MASTER

EXPLORATION OF VOLCANIC GEOTHERMAL ENERGY
RESOURCES BASED ON RHEOLOGICAL TECHNIQUES

Final Report

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Brief Statement of Objectives

Elementary considerations indicate that rheological methods may be useful in the detection and delineation of volcanic geothermal resources, in particular, those connected with subsurface molten or quasi-molten plutons such as may possibly be the case in the Oregon-Washington Cascades. The processes of main interest are the deformations or flows caused by tidal strain and seasonal snow-ice loads. We have carried out an initial physical-analytic and field study of the general applicability of the rheidity sensing techniques in the Oregon-Washington Cascade region.

Abstract

The purpose of the present study is to investigate whether tidal strain and tilt fields are modified or perturbed by local geological structure. An observable modification opens up the possibility of using tidal strain/tilt measurements as an exploration tool.

On the present study strain and tilt field observations were carried out during the period February 1978 to December 1979 at the Klamath Graben and Newberry Caldera in Oregon and at Krafla in Northern Iceland. Moreover, tilt observations were made at Mt. St. Helens, Washington, during the summer of 1980. Two strainmeters of the same type as now in use by the U.S. Geological Survey were applied in the strain work. Tilts were measured by two Kinematics model TM-1B biaxial tilt meters. The instruments were placed at depths of approximately one to two meters below the ground surface.

Both strain and tilt fields turn out to be heavily contaminated by noise that is mostly of thermoelastic origin. In spite of considerable efforts, it has not been possible to process the strain field data to obtain sufficiently clear tidal signals.

The tilt data are less contaminated and rather clear tidal signals were observed at Newberry in Oregon and Krafla in Iceland.

A local magnification by a factor of about 3 of the EW component of the theoretical solid earth and ocean load tilt was observed at one station at Krafla. Moreover, the tidal tilt component across the ring fault at Newberry appears to be magnified by a factor of 1.4 to 1.9.

The phenomena at the Krafla may possibly be due to a local magma chamber. These results are a clear indication of a tilt field modification by local structure and indicate the possibility of using tilt data to locate subsurface magma bodies.

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(1) INTRODUCTION

The following is the final report on a project entitled "Exploration of Volcanic Geothermal Energy Resources based on Rheological Techniques" that has been supported by the U.S. Department of Energy for the period April 1, 1978 to December 31, 1979 under Contract No. EY-76-S-06-2227, TA #37 including Modification No. A002.

During the summer of 1979, the project received partial support by the National Energy Authority of Iceland to conduct a brief strain and tilt survey in the Krafla geothermal area in Northern Iceland. Moreover, the emplacement of tiltmeters at Mt. St. Helens, Washington, during the summer of 1980 was supported by the Mazama Society of Portland, Oregon.

(2) PERSONNEL

The project has been under the direction of Gunnar Bodvarsson, professor of Geophysics and Mathematics, Oregon State University. The field work has been supervised by Ansel Johnson, Chairman, Department of Earth Sciences, Portland State University and the data processing has been carried out by Gudni Axelsson, Graduate Research Assistant, Geophysics Group, Oregon State University.

(3) PROJECT RATIONALE

Due to the growing importance of non-fossil energy resources, there is an enhanced interest in geophysical techniques for the exploration of volcanic geothermal systems and for the direct detection of subsurface magma bodies. Moreover, the eruption of Mt. St. Helens and the possibilities for further eruptions in the Oregon-Washington Cascades has prompted an interest in volcano surveillance and monitoring. In addition to the

conventional well established techniques such as the heat flow, electrical, potential and seismic methods, there are a number of other much less tested field techniques that may find important applications in these areas. In particular, the elastomechanical techniques based on a probing of the rheidity of the subsurface formations offer some possibilities that have yet to be explored both theoretically and in the field. These involve primarily the observing of transient elastic strain and tilt fields caused by (1) solid earth and ocean tides, (2) barometric transients, (3) subsurface fluid (magma/water) pressure transients and (4) annual snow/ice load cycle at the surface. Such elastomechanical methods may also find application in the field of fossil energy resources and related areas.

(4) STRAIN/TILT SIGNALS DUE TO SOLID EARTH TIDES - (G. Bodvarsson)

Although the average volume strain due to solid earth tides is very small, many of the resulting effects are of a significant magnitude. For example, at middle to lower latitudes the tidal strain is of the order of 10^{-8} and the resulting pressure amplitude in solid igneous rock is then of the order of 10^3 Pa (Pa = Pascal = Newton/m^2). Due to lower bulk modulus, the amplitude in porous water-saturated sediments is smaller, but values of the order of a few hundred Pa are possible. This pressure oscillation can be observed by monitoring the water level in boreholes. Tidally induced amplitudes of about 10 centimeters have frequently been reported in the literature. The theory of this phenomenon has been discussed by Bredehoeft (1967), Bodvarsson (1970), Johnson and Nur (1978) and by Rhoads and Robinson (1979).

More significant, however, is the fact that a considerable enhancement or integration of the tidal response may occur in nature. The ocean tides

are, of course, the most obvious example where a global scale integration causes a macroscopic effect. Enhancement of the solid earth tidal deformation occurs where there are volumes or regions with reduced bulk modulus and/or rigidity. All such regions will "breathe" in response to the tidal excitation. The most interesting possibilities are the following.

(4.1) Open fractures. Consider a long open fracture of depth d and width w as shown in Figure 1.

Figure 1. The open fracture.

Let the tidal stress amplitude across the fracture be s and the resulting change in fracture width be Δw . Referring to our results in section (6.i) (equation (30)) below, we estimate the amplitude of Δw as follows,

$$\Delta w = 1.2 ds/\mu \quad (1)$$

where μ is the modulus of rigidity of the surrounding formations.

Associated with the breathing is a tilt Δt that measured at the fracture edge is estimated (including effects of Poisson contraction) at

$$\Delta t_A = 0.85 s/\mu \quad (2)$$

Inserting, for example, $s = 10^3$ Pa and $\mu = 2 \times 10^{10}$ Pa we obtain an estimate

$$\Delta t_A = 4.3 \times 10^{-8} \quad (3)$$

(4.ii) Fracture zone with a gouge. We assume that the gouge has a similar bulk modulus as the surrounding rock but that it is also permeable to water. This situation leads to fracture zone breathing where the gouge pore water oscillates up/down in response to the tidal stress as indicated in Figure 2 below.

Figure 2. Fracture zone containing a permeable gouge.

This model is more complex than the open fracture under (i) and a detailed discussion of the response to tidal stress is quite involved. Moreover, the situation is further complicated by the lack of data on fracture gouge parameters. In the present context, the most interesting cases involve active fracture zones with a highly fractured gouge such that there is a high permeability. Referring to the results in section (6.ii) below, we can then apply the estimates (1) and (2) with the modification that d is now to be identified with the vertical hydraulic skin depth of the fracture gouge.

To obtain some indication of the order of magnitude involved, we will assume a fracture gouge of permeability $10^{-12} \text{ m}^2 = \text{one darcy}$, leading to a skin depth at M_2 -frequency of about $d_s = 500 \text{ m}$, $s = 10^3 \text{ Pa}$ and $\mu = 2 \times 10^{10} \text{ Pa}$. Equation (1) gives then $\Delta w = 3 \times 10^{-5} \text{ m} = 0.03 \text{ mm}$. The estimate given by (2) and (3) remain unchanged.

(4.iii) Geothermal systems. Due to reduced rigidity, vapor dominated and some highly fractured geothermal systems may also generate measurable tidal strain and tilt anomalies. Quite frequently, geothermal systems are controlled by fracture zones and we can then apply the above estimates for the elastomechanical anomalies.

(4.iv) Magma bodies. Provided the tidally induced stresses exceed any Bingham type yield strength of the magma, subsurface magma bodies are likely to be deformed under tidal loading. We can thus expect that large sufficiently shallow magma bodies can cause strain/tilt anomalies that are observable at the ground surface. Mafic magma is quite often intruded into discontinuities such as fractures and we may then be able to estimate the associated anomalies with the help of the above simple formulas.

(5) LONGER PERIOD TILT FIELDS - (G. Rodvarsson)

In the present context, two types of long-period tilting are of particular interest, that is, tilt fields caused by (1) the annual snow/ice load on high mountains such as, for example, the Oregon-Washington Cascade cones, and (2) volcanic processes such as the pressure of slowly upwelling magma.

Snow/ice load on high mountains is likely to cause small but possibly still measurable rates of tilting. For example, the annual load on Mt. Hood in Oregon is likely to amount to an average of about 1 meter of water distributed over a disk of a radius of 10 km. As indicated in section (6.iv) below purely elastic response tilt rates at the mountain are likely to amount to 0.01 to 0.02 $\mu\text{rad/day}$ and total yearly peak to peak amplitudes may be about one μrad . This signal may be observable with modern tiltmeters. Moreover, any appreciable subsurface rheidity due to temperature effects such as shallow magma chambers would probably enhance the tilt amplitude significantly.

Referring to estimating techniques set forth in section (6.iv) below, we conclude that a viscous anelastic response to the snow/ice load should become noticeable when the average subsurface formation viscosity falls below $10^{16} - 10^{17}$ Pas*depending somewhat on conditions. Various studies of the normal viscosity of the uppermost mantle indicate values of 10^{20} to 10^{21} Pas (Cathles, 1975) and we can thus conclude that a reduction by a factor of 10^3 to 10^4 would lead to the viscous anelastic effects. Using an exponential viscosity - temperature relation this reduction would correspond to an increase in the average subsurface temperature of very roughly 200°C . According to Blackwell (1979) the supernormal temperature below the Oregon-Washington Cascades is at or above this value.

* Pas = Pascal seconds.

It is of considerable interest to note that large anelastic snow/ice load effects have been observed by Tryggvason (1973) at the Myrdalsjokull volcanic complex in southern Iceland. The vertical displacement amplitude at the volcano in response to the annual snow/ice cycle is more than one order of magnitude larger than the estimated purely elastic response of the crust.

We have no reliable models for estimating the possible magnitude of volcanic tilts. Observations at both the Krafla in Iceland (Tryggvason, 1980) and Mt. St. Helens in the Cascades (see below), reveal tilt events with periods of a few months that proceed at rates of 0.5-1.0 $\mu\text{rad/day}$. During sudden deflation events at the Krafla, the rate is orders of magnitude larger.

(6) SOME THEORETICAL CONSIDERATIONS - (G. Bodvarsson)

(6.1) The open fracture. The derivation of the response of the open fracture in Figure 1 to tidal stresses leads to a very complex mathematical problem that has apparently not been given any attention. Since we are only interested in order of magnitude estimates, we will here rely on three main approximations that serve our purpose (see also Bodvarsson, 1980). First, we add an upper half space to the model in Figure 1 and extend the fracture symmetrically into the upper space. In other words, the fracture is extended to a flat rectangular cavity of a very small aperture in a homogeneous/isotropic Hookean space. We assume that the response of the center axis cavity width w to the cross-fracture stress s is about the same as the response of the fracture in Figure 1. Second, we assume that the response to the external stress s is of the same order as the

response to an internal fluid pressure $p = s$. The fracture is thus treated as a pressurized flat cavity. Third, in deriving the response of the cavity to p , we use the constrained displacement approximation (CDA) where the displacement vector is assumed to involve only a component perpendicular to the plane of the cavity.

The constrained displacement approximation. Essentially, the derivation of the stress/displacement fields around pressurized flat cavities of a very small aperture and arbitrary contour involves the solution of the fundamental equation of elasticity for Hookean half-spaces with a rather complex surface stress conditions of the mixed type. A useful approach can be obtained in the following way.

The basic equations for the displacement vector $\bar{u}$ are

$$\mu \nabla^2 \bar{u} + (\lambda + \mu) \nabla \nabla \cdot \bar{u} = 0 \quad (4)$$

where λ and μ are Lamé's constants and the z -axis is into the half-space. In the present context, it is sufficient to assume Poisson's relation $\lambda = \mu$. The most important half-space solution of (4) is the well known Boussinesq solution (Love, 1927) that is obtained when the surface loading is pure pressure p . The resulting relations are conveniently expressed in the Fourier transformed form. We introduce a coordinate system in the surface such that the (x, y) plane coincides with the surface and the z -axis is positive into the half-space. Let $\hat{p}$ be the transform of p and $\hat{w}_0$ be the transform of the displacement at the surface $z = 0$. Taking that $\lambda = \mu$ we have thus in the one-dimensional case $p = p(x)$, the Boussinesq solution

$$\hat{w}_0 = 0.75 \hat{p} / k\mu \quad (5)$$

where k is the wave number of the pressure load. A smaller horizontal displacement is associated with w_0 .

On the CDA we assume that the pure pressure loading on the half-space results mainly in a vertical displacement such that in the case of one-dimensional loading we can approximate the displacement vector by

$$w = (0, 0, w(P)) \quad (6)$$

where $P = (x, z)$ is the general field point. On the CDA, the basic equations then reduce to

$$\mu \partial_{xx}^2 w + (\lambda + 2\mu) \partial_{zz}^2 w = 0 \quad (7)$$

which with $\lambda = \mu$ becomes

$$\partial_{xx}^2 w + 3 \partial_{zz}^2 w = 0 \quad (8)$$

This equation is easily solved with the help of a Fourier transformation with respect to x and we find that

$$\hat{w}_0 = 0.58\beta/k\mu \quad (9)$$

Relative to the exact solution (5) this is an underestimate by almost 25%. It is noted that the two solutions deviate only by a constant factor and the approximation is therefore independent of the wave-number. In the present context where there is great uncertainty with regard to underlying models and even the rock parameters, we tend to regard this approximation as satisfactory, in particular, since we have a measure of the degree of approximation and can then make some useful corrections. The method will now be applied to the case of the flat cavity of an arbitrary contour where no exact solutions are available.

Flat cavity parameters. Consider a flat cavity of a very small aperture and with a given contour B in the cavity plane. To apply the CDA in this case, we assume that the displacement in the cavity walls is of the form (6) where $P = (x, y, z)$. Moreover, a boundary condition

$$w(x, y, 0) = 0 \text{ for } (x, y) \text{ on } B \quad (10)$$

is imposed on the displacement.

Equation (4) with $\lambda = \mu$ now reduces to

$$\nabla_2^2 w + 3\partial_{zz} w = 0 \quad (11)$$

where $\nabla_2^2 = (\partial_{xx} + \partial_{yy})$ and the pressure condition at the surface $z = 0$ is

$$p = -3\mu\partial_z w \Big|_{z=0} \quad (12)$$

Equation (11) with (10) is easily solved with the help of a standard eigenfunction technique.

We assume thus that

$$w = \sum_j b_j \phi_j \exp(-a_j z) \quad (13)$$

where b_j are the expansion coefficients, $\phi_j(x, y)$ are the normalized eigenfunctions satisfying

$$-\nabla_2^2 \phi_j = \lambda_j \phi_j \quad (14)$$

with the eigenvalues λ_j and the boundary condition

$$\phi_j = 0 \text{ for } (x, y) \text{ on } B \quad (15)$$

Inserting (13) in (11) and (12), we obtain the relation

$$a_j = (\lambda_j/3)^{\frac{1}{2}} \quad (16)$$

and

11

$$b_j = [1/\mu(3\lambda_j)^{\frac{1}{2}}] \int_C p \phi_j dx dy \quad (17)$$

where the integral is over the cavity wall C. When the pressure is due to a cavity fluid, that is, p is constant over the wall

$$b_j = p h_j / \mu(3\lambda_j)^{\frac{1}{2}} \quad (18)$$

where

$$h_j = \int_C \phi dx dy \quad (19)$$

Knowing p, we obtain the wall displacement w and the cavity volume increment ΔV due to p on the basis of (13).

$$\Delta V = \sum_j b_j h_j = (p/\mu) \sum_j h_j^2 / (3\lambda_j)^{\frac{1}{2}} \quad (20)$$

and hence the volume elastance

$$E_V = (1/\mu) \sum_j h_j^2 / (3\lambda_j)^{\frac{1}{2}} \quad (21)$$

It is to be expected that the above approximation will underestimate the displacement w by 25% to 50% depending on conditions. It is prudent to allow for the error by increasing computed results by this amount.

Response of a rectangular cavity. Let the (x,y) plane coincide with the cavity plane, where x is the vertical axis and y the longitudinal horizontal axis. The vertical and horizontal edge lengths are 2H and L respectively.

