilon_0 r_0 \left(1 + \frac{\delta r}{r_0}\right)}$$

Expanding the denominator of the last term and rearranging terms gives

$$F = -Kr_0 + \frac{Ze\lambda}{2\pi\epsilon_0 r_0} - \frac{\delta r}{r_0} \left(Kr_0 + \frac{Ze\lambda}{2\pi\epsilon_0 r_0} \right)$$

Employing the equilibrium condition, the first two terms on the right hand side sum to zero, and we are left with

$$F = -2K\delta r$$

The beam will oscillate around the equilibrium radius with angular frequency

$$\omega = \sqrt{\frac{2K}{M}} = \sqrt{2}\omega_\beta$$

which is the desired result. Note that derivation of this result is not dependent on the ordered nature of the beam. The frequency is characteristic of a cold, space-charge dominated beam of uniform density.

Figure 3-6 *Quadrupole (x') and monopole ($+$) periods, in units of the betatron period, versus temperature in units of $1/\Gamma$. The system has $\lambda=26$.*

Referring again to Figure 3-6, we note that the two mode frequencies are changing rapidly with temperature in the region between $\Gamma=1$ and $\Gamma=10$, and that they reach their cold beam values at temperatures well above the phase transition at $\Gamma=180$. Measurement of these frequencies could thus serve as an experimental indication of the onset of order [SCH3]. The monopole mode is probably not

directly excited by the magnets of a real storage ring. Pickups have been constructed to detect the coherent quadrupole oscillations in "hot" beams in the CERN and Fermilab antiproton storage rings [MAR1,CAR1]. Such a device could perhaps be applied to a cold beam if the sensitivity could be made high enough to detect the small oscillation amplitudes. Real-time measurement of the beam size by laser induced fluorescence is also an intriguing possibility [POU1].

The above considerations address the question of what happens to a static ordered system if it is subjected to alternating focusing. A more realistic question is whether a real beam, which is initially "hot", can be cooled to the space-charge dominated regime. The MD method is limited to temperatures less than about $\Gamma=0.1$. Above that, the large particle velocities necessitate the use of shorter time steps, leading to unrealistic requirements for computing time. Within the regime of applicability of the MD simulations, the use of alternating focusing does not seem to present any fundamental limitation to the achievement of strongly coupled OCP's in a storage ring [SCH3]. Experiments in ion traps containing few-particle Coulomb crystals have been carried out to try to simulate the alternating focusing force in a storage ring [BLÜ2]. The experimenters concluded that, for simple Coulomb systems, alternating focusing was not a severe obstacle to the formation of Coulomb crystals.

Section 3.3.2 More Realistic Cooling

The "cooling mechanism" applied in most MD simulations involves continuous rescaling of the particle velocities in all three dimensions to control the temperature to the specified range. Electron and laser cooling systems in real storage rings act predominantly on the longitudinal velocities of the particles. In addition, they are localized to a specific section of the ring; the particles experience a cooling force only in a fraction of their orbit.

MD simulations have also been run to try to simulate the experimental situation more closely [SCH3]. The "once per turn" condition can be met by renormalizing the velocities periodically on the time scale corresponding to an orbit period. The longitudinal cooling is imposed by renormalizing only the longitudinal velocities. Again, within the temperature regime accessible to the MD simulations, the combination of realistic cooling and alternating focusing does not seem to prevent attainment of a 3-dimensionally strongly coupled beam [SCH3]. The transverse thermal energy of the beam is removed by Coulomb-coupling to the

longitudinal motion [SCH3]. The coherent transverse shape oscillations of the beam are slowly damped by the same mechanism [SCH3]. The ability to cool a beam in three dimensions using only longitudinal cooling is of supreme importance to the experimental effort to obtain a uniformly cold beam, as there exist no external transverse cooling systems capable of reaching large Γ . It is important to note, however, that the maximum transverse temperatures studied with the MD simulations ($\Gamma \sim 0.1$) are still far below what has been achieved in a storage ring ($\Gamma \sim 10^{-4} - 10^{-6}$) [MÖL1].

Section 3.3.3 Shear in Storage Rings

The particle orbit in a storage ring contains regions with curvature, as the ions pass through the dipole magnets. For a beam which has horizontal extent, the particles towards the outside of the machine have larger path lengths through the dipoles (Figure 3-7). However, the purpose of the cooling system is to give the particles, as nearly as possible, precisely the same longitudinal velocity. Indeed, the existence of an ordered system implies that, in the particle frame, adjacent ions are nearly at rest relative to one another. An ordered system entering a dipole magnet will experience a horizontal "shearing" as the outside particles lag behind the inner ones [RAH1].

Figure 3-7 An illustration of shear in a dipole magnet, showing inner and outer particle trajectories and the longitudinal velocity gradient imposed by the MD model.

The implications for crystalline beams of the dipole shear have been studied using the MD simulations. The concern is whether the elasticity of the ordered system will prevent slippage between adjacent rows of particles, which would lead to heating of the beam [RAH1] and destruction of the order. The shear was studied by starting with a cold system and subjecting it to a period of shearing. The shear is imposed in the MD calculation by adding a horizontal gradient of longitudinal velocity across the beam (Figure 3-7). The beam is found to undergo "shear oscillations" after the imposed shear is stopped [SCH3]. The frequency of the shear oscillations has been determined as a function of linear charge density λ for the case of static confinement [SCH3]. These shear oscillations will be driven at the revolution frequency of the machine. Taking the betatron frequency as the measure of the elastic strength of the cold systems, Schiffer finds that ordered systems in the current generation of low-energy storage rings will not elastically resist the shearing force. The particles will slip freely, destroying the order [SCH3].

The shear seems to be a fundamental limitation to the attainment of three-dimensional order in existing machines. One possible solution is to use the cooling system to impose a slight longitudinal velocity gradient across the beam [SCH3]. This so-called "graded cooling" may be feasible with laser cooling [POU1]. Of course, for particle densities below the "string" threshold, the cold beam has no transverse extent and shear is irrelevant. It is for this reason that the string is of such experimental importance. The reduced coupling between particles for the string configuration unfortunately makes cooling the transverse motion using only longitudinal cooling problematic.

Section 3.4 Impediments to the Attainment of a Crystalline Beam

The molecular dynamics model considers the dynamics of very cold and dense beams. The MD simulations ignore the possible interaction of the beam with its surroundings and the possibility of transferring energy from the large laboratory kinetic energies of the particles into the thermal motion of the beam. There are several dynamical phenomena which may limit the performance of a storage ring at lower densities and much higher temperatures than simulated in the MD models. Many of these are covered in the excellent review by Möhl [MÖL1]. Here we deal with a few that have relevance to the experimental results reported in Chapters 6, 7, and 8 of the thesis.

Section 3.4.1 Coherent Longitudinal Instability

A dense cold beam may have liquid-like behavior, exhibiting correlations in the motion of its particles. Indeed, a beam must be "condensed" before it can be "crystallized". Such beams can support longitudinal charge density waves. The question of stability arises: are fluctuations in the longitudinal charge density damped naturally, or is the beam plasma unstable against these fluctuations? An approximate stability criterion has been obtained by Keil and Schnell [KEI1]. The natural damping mechanism is Landau damping due to the finite momentum spread in the beam [MÖL1]. Keil and Schnell considered the self-consistent electric fields due to a charge density fluctuation described by

$$\rho(s, t) = \rho_o + \Delta\rho e^{i(n\omega_r t \pm ns/R)}$$

where ρ_o is the unperturbed density, ω_r is the revolution frequency of the beam, R is the average radius of the machine, and n is the mode number of the fluctuation (the number of nodes the fluctuation has around the ring circumference.)

The response of the beam to the fields from any such fluctuation can be characterized by a generalized longitudinal impedance Z_n . The impedance Z_n for a low energy storage ring is dominated by the space charge term, given by [HOF1]

$$\frac{Z_n}{n} = \frac{Z_o}{2\beta\gamma^2} \left(1 + 2 \ln \frac{b}{a} \right)$$

where Z_o is the vacuum impedance (377 Ω), b is the vacuum chamber radius, a is the average beam radius, and β and γ are the relativistic factors. With these definitions, the Keil-Schnell criterion may be written [HOF1]

$$\frac{Z_n}{n} \leq \frac{mv^2 |\eta| \gamma}{(Ze)^2 N \omega_r} \left(\frac{\delta p}{p} \right)^2$$

η is the "off-momentum function"

$$\eta = \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} \right)$$

where γ_t is the transition energy (defined in Chapter 2). N is the number of stored particles. That this criterion spells trouble for crystalline beams can be seen immediately. For a given energy, stability is encouraged by large momentum spread, small particle number, and large beam size. This criterion only tells us if charge density fluctuations are damped or not. The implication is that, for an unstable beam the collective fluctuations will persist until dissipation of their energy increases the $\delta p/p$ (i.e. the longitudinal temperature) to the point where the above criterion is satisfied again.

The Keil-Schnell criterion will be employed in the discussion of the experimental results in Chapter 5. It should be noted that in the presence of a longitudinal cooling force, what matters is the growth time of the instability versus the cooling time of the cooling process. Growth times for the longitudinal coherent instability in a dense beam may be <1 ms [MÖL1].

Section 3.4.2 Space-Charge Tune Shift and Transverse Coherent Instability

As seen in the discussion of molecular dynamics and transverse oscillatory modes, the electrostatic forces due to the space charge of a dense beam can be of comparable strength to the confining force. Since the space charge force is repulsive, the effect is to reduce the effective confining force. In storage ring terminology, this is known as the space-charge tune shift [LAS1]. Accelerator physicists distinguish between two types of tune shift, coherent and incoherent [ZOT1]. The coherent shift applies to oscillations of the beam center (dipole oscillations). The incoherent shift applies to oscillations of single particles in the beam. The coherent shift depends on the beam spatial distribution and on the configuration of the vacuum chamber, because the image charges must be considered in the force calculation. The incoherent shift depends only on the beam's transverse spatial distribution.

For the cooled, low emittance beams which are under consideration, the coherent shift is small and can be ignored relative to the incoherent shift [MÖL1]. The incoherent shift may be written

$$\Delta Q_{x,y} = \left(\frac{(Ze)^2}{4\pi\epsilon_0 mc^2} \right) \frac{N}{\epsilon_{x,y} (1 + \sqrt{\epsilon_{y,x} / \epsilon_{x,y}}) \beta^2 \gamma^3}$$

Although we will deal only with non-relativistic beams here, the formula is written in general form to emphasize the energy dependence. At high energy, $\gamma \gg 1$, the tune shift becomes small because the force on the beam due to its own magnetic field begins to cancel the space-charge repulsion. Low energy storage rings are thus particularly susceptible to the tune shift. A zero temperature beam, collapsed to the radius r_o (Section 3.3.1 above) has an incoherent tune-shift equal to the tune.

The tune shift is problematic because even if the design tunes (for low beam density) are carefully chosen to keep the machine free from transverse resonances (Chapter 2), high beam density may cause tune-shifted particles to encounter resonance lines, leading to emittance blow-up or particle loss. Conventional wisdom states that the tune shift must be limited to be less than 0.01 [MÖL1] for a beam with no cooling. For a beam being cooled to the strongly coupled regime, resonance crossing is inevitable. The relevant concern is whether the cooling system response is fast enough to damp the emittance blow up. This question must be answered experimentally, but particles "locked" to a low order resonance are not likely to be damped by a cooling system, as the growth time may be of order of 10^{-5} s. The situation is particularly troubling for laser-cooled beams, where the only transverse cooling mechanism is the coupling to the longitudinal motion.

In addition to the incoherent process described above, a cold, dense beam may experience coherent instabilities, characterized by oscillation of the beam center around its nominal position (dipole motion). The natural damping mechanism for such motion is Landau damping due to the spread in tune in the beam (due to the chromaticity). Very cold beams have very small tune spreads, so the Landau damping is reduced. An approximate stability criterion can be obtained by requiring that the incoherent tune shift be smaller than the tune spread [MÖL1] in the beam. This yields, for the horizontal plane,

$$\left(\frac{(Ze)^2}{4\pi\epsilon_0 mc^2} \right) \frac{N}{\epsilon_x (1 + \sqrt{\epsilon_y / \epsilon_x}) \beta^2 \gamma^3} \leq \frac{1}{\pi} \left(\frac{\delta p}{p} \right) |(n - Q_x) \eta - \xi Q_x|$$

Recall that the chromaticity ξ gives the momentum dependence of the tunes. Large chromaticity may be desirable to stabilize "condensed" beams against

transverse coherent instability. In some storage rings, octupole magnets are introduced to make the tune dependent on a particle's betatron oscillation amplitude, further increasing the tune spread and providing better Landau damping [WIL1]. In addition to these passive damping mechanisms, many storage rings utilize an active feedback system to detect coherent oscillations and damp them [LAM1]. Such a system is probably essential for reaching the strongly-coupled regime.

Section 3.4.3 Intrabeam Scattering

For a very cold OCP, the Coulomb interactions among the particles lead to formation of an ordered system. For a hot beam, the Coulomb interactions between particles take the form of binary collisions, and the process is known as intrabeam scattering [PIW1]. As in a plasma, the intrabeam scattering process for a dense beam can lead to energy exchange between degrees of freedom, i.e. equilibration. Unfortunately, in a storage ring lattice, the intrabeam scattering can also lead to transfer of the beam's longitudinal kinetic energy into the relative motion [BJO1]. This "beam heating" must be counteracted by the cooling system.

We will see in Chapter 5 that the equilibration times for a dense beam can be quite rapid (of order 10^{-3} sec). The energy transfer rate from the longitudinal motion depends on the details of the storage ring lattice and must be simulated by computer [MÖL1]. Change in a particle's longitudinal velocity due to intrabeam scattering also has implications for the laser cooling process, which depends on resonant, Doppler-tuned interaction of the laser and beam particles (Chapter 4). An analytical model for intrabeam scattering will be discussed in Chapter 6. We note here only that the emittance growth rate for intrabeam scattering scales as [MÖL1]

$$\frac{1}{\tau} \propto \frac{N(Z^2 / A)^2}{\beta^3 \gamma^4 a_x^2 a_y^2 C \frac{\delta p}{p}}$$

where a_x and a_y are the transverse beam sizes and C is the ring circumference. We can use this expression to scale experimental measurements on hot beams to the cold beams, but care must be taken not to apply intrabeam scattering theory to the regime in which the multiparticle interactions begin to dominate the dynamics.

Section 3.5 The NAP-M Experiment

Armed with the results of the MD simulations and the terminology of storage ring dynamics, we can now consider the NAP-M experiment in greater detail. The Novosibirsk experimenters studied the Schottky spectra for proton beams with and without electron cooling [DEM1]. Specifically, they measured the total Schottky power in a revolution band as a function of current for cooled and uncooled beam. The results are shown in Figure 3-8 [DEM1].

As discussed in Chapter 2, for a hot beam, free of correlations, the Schottky power in a band should be proportional the particle number. The width of the band is proportional to the momentum spread (the longitudinal temperature), but the total power is independent of the longitudinal beam temperature. This is illustrated by curve 1 in the figure, recorded for an uncooled beam of momentum spread $\delta p/p \sim 10^{-4}$.

For the cooled beam, the measured behavior is quite different (curve 2). The absolute magnitude of the noise power at a given current is several orders of magnitude lower than in the hot beam case. There is also a threshold in current, below which there is no dependence of Schottky power on beam current. The proposed explanation [PAR1] was that the fluctuations in the beam density - which are the source of the longitudinal Schottky signal - were suppressed due to the interparticle interactions in the beam. In the (translated) words of the authors, "The mutual repulsion of the beam particles (if it does not lead to collective beam instabilities) blocks the development of density fluctuations since the interaction energy of the particles that form the interaction cannot be significantly higher than the beam temperature" [PAR1]. Stated another way, the cold beam is close to the point where the thermal and interparticle Coulomb energies are comparable, i.e. $\Gamma_{\parallel} \sim 1$.

In the cold beam case, the Schottky power is not a measure of the particle number but a measure of the thermal noise - the temperature - of the beam [PAR1]. Curve 2 in the figure thus shows that the temperature of the cold beam increases with beam current. The explanation for this relationship is intrabeam scattering [DIK1]. For a beam with much larger transverse than longitudinal temperature, energy transfer can occur to heat the longitudinal motion. Competition between this heating and the cooling from the electron cooler leads to a current-dependent minimum temperature [DIK1].

Figure 3-8 Longitudinal Schottky power vs. beam current for electron cooled (Curve 2) and uncooled (Curve 1) proton beam in the NAP-M ring (from [DEM1]).

The problem with this interpretation is the threshold behavior of the noise vs. current curve [DIK1]. No such behavior is predicted by the theory of intrabeam scattering; there should just be a smooth fall of temperature with current. The proposed explanation, suggested several years after the original experiments, was that longitudinal spatial ordering of the particles led to suppression of intrabeam scattering [DIK1]. The experimenters envisioned the particles as "disks" of charge, due to their large transverse betatron amplitudes. The beam of N particles would

consist of N such disks, equally spaced at distance C/N . The criterion for order derived by the Novosibirsk experimenters was that the ratio of the average distance between particles to the Debye shielding length for the beam be greater than 1 [DIK1]. Mathematically, this criterion may be written

$$\frac{(C/N)}{\sqrt{\frac{KT}{4\pi\rho e^2}}} \geq 1$$

which is not the same as the $\Gamma > 180$ criterion for the OCP. (Squaring the left-hand side of the above relationship gives a parameter similar to Γ but with a quite different length scale.) The Novosibirsk scientists were the first to realize the potential for order in a storage ring and did refer to such a state as a "crystal" [DIK1].

It has been suggested [SCH3], that the threshold in Schottky noise observed in NAP-M may be consistent with the beam current at which the transition from a "string" to a state with transverse extent would take place. The weak focusing of the NAP-M ring could certainly not overcome the slippage due to shear, thus leading to longitudinal heating and an increase in Schottky noise power. Given the low longitudinal temperature (1 K) and very small beam radius ($a = .01$ cm) reported [DIK1], this interpretation may not be unreasonable. Such speculations await experimental confirmation in today's rings.

Molecular dynamics simulations of sparse beams with anisotropic velocity distributions (high transverse and low longitudinal temperature) point to another effect which must be considered in equilibration of such systems due to Coulomb scattering [SCH5]. The transverse betatron motion in the storage ring is not random motion; it is quasi-harmonic. In the case of cylindrically symmetric harmonic confinement, different particles oscillate with different amplitude and phase, but with the same frequency. In the co-moving frame, the general particle trajectory maps out an ellipse in the x - y plane (or a straight line for the special case in which the phase difference between the x and y motion is 0 or π). For two longitudinally adjacent particles, the differences in their transverse coordinates also map out an ellipse. It is this transverse motion that determines the distance between the two particles, and therefore the strength of their Coulomb interaction, under the assumption $T_{\perp} \gg T_{\parallel}$.

For the general case in which the focusing constants K_x and K_y are not equal, the relative motion is not described by an ellipse. The character of the motion depends on the ratio of the two betatron frequencies. If this ratio is a ratio of integers (a situation to be avoided for stability), the relative motion traces out a familiar Lissajous pattern (Figure 3-9). When the ratio is an irrational number, the motion will eventually map out the entire rectangular area described by the maximum betatron amplitudes (Figure 3-9). The MD simulation has been used to study transverse to longitudinal equilibration in the static focusing approximation for the case $K_x \neq K_y$ [SCH5]. The equilibration is found to depend sensitively on the ratio of the betatron frequencies. Figure 3-10 shows energy transfer for three separate cases. Computer modeling of intrabeam scattering in a storage ring lattice and simple plasma models of equilibration do not model the individual particle dynamics in such detail. They can not reproduce such a subtle effect for cold beams.

The implication for cooling of beams to low temperature is clear. The effectiveness of removing heat from the transverse motion using a longitudinal cooling system depends sensitively on the betatron frequencies, at least in the density regime close to string density. The tune of a storage ring is adjustable, within reason, around the design values. The possibility of changing the tunes slowly during a cooling cycle in order to optimize the energy transfer at different densities is not unreasonable. The general problem of energy transfer in anisotropic cold systems deserves further study, particularly in light of the NAP-M results.

Figure 3-9 Trajectory describing the transverse separation between two particles for three different cases. The asymmetry is the ratio of betatron frequencies for the two transverse planes.

Figure 3-10 Longitudinal kinetic energy versus time (in betatron periods) during relaxation of a beam with $\lambda=0.4$. Three different asymmetries are shown. One unit of energy represents 6×10^{-3} of the transverse kinetic energy.

CHAPTER 4

Laser Cooling of Ion Beams

Section 4.1 Introduction

Laser cooling [HÄN1, WIN1] is the result of the velocity-selective transfer of photon momentum from a laser beam to a moving atom or ion. In the most basic laser cooling scheme, known as Doppler cooling, particles having a closed transition (i.e. the population is confined to two levels) between internal energy levels are utilized. Those particles which are in resonance with a laser beam absorb photons. Each absorbed photon transfers momentum of magnitude $h\nu/c$ to the particle, which recoils in the direction of the laser beam propagation. When a photon is spontaneously emitted by the excited particle, the particle again recoils, but the average momentum transfer to a particle after many spontaneous emissions is negligible, because the angular distribution of the emission is symmetric. There is thus a net radiation pressure force, directed along the laser beam, on resonant particles. Due to the small magnitude of the photon momentum (optical photons have at most a few eV/c), it is necessary for an atom to absorb many photons to achieve macroscopically significant acceleration. It is for this reason that the optical transition must be closed. The excited particle must decay to the original state, so that it may absorb another laser photon.

The force acts only on those particles which, in their rest frames, see laser photons whose frequencies are resonant with the frequency of the atomic (or ionic) transition. The force is thus dependent on the velocity of the particle, by virtue of the Doppler shift. By tuning the frequencies of two counterpropagating lasers to accelerate slow particles and decelerate fast ones, it is possible to reduce the velocity spread (in one dimension) of a collection of particles; hence the name "cooling". The low temperature limit of Doppler cooling is given by (assuming no external heating mechanisms)

$$kT \sim \frac{\hbar\Gamma}{2}$$

where Γ is the natural width of the cooling transition (not to be confused with the plasma parameter). To appreciate the magnitude of this limit, the ${}^7\text{Li}^+$ ions used for the experiments reported in Chapters 6 and 8 have a Doppler cooling limit of 62 μK . Laser cooling of particles in ion traps involves up to six laser beams to cool in three spatial dimensions. The geometry of a storage ring limits cooling to the longitudinal motion.

This short chapter gives a brief overview of laser cooling and its application to the storage ring. Attention is confined to the Doppler cooling mechanism. Other schemes have been employed in ion traps to cool particles to well below the Doppler limit, but these will not concern us here (see [CHU1] for a review.)

Section 4.2 Laser Cooling Force and Laser-induced Diffusion

The above simple picture of laser cooling actually describes most of the physical processes that are relevant to the interaction of laser light with ions in a storage ring. To introduce the concepts in a more quantitative way, and to obtain an analytical description of the radiation pressure force, we follow the treatment of Stenholm [STE2]. We consider only the one-dimensional problem for free particles.

The particle is taken to be a two-level atom or ion. The transition between levels is assumed to be an electric dipole transition with energy $\hbar\omega$. The lower level is the stable ground state, and the transition has natural linewidth Γ (lifetime $1/\Gamma$). The electric field of the laser drives the dipole transition at the Rabi frequency, given by

$$\Omega_r = \frac{\mu E}{\hbar}$$

where μ is the transition dipole moment and E is the field amplitude. Spontaneous decay from the upper level conserves energy within the uncertainty $\hbar\Gamma$. This may be described empirically by introducing a dimensionless lineshape function in the form of a Lorentzian:

$$L(\Delta + vk) = \frac{\Gamma^2}{(\Delta + vk)^2 + \Gamma^2}$$

where Δ is the detuning of the laser relative to the transition, k is the photon wavevector in the lab frame, and v the ion velocity; vk is thus the Doppler shift (angular frequency units).

The saturation parameter I is defined as

$$I = \left[\frac{\mu E}{\hbar \Gamma} \right]^2$$

It is the squared ratio of the laser-induced level flipping rate to the spontaneous decay rate. The momentum transfer from the laser to the ion is optimized when $I \approx 1$, that is, each stimulated absorption is followed by a spontaneous emission, on average. The velocity-dependent force is simply the rate of momentum transfer to the ion. This can be written

$$F = \hbar k I \frac{\Gamma^3}{(\Delta + vk)^2 + \Gamma^2}$$

This expression is the equivalent continuous force representing many discrete processes. It is therefore often referred to as a semiclassical expression. The effect of the spontaneous emission on the motion of the ion is that the ion will receive random kicks as it recoils from the emission. This may be described by introducing a diffusion coefficient D , defined as

$$\langle v(t)^2 \rangle = 2Dt$$

To estimate D , we can assume that for N spontaneous emissions

$$\langle v(t)^2 \rangle = v_r^2 N$$

where v_r is the recoil velocity ($= \hbar k / M$) of the ion. Taking $I = 1$ and $\Delta = 0$ gives

$$D \approx \frac{\hbar^2 k^2 \Gamma}{2M^2}$$

for the diffusion coefficient.

For a standing-wave laser field formed by two counterpropagating lasers (see also Chapter 6), the force may be written simply as a sum of two components with opposite direction and opposite sign of the Doppler shift

$$F = \hbar k I \Gamma \left[\frac{\Gamma^2}{(\Delta + vk)^2 + \Gamma^2} - \frac{\Gamma^2}{(\Delta - vk)^2 + \Gamma^2} \right]$$

The dependence of this force on velocity, for several positive values of the detuning, is shown in Figure 4-1. Note the short range, in velocity, of the force, which is of order Γ/k . Using the $^7\text{Li}^+$ ion again as an example, the range of the force is about 2 m/s. This may be compared to the velocity spread of a stored beam of these ions (Chapter 6), which can be of order 10^3 m/s. The force is asymmetric in v and its sign is opposite to that of v for positive Δ . It may therefore be correctly termed a cooling force. Expanding the above expression for the force for small velocities yields

$$F = -\beta v + O(v^3)$$

where

$$\beta = \frac{4\hbar k^2 \Gamma^3 I}{(\Delta^2 + \Gamma^2)^2} \Delta$$

is the friction coefficient. The maximum friction coefficient occurs for $\Delta = \Gamma / \sqrt{3}$ which, for $I=1$ gives $\beta \approx \hbar k^2$.

Figure 4-1 Dependence of the laser cooling force on velocity for four different values of the detuning. The two components of the force are due to the two counterpropagating components of a standing wave (see text). The shapes are calculated for $\Gamma = 2\pi \times 3.7$ Mhz, $\lambda = 540$ nm, characteristic of the ${}^7\text{Li}^+$ ion. The saturation parameter I is 1. No vertical scale is given; only the shape and velocity range of the force are illustrated

The above results for the force and diffusion coefficient are actually only valid in the limit of small saturation parameter, that is, for weak fields. We have also ignored polarization effects and the possible existence of fine structure in the transition. The above arguments are presented to give an intuitive picture of the laser cooling force. For strong fields nonlinear effects such as power broadening of the transition lineshape must be considered. The approximation of a two-level atom may also break down, as a strong field may excite transitions which are far away in energy [STE3]. A quantum mechanical treatment of the interaction of the ion with the modes of the quantized radiation field is necessary. The experiments to be described in Chapters 6 and 8 are very strong field experiments (saturation parameters up to 100). Where necessary, quantum theoretical calculations of the force and velocity-dependent diffusion coefficient have been carried out by our

colleagues [MØLM1,BER2]. These are discussed briefly in Chapter 6; the underlying physics is beyond the scope of this thesis.

Section 4.3 Temperature Limit of Doppler Cooling: the Fokker-Planck Equation

The laser-induced effects were described in the previous section as a velocity-dependent damping force and a stochastic diffusion in velocity. The natural question to consider is: What is the equilibrium velocity distribution of a group of particles subjected to these forces? The time evolution of the momentum distribution of such a system may be described by the Fokker-Planck equation. This may be written

$$\frac{\partial w(p,t)}{\partial t} = \frac{\partial}{\partial p} [F(p)w(p,t)] + \frac{\partial^2}{\partial p^2} [D(p)w(p,t)]$$

where $w(p,t)$ is the momentum distribution function. Writing this for the laser-induced forces and changing to a velocity distribution function gives [STE2]

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial v} \left[\frac{\beta v}{M} w + D \frac{\partial w}{\partial v} \right]$$

where the momentum dependence of D is ignored. If we consider the steady-state solution, $\frac{\partial w}{\partial t} = 0$ for the final stage of cooling, the velocity distribution is easily seen to be

$$w(v) \propto \exp \left[-\frac{\beta v^2}{2MD} \right]$$

This is a Gaussian distribution with

$$\sigma_v = \sqrt{\frac{MD}{\beta}}$$

Defining a temperature

$$\frac{1}{2}KT = \frac{1}{2}M\sigma_v^2$$

and using the approximate expressions for D and β derived in the previous section, we obtain

$$KT \sim \hbar\Gamma / 2$$

for the temperature limit of Doppler cooling.

The interpretation of this result is straightforward. The final kinetic energy spread cannot be less than the energy uncertainty in the state of the atom. We have implicitly worked under the assumption that the transition energy width $\hbar\Gamma$ is large compared to the recoil energy of the ion - this is the condition that the ion undergo many-photon scattering events before Doppler-shifting out of resonance. Transitions not satisfying this condition can be used for cooling to temperatures below the Doppler limit; the next limit being the recoil energy of the ions (See [CHU1].)

Section 4.4 Laser Cooling in Storage Rings

In a storage ring, laser cooling is achieved by using two laser beams, overlapping the ion beam for up to a few meters in the machine. This leads to an important modification in the forces derived above, as the particles are subjected to the cooling force for only a portion of their orbit. The cooling is currently limited to longitudinal cooling, as it is very difficult to achieve physical overlap of transverse laser beams with the ion beam over a sufficient length of the storage ring orbit. One laser propagates opposite to the ions' direction of travel and can decelerate particles with velocities above the mean. The second laser is copropagating and acts to accelerate slow particles. (An induction accelerator, a device for making small changes in the energy of the stored beam, has been substituted for one of the lasers in experiments in the TSR ring in Heidelberg [HAB4].) The experimental situation is thus very much like that depicted in Figure 4-1. The initial energy spread in a stored beam is usually much higher than the linewidth of the cooling transition. The lasers must then be tunable over a wide

frequency range in order to interact with all of the ions. The experimental details of laser cooling in a storage ring will be discussed in great detail in Chapters 5 to 8.

Ions in a storage ring may rarely be considered as satisfying the free-particle model used to arrive at the Doppler temperature limit above. Intrabeam and external forces (Chapters 2 and 3) will often represent sources of heat to compete with the laser cooling force. In the case that the intrabeam interactions may be described by a diffusive velocity spreading, the system may be modeled using the Fokker-Planck formalism described above [MØLM2]. Some preliminary work has been done in this regard [MØLM3].