The eigenfunctions of (14) for the whole cavity are then

$$\phi_j = (2/HL)^{\frac{1}{2}} \sin(m\pi x/2H) \sin(n\pi y/L) \quad (22)$$

and the eigenvalues

$$\lambda_j = \pi^2 [(m/2H)^2 + (n/L)^2] \quad (23)$$

which are then to be ordered such that

$$\lambda_1 < \lambda_2 < \dots \quad (24)$$

The integral (19) has non-zero values only when both m and n are odd

$$h_j = \int_C \phi_j dx dy = 8(2HL)^{\frac{1}{2}} / mn\pi^2 \quad (25)$$

When $L \gg H$, we can approximate (18) by

$$b_j = 16p(2H^3L)^{\frac{1}{2}} / 3^{\frac{1}{2}} m^2 n \mu \pi^3 \quad (26)$$

and hence for m and n odd

$$b_j \phi_j = [32pH/3^{\frac{1}{2}} m^2 n \mu \pi^3] \sin(m\pi x/2H) \sin(n\pi y/L), \quad (27)$$

In view of (27), the pressure p increases the cavity width at the center $x = H$, $y = L/2$ by

$$w_o = 2 \sum_j b_j \phi_j(H, L/2) \quad (28)$$

Using (27), summing up to the third order and using the well known series

$$\pi/4 = 1 - (1/3) + (1/5) \dots \quad (29)$$

we obtain

$$w_o = 0.93 \cdot 16pH/3^{\frac{1}{2}} \pi^2 \mu = 0.87pH/\mu \quad (30)$$

This is an underestimate that should be multiplied by a factor of 1.3 to 1.5. Reverting to the previous notation, and multiplying by 1.4, we obtain then for the fracture in Figure 1 the estimate given by equation (1) above.

(6.ii) Fracture zones with gouge. The complication due to the gouge that is assumed to be permeable to water result from the problem of estimating the active depth of the fault zone, or, in other words, the depth range over which the fault breaths in response to the tidal stress. An expression for this depth is obtained with the help of the following simple procedure (see Bodvarsson, 1970).

Let c be the fluid conductivity and s the capacitivity of the gouge, a be the corresponding diffusivity and ω be the angular frequency of the tidal oscillations. Moreover, let b be the tidally induced volume dilatation and ϵ the matrix coefficient such that ϵb is the effective pore dilatation. Assuming a one-dimensional situation, the pore pressure p is then governed by the equation

$$\partial_t p - a \partial_{xx} p = -(\epsilon/s) \partial_t b \quad (31)$$

where x is the coordinate down the gouge and $x = 0$ at the free pore fluid surface. The boundary condition at $x = 0$ is

$$x = 0, \quad (1/g) \partial_t p - c \partial_x p = 0, \quad (32)$$

and

$$p = 0 \quad \text{for } x \rightarrow \infty \quad (33)$$

Assuming that $p \propto \exp(i\omega t)$ the solution for the amplitude of p is

$$p = A \exp[x(1+i)/d_s] - (\epsilon b/s), \quad (34)$$

where A is an integration constant and d_s is the skin depth given by

$$d_s = (2a/\omega)^{\frac{1}{2}} = (2c/\rho s \omega)^{\frac{1}{2}}, \quad (35)$$

where ρ is the fluid density. A highly fractured gouge can easily have

a permeability of 10^{-12} m^2 = one darcy and we find on this basis of reasonable values for s that the skin depth for M_2 tides is of the order of 500 meters.

(6.iii) The cavity-vent system. Consider the tidal response of the simple system shown in Figure 3 consisting of a cavity at some depth in a formation of negligible permeability and connected to an open vent or bore-hole

Figure 3. The cavity-vent system.

Tidal forces cause a periodic straining of the formation resulting in a periodic oscillation of the cavity volume and thereby an oscillation of the free liquid level in the vent.

Elastance/dilatance. To illustrate the concepts to be applied in the following, we consider a bounded Hookean elastic body that includes an open internal cavity of given dimensions. The external dimensions are assumed to be very large compared to the cavity dimensions. The body can be deformed by forces acting on the exterior and/or the interior surfaces. For the present purpose it is adequate to consider only the special case where the surfaces are in contact with ideal fluids such that the forces

are along the normals and are uniform over the surfaces. Let V be the volume of the cavity, p be the pressure of the cavity fluid and σ (stress) be the negative pressure of the external fluid. We will then make the following definitions.

Let L be a characteristic dimension of the cavity. Assuming $\sigma = 0$, the cavity elastance relative to the length is given by

$$E_L = dL/dp \quad (36)$$

and the corresponding elasticity

$$e_L = (1/L)dL/dp \quad (37)$$

Moreover, the cavity dilatance relative to the same length at $p = 0$ is

$$D_L = dL/d\sigma \quad (38)$$

and the dilaticity

$$d_L = (1/L)dL/d\sigma \quad (39)$$

The same set of parameters apply to the cavity volume, that is, the volume elastance

$$E_V = dV/dp \quad (40)$$

and the volume elasticity

$$e_V = (1/V)dV/dp \quad (41)$$

Moreover, the volume dilatance at $p = 0$

$$D_V = dV/d\sigma \quad (42)$$

and the dilaticity

$$d_V = (1/V)dV/d\sigma \quad (43)$$

In the case of Hookean solids the above parameters E , e , D and d are constant. Under both internal and external stress, the cavity volume increment is

$$\Delta V = pE_V + \sigma D_V \quad (44)$$

Let σ be the (isotropic) formation stress, p the liquid perturbation pressure in the cavity, ρ the density of the liquid, V the cavity volume at $p = 0$, h the liquid level in the vent as measured relative to the average level and f be the cross section of the vent. Neglecting inertia forces and flow resistances, we have then on the basis of the above notation

$$p = \rho gh \quad (45)$$

and the conservation of liquid volume (C_F = liquid compressibility)

$$\sigma D_V + p(E_V + C_F V) + fh = 0 \quad (46)$$

We abbreviate the expression in the parenthesis by the cavity capacitance

$E_c = (E_V + C_F V)$ and insert p from (45) such that

$$h = -\sigma D_V / (\rho g E_c + f) \quad (47)$$

which can be rewritten

$$h = -h_o / (1+T) \quad (48)$$

where

$$h_o = \sigma D_V / f \quad (49)$$

is the cavity volume increment measured as a liquid level and T is the tidal factor

$$T = g\rho E_c / f \quad (50)$$

It is sometimes convenient to restate (48) as

$$h = -h_1 T / (1 + T) \quad (51)$$

where h_1 is now to be taken as a pressure amplitude measured in terms of a liquid level defined by

$$h_1 = \sigma D_V / \rho g E_c = \sigma \gamma / \rho g \quad (52)$$

Equations (48) and (51) are the main results. Two extreme cases can now be considered. Let $T \gg 1$, that is, the cavity be very soft, and we obtain then $h = h_1$, that is, the liquid level furnishes a measure of the formation stress (if e_V known). In the case of flat cavities of a very small aperture, D_V is about equal to E_c and then $h = h_1 = -\sigma/g\rho$. The cavity-vent system with $T \gg 1$ therefore acts as a stress-or pressometer.

Considering, on the other hand, the very stiff cavity where $T \ll 1$, we find that $h = -\sigma D_V / f$ and the liquid level is then directly proportional to the negative volume increment of the cavity. The borehole-cavity system with $T \ll 1$ acts then as a dilatometer. The difference between these two cases is important and indicates the type of data that can be obtained on the basis of tidal testing.

In the case of a borehole-cavity system it is of interest to obtain some picture of the cavity dimensions resulting in $T = 1$ that is intermittent between the above extremes. The volume elastance of the penny-shaped cavity of radius R is given by $E_V = 2R^3/\mu$ (Sneddon and Lowengrub, 1969). We assume that the borehole is of standard dimensions with a diameter of 0.25 m and hence $f = 0.05 \text{ m}^2$. On the basis of (50) the case $T = 1$ then implies $E_c = 5 \times 10^{-6} \text{ kg/s}^2$.

Assuming a small cavity aperture such that fluid compressibility can be neglected, $E_c = E_v = 2R^3/\mu$. Let $\mu = 2 \times 10^{10}$ Pa and we find then that $R = 37$ m. Because of the third power of the radius, cavities with $R \leq 25$ m will be stiff whereas cavities with $R \geq 50$ m will be soft.

It is frequently convenient to rewrite equation (50)

$$T = MS \quad (53)$$

where $M = \rho E_c$ is the total cavity mass elastance and

$$S = g/f \quad (54)$$

is the stiffness factor.

(6.iv) Response to snow/ice load. We consider the case of Mt. Hood in Oregon where the load amounts to an average of about 1 meter of water over a disk of radius $R = 10^4$ meters. Using the Boussinesq solution for the vertically loaded homogeneous/isotropic Hookean half-space, we obtain the elastic response of the vertical displacement at the center of the disk

$$w_o = (1-\sigma)pR/\mu \quad (55)$$

where σ is Poisson's constant, p the load and μ the modulus of rigidity. Taking the simple case of an half-space of common igneous rock, we find on the basis of $\sigma = 0.25$, $\mu = 2 \times 10^{10}$ Pa, $R = 10^4$ m and $p = 10^4$ Pa that $w_o = 4 \times 10^{-3}$ m or 4 millimeters. Because of lower elastic moduli in surface layers, the actual elastic response at Mt. Hood is more likely to be of the order of 10 millimeters and the maximum tilts associated with the downwarping may then be of the order of one μ rad.

To obtain an estimate of the maximum average half-space viscosity that would generate a noticeable viscous anelastic response, we select the following

simple procedure that is sufficient for the present purpose.

In the case of a finite homogeneous viscosity ξ , we assume that the loaded disk responds to the total snow/ice weight $S = \pi R^2 p$ by sinking downward at a uniform velocity u . The viscous resistance to this movement can be estimated on the basis of a Stokes type equation (Lamb, 1932) which we state

$$u = S/10\pi\xi R = pR/10\xi \quad (56)$$

Let the total time available for sinking be τ such that the total vertical displacement during one cycle is

$$w_s = pR\tau/10\xi \quad (57)$$

We take now that the viscous effects become significant when w_s is of the same order as w_0 given by equation (55). On this basis, an estimate of the maximum viscosity ξ is obtained as

$$\xi_{\max} = \tau p/10 \quad (58)$$

In the case of the snow-cycle $\tau \sim 10^7$ s and assuming again that $\mu = 2 \times 10^{10}$ Pa, we find $\xi_{\max} = 2 \times 10^{16}$ Pas. The factor 2 is of little significance, and we conclude that the viscous anelasticity becomes significant when $\xi = 10^{16}$ to 10^{17} Pas. Current estimates of uppermost mantle viscosity or rather pseudoviscosity give values of the order of 10^{20} to 10^{21} Pa (Cathles, 1975). Since magmas have viscosities of the order of only 10^3 to 10^7 Pas, it is quite obvious that viscous anelasticity effects are quite possible at and around volcanic centers.

7. INSTRUMENTATION - (G. Axelsson)

Model TM-1B biaxial borehole tiltmeters, built by Kinemetrics, Inc., were used to measure the NS and EW components of tilt.

In principle, the tiltmeter sensor is a container partially filled with a conducting fluid leaving a mobile bubble. Five platinum electrodes, forming two resistance bridges, are used to sense the position of the bubble. When the instrument is tilted the bubble moves and one or both bridges are unbalanced. This introduces output voltages, for each principal axis, which are functions of the magnitude and the direction of the tilt.

According to the manufacturers the resolution limit of the tiltmeter sensor is $0.008 \mu\text{rad}$.

A low-pass filter having a cutoff frequency of 0.008 Hz was used in this study.

A sketch showing the installation used here is presented in Figure 4. The tiltmeter is placed in a vertical steel pipe that is filled with fine grained sand.

Two strainmeters built according to information from the U.S.G.S. were used to measure components of strain. The strainmeter consists of a 10m long invar wire encased in a plastic pipe (see figure 4) approximately 1m below the surface. The strainmeter wire is fixed to the steel pipe on one side and to a sensing instrument at the other end. Relative strain is detected using a differential capacitance transducer by transforming a horizontal displacement to a vertical displacement with a simple mechanical lever.

Both tilt and strain signals were recorded continuously by Rustrak strip chart recorders advancing at a speed of 10 to 15 mm/hr.

The average of nine tiltmeter calibrations is $0.340 \mu\text{rad}/\text{mm}$ with a standard deviation of 6%.

The strainmeter calibrations are in the range $2 \times 10^{-8}/\text{mm}$ to $10^{-7}/\text{mm}$.

Figure 4. Design of a typical field strain/tilt station.

(8) DATA ANALYSIS - (G. Axelsson)

(8.i) Digitization. The data from the strip chart recorders are digitized by hand at hourly intervals to an accuracy of 0.1 mm. Long period drifts cause the recorders to go off scale from time to time, which results in discontinuities in the original data. These and other similar disturbances are removed during digitization. The digitized data are then read into a computer and stored on magnetic tapes for subsequent analysis.

(8.ii) Detrending and smoothing. A pronounced long term drift of nontidal origin is apparent in most of the records. To remove this, a sequence of N-term running means of the original series is computed. This is repeated M times and the resulting time series is subtracted from the original one. If $x(t)$ is the original sequence then the detrended series is given by ($h = (N-1)/2$):

$$y(t) = x(t) - \frac{1}{N^M} \sum_{i=t-h}^{t+h} \sum_{j=i-h}^{i+h} \dots \sum_{s=r-h}^{r+h} x(s) \quad (59)$$

where the sum is repeated M times (see Godin, 1967). This corresponds to a high pass filter having the response function:

$$H(\omega) = 1 - \left[\frac{\sin(N\omega\Delta t/2)}{N\sin(\omega\Delta t/2)} \right]^M \quad (60)$$

where ω is the radial frequency and Δt the digitization interval. In our analysis we most frequently use $N = 23$ and $M = 1$ or 2 . Figure 5 shows the response function for $M = 2$ and $N = 23$.

Some high frequency noise (0.1 cph or more) is present in the data. By taking n term running averages m times, the power of this noise can be reduced. The smoothed series is then given by

$$y(t) = \frac{1}{n^m} \sum_{i=t-h}^{t+h} \sum_{j=i-h}^{i+h} \dots \sum_{s=r-h}^{r+h} x(s) \quad (61)$$

where $h=(n-1)/2$ and the summation is repeated m times. This corresponds to a smoothing operator having a response function:

$$K(\omega) = \left[\frac{\sin(n\omega\Delta t/2)}{n\sin(\omega\Delta t/2)} \right]^m \quad (62)$$

Most frequently we use $n = 3$, $m = 1$. $K(\omega)$ for these values is presented in Fig. 6.

Combining these operations reduces the power of frequency components outside the frequency band of interest, i.e. 1 cycle/day to 2 cycles/day. If $F(\omega)$ is the frequency spectrum of the detrended and smoothed data series, we have:

$$F(\omega) = H(\omega)K(\omega)A(\omega) \quad (63)$$

where $A(\omega)$ is the frequency spectrum of the original data sequence. Or if $F(\omega)$ is known

$$A(\omega) = F(\omega) / (H(\omega)K(\omega)) \quad (64)$$

$H(\omega)K(\omega)$ for the five most important tidal frequencies is given in Table I ($N = 23$, $M = 2$, $n = 3$, $m = 1$)

Table I

ω (rad/hr)	$H(\omega) K(\omega)$
O_1 , 0.23385	0.956
K_1 , 0.26252	0.976
N_2 , 0.49637	0.911
M_2 , 0.50587	0.911
S_2 , 0.52360	0.909

Figure 5. Response function of the detrending operator.

Figure 6. Response function of the smoothing operator.

Although more efficient filtering methods are available, this running mean process is quite adequate for our purpose. The short data sets and the digitization interval used in this study limit the use of many other filtering methods.

(8.iii) Thermal noise removal. The most serious difficulty encountered, in the analysis of our data is caused by the varying surface temperature. Since the temperature directly affects the length of the invar wire, the thermal noise is more serious in the case of the strain measurements even though there is partial compensation. Most of the thermal noise in the tilt data is due to thermoelastic tilting of the surface (see Berger, 1975).

Since the thermoelastic tilt depends on the spatial derivatives of temperature a few temperature records around the tiltmeter sites would in principle enable us to remove a substantial part of the power of thermal noise. However, only one temperature recorder was used in this study and we will therefore proceed by assuming that the thermal noise depends linearly on temperature, that is,

$$h(t) = b(t) + k \cdot T(t + \Delta_t) \quad (65)$$

where $h(t)$ is the measured signal, $b(t)$ the tidal signal, $T(t)$ the temperature at the surface, k a constant we can call thermal efficiency and Δ_t is the phase difference between the surface temperature and the thermal noise. For the EW tilt Δ_t will be different from zero, and this is clearly what is being observed.