The theoretically achievable limiting temperature of Doppler cooling is very attractive in the context of the desire to produce a strongly coupled OCP in a storage ring. A system of singly-charged ions having a temperature of $60\text{ }\mu\text{K}$ need have a spatial density of only $10^2/\text{cm}^3$ to reach the crystallization limit. The temperature limit is several orders of magnitude lower than that achievable with electron cooling or stochastic cooling. This potential for creating very cold beams using laser cooling was part of the motivation for the construction of the ASTRID storage ring at Aarhus and the TSR ring in Heidelberg. Early experiments in ASTRID are described in Chapters 6,7, and 8 of the thesis.

CHAPTER 5

Description of ASTRID - the Experimental Setup

The Aarhus storage ring device, or ASTRID, is a small (40 meter circumference) storage ring designed to store beams of electrons or ions [MØL1]. The electrons can be used as a source of synchrotron radiation in the XUV/VUV regime. Positive or negative ions can be stored for studies in atomic physics, molecular physics, or beam dynamics. The ring was designed to facilitate the use of lasers for beam cooling and atomic and molecular physics experiments. The first laser cooling of a stored beam in ASTRID was achieved in May of 1990 [HAN1]. The following sections describe the physical layout and operation of the storage ring and its associated experimental hardware.

Section 5.1 The Storage Ring

ASTRID (Figure 5-1) is a four-sided storage device formed by four 90 degree dipole magnets. Each 90 degree bending magnet is actually a pair of 45 degree iron cores excited by a common coil. Four families of four quadrupole magnets provide the transverse focusing forces for the ions. The magnetic lattice consists of four superperiods, each having mirror symmetry. There are 16 combination sextupole/correction dipole magnets for chromaticity and orbit control. Table 5-1 gives some of the important design parameters of ASTRID. The beam envelope functions, β_x and β_y , and momentum dispersion function η_x are illustrated in Figure 5-2.

In order to achieve long storage times for the ions, the ASTRID vacuum system is designed to reach pressures as low as 10^{-12} torr. The ring vacuum chamber can be baked *in situ* to 300 C. Pumping is achieved by 20 ion pumps and 24 sublimation pumps. Average pressures for the experiments described below

Figure 5-1 The storage ring ASTRID

Table 5-1
ASTRID Design Parameters

Circumference	40 m
Length of Straight Sections	7.83 m
Vacuum Chamber Diameter	100 mm
Maximum Magnetic Rigidity	1.87 T-m
Betatron Tunes: Vertical	2.73
Horizontal	2.29
Transition γ	4.58
Chromaticities: Vertical	-7.5
Horizontal	-3.4
Betatron Acceptances: Vertical	60π mm-mrad
Horizontal	320π mm-mrad

Figure 5-2 ASTRID lattice functions for one superperiod (one-fourth of the ring) ; solid line $-\beta_x$, dot-dashed line $-\beta_y$, dotted line η_x

ranged from 10^{-10} to 10^{-11} torr, depending on the time elapsed since the last sublimation. These pressures were higher than the design capability, as the ring had only been baked to 150 C.

Section 5.2 The Ion Source and Injector

A variety of ion sources can be used in ASTRID. The present experiments utilized a hot filament plasma ion source to produce the singly charged ions ${}^7\text{Li}^+$ and ${}^{166}\text{Er}^+$. This source is chosen to provide a low emittance beam with divergence less than 1 mrad, corresponding to an initial transverse temperature of a few hundred K. The beam is accelerated electrostatically by a precision high voltage accelerator. Stability of the acceleration voltage is < 1 V (RMS) at 100 kV. The accelerator can operate at energies from 10 to 150 keV; the bulk of the current work was carried out using ions with kinetic energy 100 keV. Initial energy spread of the beam was thus of order $\delta E/E < 10^{-5}$. The ions are mass-separated in a 45 degree analyzing magnet and then pass through the injection beam line. Two sets of electrostatic quadrupoles in this beam line serve to match the source to the ring, thereby minimizing the emittance dilution in the storage ring. Magnetic correction dipoles provide horizontal and vertical fine control over the beam position and angle at injection. A fast electrostatic chopper is used to select the desired length of beam to inject. The chopper has a rise or fall-time of about 200 ns; this may be compared to revolution periods of about 24 μs and 118 μs for ${}^7\text{Li}^+$ and ${}^{166}\text{Er}^+$ ions, respectively. The minimum chopper duration is ~ 3.8 μs ; thus the number of ions injected, and the initial length of the injected pulse, can be controlled over a wide range.

The beam is deflected into the storage ring using a magnetic septum magnet. An electrostatic kicker, located diametrically opposite to the septum, kicks the beam onto the closed orbit. Both kicker and septum operate in the horizontal plane. The first 1/2-turn in the ring thus has large horizontal position oscillations. The injection devices use triggers which are synchronized to the 50 hz line frequency in order to minimize pulse-to-pulse variations in beam position at injection. The minimum time between injections is thus about 20 ms. The injection system may also be operated in a DC mode, providing a continuous stream of particles which makes about two and one half turns before hitting the vacuum chamber wall. This mode is useful for initial setup of laser frequency

and optimization of the spatial overlap between the laser and ion beams. Figure 5-1 also indicates the position of a radio frequency (RF) cavity which can be used for bunching or acceleration of the ions. This device did not play a role in the experiments described here.

Section 5.3 Beam Diagnostic Systems

The ASTRID complex incorporates several systems for monitoring the performance of the ion source and storage ring (Figure 5-3). The ion source current can be measured by several Faraday cups, located at various positions in the injection channel. These cups, which have a resolution of less than 100 nA, can be remotely inserted or removed from the beam path. The cup furthest downstream, designated Cup 2, provides the best measurement of current immediately before injection into the ring. A device to directly measure the circulating beam current does not exist. The range of currents used in these experiments (1-20 μ A) make direct measurement difficult. ASTRID is equipped with a commercial beam current transformer, but its resolution (>5 μ A) is not sufficient.

The transverse position of the beam in the storage ring can be measured using 8 vertical and 8 horizontal position pickups, located near the ends of the straight sections. A schematic diagram of a pickup is shown in Figure 5-4. The beam induces a voltage on each electrode. The sum of the two voltages is proportional to the beam current. The difference of the two signals is proportional both to the distance of the beam centroid from the center of the chamber and to the beam current (i.e. to the dipole moment of the transverse charge distribution). The ratio of the two signals thus provides a beam position measurement. In practice such detectors are calibrated, using a signal wire stretched through the aperture, before they are installed. The resolution of the ASTRID system is approximately 0.2 mm.

Figure 5-3 ASTRID diagnostic systems

Figure 5-4 Schematic cross section of a vertical beam position pickup.

Figure 5-5 shows signals from an ASTRID horizontal position pickup. The traces are from the first 8 revolutions of an injected pulse of 100 keV ${}^7\text{Li}^+$ beam. The top trace (Channel 1) is the sum signal; the bottom is the difference (note the difference in vertical scale). The gap between turns is the hole due to the chopper rise and fall time. (The apparent "decay" of the signals is a relic of the amplifying electronics; note that the relative signal heights, from notch to flat top, remain constant.) The signals from these pickups are particularly useful in optimizing the injection into the ring. The sum signal indicates if beam is being lost from turn to turn. The difference signal indicates transverse position errors. The ASTRID control system includes the software necessary to analyze distortions in the closed orbit and calculate correction element currents required to "flatten" the orbit. A correct and reproducible orbit is essential in achieving positive results with laser cooling experiments.

Figure 5-5 Signals from a horizontal beam position monitor. The first eight turns in the machine are shown. Top trace - sum signal; Bottom trace - difference signal.

ASTRID is equipped with a longitudinal Schottky pickup for measuring the longitudinal charge density fluctuations in the beam. The capacitance of the pickup and the inductance of the amplifier input form a resonant circuit tuned to a harmonic of the beam's revolution frequency (harmonic number $h=2$ in the case of ${}^7\text{Li}^+$ and $h=11$ for ${}^{166}\text{Er}^+$). The amplified signals are analyzed with a

Hewlett-Packard 4195A network/spectrum analyzer. Figure 5-6 shows a typical longitudinal Schottky spectrum for ${}^7\text{Li}^+$ beam. For uncorrelated particles, the power spectral density is proportional to the number of particles whose revolution frequencies lie in the frequency band. The spectrum may thus be thought of as the momentum distribution of the stored particles (Chapter 2). Figure 5-7 shows a Gaussian fit to the measured spectrum. The fit gives a momentum width of $\delta p/p \sim 0.0027$ (FWHM), corresponding to a longitudinal temperature of ~ 3100 K.

Figure 5-6 Spectrum analyzer plot of the longitudinal Schottky spectrum for ${}^7\text{Li}^+$ beam in ASTRID. Two representations are shown, with linear (narrow peak, 20 nV/div) and logarithmic (3 dBm/div) vertical scales.

Figure 5-7 Gaussian fit to the momentum distribution of ${}^7\text{Li}^+$ beam measured with the Schottky pickup. Solid line - fitted curve; +'s - measured points. The units of Schottky power are relative.

The Schottky signal can also be used to measure the beam storage lifetime. By tuning the spectrum analyzer to a fixed frequency band, centered on the momentum spectrum and wide enough to span it, one obtains a signal proportional to the ring current. Figure 5-8 shows this signal plotted as a function of time for ${}^7\text{Li}^+$ beam. The beam is injected at $t=0$. The large initial signal is due to the hole left in the beam by the chopper; the Schottky signal is coherent. As the hole fills in due to the momentum spread in the beam ("debunching"), the coherence disappears and the signal is a true noise signal. The interpretation of the Schottky signal is thus not straightforward for times earlier than the debunching time. Figure 5-9 shows, on a linear scale, an exponential decay curve fitted to the experimental signal. The storage lifetime is ~ 8.6 s.

Figure 5-8 Longitudinal Schottky power vs. time after injection for lithium beam. The vertical scale is 2 dBm per division. The time axis spans about 30 s. Coherence in the Schottky signal is evident in the first 300 ms.

Figure 5-9 Fit to lifetime measured by the longitudinal Schottky pickup. The solid curve is the fit to the incoherent region of the spectrum; the points are the measured data. The storage half-life is 8.6 s.

Some measurements to be described in Chapter 8 made use of a detector for neutral particles. The detector is located in a section of the vacuum chamber that extends beyond a dipole magnet at the end of a straight section (Figure 5-3). The detector package consists of a microchannel plate and a semiconductor detector, either of which can be positioned to observe the straight section upstream [AND1]. (For the present experiments the channel plate was used.) The detectors are designed to measure neutral particles produced in the straight section by, for example, charge-exchanging collisions between the beam particles and molecules of the background gas. This device has proven valuable as a monitor of relative beam intensity.

A final conventional diagnostic device in the ASTRID ring is the beam scraper system. The scrapers are essentially motorized absorber blocks which can be remotely moved into the vacuum chamber to intercept the circulating beam. In conjunction with a beam current monitor such as the longitudinal Schottky pickup, the scrapers can be used to measure - albeit destructively - the transverse extent of the circulating beam. ASTRID is equipped with two pairs of scrapers, designated top, bottom, left and right. The scraper operation will be discussed in greater detail in Chapter 8.

Section 5.4 Lasers and Optical Diagnostics

The unique design of ASTRID was motivated to a large degree by the desire to permit the interaction of laser light with the stored ions. Each of the four long straight sections can, in principle, be equipped with laser windows at both ends. Each section employed provides almost 8 m of overlap between laser and ion beam. The typical experimental setup is illustrated schematically in Figure 5-10.

The lasers employed are tunable ring-dye lasers, pumped by argon-ion lasers. The wavelengths of interest are optical, in the range 540 to 550 nm. Using the dye rhodamine 110, the lasers typically produce several hundred mW of light when pumped with 5 to 6 W of argon-ion laser light. For cooling experiments, two lasers, one copropagating with the ion beam, the other counterpropagating, are used. For probing of the longitudinal velocity distribution, one laser is sufficient. The lasers operate CW and are switched by acousto-optic modulators (A-O), which divert the beams into stops. Wavelength

measurement is performed by a phase locked Michelson wavemeter, which has a resolution of $0.0005 \text{ \AA}$

Laser photons absorbed by beam particles and subsequently re-emitted can be detected by a photomultiplier tube (PMT) oriented with its axis perpendicular to the beam path [Figure 5-10]. A system of lenses collects light emitted from the ions and focuses it onto the phototube window. Because of the Doppler effect, a narrow band interference filter (IF) can be used to discriminate between laser-induced fluorescence (LIF) photons and laser photons scattered from the windows or walls of the vacuum chamber. Since the LIF photons exhibit a dipole radiation pattern, the polarization vectors of the laser beams are oriented vertically to maximize the number of photons striking the phototube. Some experiments used a post-acceleration tube (PAT) to change the energy of the ions in a short section immediately upstream of the detector. Use of this tube can limit the resonant interaction of the ions and laser beam to a small spatial region, thus minimizing the perturbative effect of the laser on the ions' velocity distribution and allowing the laser to act as a velocity probe [KRI1].

Data collection and accumulation are performed by a computer-controlled gated photon counter. Measurement timing and laser frequency scanning are performed by a microprocessor-controlled system designed specifically for these experiments [NIE1].

CHAPTER 6

First Experiments with ${}^7\text{Li}^+$ beam

The first experiments using ${}^7\text{Li}^+$ beam in ASTRID were carried out immediately after the commissioning of the machine. This section describes the results of those first attempts at laser cooling and laser diagnostics with stored ions. The experiments took place in the spring of 1990.

Section 6.1 Characteristics of ${}^7\text{Li}^+$ beam

The beam consisted of ${}^7\text{Li}^+$ ions, which are helium-like. A fraction of the ions from the ion source is produced in the $1s2s^3S_1$ state, which is metastable and has a lifetime of about 50 seconds. This state lies about 60 eV above the ground state, which has quantum numbers $(1s)^2\,{}^1S_0$. The fraction of ions produced in the metastable state is estimated to be less than 10^{-3} . The metastable state is connected to the $1s2p^3P_2$ state through allowed optical transitions having wavelength of about 548.6 nm. The lifetime of the upper state is 43 ns, corresponding to a homogeneous linewidth of $2\pi \times 3.7$ MHz. Table 6-1 summarizes the optical and dynamical properties of the Li beam.

One of the lines connecting the upper ($7/2$, $5/2$, $3/2$, or $1/2$) and lower ($5/2$, $3/2$, or $1/2$) hyperfine multiplets (the nuclear spin is $3/2$) is closed, with less than 10^{-5} branching to the ground state (Figure 6-1). This line, $F'=5/2 \leftrightarrow F''=7/2$, is well-suited for use in laser cooling experiments. The 43 ns lifetime implies that of order 10^2 photons can be scattered by a resonant ion during one traversal of a straight section.

Table 6-1
 $^7\text{Li}^+$ Beam Parameters

Ion Mass	M	$1.15 \times 10^{-26} \text{ kg}$
Ion Kinetic Energy	E	100 keV
Initial Energy Spread	$\delta E/E$	$< 10^{-5}$
Initial Transverse Temp.	$T_{\perp}$	$\sim 1000 \text{ K}$
Typical Source Current	I	1 - 20 μA
Conversion Factor	(stored beam)	$1 \mu\text{A} \equiv 1.5 \times 10^8 \text{ ions}$
Velocity	v	$1.67 \times 10^6 \text{ m/s}$
Relativistic Factor	β	0.0056
Transition Wavelength	λ	548.6 nm
Photon Momentum	p_{γ}	2.25 eV/c
Ion Momentum	p_i	$3.6 \times 10^7 \text{ eV/c}$
Ion Recoil Velocity	v_r	0.105 m/s
Doppler Shifts	$\Delta\lambda$	3.0 nm
	$\Delta\nu$	$3.1 \times 10^{12} \text{ Hz}$
Transition Lifetime	τ	43 ns
Natural Linewidth	Γ	$2\pi \times 3.7 \text{ MHz}$
Saturation Intensity		8.8 mW/cm ²
Doppler Cooling Limit	T_D	62 μK
Metastable Fraction	f	$10^{-5} < f < 10^{-3}$

Figure 6-1 7Li^+ hyperfine levels and allowed transitions

The drawback of using the Li beam is the small fraction of ions available for laser probing or manipulation. In practice this proves not to be a limitation for probing experiments. As long as the laser does not perturb the velocity distribution of the metastable ions, their behavior may be taken as representative of the beam as a whole. As will be seen, the fluorescence signal from 10^4 to 10^6 metastable ions is easily detected. The cooling process, of course, acts only on the metastable particles. The vast majority of the ions are unaffected and under many circumstances may disrupt the cooling process via intrabeam interaction. Despite this drawback, much useful information on laser cooling has been obtained from the lithium system.

Section 6.2 Laser diagnostics

Section 6.2.1 Longitudinal Beam Heating

A single dye laser can be used as a sensitive probe of the longitudinal velocity distribution of the stored ions. A measurement of this distribution is made by tuning the laser to be in resonance with a particular ion velocity class and recording the intensity of the fluorescence signal over a short period of time. The time must be short enough that the laser does not depopulate the velocity class due to multiple photon recoils. Stepping the laser frequency through the velocity range of interest yields the distribution. In ASTRID, this probing was accomplished by measuring a single velocity interval for each new injection of beam into the ring. The results thus depend on the stability of the accelerator and ion source over time scales of several minutes. This stability has been investigated by direct measurement of the resonance wavelength using the wavemeter. Stability of these components is not a limitation for velocity distribution probing.

A typical result of a velocity distribution measurement is shown in Figure 6-2. The figure is a plot of fluorescence intensity versus laser frequency. The laser is scanned 1 GHz, approximately centered on the frequency resonant with the mean particle velocity. The scan consists of 401 measured points; each point represents 200 ms of photon counting. The curve is an accumulation of 10 scans. This particular measurement was made on the DC beam described in Section 5.2. The beam has traveled through the injection line and made 3/4 of a revolution in the ring at the measurement point. The longitudinal beam temperature in the co-moving frame is about 3 K.

The measurement technique described above was used to study the time evolution of the longitudinal velocity distribution [HAN1]. Scans such as the one in Figure 6-2 were taken for times ranging from 0 to just over 200 ms after injection. The results are plotted in Figure 6-3. The injected beam current was about 7 μA , corresponding to about 10^9 particles in the ring.

Figure 6-2 Laser fluorescence scan of beam velocity distribution, measured after 3/4 of an orbit in the ring.

Figure 6-3 Longitudinal beam temperature in the co-moving frame versus time after injection for 100 KeV ${}^7\text{Li}^+$ beam,

The beam, which is injected with a very low longitudinal temperature, rapidly heats up to several hundred K in 200 ms. The heating appears to be slowing towards a possible equilibrium, but the measurements do not extend far enough in time to determine if this is the case. The ASTRID ion source and accelerator were specifically designed to produce a very low temperature beam. The rapid heating of the beam upon injection to the ring was thus unwelcome. The latest measurable time is limited by the detection system. As the velocity distribution blows up, the number of particles in the power-broadened linewidth is decreasing. It thus becomes necessary to accumulate data for a longer period of time at each laser frequency. However, holding the laser at fixed frequency in the circulating beam will depopulate the velocity class being measured after a few tens of ms. If there is additionally any transverse blowup in the beam size, the signal will decrease due to the density change.

Section 6.2.2 Coulomb Scattering and Beam Relaxation

The longitudinal heating may be explained by intrabeam Coulomb scattering (Chapter 3). The velocity distribution of the injected ions is "flattened" by the electrostatic acceleration, i.e. the injected beam has $T_{\parallel} \ll T_{\perp}$. The transverse betatron motion thus represents a large energy source. This energy can be transferred to the longitudinal motion by Coulomb scattering in the dense beam plasma. In other words, the beam attempts to come to thermal equilibrium via the interparticle collisions. This phenomenon is well known in the physics of the transport of electron beams, and is often referred to as the Boersch effect [LAW1].

An analytical model [SØR1] for intrabeam scattering has been employed to test the validity of the above supposition. The model starts with an anisotropic velocity distribution, given by

$$f(v)dv = \frac{n}{\pi\sqrt{2}\pi\Delta_{\perp}^2\Delta_{\parallel}} e^{-(v_{\parallel}^2+v_{\perp}^2)/\Delta_{\perp}^2} e^{-v_{\parallel}^2/2\Delta_{\parallel}^2},$$

where $\Delta_{\parallel}$ and $\Delta_{\perp}$ are the longitudinal and transverse velocity spreads, and by assumption $\Delta_{\parallel} \ll \Delta_{\perp}$; n is the ion number density. The model then calculates the relaxation rate due to binary Coulomb scattering in the beam plasma. The energy transfer rate may be expressed as

$$\frac{dE_{\parallel}}{dt} = 4\pi(Ze)^4 \frac{n}{2m\Delta_{\perp}} L$$

The left hand side is the energy transfer to the longitudinal degree of freedom. The factor L , known as the "Coulomb logarithm", is the logarithm of the ratio of the maximum to minimum distance over which the Coulomb force acts in the problem. (Since the Coulomb force has infinite range and is singular for zero separation, appropriate limits must be chosen to prevent the integral over all possible collisions from diverging.) The minimum distance is taken as the distance of closest approach for a head-on collision of two particles with velocity $\Delta_{\perp}$. The maximum distance may be taken as the Debye shielding distance for the beam plasma, or as the beam transverse size, whichever is smaller [SØR2]. The slowly-varying factor L will be of order 10 for most situations encountered in storage rings.

Since the transverse velocity width $\Delta_{\perp}$ and the ion number density will change as energy is transferred to the longitudinal motion, we first consider the initial heating rate in the beam immediately after injection. In order to numerically evaluate the heating rate, the particle number density and the transverse velocity scale $\Delta_{\perp}$ must be known. Both of these can be determined by a measurement of the transverse beam size. In the first experiments, this measurement was accomplished using the laser. A narrow slit was scanned vertically across the fluorescence image of the beam at the phototube. A vertical beam profile was thus obtained. The vertical beam size at the center of the long straight section was found to be 5 mm FWHM, for the time immediately after injection. Using the machine lattice functions and the considerations of Chapter 2, the average beam density and transverse temperature were determined. An additional assumption was that the beam was initially symmetric in the x and y velocity distributions. The plasma source and injection system should produce such a distribution.

The analytical model yields a heating rate of 2 ± 1 eV/s from the experimental parameters. The initial slope of the heating curve in Figure 6-3 is 1.4 ± 1 eV/s [HAN1]. The agreement between theory and experiment is thus quite good, but the size of the experimental errors and the simplicity of the model prohibit the conclusion that the intrabeam scattering is the *only* mechanism contributing to the initial heating. Other possibilities, such as coherent interaction of the beam with the storage ring environment, can not be completely ruled out.

The molecular dynamics simulations were also used to investigate the equilibration process. The simulations began with a beam that was "hot" in all three dimensions. After the system had reached equilibrium, the longitudinal motion of the particles was stopped, simulating the condition $\Delta_{\parallel} \ll \Delta_{\perp}$. The system was then followed in time to determine the transverse to longitudinal energy transfer rate. Due to limitations in computer size, the transverse temperature and particle density of the ASTRID experiment can not be simulated directly. Four simulations were carried out for the equivalent of 10^7 ions in ASTRID with transverse temperatures ranging from one to three orders of magnitude lower than measured [HAN1]. The simulations were then compared to the predictions of the analytical model. In all cases the two models agreed to within 50%.

The experiment described above represents the first use of resonant laser spectroscopy to study complex dynamical processes in a storage ring. This method has great potential for further application. As will be illustrated in Chapter 8, the laser may be used to directly probe the transverse velocity distribution of the ions as well.

Section 6.3 Laser cooling of $^7\text{Li}^+$ beam in ASTRID

Two lasers were employed in a single straight section of the ring in an attempt to laser cool the metastable lithium ions. The first experiments illustrated the necessity of achieving precise spatial overlap of the ion and laser beams. The common experience of the TSR group in Heidelberg suggests that the best way to achieve this overlap is by designing moveable apertures into the vacuum chamber [HAB1]. In the early ASTRID experiments the overlap was optimized using the beam position monitoring system and the orbit control trim dipoles. The orbit control system was not yet fully commissioned, so overlap could not be guaranteed over the entire straight section.

The cooling process begins at injection with the co- and counterpropagating lasers detuned about 1 GHz red of the resonance frequency (in the beam frame) of the central beam velocity. The copropagating laser is in resonance with the slowest particles, the counterpropagating with the fastest. Note by comparison to Figure 6-2 that the lasers, spaced 2 GHz apart, "surround" the initial velocity distribution (FWHM of ~ 300 MHz) of the injected ions. Beginning at injection, the counterpropagating laser was swept to higher frequency at a constant rate of about

625 MHz/s. The faster ions are thus decelerated by the scanned laser until they come into resonance with the fixed-frequency laser. The scanned laser should pile up particles in front of it, forming a very narrow velocity distribution just before the laser frequencies cross. After the laser frequencies cross, the cooling force becomes a heating force and the ions are accelerated away.

Figure 6-4 depicts the fluorescence signal recorded during such a cooling scan. The sharp peak to the right of the figure represents the cooled distribution. The fluorescence rises sharply as the swept ions come into resonance with both lasers. The sharp drop in fluorescence occurs when the laser frequencies cross and the ions are accelerated out of resonance. The scan in Figure 6-4 represents an accumulation of data over many injection cycles in order to achieve good statistics. The on-resonance saturation parameters for the two lasers were 76 (swept laser) and 66 (fixed laser) [HAN1]. The beam current at injection was 8 μA .

Figure 6-4 Laser cooling scan for 100 keV ${}^7\text{Li}^+$ beam.

The fluorescence spectrum of the cooled ions may not be straightforwardly interpreted as the velocity distribution of the particles. The scan represents a dynamic situation, in which the velocity distribution is changing as the laser frequency is stepped. Experimentally we find that the width of the cold peak is determined by the power broadened linewidth of the transition; there is no measurable Doppler contribution from the ion temperature. This was determined by using the cooling lasers as probes. After each frequency step in the cooling scan, the laser power was reduced to a fraction of that used for cooling. The width of the cooled distribution was then investigated as a function of the reduced power. For powers between 10 and 100 times the saturation intensity, the width of the cold peak is equal to that expected from power-broadening alone [HAN2]. If the experimental error in determining the peak width is taken as an upper limit on the Doppler contribution, the longitudinal ion temperature would be less than 1 mK. To more confidently determine the temperature, the cooling experiment has been theoretically modeled by Mølmer and Berg-Sørensen [MØLM1]. The Coulomb interactions of the ions and other "heating" mechanisms are neglected in the following.

The two lasers have frequencies ω_1 (swept laser) and ω_2 (fixed laser) in the laboratory frame. In a frame moving relative to the laboratory with velocity $v_{st} = (\omega_1 + \omega_2) / 2k$ (the difference in wavenumbers is neglected.), the lasers have the same frequency $\omega_{st} = (\omega_1 + \omega_2) / 2$. If the two lasers have the same polarization and intensity, they produce a standing wave in this frame. Working in the standing wave frame, we assume that the ion velocity distribution at any time is the stationary one produced by the very strong laser cooling force, that is, we ignore the time it takes for the ions to come to equilibrium. This assumption is supported experimentally, as the intensity and shape of the cooled peak were independent of the laser sweeping rate. Modeling of a cooling scan thus reduces to calculating static velocity distributions for a series of standing wave frequencies.

The cooling force is derived by the continued fraction method [MIN1]. The diffusion coefficient, representing fluctuations in the cooling force, is calculated using a procedure due to Minogin [MIN2]. The velocity distribution is obtained as the stationary solution to the Fokker-Planck equation. Taking the on-resonance Rabi frequency to be 6Γ [BER1], the minimum longitudinal ion temperature of $T_{||} \sim 1$ mK occurs at a detuning of -2.5Γ . The laser fluorescence line shape predicted by the calculation is in good agreement with the measured curve.

At the time of publication, this inferred longitudinal temperature was the lowest ever reported for a storage ring [HAN1]. The existence of such a low temperature distribution of metastable ions in the background of uncooled ground state ions was puzzling. The measured lineshape agreed very well with a theoretical model which assumed that there were *no* intrabeam collisions, yet the analytical model for intrabeam scattering predicted significant diffusion for a beam having the density inferred from the injected current and the measured beam size. The cause of these apparent contradictions was determined by later experiments and will be discussed in Chapter 8.

Another interesting feature of the cooling spectrum (Figure 6-4) is the fluorescence structure to the left of the figure. The fluorescence rises sharply in less than 10 ms after injection. There is then a slow decay, followed by a rapid drop. The sharp rise in the beginning may be understood as ions coming into resonance with the two lasers due to the rapid longitudinal heating described above. (Recall that the lasers begin 2.5 GHz apart, and the injected velocity distribution is 300 MHz FWHM.) The slow decay is consistent with the storage lifetime of the beam, which was of order 1.5 seconds in the early experiments due to a slight vacuum leak in the machine. The sharp drop in fluorescence was, however, not explainable by any mechanism yet identified in ASTRID.

The explanation for the fluorescence drop must again await the results of later experiments, but several features of this phenomenon are worth noting. The fluorescence drop was studied over a wide range of laser and beam parameters [BER1]. The location of the drop was found to be independent of laser intensity, laser scan rate, laser detuning, and beam current. The drop occurred at a fixed time after injection (~ 1.1 sec), rather than at fixed laser frequencies. It was impossible to form the cold distribution (by sweeping the lasers at a higher rate) in the period from 0 to 1.1 sec after injection. After the fluorescence drop, the cold distribution could be formed very rapidly. In the absence of any competing mechanisms, the lasers should be able to cool the metastable fraction to the Doppler limit in less than 10 ms.

The initial speculation was that the high initial fluorescence was due to strong intrabeam interactions which overwhelmed the laser cooling force [HAN1]. Due to the short range (in velocity) of the laser force, the small number of metastables, and the existence of cooling in only one section of the ring, the lasers could not maintain the cold, injected velocity distribution. The fluorescence drop suggested that the competing interactions were perhaps due to a coherent

instability, and that the beam crossed the stability threshold at 1.1 s due to particle loss and/or the blow-up of the longitudinal velocity spread. The best candidate for such a process was thought to be the longitudinal microwave instability (Chapter 3). These considerations will be concluded in Chapter 8, with the discussion of the second round of lithium experiments in ASTRID.

In spite of the many unanswered questions raised by these early experiments, demonstration of the ability to laser cool the metastable lithium ions to the very low temperature of 1mK was an important development . To put this temperature in perspective, if it were possible to cool the ions to this temperature in all three dimensions, the beam would have a Γ of over 200. Of course this assumes that the transverse energy could be removed by longitudinal cooling; the number is quoted just for illustration.

CHAPTER 7

Experiments with $^{166}\text{Er}^+$ Beam

As discussed in the previous chapter, the two-component nature of the lithium beam limits the laser cooling to a small fraction of the stored ions. The presence of the uncoolable ground state ions represents a large energy source which can transfer energy to the coolable ions by Coulomb interaction. The interpretation of any experimental result is slightly clouded by the complex interactions of the two distinct components. The linear density of metastable lithium ions produced by the plasma discharge ion source is additionally not high enough to attain an ordered system with transverse extent. If the beam could be cooled in three dimensions, only a string (Chapter 3) could be formed with so few ions[HAN1].