The problem is now reduced to finding k and Δ_t and we will below describe a few methods for this purpose. Since the thermal noise has frequencies of 1 and 2 cycles per solar day it is difficult to separate the tides and the thermal noise by using spectral analysis (at a signal to noise ratio of 10 about 4 months of data would be needed to separate K_1 and the diurnal temperature variations).

Method A. By plotting $h(t)$ vs $T(t+\Delta_t)$ (assuming a fixed Δ_t) any linear dependence between $h(t)$ and $T(t)$ would show up as linear segments separated by otherwise random variations. The slopes of these segments could then be used to estimate k . An example of this is presented below.

Method B. If the temperature noise dominates the tidal signal the method of least squares can be used to find the k and Δ_t that best fit the data. We assume

$$h(t) = kT(t+\Delta_t) + a \quad (66)$$

where a is a constant, and minimize

$$\sum_t [(h(t) - kT(t + \Delta_t) - a)^2] \quad (67)$$

The maximum correlation coefficient is expressed by

$$r^2(\Delta_t) = \frac{(\sum h(t)T(t+\Delta_t))^2}{\sum h^2(t) \sum T^2(t+\Delta_t)} \quad (68)$$

and the corresponding k is obtained by

$$k = \frac{N \sum h(t)T(t+\Delta_t) - \sum h(t) \sum T(t+\Delta_t)}{N \sum h^2(t) - (\sum h(t))^2} \quad (69)$$

The above approach will not give reliable estimates of k and Δ_t when the signal and the thermal noise are of the same order of magnitude. However, by subtracting the theoretical tidal signal from the data we can improve these estimates.

Method C. This method is based on the assumption that some of the long period variations in the data are also of thermal origin. The data and temperature records are first low pass filtered to remove diurnal and higher frequency variations. Any linear trends are subsequently removed from both records and k can then be estimated with the help of the method of least squares as before. In using this method we assume k to be frequency independent which is questionable.

Method C is used by Rhoads and Robinson (1979) and Robinson and Bell (1971), in estimating barometric efficiency, when analyzing well tides. In this study the effects of barometric pressure were neglected.

As an example of thermal noise removal we will take the N-S component in the tilt records from site K-3 at the Krafla, Iceland. This data is presented in Figs. 7 and 8. Applying method A we obtain the results shown in Fig. 9. The average slope of the linear segments is $0.104 \text{ } \mu\text{rad}/^\circ\text{C}$ and the standard deviation is 0.062. Method B gives $k = 0.096 \pm 0.006$ with a correlation coefficient of $r = 0.753$. This is quite consistent with method A. Method C gave no results since the long period drift did not appear to be correlated with the long period temperature variations.

(8.iv) Spectral analysis. In order to give some qualitative information on the spectra, the standard Fourier transform (FT) of the detrended data, was computed for most of the records. The algorithm used here is a modification of the fast FT algorithm given by Claerbout (1976). The formula for the discrete FT at frequency f is

$$F(f) = \frac{1}{N} \sum_{j=0}^{N-1} x(j\Delta t) \exp(-i2\pi j\Delta t f) \quad (70)$$

Figure 7. SN component of tilt at K-3 after detrending and smoothing.

Figure 8 . Detrended temperature at K-3 for the same period as in Figure 7.

where $x(t)$ is a time series of length $T = N\Delta t$. The amplitude of a harmonic component of frequency f is then $2|F(f)|$. In the FFT we use $N = 2^n$, where n is an integer and $f = k/N\Delta t$ $k = 0, 1, \dots, N-1$.

Primarily due to the limited data length the FT method is not very accurate in the present case. To obtain better estimates of the amplitudes and phases of the various tidal components, it is appropriate to make use of the fact that the frequencies involved are known to a high accuracy. We can furthermore include in our analysis the effects of a finite data length.

We will now outline a spectral analysis method developed by J. Hanson (1979, 1980, personal comm.), based on the above ideas. His work is an extension of a paper by Munk and Hasselman (1961):

Given a finite time series of length L , that is,

$$h(t) = b(t) + \eta(t), \quad -L/2 \leq t \leq L/2 \quad (71)$$

where

$$b(t) = \sum_{n=1}^N a_n \cos(\omega_n t + \phi_n) \quad (72)$$

is the time sequence with known frequencies, and $\eta(t)$ is the noise. We want to estimate a_n and ϕ_n for $n = 1, \dots, N$, i.e. the amplitudes and phases of the various tidal constituents. Taking the Fourier transform of the time series (neglecting noise) we obtain:

$$F(\omega_k) = \frac{2}{L} \int_{-L/2}^{L/2} b(t) e^{-i\omega_k t} dt. \quad (73)$$

or using (72)

$$F(\omega_k) = \frac{2}{L} \int_{-L/2}^{L/2} \sum_n a_n \cos(\omega_n t + \phi_n) e^{-i\omega_k t} dt$$

$$\begin{aligned}
&= \frac{1}{L} \sum_n a_n e^{i\phi_n} \int_{-L/2}^{L/2} e^{i(\omega_n - \omega_k)t} dt \\
&+ \frac{1}{L} \sum_n a_n e^{-i\phi_n} \int_{-L/2}^{L/2} e^{-i(\omega_n + \omega_k)t} dt \\
&= \frac{2}{L} \sum_n a_n e^{i\phi_n} \frac{\sin(\frac{L}{2}(\omega_n - \omega_k))}{(\omega_n - \omega_k)} \\
&+ \frac{2}{L} \sum_n a_n e^{-i\phi_n} \frac{\sin(\frac{L}{2}(\omega_n + \omega_k))}{(\omega_n + \omega_k)}
\end{aligned} \tag{74}$$

If we define the two vectors

$$\bar{Z} = \begin{bmatrix} a_1 e^{i\phi_1} \\ a_2 e^{i\phi_2} \\ \vdots \\ a_N e^{i\phi_N} \end{bmatrix}, \quad \bar{F} = \begin{bmatrix} F(\omega_1) \\ F(\omega_2) \\ \vdots \\ F(\omega_N) \end{bmatrix} \tag{75}$$

and the matrices

$$A^+ = \{A_{kn}^+\}, \quad A^- = \{A_{kn}^-\}$$

where

$$A_{kn}^{\pm} = \frac{\sin(L/2(\omega_k \pm \omega_n))}{L/2(\omega_k \pm \omega_n)} \tag{76}$$

we can write ($\bar{z}^* = \bar{z}$ conjg.):

$$\bar{F} = A^- \bar{Z} + A^+ \bar{Z}^* \tag{77}$$

or

$$\begin{aligned}\operatorname{Re}\bar{F} &= (A^- + A^+) \operatorname{Re}\bar{Z} \\ \operatorname{Im}\bar{F} &= (A^- - A^+) \operatorname{Im}\bar{Z}\end{aligned}\quad (78)$$

Thus

$$\begin{aligned}\operatorname{Re}\bar{Z} &= (A^- + A^+)^{-1} \operatorname{Re}\bar{F} \\ \operatorname{Im}\bar{Z} &= (A^- - A^+)^{-1} \operatorname{Im}\bar{F}\end{aligned}\quad (79)$$

where

$$\begin{aligned}\operatorname{Re}Z_n &= a_n \cos \phi_n \\ \operatorname{Im}Z_n &= a_n \sin \phi_n\end{aligned}$$

and finally

$$\begin{aligned}a_n &= ((\operatorname{Re}Z_n)^2 + (\operatorname{Im}Z_n)^2)^{\frac{1}{2}} \\ \phi_n &= \tan^{-1}(\operatorname{Im}Z_n / \operatorname{Re}Z_n) \quad n = 1, 2, \dots, N\end{aligned}\quad (80)$$

To find the amplitudes and phases we perform the following steps:

1) On the basis of the matrices A^+ and A^- we obtain $(A^- + A^+)^{-1}$ and $(A^- - A^+)^{-1}$. To compute the inverses a modification of an algorithm given by Conte and deBoor (1972) is used.

2) The vector $\bar{F}$ is found by calculating the Fourier transform given by equation (73) for each frequency ω_k . Here we used Simpson's formula to approximate the integral.

3) The operation given by equation (79) is then performed and equation (80) is used to compute a_n and ϕ_n .

In this analysis the following eight tidal components were used:

$M_m, M_f, O_1, K_1, N_2, M_2, S_2$ and M_3 . These are the most important

components, but more components can be included when analyzing longer records. If the following inequality holds, we can in general, resolve two frequencies f_1 and f_2 :

$$|f_2 - f_1| > \frac{T^{-1}}{(\text{signal/noise level})^{\frac{1}{2}}} \quad (81)$$

where T is the length of the data set (Munk and Hasselman, 1961). As an example we need to resolve M_2 and N_2

$$T = 210 \text{ hrs.} \quad \text{if} \quad S/N = 10$$

$$T = 660 \text{ hrs.} \quad \text{if} \quad S/N = 1$$

The method described above is equivalent to a least squares estimation of the amplitudes and phases (see Bloomfield, 1976).

If A and ϕ are the amplitude and phase of a certain component, including both ocean loading and solid earth tide, as estimated by the above method, the solid earth tide amplitude and phase can be computed using the formulas below

$$\phi_a = \tan^{-1} \left(\frac{A \sin \phi - b \sin \phi_b}{A \cos \phi - b \cos \phi_b} \right) \quad (82)$$

$$a = (A \cos \phi - b \cos \phi_b) / \cos \phi_a \quad (83)$$

where b and ϕ_b are for the estimated ocean loading. Putting $b = kT$, b and ϕ_b could also be the amplitude and phase of the thermal noise.

In this paragraph (8.iv) we have neglected the effect of noise superimposed on the signal. The errors caused by this noise can be estimated as outlined in (8.vi) and (8.vii).

(8.v) Direct comparison. When the data sets are too short for spectral analysis, a direct comparison is made between the observational data and the theoretical earth tides. This is done by measuring peak to peak amplitudes and estimating the ratio

$$M = \frac{(\text{peak to peak amplitude, data})}{(\text{peak to peak amplitude, theory})} \quad (84)$$

The major computational programs used are listed in the Appendix.

(8.vi) Estimation of errors in spectral analysis results. We will now outline how the variances in the real and imaginary parts of a spectral line estimate can be computed. The below derivation is based on work by Jonathan Hanson (1979a, 1979b and personal communication). The symbols used are the same as above.

We add the noise $\eta(t)$ to the signal of equation (72)

$$h(t) = b(t) + \eta(t)$$

This gives in frequency space

$$\bar{H} = A^- \bar{Z} + A^+ \bar{Z}^* + \frac{2}{L} \int_{-L/2}^{L/2} \eta(t) e^{-i\omega_k t} dt \quad (85)$$

$$= A^- \bar{r} + A^+ \bar{r}^*$$

where $\bar{Z}$ is the noise free signal and $\bar{r}$ is the measured signal, $Z_k = a_k e^{i\phi_k}$, $r_k = b_k e^{i\psi_k}$. Writing the real and imaginary parts of (85) separately gives

$$(A^- + A^+) \text{Re} \bar{Z} + \frac{2}{L} \int_{-L/2}^{L/2} \cos \omega_k t \eta(t) dt \quad (86)$$

$$(A^- - A^+) \text{Im} \bar{Z} - \frac{2}{L} \int_{-L/2}^{L/2} \sin \omega_k t \eta(t) dt \quad (87)$$

We define

$$\delta \bar{r} = \bar{r} - \bar{Z}$$

or

$$(\text{Re} \delta \bar{r})_n = \text{Re} r_n - \text{Re} Z_n \quad (88)$$

$$(\text{Im} \delta \bar{r})_n = \text{Im} r_n - \text{Im} Z_n$$

which are the errors in the real and imaginary parts respectively of the spectral estimate r_n . We have then

$$\text{Re}\delta\bar{r} = \frac{2}{L} H^+ \int_{-L/2}^{L/2} \cos\omega_k t \eta(t) dt \quad (89)$$

$$\text{Im}\delta\bar{r} = \frac{2}{L} H^- \int_{-L/2}^{L/2} \sin\omega_k t \eta(t) dt$$

by defining

$$\begin{aligned} H^+ &= (A^+ + A^-)^{-1} \\ H^- &= (A^+ - A^-)^{-1} \end{aligned} \quad (90)$$

then we obtain

$$\begin{aligned} (\text{Re}\delta\bar{r})_j^2 &= \frac{4}{L^2} \sum_k \sum_l H_{jk}^+ I_{kl}^{(1)} H_{lj}^+ \\ (\text{Im}\delta\bar{r})_j^2 &= \frac{4}{L^2} \sum_k \sum_l H_{jk}^- I_{kl}^{(2)} H_{lj}^- \end{aligned} \quad (91)$$

where

$$I_{kl}^{(1)} = \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \eta(t) \eta(t') \cos\omega_k t \cos\omega_l t' dt dt' \quad (92)$$

and

$$I_{kl}^{(2)} = \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \eta(t) \eta(t') \sin\omega_k t \cos\omega_l t' dt dt'$$

and we have used the fact that $H_{j1}^+ = H_{1j}^+$, $H_{j1}^- = H_{1j}^-$.

Take the ensemble averages of $I_{kl}^{(1)}$ and $I_{kl}^{(2)}$ to obtain the expected values of $(\text{Re}\delta\bar{r})_j^2$ and $(\text{Im}\delta\bar{r})_j^2$

$$\begin{aligned} \langle (\text{Re}\delta\bar{r})_j^2 \rangle &= \frac{4}{L^2} \sum_k \sum_l H_{jk}^+ \langle I_{kl}^{(1)} \rangle H_{lj}^+ \\ \langle (\text{Im}\delta\bar{r})_j^2 \rangle &= \frac{4}{L^2} \sum_k \sum_l H_{jk}^- \langle I_{kl}^{(2)} \rangle H_{lj}^- \end{aligned} \quad (93)$$

where

$$\begin{aligned}\langle I_{kl}^{(1)} \rangle &= \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \cos \omega_k t \cos \omega_l t' \langle \eta(t) \eta(t') \rangle dt dt' \\ \langle I_{kl}^{(2)} \rangle &= \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \sin \omega_k t \sin \omega_l t' \langle \eta(t) \eta(t') \rangle dt dt'\end{aligned}\quad (94)$$

If $\eta(t)$ is ergodic, we can replace the ensemble average with a time average, and if $\eta(t)$ is stationary, the correlation function is independent of time.