The above factors led to consideration of another ion with a closed two-level transition having a frequency in the range of the optical dye laser. The spectrum of the ion $^{166}\text{Er}^+$ had been studied previously at Århus [BEN1], and was known to contain a closed transition at 5416 Å. The main reason for interest in $^{166}\text{Er}^+$ was that the lower level of the transition is the ground state. It should thus be possible to cool the entire beam. Another intriguing fact was that the lifetime of the transition, which had not been directly measured, was estimated to be about 1 μs . The Doppler limit of temperature should thus be about 1/20th of that for $^7\text{Li}^+$. This chapter describes the results of ASTRID experiments with erbium beam. The experiments took place in February, March and July of 1991.

Section 7.1 Characteristics of $^{166}\text{Er}^+$ Beam

The cooling transition is between the ground state ($J=13/2$) and an excited state ($J=15/2$) which lies 18463 cm^{-1} above. The nuclear spin is zero, so there is

no hyperfine structure in the spectrum. As shown in Figure 7-1, there are in addition two open transitions, reachable with the dye laser, which can in principle be used for probing the velocity distribution of the ions without exerting a cooling

Figure 7-1 Cooling and probing transitions for the $^{166}\text{Er}^+$ ion. Energy levels are in cm^{-1} .

force. The beam kinetic energy was 100 keV. The beam and optical parameters are listed in Table 7-1. Laser cooling parameters, such as the saturation intensity and the limit of Doppler cooling, are not listed due to the uncertainty in the lifetime. Typical beam currents for the erbium beam were about $1 \mu\text{A}$ ($\equiv 7.4 \times 10^8$ ions) of ^{166}Er for natural erbium, or a few μA for isotopically enriched ^{166}Er . The storage lifetime for the erbium beam was about 18 s for vacuum pressure of 5×10^{-11} torr.

Referring to Table 7-1, note that the Doppler shift for laser photons due to the beam velocity is only 0.6 nm, compared to about 3 nm for the $^7\text{Li}^+$ beam. The passband of a filter of the type used in the lithium experiment (FWHM 0.6 nm) to discriminate between laser-induced fluorescence photons and scattered laser photons is therefore too large for the $^{166}\text{Er}^+$ beam. However, since the lifetime of the cooling transition in $^{166}\text{Er}^+$ is long, it was possible to use a delayed

fluorescence technique to reduce the background from scattered photons. Using the acousto-optic modulator and the computerized timing system, the laser was switched on and off at 1 μ s intervals.

Table 7-1
 $^{166}\text{Er}^+$ Beam Parameters

Ion Mass	M	2.76×10^{-25} kg
Ion Kinetic Energy	E	100 keV
Initial Energy Spread	$\delta E/E$	$< 10^{-5}$
Initial Transverse Temp.	$T_{\perp}$	~ 1000 K
Typical Source Current	I	1-4 μ A
Conversion Factor	(stored beam)	1 μ A $\equiv 7.4 \times 10^8$ ions
Velocity	v	3.41×10^5 m/s
Relativistic Factor	β	0.0011
Transition Wavelength	λ	541.6 nm
Photon Momentum	p_{γ}	2.29 eV/c
Ion Momentum	p_i	1.7×10^8 eV/c
Ion Recoil Velocity	v_r	0.004 m/s
Doppler Shifts	$\Delta\lambda$	0.6 nm
	$\Delta\nu$	6.3×10^{11} Hz

The detection system was gated off while the laser was on, so that only fluorescence photons were detected. It was therefore possible to probe the velocity distribution with very good signal-to-noise ratio. Compared to $^7\text{Li}^+$, the longer transition lifetime and lower duty cycle decrease the fluorescence rate; this is more than compensated by the far greater number of particles which are excitable. A very narrow-band, temperature-tuned filter was eventually procured, and it was then possible to operate the probing laser CW.

Section 7.2 Beam Relaxation due to Coulomb Scattering

Using the gated fluorescence technique, we measured the time development of the longitudinal temperature of the $^{166}\text{Er}^+$ beam. A rapid heating, as observed in the $^7\text{Li}^+$ beam, was again observed. This is depicted in Figure 7-2. Due to the improvements in the measurement technique and the greater number of optically active particles, it was possible to follow the time development until times as late as 1.1 s after injection, compared to 200 ms in the first $^7\text{Li}^+$ measurements. The initial heating rate is 4 ± 2 eV/s. The plasma model predicts a heating rate of 1.8 eV/s for the density and transverse temperature inferred from beam size measurements made using the beam scraper (see Chapter 8 for a discussion of the measurement technique.) The simple intrabeam scattering model for energy transfer from transverse to longitudinal motion thus seems to give a reasonable description of the early blowup in both erbium and lithium. The longitudinal temperature at 1.1 s after injection is, however, slightly higher than what one would expect if the only heating mechanism were intrabeam equilibration. There may thus be another process active.

Figure 7-2 Laser fluorescence measurement of the longitudinal temperature vs. time after injection for $^{166}\text{Er}^+$ beam. The beam current was $2.7 \mu\text{A}$, equivalent to 2×10^9 ions injected.

Section 7.3 Laser Cooling and Schottky Diagnostics

During the period after the first experiments with the ${}^7\text{Li}^+$ beam, a system for longitudinal Schottky diagnostics of the ${}^{166}\text{Er}^+$ ions was developed. Operating at the 11th harmonic of the revolution frequency, the system can provide resolution of about 8×10^{-5} in $\delta p/p$. This is not fine enough resolution to measure the momentum distribution of the coldest laser cooled beam ($\delta p/p < 10^{-6}$) but it is adequate for monitoring the progress of cooling from a hot initial state. A sample spectrum is shown in Figure 7-3. Momentum spectra taken after the coherence in the injected beam has died away (approx. 2 seconds after injection) revealed that the momentum spread had risen to $\frac{\sigma_p}{p} = 1 \times 10^{-3}$, corresponding to a longitudinal temperature of about 2100 K. This observation confirms that transverse to longitudinal Coulomb relaxation is not the only heating mechanism for the longitudinal motion. This point will be reconsidered shortly.

Figure 7-3 Erbium momentum spectrum at 10 seconds after injection. Measured longitudinal Schottky power vs. revolution frequency at harmonic $h=11$. The solid curve is a Gaussian fit to the measured points. The units of Schottky power are arbitrary. The momentum spread is $\frac{\sigma_p}{p} \approx 9 \times 10^{-4}$.

Modifications to the ring enabled overlap of co- and counterpropagating laser beams with the ion beam in two straight sections of ASTRID for the erbium experiments. The much longer storage time of erbium (up to 20 s half-life, compared to 8 s for ${}^7\text{Li}^+$) also permitted longer cooling time. As noted above, laser probing of the velocity distribution for the uncooled beam was effective until about 1.1 s after injection.

Cooling scans analogous to those described in the previous chapter were attempted with the erbium beam. No fluorescence signature of a cooled velocity distribution was seen. Given the longer transition lifetime, various laser scanning rates were tried, ranging from a few seconds to about 2 minutes to scan one laser across the full velocity distribution. Still no optical evidence of cooling could be seen.

Lacking an optical signal, we focused on the Schottky diagnostic system. The hope was that a cooled distribution of particles existed, but that the particles were too few to detect optically. The TSR group in Heidelberg had observed longitudinal coherent behavior, generating a large Schottky signal, in their laser cooled ${}^7\text{Li}^+$ beam [SCHR1]. The laser scans were repeated while monitoring the width of the momentum distribution with the Schottky system. No statistically significant change in the width of the momentum distribution was detected; the same large range of scanning rates was explored.

Postulating that heating due to intrabeam scattering was overwhelming the laser cooling, we varied the intensity of the beam from 10^9 particles injected down to less than 10^7 - the minimum with which the Schottky diagnostics could reasonably measure the momentum width. Still there was no evidence for cooling. An attempt to depopulate a region of the velocity distribution with a single laser operated at fixed frequency for up to two minutes also failed. The only indication of a mechanical light force on the ${}^{166}\text{Er}^+$ beam was obtained by using one laser, operating CW, at a time. The laser would be tuned to be in resonance with the center of the velocity distribution and left at that frequency for up to 150 seconds. Comparing the effect of the copropagating laser with the counter propagating laser revealed a statistically significant shift of the center of the velocity distribution. The effect was a small one: the difference in the central momenta resulting from 120 seconds of illumination with the co- and counterpropagating lasers was $\delta p/p \approx 2 \times 10^{-4}$.

The inability to perturb the shape of the erbium beam's velocity distribution led us to consider suggested explanations for such behavior. The first was that the lifetime of the optical transition was much longer than $1\ \mu\text{s}$. This was investigated using the delayed fluorescence technique. The beam was illuminated with a short pulse of laser light, and the fluorescence rate was monitored as a function of time after the pulse. This measurement yielded a lifetime of $\geq 10\ \mu\text{s}$. The cooling rate was thus at least 10 times lower than anticipated. The power-broadened line width of the transition, for the highest laser powers, was thus only of order 1 Mhz. Given the momentum width of the beam, even at the peak of the momentum distribution only about 10^{-4} are in resonance.

The second suggestion was that the intrabeam Coulomb coupling was very strong, leading to frequent collisions energetic enough that particles resonant with the laser would be scattered out of resonance before they could absorb multiple photons. This possibility was investigated qualitatively. Using a beam scraper located in a region of high dispersion in the storage ring lattice, it was possible to scrape away the lowest momentum particles without significantly reducing the beam spatial density. The scraper could then be rapidly withdrawn and the development of the velocity distribution followed in time. A sample measurement is shown in Figure 7-4. The tail in the velocity distribution was repopulated in a time that was at most the time it took for the spectrum analyzer to sweep the frequency range of interest - about 1 second. From this we can infer an acceleration rate of about $10^5\ (\text{eV}/c)/\text{s}$. If we assume that a particle remains in resonance and in physical overlap with the lasers in two straight sections of the ring, the maximum laser acceleration is about $4 \times 10^4\ (\text{eV}/c)/\text{s}$. Whatever the diffusive mechanism in the beam may be, it is of comparable strength with the laser-cooling force. This was further verified by tuning the copropagating laser to be resonant with the tail of the distribution and repeating the above experiment. If the laser force were strong enough to overcome the beam diffusion, it would prevent the filling-in of the tail of the distribution. No difference was observed in the results with or without the laser.

Section 7.4 Conclusions

Based on the above results, we elected to discontinue the effort to laser cool $^{166}\text{Er}^+$ ions. The results point to some important restrictions on the effectiveness of laser cooling in a dense beam in a storage ring. The condition that the laser

cooling force must be stronger than the intrabeam diffusive mechanisms eliminates long-lived optical transitions from consideration as systems to be used in cooling a dense beam. Although these transitions have lower Doppler cooling limits, the force is not strong enough to compete with heating mechanisms in the regime of interest to crystalline beam studies. Cooling with such a system should, in principle, be possible in a very diffuse beam. The limitation is then the ability to diagnose the cooling process with so few particles.

The continued heating of the longitudinal velocity distribution of the erbium ions past the point of equilibration with the transverse motion is most probably due to longitudinal collective phenomena in the beam plasma (Section 3.4.1). Assuming

Figure 7-4 Momentum spectra for scraped beam (*'s) and beam immediately after the scraper is removed (dashed line). The particles have filled in the gap to the low frequency side of the distribution. Note that the vertical scale is logarithmic.

that the space charge dominates the longitudinal impedance, in the absence of cooling, the Keil-Schnell criterion for the $^{166}\text{Er}^+$ beam ($I=2\mu\text{A}$) yields an approximate condition

$$\left. \frac{\delta P}{P} \right|_{FWHM} \geq 1.5 \times 10^{-3}$$

which is in good agreement with Figure 7-3, where we find

$$\left. \frac{\delta P}{P} \right|_{FWHM} = 2.2 \times 10^{-3}$$

This hypothesis could be verified by measuring the equilibrium momentum spread for different beam currents, but this was not attempted. The beam is also subject to longitudinal coherent effects at the time of injection, but the Coulomb scattering dominates the heating rate for early times.

One further observation, first made with the erbium beam, is worthy of mention at this point. Using the improved beam position monitoring system in ASTRID, we noticed that, under some conditions, the injected beam underwent vertical coherent oscillations which became unstable in time. The oscillation amplitude grew until beam loss occurred due to collision of the ions with the vacuum chamber. The growth time was slow, of the order of 1 second. The instability was seen to have a threshold in current, occurring only for the higher currents achievable with the isotopically enriched erbium. It was possible to raise the threshold by retuning the machine to have a higher vertical tune, but the machine was at best quasi-stable at high current ($>1.5 \mu\text{A}$). Small fluctuations in the ion source or the injection steering could lead to unstable growth.

The erbium instability was discovered just before the measurement of the optical lifetime led us to abandon these experiments. A careful characterization of the unstable behavior was therefore not undertaken. The instability has important implications for laser cooling in ${}^7\text{Li}^+$ beams, and will be re-examined in the next chapter.

CHAPTER 8

Further Experiments with ${}^7\text{Li}^+$ Beam

The failure to cool the ${}^{166}\text{Er}^+$ beam necessitated a return to the ${}^7\text{Li}^+$ system, the only other ionic species coolable in ASTRID with the lasers available. Several unanswered questions remained from the first lithium experiment (Chapter 6), and these needed to be investigated before the laser cooling could be studied systematically. This chapter details the second attempt at laser cooling and beam dynamics experiments utilizing ${}^7\text{Li}^+$ ions. An attempt to separate the metastable ions from the ground state ions in the ring was made and is briefly discussed. New beam diagnostic tools were employed to further characterize the storage ring environment. A recent series of measurements using transverse laser probing [WAN1] is also discussed, in the context of the earlier results. The experiments discussed in this chapter took place in the summer of 1991 and the winter of 1992.

Section 8.1 Transverse Beam Instability and Laser Cooling

Upon re-establishing lithium beam in ASTRID, operating with the same tunes as in the first experiments (Chapter 6), laser cooling spectra analogous to that in Figure 6-4 were obtained. The large initial fluorescence level and sudden drop remained. Using the vertical beam position monitoring system and the Schottky pickup, it was quickly determined that the fluorescence drop was associated with beam loss due to resonant growth of vertical beam oscillations. Retuning the machine to the tunes which stabilized the erbium beam produced cooling spectra with a very different structure.

Figure 8-1 shows the signal from a vertical beam position monitor for injected ${}^7\text{Li}^+$ beam with and without the transverse instability. The oscillations of the unstable beam grow until the edge of the beam encounters the vacuum chamber

Figure 8-1 Beam position monitor signals for stable beam (top display) and vertically unstable beam (bottom). The top trace in each case is the sum signal. The bottom is the difference signal, at ten times higher gain than the sum signal. Two consecutive injections are shown. The decay in the sum signal is due to closing of the longitudinal gap left in the beam by the injection system. The revolution period for the ions is about $24 \mu\text{s}$; individual revolutions are not resolved.

wall, at which point the signal decays due to beam loss. (Recall that the signal strength is proportional to the vertical dipole moment of the beam.) The magnitude of the beam loss can be seen in Figure 8-2, which shows signals from the longitudinal Schottky pickup for stable and unstable beam. (The Schottky system commissioned using the erbium beam was modified slightly for use at the second harmonic of the lithium revolution frequency.) The beam loss corresponds in time with the sharp fluorescence drop in Figure 6-4, both occurring just after 1 s after injection. The fluorescence drop is explainable by resonant particle loss. The inability to laser cool in the period before 1 s after injection (Section 6.3) is certainly related to the coherent beam behavior, but precisely how is not known. The coherent oscillations of the beam center undoubtedly remove some of the ions from physical overlap with the laser beam. The transverse motion may also couple into the longitudinal thermal motion of the ions, leading to diffusion of ions past the cooling lasers. The longitudinal Schottky signal displays coherence in the period before the particle loss (Figure 8-2), which may indicate transverse to longitudinal coupling, or an independent longitudinal instability. The spatial density of the stable beam is at least a factor of 6 greater than that of the unstable beam, due to the loss of particles (Figure 8-2). The unstable beams' density is probably much lower than this, as the transverse emittances almost certainly blow up as a result of the coherent oscillation. The remaining beam current for the unstable beam was too small to accurately measure the beam size with the scraper (Section 8.4).