Thus the noise autocorrelation function is

$$\begin{aligned}\langle \eta(t) \eta(t + \tau) \rangle &= \frac{1}{L} \int_{-L/2}^{L/2} \eta(t') \eta(t' + \tau) dt' \\ &= \frac{1}{L} \int_{-\infty}^{\infty} h(t') \eta(t') h(t' + \tau) \eta(t' + \tau) dt'\end{aligned}\quad (95)$$

where

$$\begin{aligned}h(t) &= 1 & |t| < L/2 \\ &= 0 & \text{otherwise}\end{aligned}$$

Using the inverse Fourier transform

$$h(t) \eta(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} N(\omega) e^{i\omega t} d\omega \quad (96)$$

where

$$N(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h(t) \eta(t) e^{-i\omega t} dt \quad (97)$$

Using equations (95) through (97) gives

$$\begin{aligned}\langle \eta(t) \eta(t + \tau) \rangle &= \frac{1}{2\pi L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} N(\omega) e^{i\omega t} N(\tilde{\omega}) e^{i\tilde{\omega}(t' + \tau)} d\omega d\tilde{\omega} dt' \\ &= \frac{1}{2\pi L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} N(\omega) e^{i\tilde{\omega}\tau} N(\tilde{\omega}) e^{it'(\omega + \tilde{\omega})} d\omega d\tilde{\omega} dt'\end{aligned}\quad (98)$$

and since

$$\int_{-\infty}^{\infty} e^{it'(\omega+\tilde{\omega})} dt' = 2\pi\delta(\omega+\tilde{\omega}) \quad (99)$$

we obtain

$$\langle \eta(t)\eta(t+\tau) \rangle = \int_{-\infty}^{\infty} \frac{N(\omega)N(-\omega)}{L} e^{-i\omega\tau} d\omega \quad (100)$$

Since $\eta(t)$ is real

$$N(-\omega) = N^*(\omega) \quad (101)$$

and thus

$$\langle \eta(t)\eta(t+\tau) \rangle = \int_{-\infty}^{\infty} S_L(\omega) e^{-i\omega\tau} d\omega \quad (102)$$

where

$$S_L(\omega) = \frac{|N(\omega)|^2}{L} \quad (103)$$

is the noise power spectrum based on a finite interval. By Parseval's theorem.

$$\int_{-\infty}^{\infty} S_L(\omega) d\omega = \frac{1}{L} \int_{-\infty}^{\infty} N(\omega)N^*(\omega) d\omega = \frac{1}{L} \int_{-L/2}^{L/2} [\eta(t)]^2 dt \quad (104)$$

In our case $t+\tau=t'$, so that

$$\langle \eta(t)\eta(t') \rangle = \int_{-\infty}^{\infty} S_L(\omega) e^{-i\omega(t'-t)} d\omega \quad (105)$$

Therefore

$$\langle I_{kl}^{(1)} \rangle = \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \cos\omega_k t \cos\omega_l t' \int_{-\infty}^{\infty} S_L(\omega) e^{-i\omega(t'-t)} d\omega dt dt' \quad (106)$$

$$\langle I_{kl}^{(2)} \rangle = \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \sin\omega_k t \sin\omega_l t' \int_{-\infty}^{\infty} S_L(\omega) e^{-i\omega(t'-t)} d\omega dt dt'$$

We assume "white" noise, $S(\omega) = S_o$

$$\begin{aligned}
 \langle I_{kl}^{(1)} \rangle &= 2\pi S_o \int_{-L/2}^{L/2} \cos(\omega_k t) \cos(\omega_l t) dt \\
 &= \pi S_o L (\bar{A}_{kl}^- + A_{kl}^+) = \pi S_o L (H_{kl}^+)^{-1} \\
 \langle I_{kl}^{(1)} \rangle &= 2\pi S_o \int_{-L/2}^{L/2} \sin(\omega_k t) \sin(\omega_l t) dt \\
 &= \pi S_o L (\bar{A}_{kl}^- - A_{kl}^+) = -\pi S_o L (H_{kl}^-)^{-1}
 \end{aligned} \tag{107}$$

Combining (93) and (102) we finally obtain the variance in the real part of the j th spectral line estimate

$$\langle (\text{Re} \delta \bar{r})_j^2 \rangle = \frac{4\pi S_o}{L} H_{JJ}^+ \tag{108}$$

and

$$\langle (\text{Im} \delta \bar{r})_j^2 \rangle = -\frac{4\pi S_o}{L} H_{jj}^- \tag{109}$$

that is, the variance in the imaginary part of the j th spectral line estimate.

We can also show that the covariance term

$$\langle (\text{Re} \delta \bar{r})_j (\text{Im} \delta \bar{r})_j \rangle = 0 \tag{110}$$

for white noise.

(8.v) Estimation of errors in results from data analysis. The below derivations are based on the concept of standard deviation, which we in reality can only estimate. In computing the standard deviation of a quantity V , which is a function of k variables x_j $j = 1, 2, \dots, k$, we assume the errors in the individual variables x_j to be independent. Then we can use (Beers, 1953):

$$s_v^2 = \left(\frac{\partial V}{\partial x_1}\right)^2 s_1^2 + \left(\frac{\partial V}{\partial x_2}\right)^2 s_2^2 + \dots + \left(\frac{\partial V}{\partial x_k}\right)^2 s_k^2 \quad (111)$$

to estimate the standard deviation of the quantity V , where s_j is the standard deviation of x_j . This type of addition provides for some possibility that the contributions of the x_j 's might partially compensate each other.

We have estimated the variances in the real and imaginary parts of a spectral analysis estimate given by equation (79). The above assumption of independence is consistent with assuming the noise to be "white". If we define

$$x = \text{Re}Z_n$$

$$y = \text{Im}Z_n$$

then we have by equation (80) for the amplitude and phase of a specific spectral line estimate

$$A = (x^2 + y^2)^{1/2} \quad (112)$$

$$\phi = \tan^{-1}(y/x)$$

The standard deviation in the amplitude is then

$$s_A^2 = \frac{1}{A^2} (x^2 s_x^2 + y^2 s_y^2) \quad (113)$$

and in the phase

$$s_\phi^2 = \frac{1}{A^4} (y^2 s_x^2 + x^2 s_y^2) \quad (114)$$

where s_x^2 and s_y^2 are the variances in the real and imaginary parts respectively and

$$\begin{aligned} s_x^2 &= \frac{4\pi S_0}{L} H_{jj}^+ \\ s_y^2 &= \frac{-4\pi S_0}{L} H_{jj}^- \end{aligned} \quad (115)$$

So finally

$$S_A = \frac{1}{A} \sqrt{\frac{4\pi S_0}{L} (x_{jj}^2 H_{jj}^+ - y_{jj}^2 H_{jj}^-)} \quad (116)$$

$$S_\phi = \frac{1}{A^2} \sqrt{\frac{4\pi S_0}{L} (y_{jj}^2 H_{jj}^+ - x_{jj}^2 H_{jj}^-)}$$

where we have

$$S_0 = \frac{1}{L} \int_{-L/2}^{L/2} [h(t) - \sum_n a_n \cos(\omega_n t + \phi_n)]^2 dt \quad (117)$$

the r.m.s. deviation.

We can similarly estimate the standard deviation in a and ϕ_a as computed by equations (82) and (83).

$$S_{\phi_a}^2 = \left(\frac{\partial \phi_a}{\partial A}\right)^2 S_A^2 + \left(\frac{\partial \phi_a}{\partial \phi}\right)^2 S_\phi^2 + \left(\frac{\partial \phi_a}{\partial b}\right)^2 S_b^2 + \left(\frac{\partial \phi_a}{\partial \phi_b}\right)^2 S_{\phi_b}^2 \quad (118)$$

$$S_a^2 = \left(\frac{\partial a}{\partial A}\right)^2 S_A^2 + \left(\frac{\partial a}{\partial \phi}\right)^2 S_\phi^2 + \left(\frac{\partial a}{\partial b}\right)^2 S_b^2 + \left(\frac{\partial a}{\partial \phi_b}\right)^2 S_{\phi_b}^2$$

which after some algebra gives:

$$S_{\phi_a}^2 = \frac{1}{a^4} \left[z_1^2 S_A^2 + z_2^2 S_b^2 + z_3^2 S_\phi^2 + z_4^2 S_{\phi_b}^2 \right]$$

$$S_a^2 = \frac{1}{a^2} \left[\frac{z_3^2}{A^2} S_A^2 + \frac{z_4^2}{b^2} S_b^2 + A^2 z_1^2 S_\phi^2 + b^2 z_2^2 S_{\phi_b}^2 \right] \quad (119)$$

$$z_1 = b \sin(\phi_b - \phi)$$

$$z_2 = A z_1 / b$$

$$z_3 = A^2 - A b \cos(\phi_b - \phi)$$

$$z_4 = z_3 - A^2 + b^2$$

We can assume $S_b \ll S_A$ and $S_{\phi_b} \ll S_\phi$ if b and ϕ_b relate to ocean loading.

(9) OVERVIEW OF FIELD RESULTS - G. Bodvarsson

(9.i) History. Our strain and tilt field work started on 2/18/79 on the campus of Oregon Institute of Technology in Klamath Falls, Oregon. It is well known that one of the eastern master faults of the Klamath graben passes through the OIT campus and our strain/tilt instruments were emplaced directly over the main fault zone. Field work at the site continued until June 1979.

The equipment was moved to the Krafla geothermal area in Northern Iceland at the end of July 1979 and field work was carried out there through the month of August.

Field work at the Newberry caldera in Oregon was initiated on 9/11/79 and continued until 10/14/79. The work was then discontinued until the tiltmeters were emplaced at Mt. St. Helens in July 1980.

Our field work indicated quite early that the strain data obtained on the basis of present procedures are very contaminated by thermal noise such that we have had great difficulties in separating any useful tidal signals. This is apparent from the raw field records displayed below.

The situation with regard to the tilt data is, on the other hand, considerably more encouraging, and we believe that we have had more success in this phase of our work. To give a compact overview of our tilt results examples of our raw and processed records are displayed below in Figs. 10 to 21 and the results are discussed briefly.

(9.ii) Summary of field results

Klamath Falls, Oregon. Field sites were chosen at one of the major normal (graben) faults traversing the campus of the Oregon Institute of Technology. Figs. 10 to 12 display the results from two components at one

Fig. 10. Raw tilt data from Klamath Fall, site #1.

Fig. 11. Klamath Falls tilt data from site #1 after detrending and smoothing.

tilt station, showing data before detrending and after as well as the theoretical tilt variation. Further records from nearby sites display a similar character. Obviously, there is considerable temperature noise that contaminates the solid earth tidal signal. As of now, spectral data analysis of the Klamath Falls records has failed to indicate a tilt anomaly

Krafla Geothermal Area. Fig. 13 shows the tilt sites at Krafla, Iceland. Figs. 14 and 15 show the raw data at K-1, Figs. 16 and 17 the theoretical tilt and Figs. 18 and 19 the detrended and reduced final results of our analysis for this station. We draw attention to the observed large magnification of the WE component of the tidal amplitude at this site. The SN component is apparently not affected. We find this result to be in accordance with our expectation and draw the conclusion that there is a clear modification of the local tilt amplitude. This is our most noteworthy result. As indicated in Fig. 13, the data obtained at K-3 show a moderate amplification of both components. The data obtained at K-2 and K-4 were quite contaminated by noise and have therefore been rejected for the time being.

At this juncture, we are unable to relate the results of K-1 and K-3 to a definite cause. As discussed in section (6) above, the tilt amplitude magnification may result from the effects of an adjacent open fracture and/or to the presence of the magma chamber that is known to exist at the depth of 3 to 7 km. A fracture effect would be very local, whereas, on the other hand, the magma chamber would cause anomalies over larger areas. Field data from a few additional stations would help to resolve this problem.

Fig. 12. Theoretical earth tide tilt at Klamath Falls, site #1.

Fig. 13. Map showing locations of stations used for tidal tilt measurements at Krafla, Iceland.

Fig. 14. SN comp. of tilt at K-1, raw data.

Fig. 15. WE comp. of tilt at K-1, raw data.

Fig. 16. Theoretical solid earth tide tilt and ocean loading (M_2) at K-1, SN comp.

Fig. 17. Theoretical solid earth tide tilt at K-1, WE comp.

Fig. 18. SN comp. of tilt at K-1 after detrending, smoothing and thermal noise removal.

Fig. 19. WE comp. of tilt at K-1 after detrending, smoothing and thermal noise removal.

Newberry Caldera, Oregon. Two sites were occupied near the caldera ring fault at Newberry Crater. The purpose was to record any tidally induced differential movement of the ring fault that might be caused by molten material below the caldera floor. At both sites, the signal/noise ratio turned out to be rather low and the quality of the data therefore in some doubt. However, a processing of the available material indicates a magnification of the across-fault M_2 tidal tilt amplitude by a factor of 1.4 to 1.9. As in the case of the Krafla above, the magnification may represent the local ring fracture effect that we have anticipated and/or may be associated with other phases of the tidally induced deformation of an underlying magma chamber. Further data will help to resolve this problem. In any case, the tilt amplitude magnification is a positive result.

An interesting observation was made at site 2 at Newberry Crater. A light snow fell on the site during the time the tiltmeter was operational. The thermal insulation due to the snow cover resulted in a substantial lowering of the thermal noise and an enhancement of the signal/noise ratio. The detrended tilt data from the site are shown in Figure 20. This observation indicates that advantage can be taken of snow in selecting tiltmeter sites.

Mt. St. Helens, Washington. At this moment (August, 1980), we have about 10 days of raw field data from a tiltmeter site on the southern slope of Mt. St. Helens as shown in Fig. 21. The SN component displays a deflation of the mountain at a rate of 0.5 μ rad/day. Solid earth tides are clearly displayed on the WE component. The unprocessed tidal amplitude is of normal order of magnitude. We expect that the monitoring of tilt at

Fig. 20. Detrended tilt data from Newberry Crater, site #2.

Fig. 21. Raw tilt data from Mt. St. Helens, site #3, Aug. 1980.

Mt. St. Helens over longer periods will reveal a considerable amount of information on the progress of the volcanic processes there.

(9.iii) Comments. Both the U.S.G.S. strainmeters and the Kinemetrics tiltmeters are rather simple instruments that perform quite well in the field. There are, however, severe difficulties because of noise of thermoelastic origin. The strainmeter records have not yielded useful data and we are of the opinion that this phase of our work has failed so far. The situation may possibly be improved with a much better temperature control (see also below).

As already stated, the tiltmeter records are also contaminated by noise, but the situation is more manageable. We have been able to separate useful signals out of some of the records. Our impression is that the tiltmeter technique can be improved considerably with the help of the following measures:

- (a) More careful selection of field stations,
- (b) emplacement at depths in excess of 2 meters,
- (c) more complete temperature recordings at the sites,
- (d) measurement under snow cover,
- (e) increased resolution in signal recording,
- (f) more efficient data processing techniques.

(10) DISCUSSION OF RESULTS FROM THE KRAFLA GEOTHERMAL AREA, ICELAND -
(G. Axelsson)

The final results of the tidal tilt and strain observations in the Krafla-Namafjall area are summarized in Table II. These results reveal clearly the existence of a tidal tilt anomaly in this area.

Most noteworthy is the large amplification, possibly amounting to 200%, of the EW component of tilt at site K-1. The NS component does not seem to be affected. Somewhat less pronounced and less reliable is the anomaly observed at site K-3. The tidal strain at site K-2 is apparently not magnified.

(10.i) Possible causes of anomalies. The limited number of sites occupied at Krafla makes it impossible to determine the cause of these anomalies with a reasonable certainty. There are a number of possibilities. The observed anomalies may be caused by one or more of the following effects.

(a) A single fracture will cause anomalous tidal tilts at an adjacent observation site. The order of magnitude of this effect will be estimated below. Numerous conspicuous fractures exist within the Krafla fissure swarm and because of the north-south trend of these fissures only anomalies in the EW component of tilt can be expected. The fractures do not have to be open at the surface.

(b) Since site K-1 is located within the active part of a rift zone, the anomaly observed there may be caused by a wide swarm of fissures rather than a single fracture. An anomaly of this type would mainly be the result of the broad discontinuity in the elastic properties of the upper crust associated with the swarm.

(c) Major magma chambers at depths of a few km will modify the local

TABLE II

Earth tide observations at Krafla, results.

Site, ¹ Component	Amplification of semidiurnal tide	Phase difference ²	Quality ³
K-1			
WE tilt	2.0-3.0	-21°	7
SN tilt	0.9-1.4	0°	8
K-3			
WE tilt	1.5-(2.0)	n.s. ⁴	2
SN tilt	1.6-1.8	n.s.	4
K-4 tilt			No results
K-1 WE strain			No results
K-2 WE strain	0.8-1.0	n.s.	2
K-3 WE strain			No results

¹For locations of sites see Figure 13.²Phase lag with respect to the theoretical tide.³Quality of data rated on a scale of 0 = very bad to 10 = very good.⁴n.s. = not significant.

tilt field and may generate the observed anomalies. It is an established fact that there is a local magma chamber below the Krafla caldera, estimated to be at a depth of 3 to 7 km (Einarsson, 1978). Magma may be present elsewhere beneath the Krafla fissure swarm, but probably at greater depths. Geothermal activity and surface volcanism indicate that there may be another magma chamber in the Namafjall area. Moreover, the geological discontinuity in the southeast quadrant of the Krafla caldera suggested by resistivity, aeromagnetic and gravity anomalies (Karlsdottir et al., 1978; Thorarinsson, 1980) may indicate an additional magma chamber. This could in turn result from the transection of the Krafla caldera by the Dalvík transform fault suggested by Einarsson (1976). A seismic study of the axial rift zone in southwest Iceland (Zverev et al., 1980) has outlined a local volume void of reflecting horizons, possibly in a state of partial melting, below a fracture zone active in recent time. The depth to this volume is less than 8 km. The similarity between the rift zones of north and southwest Iceland tempts us to expect similar conditions in north Iceland, and supports the possibility for a zone of magma below the entire Krafla fissure swarm. Moreover, the magma below the swarm could be a portion of a more extended zone of partial melting below the Neovolcanic zone as a whole. Recent magnetotelluric work (Beblo and Bjornsson, 1980) has indicated a low-resistivity layer at a depth of less than 10 km within the Neovolcanic zone. The layer is inferred to be at a temperature of 1000°C or more. Focal depths of earthquakes also give useful information. Within the rift zone they are mostly less than 6 km (Palmason, 1980).

The EW tilt anomaly observed at K-1 may be caused by one or more of the above effects. The Krafla magma chamber does, however, not seem to influence the (semidiurnal) NS tilt at K-1.

Since K-3 is located some 7 km east of the center of the active fissure swarm, the magnification observed at the site can probably not be explained by (a) or (b) above. An anomaly at this location may be generated by a local magma body as set forth under (c) above.

We conclude that the earth tide strain at site K-2 is not magnified, and may possibly be reduced. This can not be determined with any certainty, but an abnormally small strain amplitude would be in agreement with the location of K-2 at the rim of the active fissure swarm.

(10.ii) Estimates of quantitative cause/effect relations. The values of the theoretical stresses and strains presented in Table III can be used to establish quantitative cause/effect relations. Note that the semidiurnal theoretical EW strain is out of phase with the NS strain by 180° . As a consequence the semidiurnal EW stress and the dilatation are considerably smaller than the diurnal stresses and dilatation. This is a result of the high latitude and is of importance in this study.

(a) We can estimate the order of magnitude of the tilt anomaly on the edge of a fracture by using equation (2) above and the data of Table III. Thus we estimate the M_2 component of the EW tilt anomaly due to a single NS trending fracture to be of the order $0.006 \mu\text{rad}$. This is an order of magnitude too small to explain the observed anomaly.

(b) The effects of a multi-fracture can not be estimated with any certainty. However, a tilt anomaly caused by the fracture zone will be a function of the EW stress and hence depend on the abnormally small amplitude of the semidiurnal EW stress at this latitude.

(c) Based on the evidence for a substantial body of magma below the Krafla area, discussed earlier, we can envision a situation as sketched

TABLE III

Theoretical Tidal Strain and Stress at 65°40'N, 17°W

	Tidal Component						
	K_1	(a)		(b)		$M_2(a) \& (b) \text{ combined}$	
		$M_2, \text{ solid earth tide}$	$M_2, \text{ ocean loading}$	$M_2(a) \& (b) \text{ combined}$			
	amplitude ¹	amplitude	phase ²	amplitude	phase	amplitude	phase
Strain							
NS $\epsilon_{\theta\theta}$	4.0×10^{-9}	7.5×10^{-9}	0	1.4×10^{-9}	1.41	7.8×10^{-9}	0.17
EW $\epsilon_{\lambda\lambda}$	6.3×10^{-9}	3.0×10^{-9}	-3.14	2.1×10^{-9}	-2.86	5.1×10^{-9}	-3.03
Dilatation ³							
θ	6.9×10^{-9}	3.0×10^{-9}	0	1.9×10^{-9}	2.63	1.6×10^{-9}	0.60
Stress ⁴							
NS $\sigma_{\theta\theta}$	$1.5 \times 10^{-8} \mu$	$1.8 \times 10^{-8} \mu$	0	$3.3 \times 10^{-9} \mu$	1.81	$1.8 \times 10^{-8} \mu$	0.18
EW $\sigma_{\lambda\lambda}$	$2.0 \times 10^{-8} \mu$	$3.0 \times 10^{-9} \mu$	-3.14	$5.2 \times 10^{-9} \mu$	-3.02	$8.2 \times 10^{-9} \mu$	-3.06

¹Phase = 0²Phase lag with respect to the tidal potential maximum, in radians.³ $\theta = 2/3 (\epsilon_{\theta\theta} + \epsilon_{\lambda\lambda})$ at a free surface in a Hookean solid assuming $\lambda = \mu$ ⁴ $\sigma_{\theta\theta} = \mu(\theta + 2\epsilon_{\theta\theta})$ $\sigma_{\lambda\lambda} = \mu(\theta + 2\epsilon_{\lambda\lambda})$ assuming $\lambda = \mu$.

in Figure 22 below. A magma body of this type is likely to cause elasto-mechanical anomalies. The possible magnitude of the associated tilt anomaly can be roughly estimated with the help of the relations for a cavity-vent system presented in section (6.iii) above. We assume f to be considerably smaller than the cross-sectional area of the magma body and neglect the elastic force fields that are required to bend the solid roof rock above the upward protruding magma. The tilt at the point A in Figure 22 can then be estimated to be of the order

$$\begin{aligned} T &= h/x \\ &= \sigma D_v / (x g \rho (E_v + c_f V)) \end{aligned} \quad (120)$$

where h is given by equation (47). Considering the simple case of a cylindrical cavity we have (Bodvarsson, 1980)

$$\begin{aligned} D_v/V &= 1.5/\mu \\ E_v/V &= 1.0/\mu \end{aligned} \quad (121)$$

and assuming $c_f \mu = 1$ to hold approximately for mafic magma, where μ is the rigidity of the surrounding rock, we find

$$T = 0.75\sigma / (x g \rho) \quad (122)$$

Inserting $\sigma = k_r \theta$ with $k_r = 5 \times 10^{10}$ Pa the bulk modulus of the surrounding rock and θ the M_2 dilatation from Table III, $\rho = 3000 \text{ kg/m}^3$ and $x = 5 \text{ km}$ in (122) we obtain the amplitude estimate

$$T = 0.5 \text{ } \mu\text{rad} \quad (123)$$

If tilt to the east is defined as positive this tilt anomaly should be in

Fig. 22. The magma body-fissure swarm model.

phase with the minimum of the tidal dilatation. The above estimate is an order of magnitude larger than the EW tidal tilt anomaly observed at K-1.

We can therefore conclude that the anomaly observed could be generated by a large body of magma below the Krafla fissure swarm. Clearly, the model of Figure 22 can not be expected to constitute a faithful model of the crustal conditions in the area. It is only constructed for the purpose of obtaining the order of magnitude estimate above. Further studies of this kind, for instance occupying a profile of sites across the fissure swarm, may give information on the unknown parameters, such as dimensions, depth and elastic constants of a possible molten body below the Krafla-Namafjall area.

The M_2 EW tilt anomaly observed at K-1 is $(0.055 \pm .040)$ μ rad and lags the tidal potential by $(4.24 \pm .40)$ rad. This is rather well in phase with the minimum of the tidal dilatation (phase lag 3.74 rad, Table III). This would indicate that the site K-1 is located east of the center of the proposed magma body. Since Krafla is on the west side of the neovolcanic zone, it appears unlikely that the anomaly observed is due to a more extensive zone of magma below the entire volcanic zone as set forth under (c) above.

Due to the discontinuities in the elastic properties of the uppermost part of the crust in the Krafla area, the possibility exists that the ocean loading tilt may be underestimated. Assuming a larger EW ocean loading tilt, we can fully explain the small, but significant, phase difference between the dilatation and the anomaly observed. The possible combination of homogeneous solid earth tide EW tilt, enhanced ocean loading EW tilt and the dilatation related tilt anomaly is presented as a vector diagram in Figure 23.

a = theoretical solid earth tide tilt

b = ocean loading tilt, $x = 1.9$

$t(-\theta)$ = dilatation related tilt, $t(-\theta) = 0.065 \mu\text{rad}$

Fig. 23. A vector diagram representing the possible combination of solid earth tide tilt, enhanced ocean loading tilt and the dilatation related tilt anomaly giving the EW M_2 tidal tilt observed at site K-1, Krafia.

By interpreting the K-1 EW tilt anomaly as a dilatation-tilt coupling we would expect a diurnal anomaly about four times the semidiurnal anomaly (Table III). The fact that we have not observed this relation (see Figure 62) may result from a combination of thermal contamination and anomalous ocean loading. Alternatively, it is also possible that the observed anomaly is a combination of two effects, one in phase with the negative dilatation and another in phase with the EW stress. These effects could add up resulting in a large semidiurnal anomaly and a smaller diurnal anomaly. The above indicates the importance of obtaining reliable estimates of the O_1 component in order to compare diurnal and semidiurnal anomalies.

A likely effect of a shallow zone of partial melting under the rift zone would be a local focusing or enhancement of the tidal stress in the solid crust above the zone. The theoretical values given in Table III would therefore be too low. We could for instance, explain the EW tilt anomaly at K-1 by a single fracture if there is a tenfold stress enhancement of this type. It would be of considerable interest to attempt at determining whether this is the case.

(10.iii) Sources of errors. Several influential factors, some mentioned before, limit the quality of the data obtained in this study and influence the results described earlier. The enumeration below, although not complete, includes the most important sources of error.

(1) The limited resolution in the recording of the signals is a source of considerable uncertainty. The tilt and strain records were digitized by hand with an accuracy of less than 0.1 mm. This corresponds to a resolution of about $0.05 \mu\text{rad}$, for the tilt measurements, which is of the same order as the field amplitudes of interest. For the strain recordings this corresponds to resolution in the range 3×10^{-9} to 2×10^{-8} . The field

signals are of the order of 5×10^{-9} .

(2) The thermal noise, both of instrumental and thermoelastic origin, has been discussed earlier. Provided sufficiently good records of surface temperature and its spatial derivatives are available, it should be possible to largely remove this noise from the data. The limited resolution both in temperature and time of the temperature recordings in this study is probably partly responsible for the limited success of thermal noise removal.

(3) Uncertainties in the calibration of the tilt and strainmeters are of importance. The average of 9 tiltmeter calibrations (from Iceland and Oregon, both NS and EW components) is $0.339 \mu\text{rad/mm}$ with a standard deviation of $0.019 \mu\text{rad/mm}$ or 6%. The maximum deviation is $0.042 \mu\text{rad/mm}$ or 13%. The calibration of the strainmeters is more uncertain. Standard deviations in the range 15% to 25% were obtained.

(4) Errors in the M_2 cotidal map used will cause discrepancies between observations and ocean loading estimates. In the present case this does not seem to have been a serious problem.

(5) Even though M_2 is by far the largest ocean tidal component around Iceland, other components are of importance. But limited knowledge makes it difficult to estimate their effect. This will probably introduce some errors when direct comparisons are made, but it will mainly be of importance for the NS component of tilt.

(6) Topography will influence tidal tilt and strain to some extent (see Harrison, 1976 and Blair, 1976 for examples).

(7) Diurnal and semidiurnal atmospheric pressure variations will influence earth tide measurements as well as rainfall and other meteorological phenomena.

(8) The NS and EW components of the tiltmeter may deviate from the true NS and EW orientations. This will introduce some errors when data is compared with theory. Errors in the strainmeter azimuths should not exceed 2° .

Of the above (1) and (2) are probably mostly responsible for the limited quality of the data collected in this study. The effect of (3) is less important. Errors due to (4) and (5), presumably not very serious, can only be reduced with increasing knowledge on ocean tides. The influence of (7) can be corrected for in the same way as thermal noise can be removed. The effect of (6) can be estimated, but involves time consuming numerical computations. Errors due to (8) can be fully eliminated.

(10.iv) Improvements. Since the results of this study indicate that tidal tilt observations may be of value as a geophysical exploration method, several improvements, that may be of great value in increasing the quality of data obtained, are worth mentioning. Our work has indicated the following possibilities.

(1) Resolution

The resolution in the signal recording should be increased by an order of magnitude, if possible. This can be achieved by amplifying the signal or by using recorders of a higher sensitivity. Of course an upper limit to the possible resolution is determined by the background noise that may be reduced by the following means.

(2) Site selection

The difference between the thermal noise in the tilt records from site K-1 and K-3 indicates quite clearly that a careful selection of field stations is very important. Sites having surface layers incapable of

transmitting thermal stresses, thus acting as insulators, are desirable. In tidal tilt observations, advantage can also be taken of other natural insulators such as a snow cover. The topography of the area around the site in question should also be considered. Smooth surrounding topography will eliminate the need for tedious topographic corrections. Emplacement of the tiltmeter at greater depths than in this study would most likely help in reducing the thermal noise.

(3) Temperature and atmospheric pressure recordings

Higher resolution records of the surface temperature and atmospheric pressure should be obtained at and around the site. These variables should be recorded at a number of sites, so as to enable computations of local EW and NS derivatives of both temperature and pressure. This would most likely make thermal noise removal more successful than in this study. More sophisticated methods of meteorological noise removal should also be attempted.

Less important are the following:

(4) Insulation of instruments

The instrumental thermal noise could be reduced significantly by carefully insulating the tiltmeter and tiltmeter electronics. The thermal stability of the power sources used would be worth considering.

(5) Improved calibration

The quality of the data obtained could still be increased by using improved calibration techniques.

(6) Data processing

A shorter digitization interval would enable the application of more efficient filtering techniques. The length of records needed from each site will depend on the significance of thermal and other noise. It would be of interest to try to resolve the O_1 tidal component to confirm an agreement with the M_2 component. More efficient data processing techniques

should be tested.

(10.v) Concluding remarks on the results in the Krafla area.

(1) The long term tilt observed at Krafla (see Figures 58, 61, 64 and 67) is in a good agreement with independent observations of the inflation of the Krafla caldera (Tryggvason, 1979). This indicates that the biaxial tiltmeters are applicable to the recording of long period tilts. Other authors (Wyatt and Berger, 1980) have questioned whether long term tectonic signals can be observed with this kind of tiltmeters. It is our conclusion that a careful selection and preparation of field stations is extremely important and may in fact be essential for the instrument stability. According to our experience from Oregon and Iceland spurious long term tilts can be caused by improper preparation of the sites.

(2) The results of this study indicate that tidal tilt observations can be of use in the explorations of geological inhomogeneities such as volcanic geothermal systems. At favorable conditions quite accurate estimates of the largest tidal tilt components can be obtained from less than two week long records using the simple field techniques adapted in this study. With increased resolution, more extensive temperature recordings and a very careful selection and preparation of field stations, tilt amplitude anomalies of 20% or less should be easily observable. The variation of the tidal stresses with latitude is worth keeping in mind if stress-tilt coupling is expected. It is of interest to note that the theoretical semidiurnal EW stress vanishes at approximately 60° N and S.

(3) Due to the instrumental thermal noise the strain measurements were not as successful. In many cases we can, however, expect relative strain anomalies to be larger than relative tilt anomalies. Provided thermal noise contamination can be minimized, strain observations could become a

more suitable field procedure.

(4) The results of the tidal tilt observations in the Krafla area indicate a considerable enhancement of the east-west component of the M_2 tidal tilt at the site K-1 inside the active part of the Krafla fissure swarm. This tilt anomaly is most likely due to a large volume of magma below the fissure swarm, although a part may possibly result from an inelastic response of the rift zone to ocean loading. The north-south component of the M_2 tilt observed at this site is in a very good agreement with the theoretical solid earth tide tilt and the ocean loading tilt. The tilt data from site K-3 southeast of the caldera are less reliable but indicate an anomaly at that site.

(5) These results, although limited, are sufficiently favorable to justify further applications of the tidal tilt technique to study the Krafla volcanic complex and geothermal area. This technique could give additional information on the deep structure of the region and thus supplement the exploration results that have been obtained in past field work. A number of stations inside the caldera could probably give information on the depth and dimensions of the local magma chamber. A profile of sites across the Krafla fissure swarm may furnish information on the crustal conditions below the swarm.

(11) FIELD DATA AND SOFTWARE - G. Axelsson

The complete set of field data obtained on the project are displayed in Figs. 24 to 91. The captions are designed to give sufficient information on each figure. Software such as computational programs and cotidal maps are displayed in the Appendix.

Fig. 25. Data from figure 24 after detrending and smoothing.
 Direct comparison with theory in figure 26 gives $M^* \approx 1.0$. Spectral analysis (M_2) gives $M = 1.0 \pm 0.6$

*M = observed amplitude/theoretical amplitude

Fig. 31. Data from figure 30 after detrending and smoothing.

Direct comparison with theory in figure 32 gives a magnification $M = 1.0-2.0$ for SN.

Figure 47. Theoretical solid earth tide tilt at Klamath Falls, site #2. Same period as in Figure 45.

Figure 48. Raw strain data from Klamath Falls, site #1, EW component. Period 3/11/79 19:00 to 3/28/79 16:00 GMT. (Straight line = no data).

Figure 51. Raw strain data from Klamath Falls, site #1, EW component. Period 5/12/79 22:00 to 6/12/79 18:00 GMT. (Straight line = no data).

Figure 52. Data from Figure 51 after detrending and smoothing. Neither spectral analysis nor direct comparison gave any results for the strain from site #1.

Figure 57. Map showing locations of stations used for tidal tilt and strain measurements at Krafla, Iceland. Also shown is the amplification observed.

Figure 58. Raw tilt data from Krafla, site #1, SN component. Beginning at 08/05/79 0:00 GMT. Long term tilt 0.53 μ rad/day to the south, consistent with the inflation of the Krafla caldera.