Figure 8-2 Longitudinal Schottky power at the second revolution harmonic vs. time for stable (top) and transversely unstable lithium beam. The vertical scale is 2 dBm/div. The time axis is 29.9 s full scale. Three consecutive injections are shown. The off-scale excursion at the beginning of each cycle is due to longitudinal coherence in the beam. For the stable beam the coherence persists for about 300 ms, corresponding to the time required for the longitudinal gap in the beam to close. For the unstable beam, the coherence persists for up to 1 s. The stable beam current is roughly 6 times higher than the current remaining after the particle loss.

As in the case of the erbium beam, retuning the machine to have a higher vertical tune stabilized the beam for less than about 10^9 injected particles. A laser cooling scan for stable beam is shown in Figure 8-3. The scan parameters are identical to those for Figure 6-4. The beam current and laser powers are also comparable to those used in Figure 6-4. The only significant difference in the two measurements is that lasers were employed in two straight sections of the ring in Figure 8-3; only one straight section was used in the earlier measurement. The average cooling force is thus stronger for the cooling scan in Figure 8-3.

Figure 8-3 Laser cooling scan for 100 KeV ${}^7\text{Li}^+$ beam with no transverse instability. The injected current was $9.8 \mu\text{A}$. Cooling took place in two straight sections. The sweep duration was 4 seconds.

The sharp fluorescence drop characteristic of particle loss is no longer present. For the same injected current, the cooled peak is larger and broader than in the case of unstable beam. Even with approximately twice the cooling force, we find that there is now a Doppler contribution to the width of the cold peak - the

peak width is not fully accounted for by power-broadening of the cooling transition. The systematic investigation of the beam temperature will be discussed shortly.

Section 8.2 Time Development of the Longitudinal Temperature

For this second round of lithium experiments, several improvements to the diagnostics systems allowed for improved optical detection and beam characterization. The beam orbit control system was fully functional, allowing for precise overlap of the laser and ion beams. The aperture of a moveable microwave cavity used in laser/RF double fluorescence measurements [KRI2] in the ring served as a further guide to overlapping the beams. The laser windows in the vacuum chamber had been tilted at slight angles to prevent the light reflected by the window surfaces from scattering to the phototube. The newly-installed post-acceleration tube (Figure 5-10) allowed for measurement of the longitudinal beam temperature at later times after injection.

Using the improved laser diagnostics, the longitudinal beam temperature was again measured as a function of time after injection. Both stable and transversely unstable beam were studied. The results are plotted in Figure 8-4. As in the case of the erbium beam, the temperature of the stable beam at late times rises to well above that expected merely from transverse to longitudinal temperature equilibration. Schottky spectra taken at times as late as 20 seconds after injection reveal little change from the 3300 K longitudinal temperature maximum in Figure 8-4. For 12 μ A lithium beam, the Keil-Schnell criterion gives

$$\left. \frac{\delta P}{P} \right|_{FWHM} \geq 2.3 \times 10^{-3}$$

which is in good agreement with the spectrum shown in Figure 5-6, which had $T=3100$ K, and

$$\left. \frac{\delta P}{P} \right|_{FWHM} = 2.7 \times 10^{-3}$$

In the absence of cooling the beam behavior is consistent with the standard instability criterion.

Figure 8-4 Laser fluorescence measurements of the longitudinal temperature vs. time after injection for stable (solid circles) and transversely unstable (squares) ${}^7\text{Li}^+$ beam. The injected beam current was $12\ \mu\text{A}$. The curves are drawn to guide the eye.

The sudden drop in longitudinal temperature as the transverse oscillations lead to beam loss (Figure 8-4) is also noteworthy. Since the oscillation is in the vertical plane, we do not expect *a priori* that high or low momentum particles will be selectively lost by contact with a physical aperture. This would be the case for a horizontal oscillation if the limiting aperture were in a location having momentum dispersion. More likely is that, due to the uncorrected chromaticity in the machine,

either the high or low momentum particles are more susceptible to resonant growth because of their extreme betatron frequencies.

The nature of the transverse instability was not investigated further. This may be undertaken when better diagnostics, such as transverse Schottky pickups, are available. The slow growth time (≈ 1 s) suggests that the resonance is of fairly high order. Such a slow growth should be easily damped with an active feedback system [LAM1]. Since transverse cooling of the beam will certainly lead to other resonance crossings, such a system is probably essential for a dense beam.

Section 8.3 Measurement of the Transverse Beam Sizes

In order to accurately characterize the beam and determine what dynamic processes are important, the beam spatial density and beam transverse "temperature" must be known. The transverse extent of the stored beam must therefore be measured. As mentioned in Chapter 6, the beam size can be measured using the fluorescence image of the beam. A slit can be scanned across the image, and the beam size determined from the distribution of emitted photons. The transverse intensity profile of the laser must be taken into account, and the technique is limited to beams smaller in diameter than the laser beam. In ASTRID, the measurement is limited to the vertical plane due to the detection geometry. The technique is also limited to early times after injection. As the longitudinal velocity distribution heats up, the fluorescence signal due to a fixed-frequency laser drops because fewer and fewer particles are in resonance. The laser technique was employed successfully to measure the vertical size of the $^7\text{Li}^+$ beam immediately after injection (Chapter 6) for use in calculating the heating rate due to intrabeam scattering.

After commissioning of the Schottky system, another method for measuring beam sizes became available. By moving a beam scraper (Chapter 5) into the stored beam and measuring the strength of the Schottky signal as a function of scraper position, the beam transverse sizes can be inferred. Using the known properties of the storage ring lattice, the beam sizes in the rest of the ring may then be determined. The transverse velocity distributions may also be inferred. ASTRID is equipped with both horizontal and vertical scrapers, located near the center of a straight section (Figure 5-3). Since there is momentum dispersion at this point in the ring, the momentum distribution of the beam must be factored into the horizontal size determination.

A typical scraper scan is shown in Figure 8-5. Due to the low beam currents (a few μA), the measurement is repeated many times; the plot in Figure 8-5 is the average of 100 individual measurements. Stability of the beam position at injection and stability of the ion source were monitored carefully to ensure that there was no broadening of the measured sizes due to orbit drift. Vertical and horizontal scraper scans were taken for 3 different delays: 2 s, 5 s, and 10 s after injection. The injected beam currents were in the range of 9 to 11 μA . The beam was transversely stable for all measurements.

Figure 8-5 Vertical scraper scan for ${}^7\text{Li}^+$ beam. Longitudinal Schottky power (relative units) is plotted against position of the vertical scraper. The scan begins 2 seconds after injection and takes two seconds. The injected current was about 10 μA . The circles are the measured points; the solid curve represents smoothed data.

A curve such as Figure 8-5 is an integral representation of the distribution of betatron oscillation amplitudes for the particles in the ring at the longitudinal location of the scraper. Since the scraper moves very slowly relative to the time scale for betatron oscillations, for a scraper position x , all particles with betatron amplitude $x_\beta \geq x$ are lost (cf. Figure 2-2). Since a particle with betatron amplitude x_β may be found anywhere between x_β and $-x_\beta$ on a given revolution, the real spatial distribution of the beam is the convolution of the betatron amplitude

distribution with a function expressing the relative time a particle with amplitude x_β spends between x and $x+dx$. This may be expressed

$$S(x) \propto - \int_x^\infty \frac{dI}{dx_\beta} \frac{1}{\sqrt{x_\beta^2 - x^2}} dx_\beta$$

where $S(x)dx$ is the number of particles between x and $x+dx$, and $\frac{dI}{dx_\beta}$ is the derivative of the scraper scan curve (beam current I versus betatron amplitude x_β).

Using the measured data, the right-hand side of the above expression can be numerically integrated to yield the beam spatial profile. The largest uncertainty in the calculation is in the determination of the beam center position x_c . For the current measurements this translates into an uncertainty in the rms beam sizes of about 0.5 mm. The result of a sample numerical integration is shown in Figure 8-6.

Figure 8-6 Vertical beam half-profile deduced from a scraper measurement. The scan used was the measurement at 10 second delay. The *'s are determined by numerical integration of the measured data. The solid curve is the best-fit Gaussian to the data.

Using Schottky momentum spectra taken at the same delay times as the profile measurements, and assuming that the transverse emittances and longitudinal momenta are uncorrelated, the horizontal emittance can be determined in each case (Gaussian distributions in momentum and emittance are assumed). The vertical emittances are determined directly from the measured sizes and the lattice functions (see Chapter 2). The results are summarized in Table 8-1. Within the experimental error, there is little or no emittance growth for the period studied. The vertical beam size is, however, about twice as large as that measured by the laser at injection (Chapter 6).

Table 8-1
Results of Scraper Measurements

Plane, Delay-time	RMS 1/2 size	Emittance (1σ)
Horizontal 2 s	10.2 ± 0.7 mm	8.0π mm-mrad
Horizontal 5 s	11.4 ± 0.9 mm	10.3π mm-mrad
Horizontal 10 s	11.8 ± 1.1 mm	11.1π mm-mrad
Vertical 2 s	4.6 ± 0.7 mm	6.9π mm-mrad
Vertical 5 s	4.6 ± 0.8 mm	6.9π mm-mrad
Vertical 10 s	5.1 ± 0.9 mm	8.4π mm-mrad

Using the measured emittances and the lattice functions for the entire ring, the beam sizes at any point in the ring can be inferred. Figure 8-7 shows a plot of the beam transverse area for the measurements at 2 s delay. The area is taken as $2\pi\sigma_x\sigma_y$, where σ_x and σ_y are the horizontal and vertical rms sizes, determined from the Gaussian fits to the measurements. The figure also shows the average cross-sectional area for an ASTRID beam of 10^9 ions having zero transverse temperature. The measured beam is obviously not close to the space charge dominated size. The beam size measurements can also be used to determine the transverse velocity spreads in the beam. We will return to this point in the discussion of transverse laser probing.

Figure 8-7 Measured transverse beam area (solid curve), plotted for $1/4$ of the ring circumference. The dot-dashed line represents the zero temperature size of a beam of 10^9 ions confined with the average focusing strength of the ring.

Section 8.4 The Effectiveness of Laser Scanning

To achieve effective cooling, the scanned laser must capture and accelerate (or decelerate) all of the particles with which it comes into resonance. In the case of $^{166}\text{Er}^+$ ions, the laser force was not strong or long-ranged enough to prevent particles from escaping the velocity window defined by the lasers. It was therefore of interest to determine the effectiveness of laser scanning in the lithium beam. Two separate methods were employed to gain a qualitative understanding of the effectiveness of this process.

The first method is depicted in Figure 8-8. The copropagating laser was tuned to be in resonance with ions of velocity $v_0 - \Delta v$ (v_0 is the mean ion velocity), while the counterpropagating laser was in resonance with velocity $v_0 + \Delta v$. After the beam had been injected into the ring, the lasers were swept simultaneously towards the center of the distribution, forming the cooled velocity distribution centered around v_0 . The height of the cooled peak was studied as a function of the initial velocity offset Δv . The height of the cold peak should be

proportional to the number of particles in resonance with both lasers just before they cross. The measurement thus addresses the question of whether the lasers collect a number of particles proportional to the population in the swept region.

Figure 8-8 Diagram illustrating the variable sweep range experiments. The initial detunings of the lasers are shown relative to the velocity distribution. Both lasers are subsequently swept towards the center of the velocity distribution. The velocity and detuning scales are approximate.

The equilibrium velocity distribution of the beam is very well represented by a Gaussian (Figure 5-7). The experiment was carried out for several values of Δv , ranging from 0.26σ to 2.2σ of the velocity distribution (1 to 8.5 Ghz detuning). The results are plotted in Figure 8-9. To check if the relative peak heights are consistent with those expected from scanning the Gaussian distribution, the Gaussian integrals

$$I = \int_{v_s - \Delta v}^{v_s + \Delta v} \frac{1}{\sqrt{2\pi}} e^{-(v-v_s)^2/2\sigma_v^2} dv$$

were evaluated for the range of detunings in the experiment using a numerical routine. These were then normalized to the data point for 4 Ghz detuning and are plotted as the solid curve in Figure 8-9. The agreement between the expectations and the measurements is reasonable. The laser does seem to collect a number of particles proportional to the number scanned, at least for the case in which the scan is performed after the fast longitudinal heating has subsided. An analogous measurement made immediately after injection revealed no evidence for particle accumulation.

Figure 8-9 Relative number of particles accumulated by swept lasers as a function of the width of the sweep. The measured points are normalized to the point at 4 Ghz detuning. The solid curve is that expected for a Gaussian velocity distribution. The errors are due to counting statistics and the error in current measurement. (See also Section 8.7.)

The second measurement tested the efficiency of the accumulation process. This was achieved by sweeping a single laser *twice* across the velocity distribution. Referring to Figure 8-8, the copropagating laser was kept at fixed frequency detuned about 5 GHz to the low-velocity side of the center of the velocity distribution. The counterpropagating laser was swept from its starting point at 15 GHz to the high velocity side until the two lasers crossed, forming a fluorescence peak. The counterpropagating laser was then de-energized, returned to its starting point, and the sweep was repeated. If the first scan is effective at accumulating metastable ions, the second scan should accumulate at most half as many as the first. This is because when the lasers cross, the cold distribution is disrupted and the particles are accelerated out of resonance by the lasers. Assuming equal laser strengths and equal physical overlap of the two lasers with the ion beam, half of the particles will be lost to the low velocity side of the fixed laser, where they can not be accumulated by the second scan. The swept laser continued to sweep 10 GHz past the point that the cold distribution was formed, in order to be sure to decelerate the remaining particles well out of the area where they might be captured during the second scan.

The fluorescence signature of such a double scan is shown in Figure 8-10. Contrary to the expectations of efficient sweeping, a second peak of the same intensity as the first is observed. This is particularly surprising since the second peak is produced at a time corresponding to $3/4$ of the storage half-life later than the first. There are thus roughly 40% fewer particles in the ring at the time the second peak is formed. That this is the case can also be seen in the figure. The broad peak to the left of each sharp fluorescence peak is the fluorescence signal of the uncooled velocity distribution as the laser sweeps across it. The signal has clearly decayed from one scan to the next, yet the cold peaks are comparable.

This experiment was further refined to ensure that the particles collected on the first sweep were unable to be recaptured on the second. On the first scan, the stopping (copropagating) laser was not present, so as the counterpropagating laser continued its sweep no cold distribution was formed. The particles collected were thus not subjected to the heating force caused by the lasers crossing. In addition, the momentum scraper was positioned partly into the aperture so that the ions decelerated by the sweeping laser would be lost from the storage ring. The second scan then proceeded as in the first experiment, with the stopping laser energized to form the fluorescence peak out of whatever ions could be accumulated. The control for this experiment was to de-energize both lasers for the time period of the

first scan, so that no ions would be swept out of the machine. The second scan was then performed, creating the cold distribution.

Figure 8-10 *The swept laser scans the ions' velocity distribution twice. Two nearly identical peaks are created, although the first sweep should have removed ions from the scanned region.*

The results of these measurements are shown in Figure 8-11. The sizes of the cold peaks are equal, within the experimental error. There is thus no evidence that the first laser scan has removed a significant fraction of the ions from the machine. We are forced to conclude that the laser is inefficient at accumulating and decelerating ions as it is swept. This may point to the existence of intrabeam interactions which cause velocity diffusion of the particles, enabling them to jump out of resonance with the laser before they can scatter many photons. Even with the short lifetime of the lithium transition, the velocity range of the laser cooling force (~ 10 m/s) is only about 10^{-3} of the FWHM (~ 5000 m/s) of the equilibrium velocity distribution of the uncooled ions. The scan rate of the lasers may also be too high; this has not yet been tested experimentally.

Figure 8-11 Double sweep experiments. Top: The swept laser passes over the velocity distribution once and decelerates captured particles into the scraper. The sweep is then repeated with the fixed laser energized. A cold peak is formed, indicating that metastable ions remain in the ring. Bottom: The first scan is not performed, so no metastable ions are lost. A cold peak is formed on the second scan. The heights of the cold peaks for the two experiments are comparable. The frequency scale for the swept laser is as in Figure 8-10. The jumps in background occur when the copropagating laser is switched on because its frequency is above the center frequency of the IF filter. The scattered photons can lose energy, allowing them to pass through the filter.

Section 8.5 Separation of the Metastable Ions from the Ground State Ions

In light of the uncertain influence of the large ground-state component on the cooling experiments, an attempt was made to remove them from the ring. The procedure was to use the lasers to accelerate (or decelerate) the metastable ions while the magnets of the storage ring were ramped to keep the metastables at the central energy of the ring. The ground state ions, which are not accelerated, eventually hit the vacuum chamber of the storage ring because their orbits are displaced due to momentum dispersion. The momentum scraper was partially inserted in the ring so that the energy change of the metastables could be minimized.

These attempts were partially successful. It was possible to decelerate metastable ions by up to 15 GHz in laser frequency, corresponding to about 1 keV in lab-frame energy. At the same time, the ramping of the magnets removed ground-state ions to the extent that the signal from the longitudinal Schottky pickup reached its noise level. This implies that the number of ground-state ions fell by at least a factor of 50. The fluorescence signal of the cooled distribution formed at the new energy was at best half as strong as that of the distribution formed without acceleration. However, the results of the previous section on the efficiency of laser scanning prohibit a quantitative evaluation of the effectiveness of the deceleration process. The technique shows promise and needs to be investigated with more sensitive diagnostics in the future.

Section 8.6 Transverse Laser Probing of the Stored Ion Beam

One of the dye lasers was used as a probe of the transverse velocity distribution of the ions [WAN1]. The experimental configuration is shown in Figure 8-12. The laser beam was directed at an angle of 56° with respect to the ion beam, so that the fluorescence photons would be Doppler-shifted relative to the scattered laser photons. Since there is a longitudinal component to the Doppler shift, the longitudinal velocity distribution must also be measured using a longitudinal laser. The width of the vertical velocity distribution can then be unfolded from the two measurements.

Figure 8-12 Configuration of the transverse probing experiments.

As in the case of longitudinal probing, the transverse temperature at the position of the probing laser was measured as a function of time after injection. The results are shown in Figure 8-13. The injected beam heats up from an initial temperature of ~ 500 K to about 5000 K in 3 seconds. Two points inferred from the scraper measurements of the beam size (Section 8.4) are also plotted. The agreement with the laser measurements is good. A scraper measurement at 11 seconds after injection (not plotted) indicates a further slight rise to 6100 K over the next 5 seconds.

It must be emphasized that this transverse "temperature" is associated with the mean-square vertical component of velocity at a single location in the ring. The temperature defined in this way is not a constant in the storage ring, as the beam trades transverse kinetic energy for potential energy in the alternating gradients of the magnets. The conserved quantity is the emittance (Chapter 2).

Figure 8-13 Vertical temperature vs. time after injection measured by transverse laser probing. The solid dots are inferred from the scraper scans (Section 8.4). (Courtesy of B. Wanner)

The transverse heating is best described as a blowup of the transverse emittance of the beam. The source of this blowup has not yet been identified. The above measurements were made on a beam that was, to the limit of the sensitivity of the diagnostics, free from transverse injection error (in position and angle of the beam) and coherent transverse instability.

Some possible mechanisms which would not be detected by the diagnostics available are a mismatch between the injection line and the storage ring, and incoherent growth due to betatron resonances. In the former, the orientation of the beam in the transverse phase plane is not similar to the orientation of the acceptance ellipse for the machine at the injection point. The particles do not follow similar orbits in the phase plane, leading to emittance growth by filamentation [BRY1]. The designs of the ring and injection line are known to be slightly mismatched in the vertical plane [CRA1]. This process may explain the initial fast growth, but the slower growth at later times is inconsistent with injection mismatching. The slower growth may be due to high order betatron

resonances; the chromaticity of the machine is not corrected for these measurements. As the longitudinal momentum spread increases, so does the tune spread of the beam, possibly leading to resonance crossing for some particles. Intrabeam scattering can also not be ruled out for the slow growth mechanism; at early times the energy transfer is to the longitudinal motion. The relevance of this transverse emittance blowup to the effort to reach the strongly-coupled regime will be discussed in Chapter 9.

Section 8.7 Measurements with the Neutral Particle Detector - Beam Lifetime and Fast Particle Loss

The neutral particle detector (Figure 5-3) proved to be a very useful monitor of the stored beam current. Since it measures the result of ion collisions with the rest gas, the signal is proportional to the total stored current (for constant pressure in the vacuum system). The detector signal was analyzed using a computerized multichannel scaling system. Integrating for 50 ms per channel, the detection system accumulates for about 2100 revolutions for each data point. The system is thus insensitive to the gap in the beam at early times after injection. Unlike the Schottky pickup, which is not a useful current monitor during the period that the beam is debunching, the neutral particle detector can be used immediately after injection to detect changes in beam current on the time scale of 50 ms or less.

One of the first storage lifetime measurements with this detector is plotted in Figure 8-14. The beam was observed to undergo a fast loss in the first 2.5 seconds after injection, followed by a slower, exponential decay. The slow decay agrees with the lifetime measured by the Schottky detector, but the fast loss was not observable on the Schottky due to the initial coherence from the gap. Further study showed that the existence of the fast drop was dependent on the amount of beam injected into the ring from the ion source. Below a certain threshold in current, the beam decayed smoothly with a storage lifetime determined by the ring pressure.

Figure 8- 14 Signal from the neutral particle detector for $9.4 \mu\text{A}$ current injected. An exponential fit to the data from 5 to 50 seconds after injection is also shown. The inset shows the same data on a logarithmic scale.

The behavior of the beam as a function of the injected current (as measured by the Faraday cup in the injection beamline) is depicted in Figure 8-15. Two sets of points are plotted. The first (circles) gives the measured count rate at time $t=0$ as a function of the number of particles injected into the ring. The curve is linear as expected; a least-squares fit to the data is also plotted. The other plotted points (+'s) represent the initial count rate that would be expected from the exponential fit to the decay for times 5 to 50 seconds (5 seconds is chosen as being safely after the fast loss has ended.) The two sets of points agree very well for less than about 1.2×10^9 particles injected; there is no fast particle loss. Above this level, the stable stored current does not increase with increasing injected current. The two points

with the highest injected current suffer from the vertical coherent instability, which explains why their stable current is so low. The others, however, show no evidence for coherent transverse instability.

Figure 8-15 Initial signal in the neutral particle detector vs. number of ions injected. The circles are the measured points. The +’s are the expected values based on the stored current at late times (see text).

There is thus apparently a maximum current which may be injected into the ring while maintaining stability. We must, however, consider the possibility that the fast loss of signal is simply due to the decreasing spatial density of the beam as the emittance blows up. Perhaps the flux of neutrals reaching the detector simply decreases because the beam is expanding transversely. The equilibration time for the vertical emittance was also of order 2 seconds (Section 8.6). This possibility has been eliminated by using the longitudinal Schottky pickup. Since the beam goes through the Schottky pickup, the pickup is not sensitive to any change in the transverse extent of the beam. Examining the relative strength of the Schottky

signal for times after 5 seconds yields the same result: increasing the number of injected particles above about 1×10^9 does not result in more stored current.

Whether this particle loss has a longitudinal or transverse origin is not yet known. The vertical emittance blowup recorded by the laser probing (Figure 8-13) does not result in an emittance large enough that the beam should be lost on the walls of the vacuum chamber. The horizontal beam sizes measured with the scraper are also not large enough to explain significant particle loss. The coherence in the longitudinal Schottky signal persists for only about 300 ms, compared to 2.5 seconds for the fast loss.

Whatever the source of the loss, the implications are clear. The cold, dense plasma injected into ASTRID is unstable for currents above about 7 μA . This threshold is much lower than the threshold ($>14 \mu\text{A}$) for coherent transverse instability, measured under the same circumstances. Normalization of experimental results using the current measured by the Faraday cup in the injection beamline is only valid for currents below about 7 μA . Above this, we can not say with certainty how much current remains in the ring after the fast loss, unless simultaneous measurements are made, preferably with the neutral detector, because of its insensitivity to coherence, or with the Schottky probe. Simultaneous use of the neutral detector and lasers for cooling is currently not possible, as the detector blocks one of the laser windows.

Section 8.8 Temperature of the Cold Distribution

Two attempts were made to study the structure of the fluorescence signature of the cold distribution as a function of the beam spatial density. The intent was to quantify the density dependence of the longitudinal temperature of the cold ions. The cold distribution achieved with very sparse beam (Chapter 6) had been found to be much colder than that obtained with the denser, transversely stable beam. The beam density was varied by simply allowing the stored current to decay for varying times before performing a cooling scan. Delay times out to about 20 seconds can be studied with reasonable accuracy. Beyond twenty seconds the contribution to the width of the cold peak due to drift in the laser frequencies is no longer negligible, as the spectra take a long time to accumulate to good statistics. The other method of varying the beam current, injecting different lengths of beam using the chopper, had been shown to result in strong, longitudinal collective effects in the beam (KRI1).

In light of the observed range of instability phenomena and the measured ineffectiveness of the lasers to accumulate particles during scanning, the results of these cooling characterization measurements may at best be interpreted qualitatively. Confidence in the determination of the beam spatial density would require constant monitoring of the ring by all of the diagnostic devices which have been described above. This is a practical impossibility, as some of the diagnostic methods are destructive (scrapers) and cannot be used as real-time monitors. Others (transverse laser probing, neutral particle detection) are incompatible with the use of two lasers for cooling. Experimentally, our incomplete characterization of the beam manifests itself as a degree of irreproducibility in measurements taken under apparently similar circumstances. Suggested improvements to the experimental setup will be discussed in Chapter 10.

Given the above restrictions, the results of cooling experiments permit some general conclusions. The first is that, in the absence of the vertical coherent instability, the distribution shape consistent with the low longitudinal temperature of 1 mK (Chapter 6) is not obtainable for the range of circulating currents studied (1 to 7 μA). The minimum temperature achieved for the stable beam, attributing the peak width to power broadening and Doppler broadening only, was about 50 mK. We may speculate that reducing the current is not sufficient to weaken the diffusion enough that it does not define a large equilibrium temperature. The unstable beam suffers current loss *and* transverse emittance dilution. The transverse sizes of the unstable beam have not been measured, but assuming that the beam fills the machine aperture after the instability, the vertical size and transverse temperature of the beam are at least three times larger than those of the stable beam. All of the collective or stochastic processes which can lead to growth of the beam longitudinal momentum spread (Chapter 3) are thus much weakened. This point will be discussed further in Section 9.3.

The second general feature of these experiments is that the lasers become more and more efficient at collecting particles into the cold distribution as the current in the ring decreases. This observation may also point to a weakening of the disruptive processes as the beam density decreases. There may also be a slow decrease in the width of the cold distribution, that is, in the contribution of Doppler broadening to the width, as the current decays. This effect is, however at the limit of the experimental resolution and must be studied further.

CHAPTER 9

Beam Dynamics, Laser Cooling, and Crystalline Beams in ASTRID

The results of the experiments described in chapters 6 through 8 provide information about the dynamics of the beam plasma in ASTRID and the interplay between the laser cooling force and intrabeam or external forces. ASTRID is a unique machine in that it operates in the low-energy, space-charge dominated regime of storage ring dynamics. In this chapter the laser cooling and the laser and conventional diagnostic measurements are reviewed in the context of the information they provide about the possibility of using laser cooling to create an ordered OCP in a storage ring. The first step is an orderly review of the diagnostic results and of the characterization they provide of the ASTRID stored beam.

Section 9.1 Characterization of the ASTRID Stored Beam

The ASTRID injection system is capable of providing a cold, dense beam for injection to the storage ring. The injected beam spatial density is as high as 2×10^6 ions/cm³. The injected longitudinal momentum spread is of order 10^{-6} , corresponding to a temperature in the co-moving frame of about 0.1 K. The initial transverse kinetic energy is equivalent to a temperature on the order of 1000 K. From the indications of all of the diagnostic devices examined to date, in the absence of cooling this injected beam is far from equilibrium in the storage ring. The beam rapidly heats up in all three phase planes to a state an order of magnitude less dense and several orders of magnitude hotter than the injected beam. Table 9-1 provides a summary of our understanding of the initial and equilibrium states of the beam (⁷Li⁺ is described.) The diagnostic method used to arrive at each beam parameter is also identified.

Table 9-1
Initial and Equilibrium Beam Parameters

Parameter	Value	Method
initial number of ions	up to 3×10^9	Faraday Cup
maximum stored ions	1×10^9	Faraday Cup, Neutral Det.
initial longitudinal temp.	100 mK	laser fluorescence
equil. longitudinal temp.	3000 K	Schottky pickup
initial transverse temp.	500 K	laser fluorescence
equil. transverse temp.	6000 K	laser fluor., scraper
initial vertical 1/2 size	1.7 mm	laser fluorescence
equil. vertical 1/2 size	5.7 mm	beam scraper
initial horizontal 1/2 size	1.7 mm	(assumed = vertical)
equil. horizontal 1/2 size	7 mm	scraper, Schottky
initial ion density	$2 \times 10^6/\text{cm}^3$	(derived)
equil. ion density	$10^5/\text{cm}^3$	(derived)

Section 9.2 Dynamic Phenomena Identified

In some cases, the dynamic phenomena that lead to equilibration of the injected beam in ASTRID can be confidently identified. Temperature equilibration between the transverse and longitudinal degrees of freedom due to intrabeam scattering drives the initial rise ($t \leq 1$ s) in the longitudinal temperature. The width of the equilibrium momentum distribution is in good agreement with the threshold criterion (Keil Schnell) for instability of longitudinal charge-density waves. The above arguments are consistent with the data for both lithium and erbium beam.

In other cases, the diagnostic tools available do not provide enough information to identify a cause with an effect. The transverse coherent instability (vertical dipole oscillation) is not surprising in light of the density of the beam and the estimated magnitude of the coherent space-charge tune shift, but more study is necessary to pinpoint a precise cause. The cause of beam loss for times less than 2 s and currents above $7 \mu\text{A}$, is very difficult to identify, as the beam is unstable and changing rapidly in all planes. It is thus difficult to compare thresholds for

various known instabilities with the onset of the particle loss. The beam is unstable for multiple reasons.