Figure 59. Data from Figure 58 after detrending, smoothing and thermal noise removal. Direct comparison with theory in Figure 60 gives $M = 1.3 \pm 0.2$. Spectral analysis gives magnifications for M_2 , $M = 0.9 \pm 0.3$
 $\Delta^* = 0.00 \pm 0.31$
for K_1 , $M = 1.9 \pm 0.4$
 $\Delta = -0.18 \pm 0.23$
The K_1 estimate is unreliable because of thermal noise.

* Δ : phase lag with respect to the theoretical tilt.

Figure 61. Raw tilt data from Krafla, site #1, WE component. Beginning at 08/05/79 0:00 GMT. Long term tilt 0.31 $\mu\text{rad/day}$ to the east, not consistent with the inflation of the Krafla caldera but could be caused by the influence of nearby fractures.

Figure 62. Data from Figure 61 after detrending, smoothing, thermal noise removal and subtraction of ocean loading. Direct comparison with theory in Figure 63 gives $M = 2.4 \pm 0.4$. Spectral analysis gives magnifications for M_2 $M = 3.2 \pm 1.5$
 $\Delta = -0.37 \pm 0.27$
for K_1 $M = 1.3 \pm 0.4$
 $\Delta = -0.30 \pm 0.29$
The K_1 estimate is unreliable because of thermal noise.

Figure 63. Theoretical solid earth tide tilt at K-1, WE component.

Figure 65. Data from Figure 64 after detrending, smoothing and thermal noise removal.

Direct comparison with theory in Figure 66 gives a magnification $M = 1.7 \pm 0.4$

Spectral analysis was not successful.

Figure 66. Theoretical solid earth tide and ocean loading (M_2 only) tilt at K-3, SN component.

Figure 68. Data from Figure 67 after detrending, smoothing, and thermal noise removal.

Direct comparison with theory in Figure 69 gives a magnification $M = 1.7 \pm 0.4$. This estimate is uncertain because of noise.

Figure 69. Theoretical solid earth tide and ocean loading tilt at K-3, WE component.

Figure 70. Raw tilt data from Krafla, site #4, SN component.
Beginning at 09/01/79 9:00 GMT.

Figure 73. Data from Figure 72 after detrending, smoothing and partial thermal noise removal.

Direct comparison with theory in Figure 74 gives $M = 0.9 \pm 0.2$.

Figure 77. Theoretical solid earth tide and ocean loading strain at K-1.

Figure 79. Data from Figure 78 after detrending and smoothing. The signal is dominated by thermal and other noise.

Figure 80. Theoretical solid earth tide and ocean loading strain at K-3.

Figure 83. Theoretical solid earth tide tilt at Newberry Crater, site #1. Same period as in Figure 81.

Figure 86. Theoretical tilt corresponding to the data in Figure 85.

Figure 91. Theoretical tilt corresponding to the data in Figure 90.

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APPENDIX

The cotidal maps used in estimating the ocean loading tides at Krafla in Iceland are presented below along with listings of the major computer programs used in the data processing.

Figure 92. Map showing cotidal lines for the M_2 -ocean tide. Phases in lunar hours referred to the maximum of the M_2 -tide in Reykjavik (1 lunar hour = 1 hr 2 min).

Figure 93. Map showing co-amplitude lines for the M₂-ocean tide. Amplitudes in meters.

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      GUERTD                      6/ 2/80 12.19.37      PAGE 1
1/C *****
2/C      PROGRAM GUERTD COMPUTES SOLID EARTH TIDES AS RIGID
3/C      EARTH POTENTIAL, GRAVITY, TILT OR STRAIN TIDES. TIDES
4/C      ARE CALCULATED FROM HARMONICS 2 THROUGH 3 FOR THE
5/C      LUNAR TERMS, 2 FOR THE SOLAR TERMS ( EXCEPT THAT
6/C      DEGREE 4 IS INCLUDED FOR THE LUNAR GRAVITY TIDE ).
7/C      GRAVITY TIDE IS IN MICROGALS, PLUS FOR UP ACCELERATION
8/C      RIGID-EARTH POTENTIAL IS IN METERS (M/G), PLUS FOR UP
9/C      TILT TIDE IS IN HANORADIANS
10/C     STRAIN TIDE IS IN PARTS IN E 9, PLUS FOR EXTENSION.
11/C     AZIMUTHS FOR STRAIN AND TILT ARE RECKONED POSITIVE
12/C     EAST FROM NORTH.
13/C
14/C     PROGRAM INITIATION AND MODIFICATION
15/C     SCRIPPS INSTITUTE OF OCEANOGRAPHY
16/C     LA JOLLA, CA
17/C     BY J LEVINE, K YOUNG AND B AGNEW
18/C     BASED ON PROGRAMS BY J C HARRISON.
19/C
20/C     PROGRAM MODIFIED BY J HANSON
21/C     LAWRENCE LIVERMORE LABORATORY
22/C     LIVERMORE, CA
23/C
24/C     PROGRAM MODIFIED AND CONVERTED TO ECLIPSE S/130
25/C     BY GUDNI AXELSSON
26/C     GEOPHYSICS GROUP
27/C     OREGON STATE UNIVERSITY, OR
28/C     NOVEMBER 1979.
29/C *****
30/C     REAL NU, LN, IYR, IDAY, JYR, JDAY, K, L, KR
31/C     REAL NU, NUT, MUL, NUTT, NULL, NUTL
32/C     DOUBLE PRECISION TT, TS, TE, B, HR, P120, T, T2, LL, HS, PS, ES, PSIG,
33/C     ALS, SZ, CZ, PSIS, WM, PN, HM, BL, BLS, BF, BB, TLATM, TLONGM, PLX, SINMLA,
34/C     ACOSMLA, SINMLN, COSMLN, AT1, AT2, RAM, U, SINU, COSU, LAMDA,
35/C     AZHR, YHR, CHMP, SHMP, SLS
36/C     DIMENSION SEROUT(6), AZT(3,2), AZS(3,3), TTUN(2), STNSUN(3),
37/C     ATTNUN(2), STNNUN(3), H(3), K(3), L(2), BIN(3), DEL(3), P(3),
38/C     APP(2), PPP(2), KR(4)
39/C
40/C     LOVE NUMBERS H(N), K(N) AND L(N) AND OTHER CONSTANTS
41/C
42/C     DATA H/.6114, .2891, .173/, K/.304, .09421, .043/, L/.0032, .0145/,
43/C     APP(1)/3./, A/6.37816E 08/, G1/979.828/, G2/982.822/,
44/C     ARL/1./, IFLAG/0/, IFLAG2/0/
45/C     DATA P120/62.83185387179583 00/
46/C
47/C     FIND LOVE NUMBER COMBINATIONS FOR TILT AND GRAVITY
48/C
49/C     DO 10 I=1,3
50/C     BIN(I) = 1.+K(I)-H(I)
51/C     10 DEL(I) = 1.+(2./<FLOAT(I)+1.)+H(I)-<<FLOAT(I)+2.>/<FLOAT(I)+1.>)*K(I)
52/C
53/C     INPUT
54/C
55/C     WRITE(10,60)
56/C     READ(11,140) IYR, IDAY, ZHR
57/C     WRITE(10,70)
58/C     READ(11,140) JYR, JDAY, YHR

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59:    WRITE(10,80)
60:    READ(11,170) THETA, LANDA, D
61:    WRITE(10,90)
62:    READ(11,150) ELEV
63:    ACCEPT* HNCR = *,HNCR
64:    ACCEPT* NNTL = *,NNTL
65:    ACCEPT* NMST = *,NMST
66:    IF(NNTL.EQ.0) GO TO 30
67:    WRITE(10,110)
68:    DO 20 I=1,NNTL
69:    READ(11,150) AZT(1,I)
70:    20 CONTINUE
71:    30 IF(NMST.EQ.0) GO TO 50
72:    WRITE(10,120)
73:    DO 40 I=1,NMST
74:    READ(11,150) AZS(1,I)
75:    40 CONTINUE
76:    50 CALL OUTFILE(1)
77:C
78:    60 FORMAT(' INPUT START YEAR (F5.0), DAY (F4.0), HOUR (F4.1)')
79:    70 FORMAT(' INPUT STOP YEAR (F5.0), DAY (F4.0), HOUR (F4.1)')
80:    80 FORMAT(' INPUT N.LAT.,E.LONG., TIME INTERVAL (HRS) (F6.2)')
81:    90 FORMAT(' INPUT ELEV (M) (F7.2)')
82:    110 FORMAT(' INPUT TILTMETER AZINUT(S) + CLOCKWISE FROM N (F7.2)')
83:    120 FORMAT(' INPUT STRAINMETER AZINUTH(S) + CLOCKWISE FROM N (F7.2)')
84:    140 FORMAT(F5.0,F4.0,F4.1)
85:    150 FORMAT(F7.2)
86:    170 FORMAT(3F6.2)
87:C
88:    IYR = IYR-1900.
89:    JYR = JYR-1900.
90:    THETA=90.-THETA
91:    HT=100.0+ELEV
92:C
93:C    FIND TIME IN HOURS FROM 0 HR 1 JAN 1900
94:C
95:    TS=ZHR+24.D 80*(IDAY-1)+8760.D 80*IYR+24.D 80*INT((IYR-1)/4)
96:    TE=YHR+24.D 80*(JDAY-1)+8760.D 80*JYR+24.D 80*INT((JYR-1)/4)
97:C
98:C    IF HNCR=-1, COMPUTE RIGID-EARTH POTENTIAL
99:C
100:    IF(HNCR.LT.0) IFLAG2=1
101:    HNCR=IABS(HNCR)
102:C
103:    NTOTL=HNCR+NNTL+NMST
104:    DO 180 I=1,NNTL
105:    AZT(2,I)=COS(AZT(1,I)/57.2958)
106:    190 AZT(3,I)=SIN(AZT(1,I)/57.2958)
107:    IF(NMST.EQ.0) GO TO 200
108:    DO 190 I=1,NMST
109:    AZS(2,I)=COS(AZS(1,I)/57.2958)
110:    190 AZS(3,I)=SIN(AZS(1,I)/57.2958)
111:C
112:C    EXECUTION BEGINS
113:C
114:    200 I=1
115:    TT=TS
116:    THETA=THETA+0.01745329252

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117;    BIF=0.00337*SIN(2.*THETA)
118;    THETA=THETA+BIF
119;    LAMBDA=LAMBDA+0.01745329252
120;    CTHETA=COS(THETA)
121;    STHETA=SIN(THETA)
122;    RE=A*(1.-0.003353*CTHETA**2)
123;    RE=RE+HT
124;    G=978.063*(1.+0.005244*CTHETA**2)
125;    HR=BHOB(TS,24.D 00)
126;    ETHUT=41.184+IYR-78.
127;C
128;C      COMPUTE MEAN OBLIQUITY OF THE ECLIPTIC
129;C
130;    U=.409319747D 00-.0002271107D 00*((TS+TE+24.D 00)/17532.D 02)
131;    COSU=DCOS(U)
132;    SINU=DSIN(U)
133; 210 IF(TT.GT.TE) GO TO 420
134;C
135;C      T IS EPHEMERIS TIME IN JULIAN CENTURIES FROM 12 HR 0 JAN 1900
136;C
137;    T=(TT+12.D 00+(ETHUT/3600.D 00))/876600.D 00
138;    T2=T*T
139;C
140;C      COMPUTE SOLAR CONSTANTS FOR GIVEN T
141;C
142;    HS=4.081627982402D 00+628.3319508731D 00*T+0.523598775578D-05*T2
143;    HS=BHOB(HS,PI20)
144;    PS=4.908229466993D 00+0.03000526416690D 00*T+0.790246300201D-05*T2
145;    ES=0.0167504D 00-0.00004180D 00*T-0.000000126D 00*T2
146;    PSIG=0.2617993877971D 00*(HR-12.D 00)+HS
147;    CHNP=BCOS(HS-PS)
148;    SHNP=BSIN(HS-PS)
149;    LS=HS+SHNP*ES*(2.D 00+2.D 00 00*ES+CHNP)
150;    SLS=BSIN(LS)
151;    CZ=SINU*SLS
152;    SZ=BSQRT(1.-CZ**2)
153;    PSIS=BATN2(COSU*SLS,BCOS(LS))
154;    RBARR=1.+ES*(CHNP+ES*(CHNP-SHNP)*(CHNP+SHNP))
155;    LL=PSIS-PSIG
156;    SLL=SIN(LAMBDA-LL)
157;    CLL=COS(LAMBDA-LL)
158;C
159;C      CALCULATE COSINE OF ZENITH ANGLE
160;C
161;    RU=CTHETA*CZ+STHETA*SZ*CLL
162;C
163;C      CALCULATE MEAN EQUATORIAL PARALLAX
164;C
165;    XI=4.2635E-05*RBARR*RE/A
166;    CC=2.1237E 14*XI*(A/RE)
167;C
168;C      CALCULATE SOLAR POTENTIAL CONSTANTS, LEGENDRE POLY'S, ETC...
169;C
170;    KR(2)=CC*XI*XI
171;    P(1)=0.5*(3.*RU*RU-1.)
172;    PP(1)=3.*RU
173;    NUT=-STHETA*CZ+CTHETA*SZ*CLL
174;    NUTT=-RU

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175;  NUL=-STHETA*SZ*SLL
176;  NULL=-STHETA*SZ*CLL
177;  NUTL=-CTHETA*SZ*SLL
178;C
179;C      CALCULATE SOLAR GRAVTIME
180;C
181;  IF(NHGR.EQ.0) GO TO 220
182;  GTSUM=2.*DEL(1)*KR(2)*P(1)*G1/RE
183;  GNTH=-BIN(1)*KR(2)*PP(1)*MUT*G1/RE
184;  GTSUM=GTSUM+GNTH*DIF
185;  IF(IFLAG2.EQ.1)GTSUM=1.E-08*KR(2)*P(1)
186;C
187;C      CALCULATE SOLAR TILTTIME
188;C
189;  220 IF(NHTL.EQ.0) GO TO 240
190;  DO 230 KK=1,NHTL
191;  TTSUN(KK)=- (BIN(1)*KR(2)*PP(1)/RE)*(MUT*AZT(2,KK)-MUL*AZT(3,KK)/STHETA)
192;  GTSU=DEL(1)+2.*KR(2)*P(1)/RE
193;  230 TTSUN(KK)=TTSUN(KK)*(G1/G)-GTSU*DIF*AZT(2,KK)
194;C
195;C      CALCULATE SOLAR STRAINTIME
196;C
197;  240 IF(NNST.EQ.0) GO TO 260
198;  ELL = KR(2)*L(1)*(PPP(1)*MUT*CTHETA/STHETA+(PPP(1)*MUL*MUL
199;  A+PP(1)*NULL)/(STHETA*STHETA))+H(1)*P(1))
200;  ETT=KR(2)*L(1)*(PPP(1)*MUT*MUT+PP(1)*MUT)+H(1)*P(1))
201;  ETL=2.*(KR(2)*L(1)*(PPP(1)*MUL*MUT+PP(1)*MUTL-
202;  A*CTHETA*PP(1)*MUL/STHETA))/STHETA
203;  DO 250 KK=1,NNST
204;  STNSUN(KK)=(ELL*AZS(3,KK)+AZS(3,KK)-ETL*AZS(3,KK)+AZS(2,KK)
205;  A+ETT*AZS(2,KK)+AZS(2,KK))/RE
206;  250 STNSUN(KK)=STNSUN(KK)*(G1/G2)
207;  260 CONTINUE
208;C
209;C      COMPUTE LUNAR CONSTANTS FOR GIVEN T
210;C
211;  270 CONTINUE
212;  MM=4.71996660 00+8399.78914493 00*T-0.00001980 00*T2
213;  PM=5.835151548 00+71.818041288390 00*T-.1802050-03*T2
214;  NM=4.5236815150 00-33.757146240 00*T+.36264863350-04*T2
215;C
216;C      BL,BLS,BF,GB ARE THE FUNDAMENTAL ARGUMENTS OF BROWNS THEORY
217;C
218;  BL=MM-PM
219;  BLS=MS-PS
220;  BF=MM-NM
221;  BD=MM-MS
222;C
223;C      LUNAR LAT, LONG AND PARALLAX FROM BROWN, LATTER TWO FROM
224;C      IMPROVED LUNAR EPHEMERIS, LAT FROM RAS PAPER OF 1988
225;C
226;  R1=0.18976*DSIN(BL)
227;  R2=-0.02224*DSIN(BL-2.*00)
228;  R3=0.01149*DSIN(2.*00)
229;  R4=0.00373*DSIN(2.*00)
230;  R5=-0.00324*DSIN(BLS)
231;  R6=-0.002*DSIN(2.*BF)
232;  R7=-0.00183*DSIN(2.*00L-2.*00)