The causes of these instabilities are not as important to the present study as are their implications for the effectiveness of laser cooling of a dense low-energy beam. We can generally consider the dynamic phenomena to be characterized by the degree of freedom on which they act, the time scale on which they develop, and the rate with which they transfer energy to the thermal motion of the ions in the beam. In some cases, the fraction of particles participating in the unstable behavior may also be relevant, if, for example, the mechanism is frequency selective. In the following, some of the observed longitudinal dynamical phenomena are categorized in terms of their energy transfer rates, and these transfer rates are compared, using a simple model, to the strength of the laser cooling force.

Section 9.3 Diffusion Rates of Observed Phenomena and their Implications

In light of the discussion of Chapter 4 concerning laser cooling and the Fokker-Plank equation, it seems natural to attempt to categorize some of the equilibration phenomena in the ion beam in terms of a diffusive spreading of velocities [MØLM3]. The equilibrium temperature for laser cooling was, after all, given by the balance of the damping due to the laser friction force and the diffusive spreading due to the spontaneous emission. For simple order of magnitude estimates, we can consider the diffusive mechanisms in the beam to be velocity-independent. The magnitude of the momentum-independent laser diffusion coefficient given for metastable ${}^7\text{Li}^+$ ions and the laser parameters discussed in Chapter 6 is of order $10^4 \text{ m}^2/\text{s}^3$ [JAV1, STE2].

The rate of increase of the longitudinal kinetic energy of the lithium beam immediately after injection was measured by laser fluorescence to be of order 1 eV/s . This can be converted to a longitudinal diffusion coefficient by dividing by the mass of the ion. The result in MKS units is $D \sim 10^7 \text{ m}^2/\text{s}^3$, which is 3 orders of magnitude higher than the laser diffusion rate. At times a few seconds after injection, when the lasers are swept together to form a cold distribution of particles, the measured longitudinal diffusion (for the beam without the coherent instability) has decreased to $D \sim 10^6 \text{ m}^2/\text{s}^3$ (Figure 8-4), which is still a factor of 100 higher than the laser-induced diffusion. If we take the weak-field expressions for the strength of the laser friction force (Chapter 4), the measured longitudinal diffusion at late times implies an equilibrium temperature of order 10 mK. The temperatures

corresponding to the apparent Doppler contribution to the width of the cold distribution for the isomeric fraction of the stable beam were about 50 mK, in good agreement with this rather crude estimate.

The measured diffusion rate may be used to estimate how long it takes for an ion to diffuse across the velocity range of the cooling force. Using

$$\langle v^2 \rangle = 2Dt$$

and taking the range of the cooling force to be of order $5\Gamma/k$, based on a numerical calculation performed to model the ASTRID experiment [BER2], the magnitude of the diffusion coefficient at 3 seconds after injection implies

$$t_d \approx 50\mu s$$

Considering that this is of the same order of magnitude as the revolution frequency in the storage ring, and that the particles experience no laser cooling force to counteract the diffusion in more than half of the ring circumference, it is no longer surprising that the laser is not very efficient at accumulating particles during a frequency scan. (The particles' betatron oscillations may also carry them outside of the region illuminated by the laser.) The inability of the lasers to stop the very fast blowup of the longitudinal temperature which occurs immediately after injection is also to be expected, since the diffusion is at least a factor of 10 higher at early times.

The above crude estimates are also reasonable when we consider the low temperatures (1 mK) reached for the cooling experiments with the beam which had the vertical coherent instability, leading to particle loss and emittance blow-up. We must only make the assumption that the diffusion rate is proportional to the beam density. The spatial density of the beam after the particle loss was at least 10 and perhaps 50 times lower than that of the stable beam. Such a reduction in density would reduce the diffusion coefficient to nearly the level of the laser diffusion. The reasonable agreement between measured and calculated fluorescence line shapes for the unstable beam may thus be accounted for.

We should also note that this situation will not necessarily improve by utilizing a single component beam in which all ions can be cooled. The diffusion rate for the dense beam is dominated by the beam plasma dynamics, and the

strength of the laser cooling force depends only on the characteristics of the cooling transition and the laser light. The ability of the laser to perturb the ion velocities at a given point in the ion velocity distribution will depend on the local magnitude of the diffusion coefficient at that point. A more careful calculation of the steady-state solution to the Fokker-Planck equation utilizing the measured diffusion rate, assumed to be velocity independent, predicted that the laser would not be able to locally perturb the ions' velocity distribution for currents above about $1\ \mu\text{A}$ [MØLM3].

The situation is further complicated when one remembers that we depend on the longitudinal laser cooling to remove the transverse kinetic energy as well. In order for this to occur, the diffusion rate due to coupling between the transverse and longitudinal degrees of freedom must be large enough that energy can be removed, but small enough that particles are not lost by diffusing too quickly out of range of the laser force. Careful modeling of this situation using the Fokker-Planck equation should yield limits on the tolerable diffusion rate. The question of the optimum frequency scan rate of the lasers must also be addressed. This has not yet been studied experimentally. The assumption, used in modeling of the sparse beam, that the velocity distribution comes rapidly to equilibrium with the laser cooling force at each step of the laser scan is almost certainly not valid for the denser beam. We must therefore consider time-dependent solutions to the Fokker-Planck equation in the analysis of the cooling process.

Better experimental input into the theoretical models of the nature of the intrabeam or external forces causing the diffusion is needed. Specifically, the diffusion needs to be quantified as a function of ion velocity in order to decide if the laser can macroscopically perturb the velocity distribution of a dense beam. The molecular dynamics calculations can unfortunately not help us due to the large temperatures characterizing the experimental situation. In many cases, better diagnostics in ASTRID will provide more clues to the precise cause of the observed diffusion rates. Some of these causes, such as the coherent transverse instability, have technical cures unrelated to cooling. Intrabeam scattering in the hot dense beam can only be counteracted by cooling. In regimes where the diffusion is of comparable magnitude to that measured in ASTRID, laser cooling forces having the strength and range of that in ${}^7\text{Li}^+$ may not be very efficient at accumulating ions into a cold distribution.

Section 9.4 Implications for the Attainment of Crystalline Beams

The experimental realization of the laser cooling force in hot stored beams is significantly different from the cooling simulated in cold beams using the molecular dynamics method. In the simulations the force is applied to all particles whose relative velocities become too large, that is, the force has a range larger than the velocity spread in the (already quite cold) beam. The cooling force also acts on a time scale similar to the revolution period in the storage ring. The type of force employed in the MD program is thus analogous to the laser force seen by the particles already captured by the laser and formed into the cold distribution. The MD force is not representative of the laser force sweeping across the hot distribution. This is not a fundamental limitation with the MD method, as a laser-like force could be modeled, but this has not been attempted to date. The MD simulation does also not simulate interaction of the beam with its environment, which experimentally leads to coupling of the beam's large lab frame energy into the relative thermal motion.

Nevertheless, the metastable fraction of a sparse beam of ${}^7\text{Li}^+$ has been cooled to the very low longitudinal temperature of 1 mK. Careful calculation of the laser force [BER2] and the order of magnitude estimates of Section 9.3 both support the conclusion that the diffusion due to intrabeam and external forces in this sparse beam must be of the same order of magnitude as the diffusion due to the laser alone. If we make the optimistic assumption that the lasers can tolerate diffusion 10 times larger and remain effective at cooling, we may ask how long it would take for the lasers to remove the kinetic energy associated with the transverse motion of the beam. Taking a transverse temperature of 1000 K and assuming that the transverse energy can couple to the longitudinal motion at the same rate that longitudinal energy is removed by the lasers, the time scale obtained is roughly 20 seconds per transverse degree of freedom. This is again a very crude estimate, but it illuminates the order of magnitude of the time scale involved. Of course the situation is a dynamic one, as the coupling between degrees of freedom and the diffusion rate change as energy is removed from the beam. A more careful model of the energy balance can be constructed when the nature of the diffusive mechanisms is better understood. Given the present state of affairs in ASTRID, with the two component lithium beam, the laser cooling forces are not strong enough to cool the beam to the strongly coupled regime.

Section 9.5 Conclusion

In summary, the dynamical situation in ASTRID is characterized by diffusive mechanisms which represent strong competition for the laser cooling force. These mechanisms currently prohibit the attainment of a strongly-coupled OCP in ASTRID, at least for the two-component beam with very small metastable ion content. The measured magnitudes of the diffusive growth rates are consistent with both the observed inefficiency of the laser to accumulate particles as it is swept, and with the equilibrium cold temperatures obtained under various circumstances. The measurements to date point to the need for better understanding of the nature of the disruptive diffusive mechanisms, if we are to learn how to extract the full potential of laser cooling.

Tests of laser cooling with a single-component beam having favorable cooling parameters are the obvious next step. The ion $^{24}\text{Mg}^+$ is the prime candidate for ASTRID [POU2]. This ion has a closed transition at 290 nm having a linewidth of 43 Mhz. The lasers to be used for cooling this transition are currently under development in Aarhus [NIE2]. The future of these efforts, as well as the improvements in diagnostic hardware required to fully evaluate their success, are discussed in the next chapter.

Meanwhile, progress at other laboratories has also been rapid in the past two years. The TSR in Heidelberg is equipped with both laser and electron cooling [HAB1]. Indications are that the combination of laser and electron cooling for $^7\text{Li}^+$ beam is more effective than is either process alone [HAB4]. In the TSR, electron cooling serves to cool the residual ground state ions, which the laser cannot affect, and also to reduce the transverse temperature. Laser cooling of the $^9\text{Be}^+$ ion to temperatures as low as 50 mK has also been reported [PET1]. The combination of electron and laser cooling has also been investigated in the beryllium system. More recently, the ESR storage ring in Darmstadt has come on line [SPÄ1], carrying with it the potential for electron cooling of fully-stripped heavy ions. The investigation of the interaction between very strong (because of the Z^2 enhancement) Coulomb coupling and electron cooling forces is possible in this machine. The machine proposed to be built in Italy will eventually combine electron, laser, and stochastic cooling in an attempt to create an ordered beam [BIS1].

While we have barely begun to tap the potential of laser cooling in storage rings, workers using laser cooling in a ring-shaped ion trap have successfully

produced cylindrically confined Coulomb crystals [BIR1]. They have dramatically confirmed the predictions of the MD simulations for the occurrence of the string, helix, and shell configurations (Chapter 3). Their work with the $^{24}\text{Mg}^+$ ion makes possible a new generation of laser cooling experiments in ASTRID. These experiments are discussed in the final chapter.

CHAPTER 10

Possibilities for Further Study in ASTRID

Section 10.1 Research with $^{24}\text{Mg}^+$

As mentioned in Chapter 9 the logical next step in ASTRID is to work with the $^{24}\text{Mg}^+$ ion. This is a system in which all ions can participate in the cooling, as the transition operates from the ground state. The wavelength of laser light required, 290 nm, can be achieved by frequency doubling of the light from the dye lasers used in the lithium and erbium experiments. The experimenters who produced the cylindrically-confined Coulomb crystals in an ion trap were able to produce a few mW of laser light at the proper frequency [BIR1]. A conceptual design exists at Aarhus to produce about 10 mW of light [NIE2]. This is much less than the 300-400 mW produced by the dye lasers, but the transition in $^{24}\text{Mg}^+$ has a lifetime about a factor of 10 less than that in lithium, and the laser photons carry twice as much momentum.

Research with $^{24}\text{Mg}^+$ should quickly address the issue of the competition between the laser cooling force and diffusive mechanisms in the beam. The ions can be produced by the plasma source in sufficient quantity to reach the unstable regime of the storage ring, and the storage lifetime (for stable beam) should be longer of than that of lithium [POU2]. The fact that all of the ions participate in the cooling suggests the use of Schottky diagnostics to supplement the laser fluorescence measurements.

The possibility of constructing an induction accelerator for use in ASTRID is also being considered. The induction accelerator takes the place of the swept laser. It slowly (without spatial bunching) accelerates the particles into resonance with a fixed frequency laser to form the cold distribution. The sweeping is not velocity selective, as is the laser sweeping. Such a device has been used with great success at the TSR in Heidelberg [HAB4, PET1]. An added advantage is that the

induction accelerator frees one of the cooling lasers for use as, for example, a probe of the velocity distribution in the transverse directions. The combination of two frequency-doubled dye lasers and the induction accelerator will provide the most experimental flexibility for $^{24}\text{Mg}^+$ research in ASTRID.

Section 10.2 Improvements to Diagnostics

Many of the limits to our current understanding of the interaction between laser cooling and the other forces active in the storage ring are due to deficiencies in diagnostic instrumentation. The deficiencies do not represent a fundamental lack of available technology; they are rather due to the relative youth of the machine. The situation can be improved by implementing some of the systems described below.

As we have seen in Chapters 6 to 8, a direct measurement of the circulating beam current is not currently possible for the low currents (1-15 μA) encountered in the laser cooling experiments. A commercial high-sensitivity current transformer with a sensitivity of about 0.5 μA could be installed and used to calibrate the longitudinal Schottky pickup, which has much higher sensitivity. The neutral particle detector could probably also be absolutely calibrated for constant vacuum in the ring. Direct measurement of the circulating current would eliminate much uncertainty about the beam density.

The existence of transverse coherent instabilities in the ring emphasizes the need for better diagnostics of the transverse motion. A resonant Schottky system, which can measure, with great sensitivity, the power in a single betatron sideband (Chapter 2), is needed for both transverse planes. A wide-bandwidth pickup, which would be sensitive to transverse coherent motion over a wide range of revolution harmonics, would also be very useful for diagnosing instabilities in the beam. A real-time monitor of transverse beam sizes (emittances) is also needed, as the scraper method (Chapter 8) is slow and destructive. A profile monitor based on detection of electrons produced by ionization of the background gas by the beam has been implemented in the TSR Heidelberg [HOC1] and could also be applied to ASTRID. This device could be supplemented by transverse laser probing of the velocity distribution (Chapter 8).

Another intriguing possibility is development of a quadrupole pickup for ASTRID. Such a device could be used to detect a mismatch between the injection line and the storage ring, and possibly to detect the shape oscillations associated with the onset of order (Chapter 3).

Finally, an active feedback system which can sense transverse dipole oscillations of the beam and damp them is needed if we are to reach the strongly coupled regime. ASTRID already operates at the threshold of these transverse instabilities (Chapters 7,8). Matters will only get worse if the laser cooling in $^{24}\text{Mg}^+$ proves to be effective at reducing the transverse emittance as well as the longitudinal momentum spread.

Space limitations in the ring probably make implementation of all of these improvements impractical. They are merely outlined to emphasize the potential for improvement of our understanding of the character of the beam and its environment.

Section 10.3 Further Experiments with $^7\text{Li}^+$

The $^7\text{Li}^+$ ion may still be of interest if it were possible to remove the ground state ions with great efficiency. Installation of the induction accelerator would add another tool with which this might be achieved. The metastables could be held at fixed velocity with the lasers while the induction accelerator accelerates the ground-state ions out of the ring. This has been tried in the TSR with limited success [BOC1]. For these experiments to be worthwhile, the initial fraction of metastable ions in the beam must also be raised. This may be possible using another type of ion source. Another possibly significant improvement in the laser cooling of lithium involves use of an acousto-optic modulator ring [JES1]. This is a device, originally developed for use in the erbium experiments, which creates a "comb" of sidebands around the original laser frequency. In the original application, the A-O ring produced many (>20) sidebands with about 10 Mhz spacing. The capture range of the laser force is thus enhanced. Use of the laser light produced in this fashion created a large improvement in the very weak laser acceleration (Chapter 7) produced in the erbium beam. While the potential exists for further study with lithium, these endeavors are best viewed as lower priority alternatives to the more promising experiments with $^{24}\text{Mg}^+$.

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