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233;  R8=-0.001*BSIN(BL+BL8-2.*03)
234;  R9=0.00093*BSIN(BL+2.*03)
235;  R10=-0.0008*BSIN(BL8-2.*03)
236;  R11=0.00072*BSIN(BL-BL8)
237;  R12=-0.00061*BSIN(B3)
238;  R13=-0.00053*BSIN(BL+BL8)
239;C
240;  TLONGH=HM+R1+R2+R3+R4+R5+R6+R7+R8+R9+R10+R11+R12+R13
241;C
242;  R1=0.00958*BSIN(BF)
243;  R2=0.00498*BSIN(BL+BF)
244;  R3=-0.00485*BSIN(BF-BL)
245;  R4=-0.00383*BSIN(BF-2.*03)
246;  R5=0.00097*BSIN(2.*03+BF-BL)
247;  R6=-0.00081*BSIN(BL+BF-2.*03)
248;  R7=0.00057*BSIN(BF+2.*03)
249;C
250;  TLATH=R1+R2+R3+R4+R5+R6+R7
251;C
252;  R1=186.54*COS(BL)
253;  R2=34.31*COS(BL-2.*03)
254;  R3=28.23*COS(2.*03)
255;  R4=18.17*COS(2.*03)
256;  R5=3.09*COS(BL+2.*03)
257;  R6=1.92*COS(BL8-2.*03)
258;  R7=1.44*COS(BL+BL8-2.*03)
259;  R8=1.15*COS(BL-BL8)
260;  R9=-0.98*COS(B3)
261;  R10=-0.95*COS(BL+BL8)
262;  R11=-0.71*COS(BL-2.*03)
263;  R12=0.62*COS(3.*03)
264;  R13=0.6*COS(BL-4.*03)
265;C
266;  PLX=3422.45+R1+R2+R3+R4+R5+R6+R7+R8+R9+R10+R11+R12+R13
267;C
268;  RBARR=PLX/3422.45
269;  SINHLA=BSIN(TLATH)
270;  COSHLA=BCOS(TLATH)
271;  SINHLH=BSIN(TLONGH)
272;  COSHLH=BCOS(TLONGH)
273;C
274;C  CONVERT FROM CELESTIAL LAT AND LONG ACCORDING TO EXPLAN SUPPL
275;C  OF NAUTICAL ALMANAC AND LUNAR EPHEMERIS, P. 26
276;C
277;  CZ=COSHLA*SINHLH*SINU+SINHLA*COSU
278;  SZ=DSQRT(1.-CZ**2)
279;  AT1=COSHLA*SINHLH*COSU-SINHLA*SINU
280;  AT2=COSHLA*COSHLH
281;  RAM=BATH2(AT1,AT2)
282;  LL=RAM-PSIG
283;  SLL=SIN(LAMBDA-LL)
284;  CLL=COS(LAMBDA-LL)
285;C
286;C  CALCULATE COSINE OF ZENITH ANGLE
287;C
288;  NU=CTHETA+CZ+STHETA+SZ*CLL
289;C
290;C  CALCULATE MEAN EQUATORIAL PARALLAX

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GUERTD                      6/ 2/80  12.19.37      PAGE  6

291;C
292;      XI=0.0165925*RBARR*(RE/A)
293;      CC=7.8452E 06*XI*(A/RE)
294;C
295;C      CALCULATE LUNAR POTENTIAL CONSTANTS
296;C
297;      KR(2)=CC*XI*XI
298;      KR(3)=KR(2)*XI
299;      KR(4)=KR(3)*XI
300;C
301;C      CALCULATE LUNAR LEGENDRE POLYNOMIALS
302;C
303;      P(1)=0.5*(3.*MU*MU-1.)
304;      P(2)=0.5*MU*(5.*MU*MU-3.)
305;      P(3)=0.25*(7.*MU*P(2)-3.*P(1))
306;C
307;C      CALCULATE DERIVATIVES OF LUNAR LEGENDRE POLYNOMIALS
308;C
309;      PP(1)=3.*MU
310;      PP(2)=1.5*(5.*MU*MU-1.)
311;      PPP(2)=15.*MU
312;C
313;C      CALCULATE LUNAR DERIVATIVES OF MU
314;C
315;      MUT=-STHETA*CZ+CTHETA*SZ*CLL
316;      MUTT=-MU
317;      MUL=-STHETA*SZ*SLL
318;      MULL=-STHETA*SZ*CLL
319;      MUTL=-CTHETA*SZ*SLL
320;C
321;C      CALCULATE LUNAR GRAVITIDE
322;C
323;      IF(NHGR.EQ.0) GO TO 290
324;      GTHUN = 0.
325;      DO 280 J=2,4
326;      N=J-1
327;      290 GTHUN=GTHUN+DEL(N)*J*KR(J)*P(N)*G1/RE
328;      GNTN=-BIN(1)*KR(2)*PP(1)*G1*MUT/RE
329;      GTHUN=GTHUN+GNTN*BIF
330;      IF(IFLAG2.EQ.1) GTHUN=1.E-08*(KR(2)*P(1)+KR(3)*P(2))
331;C
332;C      CALCULATE LUNAR TILTTIDE
333;C
334;      290 IF(NMTL.EQ.0) GO TO 320
335;      DO 310 KK=1,NMTL
336;      TTHUN(KK)=0.
337;      DO 300 J=2,3
338;      N=J-1
339;      300 TTHUN(KK)=TTHUN(KK)-(BIN(N)*KR(J)*PP(N)/RE)*(MUT*AZT(2, KK)-
340;      ANUL*AZT(3, KK)/STHETA)
341;      GTHU=DEL(1)*2.*KR(2)*P(1)/RE
342;      310 TTHUN(KK)=TTHUN(KK)+G1/G-GTHU*BIF*AZT(2, KK)
343;C
344;C      CALCULATE LUNAR STRAINTIDE
345;C
346;      320 IF(NNST.EQ.0) GO TO 350
347;      ELL=0.
348;      ETL=0.

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GUERTD                                6/ 2/80  12.19.37    PAGE  7
349;   ETT=0.
350;   DO 330 J=2,3
351;   M=J-1
352;   ELL=ELL+KR(J)*(L(N)*(PP(N)*MUT*CTHETA/STHETA+(PPP(N)*MUL
353;   &=MUL+PP(N)*MULL)/(STHETA*STHETA))+H(N)*P(N))
354;   ETT=ETT+KR(J)*(L(N)*(PPP(N)*MUT*MUT+PP(N)*MUTT)+H(N)*P(N))
355;   ETL=ETL+2.*(KR(J)*L(N)*(PPP(N)*MUL*MUT+PP(N)*MUTL-CTHETA*
356;   &PP(N)*MUL/STHETA))/STHETA
357; 330 CONTINUE
358;   DO 340 KK=1,MNST
359;   STNMUN(KK)=(ELL*AZS(3,KK)**2-ETL*AZS(3,KK)*AZS(2,KK)
360;   &+ETT*AZS(2,KK)**2)/RE
361; 340 STNMUN(KK)=STNMUN(KK)*(G1/G2)
362; 350 CONTINUE
363;C
364;C      CALCULATE NORMALIZED TOTAL TIDES
365;C
366;C      IF(NHGR.EQ.0) GO TO 368
367;C
368;C      GRAVITY (MICROGALS)
369;C
370;C      SEROUT(1)=(GTSUN+GTNUM)*1.E 06
371; 360 IF(NHTL.EQ.0) GO TO 380
372;C
373;C      TILT (HANOGRADIANS)
374;C
375;C      DO 370 KK=1,NHTL
376;C      IDB=NHGR+KK
377; 370 SEROUT(IDB)=(TTSUN(KK)+TTNUM(KK))*1.E 09
378; 380 IF(NHST.EQ.0) GO TO 400
379;C
380;C      STRAIN (PARTS IN E 9)
381;C
382;C      DO 390 KK=1,MNST
383;C      IDB=NHGR+NHTL+KK
384; 390 SEROUT(IDB)=(STNSUN(KK)+STNMUN(KK))*1.E 09
385;C
386;C      OUTPUT COMPUTED TIDES
387;C
388; 400 I=I+1
389;   WRITE(1,410)(SEROUT(J),J=1,NTOTL)
390; 410 FORMAT(6(F6.2,1X))
391;C
392;C      TT = TT+D
393;C      HR=HR+D
394;C      GO TO 210
395;C
396; 420 CALL CLOSE(1)
397;   END

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      GUTID6                      6/ 2/80  12.19.37      PAGE 1
1/C*****
2/C      PROGRAM GUTID6 EXTRACTS THE AMPLITUDE AND
3/C      PHASE INFORMATION FROM A TIME SERIES Y(J)
4/C      WITH KNOWN FREQUENCIES. THE PROGRAM
5/C      CALCULATES THE FOURIER TRANSFORM OF Y(J) FOR
6/C      THESE FREQUENCIES AND CORRECTS FOR THE
7/C      FINITE DATA LENGTH.
8/C
9/C      XL IS THE LENGTH OF Y(J) IN HRS.
10/C     XL = DELT(N - 1), WHERE N IS THE NUMBER OF POINTS
11/C
12/C     SUBROUTINES USED.
13/C             GUSIMP
14/C             GUINV
15/C
16/C             GUDNI AXELSSON MAY 1979.
17/C*****
18/C     COMPILER DOUBLE PRECISION
19/C     DIMENSION Y(500),YS(500),YC(500),HPL(10,10),HMI(10,10),Z1(10),Z2(10),
20/C     IM(10),R1(10),R2(10),A1(10,10),A2(10,10),F(500),AMP(10),PH(10)
21/C     COMMON A(10,10),AINV(10,10)
22/C     PI = 3.141592654
23/C
24/C     INPUT
25/C
26/C     ACCEPT* DELT = *.DELT
27/C     CALL INFILE(1)
28/C     I = 1
29/C     10 READ(1,101,END=9) Y(I)
30/C     I = I + 1
31/C     GO TO 10
32/C     9 N = I - 1
33/C     WRITE(10,107) N
34/C     PAUSE
35/C     XN = N
36/C     XL = (XN - 1.)*DELT
37/C
38/C     8 KNOWN FREQUENCIES.
39/C
40/C     NFR = 8
41/C     LONG PERIOD.....N(N),N(F)
42/C     U(1) = 0.00151215
43/C     U(2) = 0.00305009
44/C     DIURNAL.....01,K1
45/C     U(3) = 0.03073066
46/C     U(4) = 0.04170075
47/C     SEMIDIURNAL.....N2,N2,S2
48/C     U(5) = 0.07899925
49/C     U(6) = 0.00051148
50/C     U(7) = 0.00333333
51/C     TERDIURNAL.....N3
52/C     U(8) = 0.12076710
53/C
54/C     CONSTRUCTING H+=(A++A-)INV AND H-=(A+-A-)INV
55/C
56/C     DO 1 J=1,NFR
57/C     DO 1 I=1,NFR
58/C     IF(I-J) 57,300,57

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GUTIBES                      6/ 2/80 12.19.37      PAGE 2
59: 57 A2(I,J) = (SIN((U(I)-U(J))*XL*PI))/(U(I)-U(J))/(XL*PI)
60: 63 A1(I,J) = (SIN((U(I)+U(J))*XL*PI))/(U(I)+U(J))/(XL*PI)
61: 1 A(I,J) = A1(I,J) + A2(I,J)
62:   CALL GUINV(NFR)
63:   DO 2 J=1,NFR
64:   DO 2 I=1,NFR
65:   2 HPL(I,J) = AINV(I,J)
66:C
67:   DO 3 J=1,NFR
68:   DO 3 I=1,NFR
69:   3 A(I,J) = A1(I,J) - A2(I,J)
70:   CALL GUINV(NFR)
71:   DO 4 J=1,NFR
72:   DO 4 I=1,NFR
73:   4 HMI(I,J) = AINV(I,J)
74:C
75:C      FIND SPECTRAL ESTIMATES R(J) = R1(J) + IR2(J)
76:C      J = 1,...,NFR
77:   DO 6 J=1,NFR
78:   DO 5 I=1,N
79:   XI = I - 1
80:   T = (XI - XL/2.)*DELTA
81:   YC(I) = Y(I)*COS(U(J)*T*2.*PI)
82:   5 YS(I) = Y(I)*SIN(U(J)*T*2.*PI)
83:   CALL GUSIMP(XL,N,YC,H1)
84:   CALL GUSIMP(XL,N,YS,H2)
85:   R1(J) = 2.*H1/XL
86:   6 R2(J) = 2.*H2/XL
87:C
88:C      CORRECT FOR FINITE LENGTH
89:C
90:   DO 8 J=1,NFR
91:   Z1(J) = 0.
92:   Z2(J) = 0.
93:   DO 7 I=1,NFR
94:   Z1(J) = Z1(J) + HPL(J,I)*R1(I)
95:   7 Z2(J) = Z2(J) + HMI(J,I)*R2(I)
96:   AMP(J) = SQRT(Z1(J)**2+Z2(J)**2)
97:   PH(J) = ATAN2(Z2(J),Z1(J))
98:   WRITE(12,103)
99:   WRITE(12,104)
100:  WRITE(12,102) U(J),R1(J),R2(J)
101:  WRITE(12,105)
102:  WRITE(12,106)
103:  8 WRITE(12,102) U(J),AMP(J),PH(J)
104:C
105:C      ESTIMATE ERROR IN SPECTRAL ESTIMATES
106:C
107:   WRITE(12,205)
108:   DO 201 J=1,N
109:   XJ = J - 1
110:   T = (XJ - XL/2.)*DELTA
111:   SUN = 0.
112:   DO 202 I=1,NFR
113:   ARG = 2.*PI*T*U(I) + PH(I)
114:   202 SUN = SUN + AMP(I)*COS(ARG)
115:   RES = Y(J) - SUN
116:   201 F(J) = RES*RES

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      GUTID66                      6/ 2/80 12.19.37      PAGE 3
117;      CALL GUSIMP(XL,N,F,HH)
118;      DO 203 I=1,NFR
119;      B1 = Z1(I)**2*HPL(I,I) - Z2(I)**2*HNI(I,I)
120;      AMER = SQRT(HH*4.*PI*B1)/(XL*AMP(I))
121;      PHER = AMER/AMP(I)
122;      203 WRITE(12,204) AMER, PHER
123;      204 FORMAT('0',2E14.6)
124;      205 FORMAT('1',' ERRORS IN THE ABOVE SPECTRAL ESTIMATES, ',/,
125;      1'  AMPLITUDE.      PHASE. ')
126;C
127;      101 FORMAT(F6.2)
128;      102 FORMAT(' ',3E14.6)
129;      103 FORMAT('0',' UNCORRECTED SPECTRAL ESTIMATES, ')
130;      104 FORMAT(' ',, FREQUENCY.      RER.      IMR.  ')
131;      105 FORMAT(' ',, CORRECTED SPECTRAL ESTIMATES, ')
132;      106 FORMAT(' ',, FREQUENCY.      AMPLITUDE:      PHASE.  ')
133;      107 FORMAT(' ',, N =,(N MUST BE ODD) ',I4)
134;      CALL CLOSE(1)
135;      GO TO 301
136;      300 A2(I,J) = 1.
137;      GO TO 63
138;      301 END

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      GUOCTIL                      6/ 2/80 12.19.37      PAGE 1
1/C*****
2/C      PROGRAM GUOCTIL COMPUTES OCEAN LOADING TILT USING
3/C      GREEN'S FUNCTIONS CALCULATED BY FARRELL (1972).
4/C      PROGRAM CONVOLVES OCEAN TIDAL DATA WITH GREEN'S
5/C      FUNCTION. PROGRAM CALCULATES AMPLITUDE AND PHASE AND
6/C      GENERATES A TIME SERIES.
7/C
8/C      NUMBER OF GRIDPOINTS IS N X N
9/C      LIMITS OF INTEGRATION THMIN AND THMAX (DEG)
10/C     FREQUENCY IS W (RAD/HR), IF FOR N2 W=1.
11/C
12/C     SUBROUTINE USED,
13/C         GUTILTE
14/C
15/C         GUDNI AXELSSON MARCH 1980.
16/C*****
17/C     COMPILER DOUBLE PRECISION
18/C     DIMENSION Z(40,30),P(40,30)
19/C
20/C         INPUT
21/C
22/C     ACCEPT* N = ",N
23/C     ACCEPT* M = ",M
24/C     ACCEPT* THMAX = ",THMA
25/C     ACCEPT* THMIN = ",THMI
26/C     PI = 3.1415926
27/C     ACCEPT* W = ",W
28/C     IF(W.EQ.1) W = 0.503068
29/C
30/C         NUMERICAL CONSTANTS
31/C
32/C     A = 4.1466188E 16
33/C     XME = 5.976E 24
34/C     R = 6.371E 06
35/C     K = 1
36/C
37/C         INPUT DIGITIZED COTIDAL MAP
38/C
39/C     TYPE* Z(J,I) FIRST *
40/C     CALL INFILE(1)
41/C     DO 10 J=1,M
42/C 10 READ(1,101)(Z(J,I),I=1,N)
43/C     CALL CLOSE(1)
44/C     TYPE* THEN P(J,I) *
45/C     CALL INFILE(2)
46/C     DO 20 J=1,M
47/C 20 READ(2,101)(P(J,I),I=1,N)
48/C     CALL CLOSE(2)
49/C
50/C         CONVOLUTION
51/C
52/C     XLN = ALOG(THMI)
53/C     BLNT = (ALOG(THMA)-XLN)/FLOAT(N)
54/C     BP = 2.*PI/FLOAT(N)
55/C     TEW1 = 0.
56/C     TEW2 = 0.
57/C     TNS1 = 0.
58/C     TNS2 = 0.

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GUOCTIL

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PAGE 2

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59:      DO 30 I=1,N
60:      XLNTH = XLM + (FLOAT(I)-.5)*DLNT
61:      THE = EXP(XLNTH)
62:      CALL GUTILTE(THE,TE,K)
63:      TE = TE*1.E-12/R/R
64:      THE = THE*PI/180.
65:      YY = A*SIN(THE)*DLNT*BP*(TE/THE+THE*COS(THE/2.)/4./XNE/(SIN(THE/2.))**2))
66:      DO 30 J=1,M
67:      PSI = (FLOAT(J)-.5)*BP
68:      H1 = Z(J,I)*COS(0.923599*P(J,I))
69:      H2 = Z(J,I)*SIN(0.923599*P(J,I))
70:      XX1 = YY*H1
71:      XX2 = YY*H2
72:      TEW1 = TEW1 + XX1*SIN(PSI)
73:      TEW2 = TEW2 + XX2*SIN(PSI)
74:      TNS2 = TNS2 + XX2*COS(PSI)
75:      TNS1 = TNS1 + XX1*COS(PSI)
76: 30 CONTINUE
77:C
78:C      COMPUTING AMPLITUDE AND PHASE
79:C
80:      PHN = DATAN2((-1.)*TNS2,TNS1)
81:      AN = SQRT(TNS1**2+TNS2**2)
82:      PHE = DATAN2((-1.)*TEW2,TEW1)
83:      AE = SQRT(TEW1**2+TEW2**2)
84:      WRITE(10,102) AN,PHN,AE,PHE
85: 101 FORMAT(2(10F5.2,/,),10F5.2)
86: 102 FORMAT(' SN AMPLITUDE = ',E14.6,' SN PHASE = ',E14.6,/,
87: 0' VE AMPLITUDE = ',E14.6,' VE PHASE = ',E14.6)
88:C
89:C      COMPUTING TIME SERIES
90:C
91:      TYPE = GENERATE OCEAN LOADING TIME SERIES ?
92:      ACCEPT = YES = 1 ,L
93:      IF(L.NE.1) GO TO 99
94:      TYPE = NS COMP.
95:      CALL OUTFILE(1)
96:      TYPE = EV COMP.
97:      CALL OUTFILE(2)
98:      ACCEPT = T0 = ,T0
99:      ACCEPT = TH = ,TH
100:      ACCEPT = BT = ,BT
101:      T = T0
102: 40 X = AN*COS(U*T+PHN)
103:      X = X*1.E 09
104:      WRITE(1,103) X
105:      T = T + BT
106:      IF(T.GT.TH) GO TO 50
107:      GO TO 40
108: 50 T = T0
109: 60 X = AE*COS(U*T+PHE)
110:      X = X*1.E 09
111:      WRITE(2,103) X
112:      T = T + BT
113:      IF(T.GT.TH) GO TO 70
114:      GO TO 60
115: 70 CALL RESET
116: 103 FORMAT(' ',F7.3)
117: 99 END

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      GUTILTE                      6/ 2/80  12:19:37      PAGE 1
1/C*****
2/C      SUBROUTINE GUTILTE COMPUTES ELASTIC PART OF OCEAN
3/C      LOADING TILT GREEN'S FUNCTION FROM DATA TABULATED BY
4/C      FARRELL (1972) USING LINEAR INTERPOLATION. DATA IS
5/C      FOR OCEANIC MANTLE.
6/C
7/C      THE IS THE ANGULAR DISTANCE FROM THE LOAD
8/C      TE IS THE VALUE OF THE GREEN'S FUNCTION
9/C
10/C                      GUDNI AXELSSON MARCH 1980.
11/C*****
12/C      COMPILER DOUBLE PRECISION
13/C      SUBROUTINE GUTILTE(THE,TE,K)
14/C      COMMON/XY/X(16),Y(16)
15/C      DATA X(1)/.5/X(2)/.6/X(3)/.8/X(4)/1./X(5)/1.2/
16/C      1X(6)/1.6/X(7)/2./X(8)/2.5/X(9)/3./X(10)/4./X(11)/5./
17/C      2X(12)/6./X(13)/7./X(14)/8./X(15)/9./X(16)/10./
18/C      DATA Y(1)/16.59/Y(2)/16.71/Y(3)/16.93/Y(4)/17./Y(5)/17./
19/C      1Y(6)/16.49/Y(7)/15.69/Y(8)/14.50/Y(9)/13.24/Y(10)/10.81/Y(11)/8.71/
20/C      2Y(12)/7.10/Y(13)/5.94/Y(14)/5.14/Y(15)/4.61/Y(16)/4.24/
21/C      DO 10 I=K,15
22/C      IF(THE.GT.X(I).AND.THE.LE.X(I+1)) GO TO 20
23/C      10 CONTINUE
24/C      GO TO 30
25/C      20 TE = Y(I) + (Y(I+1)-Y(I))/(X(I+1)-X(I))*(THE-X(I))
26/C      K = I
27/C      30 RETURN
28/C      END

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      GUOCSTR                      6/ 2/80 12.19.37      PAGE 1
1)C*****
2)C      PROGRAM GUOCSTR COMPUTES OCEAN LOADING STRAIN AS WELL
3)C      AS VERTICAL DISPLACEMENT USING GREEN'S FUNCTIONS
4)C      CALCULATED BY FARRELL (1972). PROGRAM CONVOLVES
5)C      OCEAN TIDAL DATA WITH GREEN'S FUNCTIONS. PROGRAM
6)C      COMPUTES AMPLITUDE AND PHASE FOR STRAINMETER AZINUTH
7)C      BETA AND GENERATES A STRAIN TIME SERIES.
8)C      NUMBER OF GRIDPOINTS IS N X N
9)C
10)C      LIMITS OF INTEGRATION THMIN AND THMAX (DEG)
11)C      FREQUENCY IS W (RAD/HR), IF FOR M2 W=1.
12)C
13)C      SOUBROUTINE USED.
14)C          GSTR
15)C
16)C          GUBNI AXELSSON APRIL 1980.
17)C*****
18)C      COMPILER DOUBLE PRECISION
19)C      DIMENSION Z(40,30),P(40,30),E(2,2),T1(2,2),T2(2,2)
20)C
21)C          INPUT
22)C
23)C      ACCEPT " N = ",N
24)C      ACCEPT " M = ",M
25)C      ACCEPT " THMAX = ",THMA
26)C      ACCEPT " THMIN = ",THMI
27)C      PI = 3.1415926
28)C      ACCEPT " W = ",W
29)C      IF(W.EQ.1) W = 8.585868
30)C
31)C          CONSTANTS
32)C
33)C      A = 4.1466188E 16
34)C      R = 6.371E 06
35)C      K = 1
36)C
37)C          INPUT DIGITIZED COTIDAL MAP
38)C
39)C      TYPE " INPUT Z(J,I) "
40)C      CALL INFILE(1)
41)C      TYPE " INPUT P(J,I) "
42)C      CALL INFILE(2)
43)C      DO 10 J=1,M
44)C          READ(1,101)(Z(J,I),I=1,N)
45)C      10 READ(2,101)(P(J,I),I=1,N)
46)C      CALL RESET
47)C
48)C          CONVOLUTION.
49)C      T1(I,J) AND T2(I,J) ARE THE REAL AND IMAGINARY PARTS OF THE STRAIN
50)C      TENSOR, U1 AND U2 THE REAL AND IMAGINARY PARTS OF THE VERTICAL
51)C      DISPLACEMENT
52)C
53)C      XLN = ALOG(THMI)
54)C      BLNT = (ALOG(THMA)-XLN)/FLOAT(N)
55)C      BP = 2.*PI/FLOAT(N)
56)C      U1 = 0.
57)C      U2 = 0.
58)C      DO 15 J=1,2

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59:      DO 15 I=1,2
60:          T1(I,J) = 0.
61:      15 T2(I,J) = 0.
62:      DO 30 I=1,N
63:          XLNTH = XLN + (FLOAT(I)-.5)*DLNT
64:          THE = EXP(XLNTH)
65:          CALL GISTR(THE,S1,U,V,K)
66:          THE = THE*PI/180.
67:          S1 = S1*1.E-12/R/R
68:          S2 = (U+V/TAN(THE))*THE*1.E-12/R/R
69:          YY = A*SIN(THE)*DLNT*BP/THE
70:          Y = 6.519E-03*SIN(THE)*DLNT*BP*U
71:      DO 30 J=1,M
72:          PSI = (FLOAT(J)-.5)*BP
73:          E(1,1) = S1*SIN(PSI)**2 + S2*COS(PSI)**2
74:          E(1,2) = (S2-S1)*SIN(PSI)*COS(PSI)
75:          E(2,1) = E(1,2)
76:          E(2,2) = S1*COS(PSI)**2 + S2*SIN(PSI)**2
77:          U1 = U1 + Y*Z(J,I)*COS(0.523599*P(J,I))
78:          U2 = U2 + Y*Z(J,I)*SIN(0.523599*P(J,I))
79:      DO 25 L=1,2
80:      DO 25 LL=1,2
81:          T1(L,LL) = T1(L,LL) + YY*Z(J,I)*COS(0.523599*P(J,I))*E(L,LL)
82:      25 T2(L,LL) = T2(L,LL) + YY*Z(J,I)*SIN(0.523599*P(J,I))*E(L,LL)
83:      30 CONTINUE
84:      TYPE ' VERTICAL DISPLACEMENT: '
85:      WRITE(10,200) U1,U2
86:      TYPE '          STRAIN TENSOR: '
87:      TYPE '          IN PHASE IN QUADRATURE '
88:      WRITE(10,200) T1(1,1),T2(1,1)
89:      WRITE(10,200) T1(1,2),T2(1,2)
90:      WRITE(10,200) T1(2,1),T2(2,1)
91:      WRITE(10,200) T1(2,2),T2(2,2)
92:      200 FORMAT(' ',2E14.6)
93:/C
94:/C      COMPUTING ANPLITUDE AND PHASE FOR AZINUTH BETA
95:/C
96:      35 TYPE ' INPUT STRAINMETER AZINUTH, POSITIVE CLOCKWISE FROM NORTH '
97:      ACCEPT ' BETA (DEG) = ',00
98:      B = 00*PI/180.
99:      X1 = T1(1,1)*SIN(B)**2 + 2.*T1(1,2)*SIN(B)*COS(B) + T1(2,2)*COS(B)**2
100:      X2 = T2(1,1)*SIN(B)**2 + 2.*T2(1,2)*SIN(B)*COS(B) + T2(2,2)*COS(B)**2
101:      PH = 90ATAN2((-1.)*X2,X1)
102:      A = SQRT(X1*X1+X2*X2)
103:      WRITE(10,102) 00,PH,A
104:      101 FORMAT(2(10F5.2,/,),10F5.2)
105:      102 FORMAT('  AZINUTH = ',F10.4,'  PHASE = ',E14.6,'  ANPLITUDE = ',E14.6)
106:/C
107:/C      COMPUTING STRAIN TIME SERIES
108:/C
109:      TYPE ' GENETRATE OCEAN LOADING TIME SERIES ? '
110:      ACCEPT ' YES = 1 ',L
111:      IF(L.NE.1) GO TO 99
112:      CALL OUTFILE(1)
113:      ACCEPT ' T0 = ',T0
114:      ACCEPT ' TH = ',TH
115:      ACCEPT ' BT = ',BT
116:      T = T0

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117; 40 X = A*COS(W*T+PH)
118; X = X*1.E 09
119; WRITE(1,103) X
120; T = T + DT
121; IF(T.GT.TN) GO TO 58
122; GO TO 40
123; 50 CALL CLOSE(1)
124; 103 FORMAT(' ',F7.3)
125; 99 TYPE * DIFFERENT AZINUTH ? *
126; ACCEPT * YES = 1 ",L
127; IF(L.NE.1) GO TO 97
128; GO TO 35
129; 97 END
130;
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      GISTR                      6/ 2/88 12.19.37      PAGE 1
1/C *****
2/C      SUBROUTINE GISTR COMPUTES OCEAN LOADING STRAIN AS
3/C      WELL AS DISPLACEMENT GREEN'S FUNCTIONS FROM DATA
4/C      TABULATED BY FARRELL (1972) USING LINEAR INTERPOLATION.
5/C      DATA IS FOR OCEANIC MANTLE.
6/C
7/C      THE IS THE ANGULAR DISTANCE FROM THE LOAD
8/C      S1 IS THE VALUE OF THE STRAIN GREEN'S FUNCTION
9/C      ( LINEAR STRAIN IN THETA DIRECTION )
10/C      U AND V ARE THE DISPLACEMENT GREEN'S FUNCTIONS.
11/C
12/C
13/C *****
14/C      COMPILER DOUBLE PRECISION
15/C      SUBROUTINE GISTR(THE,S1,U,V,K)
16/C      COMMON/XYZW/X(16),Y(16),Z(16),W(16)
17/C      DATA X(1)/.5/X(2)/.6/X(3)/.8/X(4)/1./X(5)/1.2/
18/C      1X(6)/1.6/X(7)/2./X(8)/2.5/X(9)/3./X(10)/4./X(11)/5./
19/C      2X(12)/6./X(13)/7./X(14)/8./X(15)/9./X(16)/10./
20/C      DATA Y(1)/-15.27/Y(2)/-14.98/Y(3)/-14.32/Y(4)/-13.60/Y(5)/-12.87/
21/C      1Y(6)/-11.48/Y(7)/-10.24/Y(8)/-8.933/Y(9)/-7.853/Y(10)/-6.274/Y(11)/-5.263
22/C      2/Y(12)/-4.623/Y(13)/-4.212/Y(14)/-3.937/Y(15)/-3.748/Y(16)/-3.590/
23/C      DATA Z(1)/-5.78/Z(2)/-5.738/Z(3)/-5.647/Z(4)/-5.531/Z(5)/-5.385/
24/C      1Z(6)/-5.032/Z(7)/-4.651/Z(8)/-4.188/Z(9)/-3.759/Z(10)/-3.031/Z(11)/-2.486
25/C      2/Z(12)/-2.186/Z(13)/-1.854/Z(14)/-1.692/Z(15)/-1.59/Z(16)/-1.526/
26/C      DATA W(1)/5.862/W(2)/5.791/W(3)/5.871/W(4)/5.876/W(5)/5.987/
27/C      1W(6)/6.288/W(7)/6.283/W(8)/6.027/W(9)/5.888/W(10)/5.12/W(11)/4.315/
28/C      2W(12)/3.491/W(13)/2.75/W(14)/2.164/W(15)/1.722/W(16)/1.398/
29/C      DO 10 I=K,15
30/C      IF(THE.GT.X(I).AND.THE.LE.X(I+1)) GO TO 20
31/C      10 CONTINUE
32/C      GO TO 30
33/C      20 A = (THE-X(I))/(X(I+1)-X(I))
34/C      U = Y(I) + A*(Y(I+1)-Y(I))
35/C      V = Z(I) + A*(Z(I+1)-Z(I))
36/C      S1 = W(I) + A*(W(I+1)-W(I))
37/C      K = I
38/C      30 RETURN
39/C      END

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